2007

Sound Spectral Analysis of UNI Carillon Bells

Kimberly Cordray
University of Northern Iowa

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A Project Submitted
In Partial Fulfillment
Of the Requirements for the Designation
University Honors

Kimberly Cordray
University of Northern Iowa
May 2007

This Study by: Kimberly Cordray

Entitled: Sound Spectral Analysis of UNI Carillon Bells

has been approved as meeting the project requirement for the designation University Honors.

5-2-07

Date

Dr. Roger Hanson, Honors Project Advisor

5/2/07

Date

Jessica Moon, Director, University Honors Program

INTRODUCTION

The campanile located in the center of the University of Northern Iowa campus has been a symbol of the university since its completion in 1926. This 100 foot tall structure was the chosen memorial celebrating the 50th anniversary of the university and dedicated to the founders and builders. But this towering structure is more than just a popular gathering point and icon, it houses 47 bells[1] and a carillon instrument[2] which plays these bells.

Bells are a unique and complex instrument. It is not hard to comprehend when you think about what a bell is. A carillon bell is this odd shaped piece of metal that, when struck, does not simply emit a percussion type noise but has a sound with a definite pitch. In fact, there are many vibrations working together to produce the recognized pitch. The UNI carillon bells provided the means to study bells and their sound first hand.

FIGURE 1 The UNI Campanile.

HISTORY

The campanile originally held 15 bells cast by Meneely and Company of Watervliet, New York and ranging in size from 225 to 5000 pounds. In comparison, the world's largest bell is the Tsar Kolokol or "Tsar of bells" located in Moscow, Russia and weighing in at a massive 216 tons. The Tsar however is not a working bell, and the title of the largest swinging bell goes to "The World Peace Bell" cast in 1999 at 66,000 lbs. and located at the Millennium Monument in Newport, Kentucky. Other famous bells include Big Ben in London, England at 15 tons and the Liberty Bell in Philadelphia, Pennsylvania weighing just over 2,000 lbs. With only 15 bells, the instrument at UNI was considered a chime as 23 or more bells (2 octaves) were required to be classified as a carillon. It is interesting to note that in 1927 there were only 23 carillons in North America, 19 of which were installed since 1922 largely due to the promotion of William Gorham Rice. While the UNI instrument was not a carillon at that time, it may be no surprise that the bell tower was built during this time. Carillon status was achieved in 1968 when the Petit & Fritzen Foundry of the Netherlands cast 36 bells for the campanile, 32 additional bells and 4 replacements. This brought the total to 47 bells, a 4 octave carillon, which is what remains today.

FIGURE 2
Some well known bells.

Tsar Kolokol

World Peace Bell

Liberty Bell

[1] See Appendix A for photos of the carillon bells.
[2] See Appendix B for details on the carillon instrument.

While new carillons continue to be installed in the U.S. at a rate of one or two each year, it is still a rather rare thing to find. Iowa has only two carillons, the Iowa State University campus being the home of the second. In fact, most states average only two carillons that have at least a four octave range. It is no wonder that the University of Northern Iowa is so often identified by this magnificent and uncommon structure.

SOUND VIBRATIONS

Sound is generated by vibrations. These vibrations can be in a string, a wooden bar, an air column, a stretched membrane or even an odd shaped piece of metal. Each of these geometries have different characteristics in their vibrations, but the basic idea of what is going on is easiest to picture by considering a string fixed at both ends.

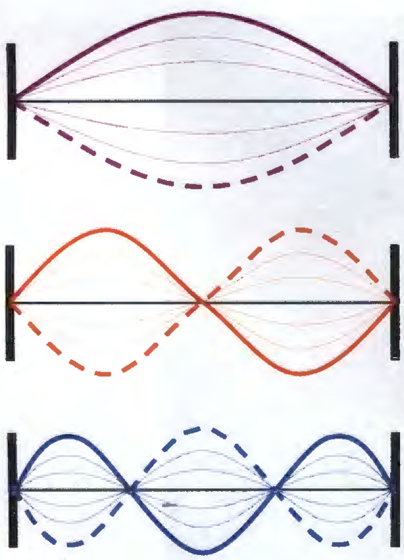

FIGURE 3 Normal modes of a string. The fundamental or first harmonic is at the top, the second harmonic is in the middle, and the third harmonic at the bottom. The solid line represents the string at one moment in time, the dashed line a half period later, and the faint lines show the shape at intermediate times.

