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On abduction, dualities and reason

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Abstract

We integrate dualistic conceptions of the real with Peirce's perspectives about reality and abduction, emphasizing the concept of reason underlying Peirce's thoughts. Peirce's abduction is related to the notions of retrogression and grounding in Hegel, later re-encountered in Hansonian-abduction. Abduction in turn is considered in relation to abstraction acquiring its fullest sense as a stage in the process of producing a theory. The process is iterative and self improving, it incorporates "turbid thinking" making it increasingly "clear" at successive iterations that incorporate the lessons taught by failed predictions, i.e., refutations. The cycle of thoughts promoted by doubts comes to rest when belief is reached. We discuss how this coming to rest depends on a criterion for cessation of doubts. The observation is illustrated with two criteria, one proposed by Mach that only demands analogy and the criteria of dualists such as Goethe and Whewell that inspire the present work. Hence, it is possible to produce, and socially accept, imperfect theories unless we demand the highest level of rationality, avoiding any leftover of the turbid thoughts that have been used in the early developments. Influenced by a constructivist, Piagetian, perspective of science, we propose and discuss conditions that are characteristic of rational abduction: rules for the rational construction of theories. We close arguing that there is an urgent need to develop a critical epistemology incorporating dualistic perspectives.
Keywords: theory building; rules of reasoning; epistemology[1]

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1 Introduction

Abduction or retroduction[2] is indissolubly linked to the name of Charles Peirce (1839-1914) who studied scientific thinking from his pragmaticist perspective. Among his influential readings, Peirce indicates Aristotle (Peirce, 1994, CP 1.22 and several other paragraphs) Kant (Peirce, 1994, CP 1.4) and Hegel, about whom he said "My philosophy resuscitates Hegel, though in a strange costume." (Peirce, 1994, CP 1.42). Peirce's works must then be contextualised within the Enlightenment. In [CP 2.191][3], he writes

> *In studying logic, you hope to correct your present ideas of what reasoning is good, what bad. This, of course, must be done by reasoning; and you cannot imagine that it is to be done by your accepting reasonings of mine which do not seem to you to be rational. It must, therefore, be done by means of the bad system of logic which you at present use.*

This is a declaration that conforms with the main requisite of the Enlightenment according to Kant's view of Illustration (Kant, 1783) and somehow reminds us of Wilhelm von Humboldt's view (von Humboldt, 1792 (1854 Printed, p. 20):

> *Whatever man is inclined to, without the free exercise of his own choice, or whatever only implies instruction and guidance, does not enter into his very being, but still remains alien to his true nature, and is, indeed, effected by him, not so much with human agency, as with the mere exactness of mechanical routine.*

Peirce's ideas about the process of grasping what knowledge means is very much exemplified in his own work, which –as much as the work of Aristotle [CP 1.22]– is *evolutionary*. This has been described as successive changes in Peirce's view of abduction (shifting occasionally the name from hypothesis making, to induction, abduction and retroduction; see e.g., (Paavola, 2005)).

Since in Peirce thought is promoted by doubt and comes into a (temporary) rest when belief is reached, we must conclude that Peirce died leaving his work unfinished. Those, like the present authors, that attempt to (somehow) continue his work must be aware of Goethe's dictum (von Goethe, 1906, #556):

> *A man does not need to have seen or experienced everything himself. But if he is to commit himself to another's experiences and his way of putting them, let him consider that he has to do with three things – the object in question and two subjects*

[2] Following Peirce, we use these words interchangeably. See however (Peirce, 1994, CP 1.65).

[3] To avoid extensive repetition, we will often quote Peirce by indicating the paragraph (e.g., [CP 1.65]) in (Peirce, 1994).

Thus, in every continuation of Peirce's thoughts there is something that comes from the reality of knowledge, something that comes from Peirce and something that is provided by the new subjects. There are at least two forms of continuation so far attempted, that have been called "Hansonian and Harmanian abduction" (Paavola, 2006).

Those authors that relate with Peirce by their inclinations as logicians emphasise the early syllogistic approach as in (Redding, 2003), where a correspondence is made among syllogisms in Aristotle, Hegel and Peirce. Authors inclined towards logic relate abduction to the following structure (Peirce, 1994, CP 5.189):

> *The surprising fact, C, is observed; But if A were true, C would be a matter of course, Hence, there is reason to suspect that A is true.*

The reasoning grants the proposal of A the name of *abductive inference*. However, to raise this expression to a definition leaves abduction abandoned to free interpretation. It must be recalled that for Peirce, [CP 2.195],

> *Logic came about for the sake of reasonableness, not reasonableness for the sake of logic.*

This is: the main concern is reason, not logic (as Paavola emphasizes in (Paavola, 2004)). This view directs us towards Hansonian abduction and away from Harman's views.