A string vibrates naturally in its normal modes. In the lowest mode of vibration the whole string appears as one segment moving up and down. This is called the fundamental mode and is the mode with the lowest frequency or fewest vibrations per second. The next mode of vibration results in two segments with an alternating up and down motion. The point in the middle that divides these two segments is a node where little or no motion occurs. The pattern continues with each higher mode of vibration producing one additional segment along with an increase in the frequency. The important relationship is that more vibrating segments correspond to a higher frequency. For strings, the frequency of a mode of vibration is always an integer multiple of the fundamental mode, such that the second mode has twice the fundamental frequency, the third mode three times and so forth (i.e. $f_n = n f_1$ where n is the number of the mode with frequency f_n). This relationship forms a harmonic series with each mode referred to as a harmonic. The first three modes of vibration of a string are represented in Figure 3.

Some instruments have modes that are not harmonically related. When this is the case acousticians refer to the modes as partials. Bells fall into this category as they have many modes of vibration with frequencies that are non-integer multiples of the fundamental. These modes of vibration, like the harmonics of a string, are characterized by vibrating segments separated by nodes. Bells have both linear, or meridian, nodes as well as circular nodes and consequently there are many ways a bell can be divided into segments. Figure 4 shows holographic images of six modes of vibration (many more are possible) of a bell. As in the case of a string, higher frequencies are associated with more vibrating segments. Figure 5 is a representation of how the bell distorts in various modes.

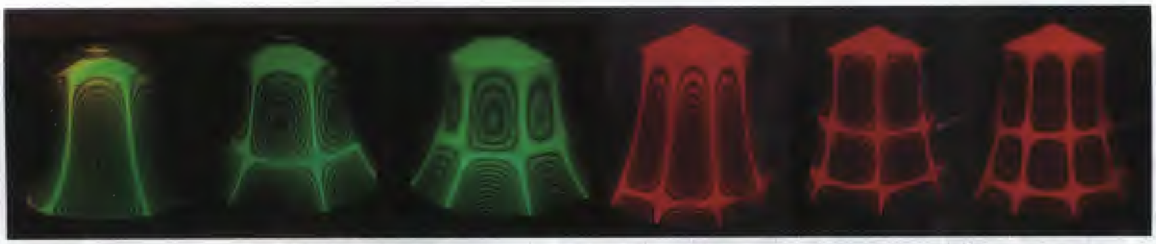

FIGURE 4 Holograms made by Richard Peterson of Bethel University representing various modes of vibration of bells. The bright lines show the locations of nodes and the dark regions the vibrating segments. As the number of segments increases from left to right so does the frequency of vibration. Although these are holograms of a handbell, they accurately depict modes of vibration of a carillon bell.

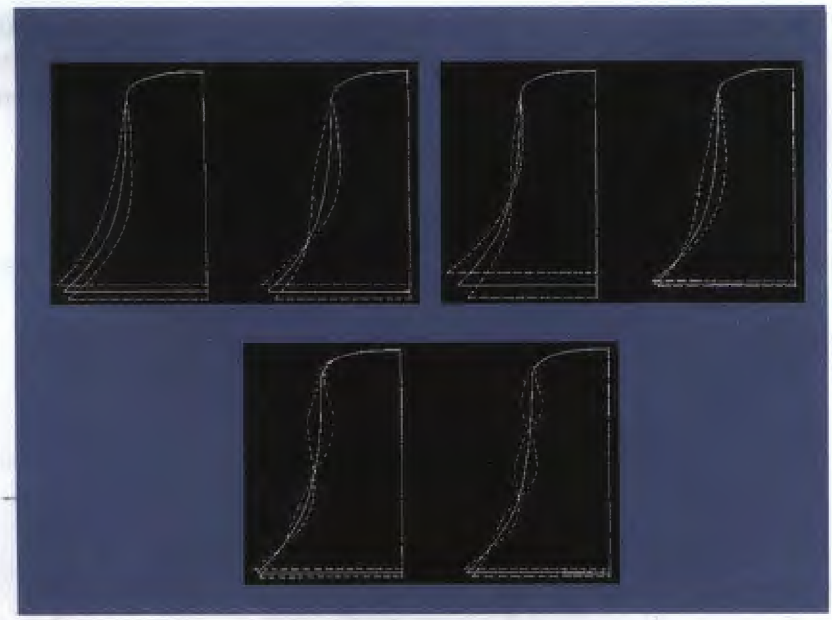

FIGURE 5 Distortion of a bell about its nodes during vibration.

When a bell is struck many of these individual modes are simultaneously excited. As a result, a carillon bell must have its individual partials tuned in order to produce the desired sound. Tuning is achieved by turning the bell on a lathe, as in Figure 6, to remove metal from the inside. In general, the first 5 partials are tuned and higher partials then fall into place. The art of tuning is an old one and in many cases has been passed down from generation to generation, the secret and skill remaining in the family. While computer technology is now used during the process, the ear of the bell tuner remains the final determining factor.

FIGURE 6 Tuning a bell.

ANALYSIS

The bells and carillon are located at the top of the campanile and accessed only by a narrow spiral staircase. The weight and bulkiness of the frequency analyzer[3] required that the bell sounds be recorded and then analyzed. The recordings were made with a hand held digital recorder[4] attached to an external microphone for improved quality. Recordings were made of all bells corresponding to a natural note (i.e. a white key on a piano). For each of these bells, four sound samples were taken using various striking strengths, a precautionary measure to avoid overloading the recorder. All the recordings were made on the same day in order to keep environmental factors constant.

The first item studied was the internal tuning of the bells. This included investigation of the partials which were present, strongest, and survived longest. As Figure 7 shows, there are many partials present as anticipated by published data. The significant partials have frequency ratios to the fundamental that match up with ideal ratios[5] and are labeled with their given names. A semilog graph is used with the vertical scale being logarithmic, thus some noise can be seen along the bottom, but if the amplitude is compared to that of the prominent partials it is many times less and was therefore ignored in the analysis.

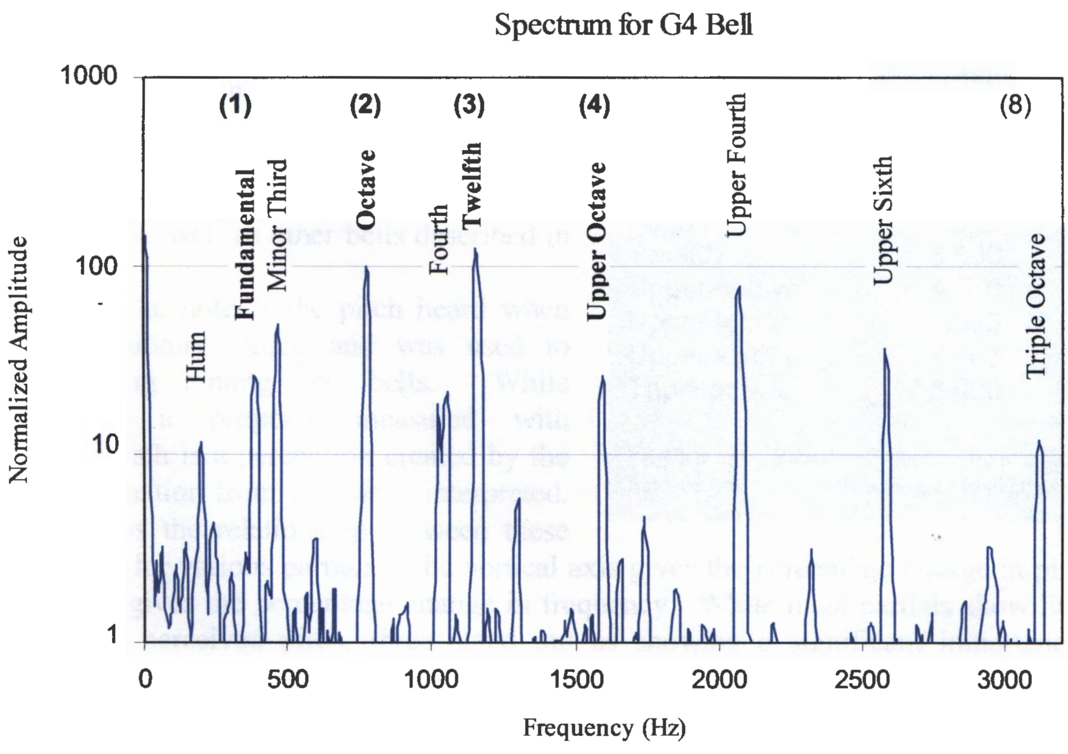

FIGURE 7 Analyzed spectrum with labeled partials. The numbers along the top indicate those partials which are harmonics of the fundamental. A vertical log scale is used.

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[3] Stanford Model SR785, see Appendix C for details.

[4] Olympus VN-3100 model, see Appendix C for limitations.

[5] In musical scales, an ideal ratio is one where the frequencies are related by whole numbers. For example, the minor third has a 6:5 ratio to the fundamental which is 1.20. The fifth has a 3:2 ratio or 1.50.

The most important partials are probably those in bold; the fundamental, octave, twelfth and upper octave. As the numbers above them indicate, these four partials form a harmonic series with the fundamental as the first harmonic. In fact, although the sound or pitch of the bell would be related to the fundamental, as the graph shows it is not necessarily the strongest partial and in some cases is absent or decays quickly. This missing or virtual fundamental is an interesting phenomenon because again, the pitch assigned to the bell would correspond to the frequency of this missing partial. Amazingly, the brain is capable of extrapolating this pitch as corresponding to the first harmonic of the harmonic series that includes the octave, twelfth and upper octave.