The adoption of explanatory hypotheses is indissolubly linked to the adoption of beliefs and the cessation of doubt, the latter being the motor of thoughts. Peirce is clear in distinguishing the beliefs he pursues from religious and other common beliefs; he offers three characteristics of them:

> *First, it is something that we are aware of; second, it appeases the irritation of doubt; and, third, it involves the establishment in our nature of a rule of action, or, say for short, a habit. As it appeases the irritation of doubt, which is the motive for thinking, thought relaxes, and comes to rest for a moment when belief is reached. But, since belief is a rule for action, the application of which involves further doubt and further thought, at the same time that it is a stopping-place, it is also a new starting-place for thought. [CP 5.397]*

All these elements are essential to Peirce's notion of belief and as such, none of them can be dropped (not even in didactic examples) when regarding abduction, which is an act of thought. Further, Peirce sustains,

> *Thought [...'s] sole motive, idea, and function is to produce belief, and whatever does not concern that purpose belongs to some other system of relations. [CP 5.396]*

Thought, doubt and belief refer to our inner senses. They are to some degree defined in relation to each other. Abduction or retroduction in Peirce is also a part of scientific thinking. He provides more insight into it:

> *These three kinds of reasoning are Abduction, Induction, and Deduction. Deduction is the only necessary reasoning. It is the reasoning of mathematics. ... Induction is the experimental testing of a theory. The justification of it is that, although the conclusion at any stage of the investigation may be more or less erroneous, yet the further application of the same method must correct the error. The only thing that induction accomplishes is to determine the value of a quantity. It sets out with a theory and it measures the degree of concordance of that theory with fact. It never can originate any idea whatever. No more can deduction. All the ideas of science come to it by the way of Abduction. Abduction consists in studying facts and devising a theory to explain them. Its only justification is that if we are ever to understand things at all, it must be in that way. (Peirce, 1994, CP 5.145) [emphasis added]*

Further, the abduction of hypotheses in Peirce has as sole goal to put the hypotheses to test, and consequently to abandon them the moment they are refuted, continuing with the search for new hypotheses. This reflects in a cyclic or iterative process (Peirce, 1994, CP 7.220) where the subsequent proposal of hypotheses occurs from a different ignorance level: At least we know that all the previous attempts were inappropriate and we know where they first failed in the testing phase. In other words, the abductive reasoning does not "stop" at the first hunch but rather when belief is attained and a new and richer theory is proposed.

Dualism, reason and retroduction Kant's dualistic view about the emergence of knowledge (Kant, 1787):

> *Understanding cannot intuit, and the sensuous faculty cannot think. In no other way than from the united operation of both, can knowledge arise*

was further elaborated by W. Whewell (Whewell, 1858) as the "fundamental dialectic"

> *In all human KNOWLEDGE both Thoughts and Things are concerned. In every part of my knowledge there must be some thing about which I know, and an internal act of me who know... Man is interpreting the phenomena which he sees. He often interprets without being aware that he does so*

and further propagates in time into the views of Piaget and García (Piaget and García, 1982)

> *Un hecho es, siempre, el producto de la composición entre una parte provista por los objetos y otra construida por el sujeto [Original version].*
>
> *A fact is always the product of the composition between one part provided by the objects and another constructed by the subject [Our translation].*

There is no evidence of Peirce being deeply aware of the intervention of the subject in the production of facts. Peirce permanently refers to "observed facts", hence ignoring the participation of the subject in the production of facts alongside what comes from the external senses. We found only an indication of awareness in paragraphs concerning self-consciousness as a result of a clash between an outward and inward motion [CP 8.41]. This absence represents a substantial drawback that needs to be repaired since it is in this process where –according to Piaget and García as well as Husserl (Husserl, 1983)– the process of ideation occurs, this is, where the observed is registered as a fact after (quite often unconscious) rationalisation. The introduction of a dualist vision of knowledge (Solari and Natiello, 2022) makes enough room to incorporate pre-rational elements such as (simple) intuition, habits, epistemological frames, pre-existing theories, phantasy, imagination and in general the elements that participate in the initial production of facts but belong to the subject and not to the object[4].

These pre-rational elements participate along with reason in the process, yet, in the ideal final product where the phases of abduction, deduction and induction as described by Peirce can be identified, the contributions of the "turbid" thoughts have been made transparent by reason, leaving only the distinctive clear thinking. There will be room as well for the flux of error and the iterative refinement of theories.