Another significant partial is that with the lowest frequency, the hum tone. This subharmonic partial is significant not for its size but for its duration. The hum tone was observed to outlive all other partials, determined by both viewing the decay on the real-time analyzer but also with the ear. As a bell was heard, after the overall sound had decayed, a faint humming continued to ring that appeared to be an octave lower than at the initial strike.

Figure 8 includes a table of data for the C4 bell (which corresponds to middle C on the piano). This table gives the frequency ratios of the partials to the fundamental and compares them with the ideal ratios. These data were encouraging since the calculated ratios were comparable to published data from other bells[6] and the differences from ideal ratios are barely detectable by the human ear. This table indicates that there are two listed partials missing; the same two partials are absent from the G4 spectrum in Figure 7 and in most of the bells that were analyzed as well as other bells described in literature.

Name of partial	Ideal	C4 Bell
Hum	0.500	0.519
Fundamental	1.000	1.000
Minor Third	1.200	1.195
Fifth	1.500	-------
Octave	2.000	1.988
Major third	2.500	-------
Fourth	2.667	2.667
Twelfth	3.000	2.969
Upper octave	4.000	4.086
Upper fourth	5.333	5.311
Upper sixth	6.667	6.618
Triple octave	8.000	7.978

FIGURE 8 Ratios of partial frequencies to the fundamental for C4 bell compared to ideal ratios.

The strike note is the pitch heard when the bell is initially struck and was used to compare tuning among the bells. While frequency is a property measured with instruments, pitch is a perception created by the brain as information from the ear is interpreted. Figure 9 shows the relationship between these two properties for various partials. The vertical axis gives the percentage change in pitch while the horizontal gives the percentage change in frequency. While most partials show little or no change in the perceived pitch, three stand out as showing a significant importance in the determination of pitch. Not surprisingly, these correspond to the octave, twelfth, and upper octave which are the second, third and fourth harmonics. Since the octave and twelfth are the most sensitive to frequency change as well as being two of the strongest partials, the average fundamental determined by these two partials was taken to be the strike note of the bell.

[6] Fletcher, N.H. and T.D. Rossing. "The Physics of Musical Instruments." Second Ed., Springer-Verlag New York, Inc., 1998, p. 682.

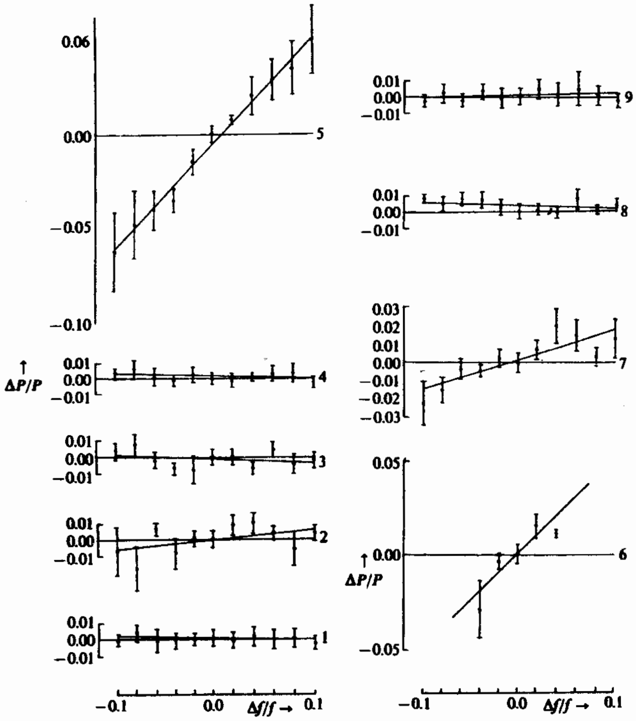

FIGURE 9 Graphical representations comparing change in pitch to change in frequency for various partials. The most noticeable change occurs in the fifth, sixth and seventh partials listed which correspond to the octave, twelfth and upper octave. (From Fletcher, N.H. and T.D. Rossing. "The Physics of Musical Instruments." 1998. p. 684)

After the strike note was determined, equal tempered[7] scaling was used to compare the bells. The equitempered scale was based on the measured value of A4. The percent difference between the measured and calculated values is given in Figure 10 and determines the possibility of a noticeable difference. Unfortunately, there is no clear agreement on what percentage change is noticeable for all frequencies. It is generally agreed upon that above 1000 Hz, the just noticeable difference is ~0.5%. The D7 bell is the only one in this frequency range that differs significantly from the equitempered scale. For frequencies below 1000 Hz, the percent change for the just noticeable difference does increase above 0.5% but depends on various psychoacoustic analyses and varies among subjects. Remarkably, the percent difference gives no evidence of the two sets of bells made at different times by different companies.

The spectra of some of the other bells[8] were also studied indicating similar properties. Frequency limitations of the recording equipment restricted investigations of higher partials of the high pitched bells.