Little can be gained however if we cannot make somewhat objective what is meant by reason. *Scientific*, for Peirce, means here what really directs us to satisfy the aims of those "to whom nothing seems great but reason" and see nature as "a cosmos, so admirable, that to penetrate to its ways seems to them the only thing that makes life worth living" [CP 1.43]. Thus, reason is essential to science and appears as associated to the making a cosmos from the chaos that reaches our senses.

This will be the focus of the present work. We highlight that reason is a mandatory part of understanding, and that understanding is a name for having a theory

[4]In this sense, (Paavola, 2005) recognises "a clearer change in Peirce's views than from evidential to methodological perspective concerned the role of instinct in abduction."

in which we can believe, recalling that belief in Peirce establishes rules of action. We will advance in the present project producing rules for the control of the rationality in retroduction. For Peirce [CP 5.384] the concept of reality (see Section 4.1.1) is his sole hypothesis, although he claims that "...the method must be such that the ultimate conclusion of every man shall be the same". Putting things together, the current logic of any human should be of such kind as to allow them to improve it and reach an intersubjective level of understanding in all matters concerning reality. Peirce closes his paragraph with

> *Experience of the [scientific] method has not led us to doubt it, but, on the contrary, scientific investigation has had the most wonderful triumphs in the way of settling opinion. These afford the explanation of my not doubting the method or the hypothesis which it supposes; and not having any doubt, nor believing that anybody else whom I could influence has, it would be the merest babble for me to say more about it. If there be anybody with a living doubt upon the subject, let him consider it.*

Peirce's optimistic view is based upon his conception of the scientist. However, much of what is labelled as scientific today does not fully meet Peirce's criteria exposed in "The scientific attitude" [CP 1.43–1.45] but rather what is described in [CP 1.45] as non scientific. Thus, reason is essential to science, and what reason means in the present context needs to be further explained if the confusion that enters our minds through interests in conflict is to be avoided[5] .

We will adopt a constructivist approach influenced by Piaget's work. A reason without consciousness of its own constructive efforts will consider its scientific activity to be the discovering of the laws of the universe, perhaps without noticing that their efforts are pre-formatted by their own rules of reasoning and their own criteria of considering an argument to be correct/convincing/acceptable. Thus, there is a level of meta-scientific criteria that needs to be explored and explained. We owe the idea regarding the existence of rules or norms to Piaget, as Gruber and Vonèche (1995, p. 739) write:

> *Rules or norms are generally considered as dependent on structures in the subject. They do not depend on the structure of physical reality for their validation but are instead entirely determined by a principle of deduction that is not empirical in nature.*

In the coming Sections, we start by connecting abduction and rationality, recalling the views of Hegel and Peirce. Further, we consider the relation of abstraction

[5]We mean the utilitarianism indicated by Peirce in [CP 1.45] as well as the careerism imposed onto academics by governing/administrative bodies (Solari et al., 2016) and the social pressure exerted by the scientific field (Bourdieu, 1999).

and idealisation with the abductive process, as well as the role of analogy. Then we present a few rules for rational retroduction as recognised by our eidetic seeing (the production of ideas by intuition Husserl (1983, Ch. One)), discussing examples in several fields. Finally, we summarise the discussion in Section 5.

2 On Hegel, Peirce and rationality

The mysterious phrase (quoted in the introduction) regarding Hegel's philosophy in relation to Peirce's own philosophy (Peirce, 1994, CP 1.42) deserves some inquire. We read in (Hegel, 2001):

> *(§ 101) ... progress in philosophy is rather a retrogression and a grounding or establishing by means of which we first obtain the result that what we began with is not something merely arbitrarily assumed but is in fact the truth, and also the primary truth.*
>
> *(§ 102) It must be admitted that it is an important consideration −one which will be found in more detail in the logic itself− that the advance is a retreat into the ground, to what is primary and true, on which depends and, in fact, from which originates, that with which the beginning is made.*
>
> *(§ 1707) So far, then, it must be said that cognition, once it has begun, always proceeds from the known to the unknown.*

These ideas correlate with (Peirce, 1994, CP 5.189) (quoted in the introduction) as follows: that "we began with" is C. Thus, progress means to find the A that makes C true and not arbitrarily assumed and that is also grounded to our pre-existing knowledge base. Further, notice the occurrence of "retrogression", the backward motion implied as well in retroduction. Later, Hegel gives precisions when discussing analytic and synthetic cognition:

> *(§ 1720)... Analytic cognition is the first premise of the whole syllogism −the immediate relation of the Notion to the object; identity, therefore, is the determination which it recognises as its own, and analytic cognition is merely the apprehension of what is. Synthetic cognition aims at the comprehension of what is, that is, at grasping the multiplicity of determinations in their unity. It is therefore the second premise of the syllogism in which the diverse as such is related.*