ACKNOWLEDGMENTS

Thank you to Dr. Roger Hanson for his guidance, support and willingness to investigate bells. This project would not have been possible without him. Thank you to Dr. Kui-Im Lee who sparked my interest in the carillon and bells when she taught me how to play the instrument. She also provided access to make the recordings of the bell sounds. Thank you to the UNI Physics Department for funding the recording equipment. Thanks also go to Jessica Moon and Brenda Hackenmiller of the UNI Honors Program for their assistance.

[7] See Appendix D for details on equal temperament.
[8] See Appendix E for additional spectra.

Bell	Strike Note (Hz)	Equitempered Scale (Hz)	Percent Difference
C4	256.5	256.026	0.185
D4	288.7	287.380	0.448
E4	322.2	322.573	-0.126
F4	345.1	341.754	0.974
G4	385.5	383.606	0.494
A4	430.6	430.583	0.000
B4	483.9	483.313	0.125
C5	517.2	512.053	0.999
D5	578.9	574.760	0.723
E5	650.7	645.146	0.856
F5	689.8	683.508	0.913
G5	773.0	767.212	0.754
A5	866.9	861.167	0.664
B5	971.6	966.627	0.510
C6	1028.9	1024.106	0.470
D6	1154.7	1149.520	0.448
E6	1292.2	1290.292	0.145
F6	1369.8	1367.017	0.206
G6	1534.8	1534.425	0.021
A6	1720.2	1722.333	-0.126
B6	1928.8	1933.254	-0.233
C7	2055.5	2048.211	0.356
D7	2281.0	2299.039	-0.785

FIGURE 10 Comparison of relative tuning among bells. The equitempered scale was based on the strike note of the A4 bell (Concert A). The percentage difference between the equitempered scale and the measured strike note indicates the possibility that the difference would be noticeable. Negative percentages indicate that the strike note was lower than the calculated scale.

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APPENDIX A: THE UNIVERSITY OF NORTHERN IOWA CARILLON BELLS

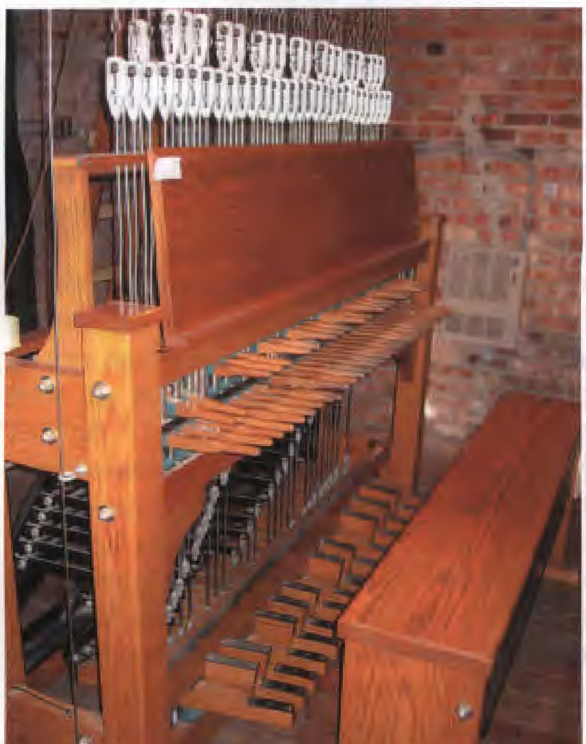
Bells from the original 1926 set.

One of the bells installed in 1968.

APPENDIX B: THE CARILLON INSTRUMENT

The traditional carillon keyboard has the appearance of a piano or organ keyboard but is played much differently. There are two keyboards, the upper to be played with the hands and a lower one to be played with the feet. The upper keyboard is made of many wooden handles, each associated with a single bell. These are generally played by holding the hand in a loose fist and striking the handle with the side of the little finger. There is a system of wires that connects each handle or foot pedal to its respective bell. The bells are stationary and do not actually move during the playing (except for vibrations). Rather, the clapper or metal piece that strikes the bell is what is put into motion.

Today, many traditional keyboards are being replaced by electronic keyboards. These keyboards look and play the same as a piano, although they may be much shorter. The UNI keyboard, shown below, continues to be the traditional style. This keyboard was installed by the Verdin Company in 1984.



Appendix C: Equipment

I. Recorder Limitations

The Olympus VN-3100 digital voice recorder was small, light and convenient to use. Its limitation was due to the fact that it had a frequency response region that was narrower than the range of the partial frequencies for all the UNI bells. The recorder had a flat gain region from 300-7200 Hz. For some of the low frequency bells, the hum and even prime frequencies fell below the 300 Hz cutoff. Likewise, the spectra of some of the high frequency bells could not be analyzed since the prime frequency was large enough that many of the higher partials had frequencies greater than 7200 Hz. A white noise generator was used to test the linearity of signal received by the analyzer from the recorder and to create a calibration curve used to correct the low frequency data.

II. Analyzer Details and Limitations

The Stanford Model SR785 frequency analyzer utilizes the Fast Fourier Transform (FFT). The Fourier Transform is a mathematical algorithm that converts a time-domain signal into a frequency spectrum. This includes an integral which can be approximated by a sum over a given number of time samples, called the Discrete Fourier Transform (DFT). This can create the need for many calculations and an extended period of time for their completion. FFT uses the fact that many calculations are identical to reduce the number needed. As little as 2% of the calculations for the DFT are required for the FFT. Even with a fraction of the calculations, there is no loss of precision when going from the DFT to FFT algorithm. There are many FFT variations.

The spectrum could be captured over the full recording time and then played back and paused at desired times to analyze the spectrum. This allowed the determination of the partial frequencies to a higher resolution. Some limitations resulted from the requirement that for greater resolution (a smaller frequency span) a longer time interval was required. This arises from the inverse relationship between the time interval (T) and the width of each filter (Δf), $T=1/\Delta f$. The Δf is given by the span of frequencies divided by the number of filters. Due to the recommendation of the manufacturer, 400 filters were used for all displays. Thus, the time interval required for a certain span was T = *400/frequency span*. For example, at a 400 Hz span 1 second of data was needed. For some of the partials beating was observed at large spans indicating that there were really two peaks very close together. In this case, if the full span was 3200 Hz there were 8 Hz between each filter. If two partials were separated by only 4 Hz they fell into the same filter and appeared as one peak that bounced as the two signals alternated between interfering constructively and destructively. Unfortunately, it was not always possible to narrow the frequency range in order to see the peak split into two because the fast decay times of some partials meant that the amount of time needed for the necessary resolution was greater than the life of the partial.

The SR785 allowed spectra to be saved on a 3.5" floppy disk[9]. It also came with a program that could be installed on a computer that would convert the files to ASCII (American Standard Code for Information Interchange) files which could then be formatted and the data viewed in a spreadsheet program such as Microsoft Excel. This also allowed easy comparison of data and calibrated wave profiles.

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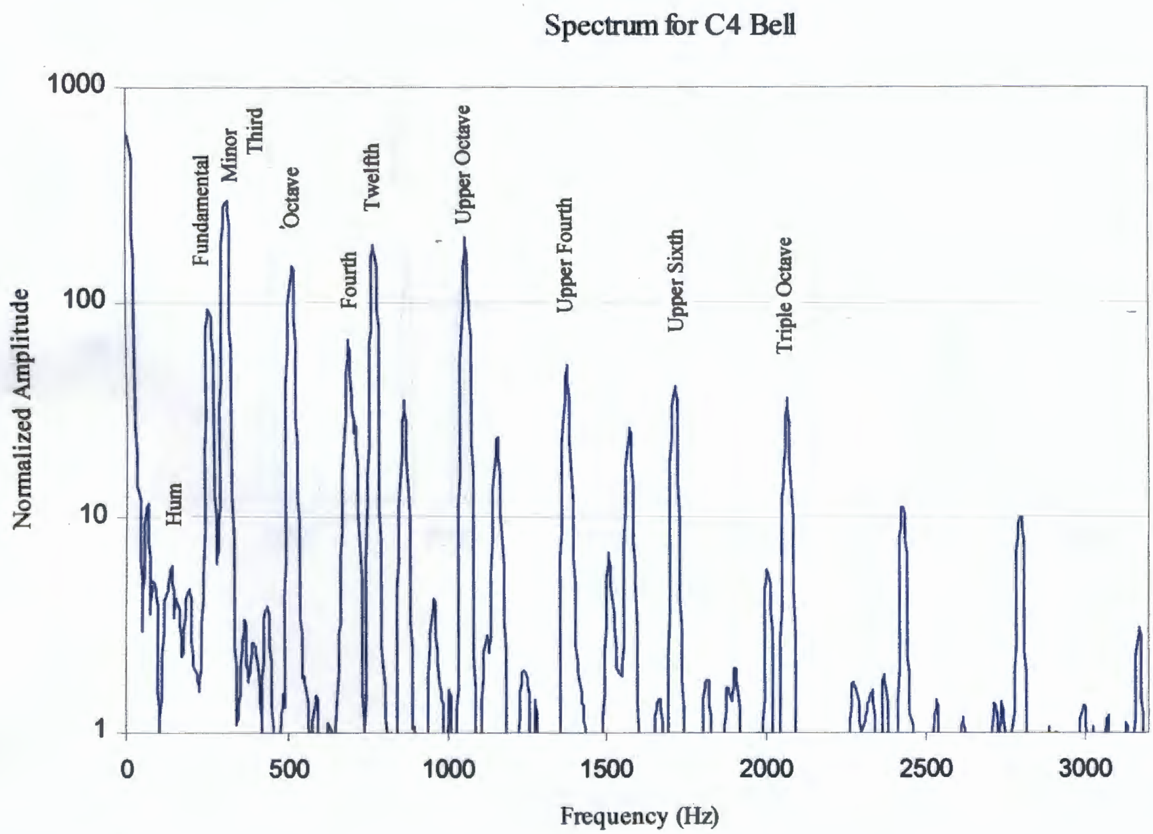
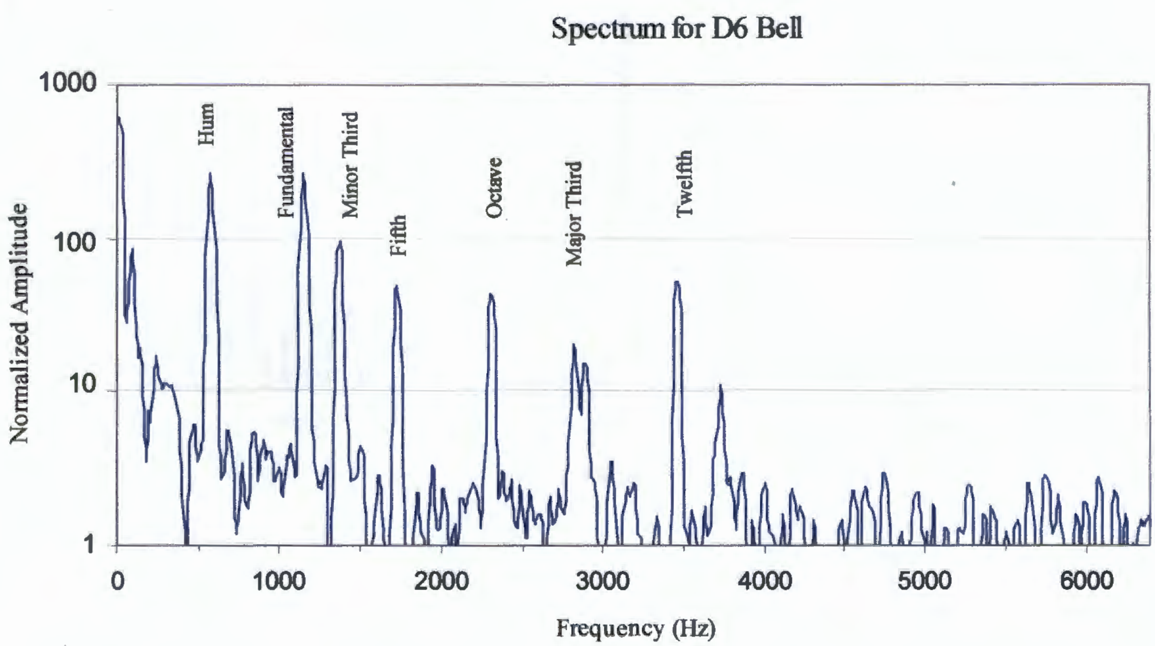
[9] Disk labeled "Carillon Bell Spectra 2007."

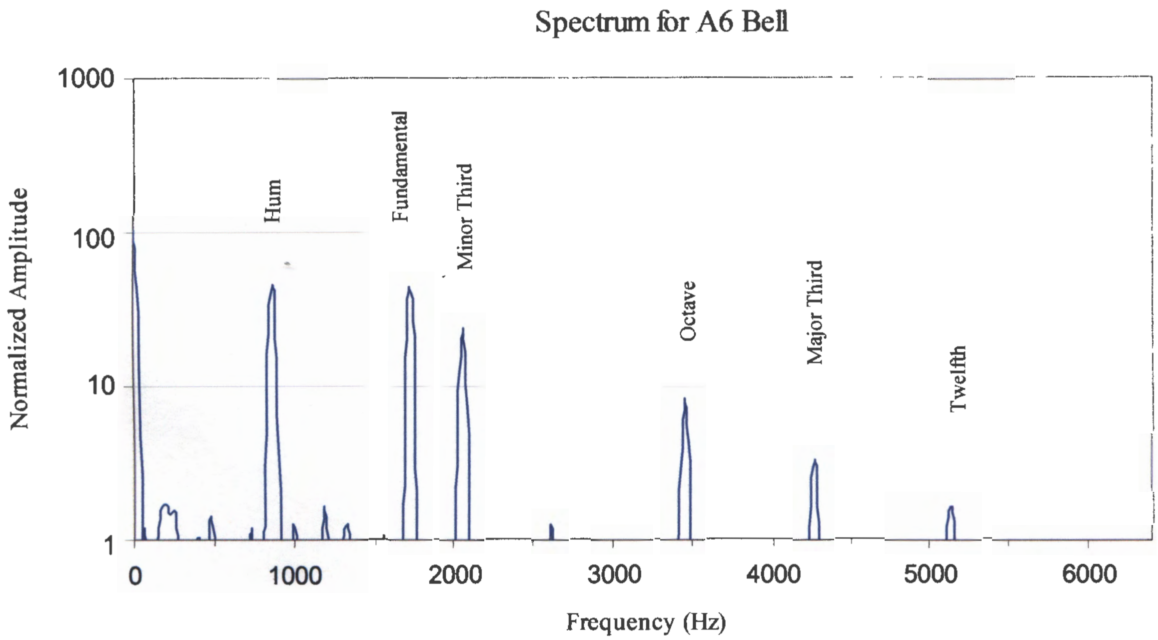
APPENDIX D: EQUAL TEMPERAMENT

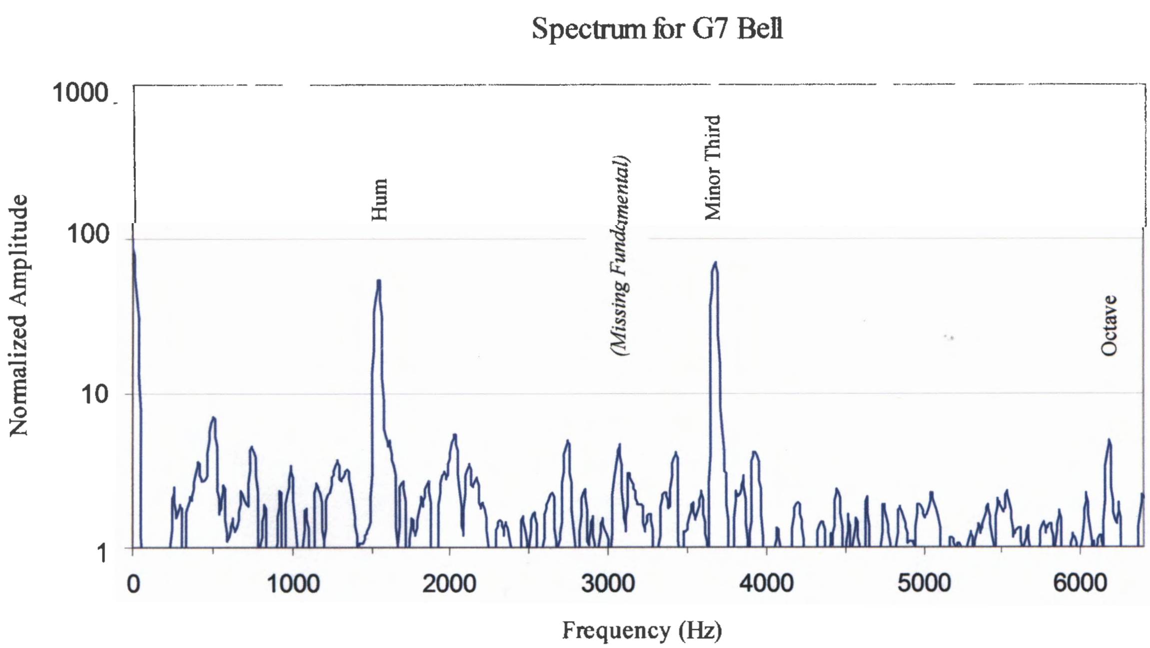
The equal temperament scale divides an octave into 12 equal semitones. The octave represents a doubling of frequency, so each semitone is a $2^{1/12}$ step, a factor of approximately 1.059. The equal temperament scale is the basis for piano tuning and a semitone corresponds to a half-step. The frequency of concert A is used to set most tuning scales, so A4=430.6 Hz was used as the base for the calculated equitempered scale in Figure 10. For frequencies higher than the A4 bell, the calculations involved multiplying the frequency of the A4 bell by $2^{1/12}$ for each half step up. For frequencies lower than that of the A4 a division of $2^{1/12}$ was taken for each half step down. Since most of the percentage differences are positive, a calculated equitempered scale based on A4 slightly greater than 430.6 Hz would result in a smaller average deviation from the base because it would bring some of the highest percent differences closer to zero (while making some of the small positive differences small negative differences).

APPENDIX E: ADDITIONAL SPECTRA






Spectrum for A6 Bell
Normalized Amplitude
1000
100
10
1
Hum
Fundamental
Minor Third
Octave
Major Third
Twelfth
0
1000
2000
3000
4000
5000
6000
Frequency (Hz)


Spectrum for G7 Bell
Normalized Amplitude
1000
100
10
1
Hum
(Missing Fundamental)
Minor Third
Octave
0
1000
2000
3000
4000
5000
6000
Frequency (Hz)