Peirce [CP 4.85] in turn will take (as much as Hegel) synthetic cognition from Kant (1787, p. 37) [6]. Along the development of his ideas, Peirce [CP 2.629]

[6]Notice that because of a difference in translations Peirce adopted "ampliative judgments" while the referenced paragraph says "augmentative judgments".

writes :

> *So, an hypothesis is really a subsumption of a case under a class*

If we consider that the analytic judgment carries no other novelty than making explicit what was already contained in the premises, the progress of science is linked to the synthetic judgment. And, if in addition, abduction is the "only logical operation which introduces any new idea" [CP 5.171], making the intersection of the claims, rational abduction is of the order of "ampliative abduction" (see for example (Aliseda, 2004)).

It is apparent that from Hegel to Peirce the grounding of the hypothesis has been lost or at least it has been de-emphasised. Explanation is not the only goal in Hegel as he wants to remove arbitrariness. An hypothesis which is arbitrary but explains the observed and is not refuted by other implied facts would not achieve the desired elimination of arbitrariness but only a translation of arbitrariness from the fact to the hypothesis. Since arbitrariness is "the quality of being based on chance rather than being planned or based on reason" (Cambridge dictionary) or "existing or coming about seemingly at random or by chance or as a capricious and unreasonable act of will" (Webster dictionary), it appears as a safe measure to request the abductive hypothesis to be rational. A requirement that only puts us at the beginning of a quest: how to determine intersubjectively the reasonability of an hypothesis?

If one is set to understand the "logic of science", which is certainly Peirce's intention, one is obliged to adopt abduction as implying not only the logical rule of [CP 5.189] but the additional requirements for hypotheses to be ampliative and rational. Peirce [CP 7.220] requests of the hypothesis first to "be capable of being subjected to experimental testing" and second it "must be such that it will explain the surprising facts we have before us which it is the whole motive of our inquiry to rationalize". Thus, our observation to Peirce is that rationalisation is not completely achieved by producing an explicative hypothesis, since such an hypothesis may very well be irrational itself.

Hanson (Hanson, 1965) distinguishes three ingredients in the logic of discovery

> *1) proceeds retroductively, from an anomaly to*
>
> *2) the delineation of a kind of explanatory H which*
>
> *3) fits into an organized pattern of concepts*

Thus, Hanson's view recovers the "grounding" present in Hegel but not so evident in Peirce. We will try in the coming sections to move towards the recovery of a self-critical rationality which is the ultimate exercise that allows us to improve our science. In section 4 we propose some minimalistic principles of rationality.

3 On the relation of abstraction and ampliative hypotheses

The word abstraction (from Latin *abstrahere* "to drag away, detach, pull away, divert;") can be read at least in two forms. A first form, procedural, in which we eliminate properties leaving a less determined, more general, idea. This procedure leads to the question: what is then left when all properties are withdrawn? Such thing has been named the "thing-in-itself", a metaphysical entity already criticised by Sartre[7] as devoid of any meaning. The problems with this procedure have been already indicated by Hegel (2001, § 22). A different connection with the etymology is to extract, to pull out from the concrete form, its essence. Consider the first form, stopping before hitting emptiness. Let us say we start with an object and stop with a general idea of the object, an *abstraction* of it. Because of its sub-determination, chances are that other objects, after a similar operation match our abstraction as well. In this way, a relation is established between them: they belong to the same class (the relation is easily shown to be transitive). For example, suppose I am holding a purring thing in my arms and say: this is my cat. The same saying can be used by every cat owner despite all the possible differences among cats. We look now to what we have achieved: to put in relation the singular with the general, the particular with the universal. A particular cat is a cat because it can be put in relation with the abstract (general/universal) idea of cats while the general idea is such because it is something that can be put in relation with all the particular forms of cats. The idea of a cat cannot pre-exist the particular cats from which it is abstracted, but the particular cats are in no form recognised as cats if there is no general idea of them. Thus, in a cognitive sense, the abstract form and the particular realisations are created at the same time. Yet, the unrecognised things exist even if we have not yet recognised them as cats. This is an example of what Solari and Natiello (2018) called "dialectical opening" in which two or more concepts are produced simultaneously because such relation is useful to organise the observable input. In the particular case of abstraction,

[7] *Force, for example, is not a metaphysical conatus of an unknown kind which hides behind its effects (accelerations, deviations, etc.); it is the totality of these effects. Similarly an electric current does not have a secret reverse side; it is nothing but the totality of the physical-chemical actions which manifest it (electrolysis, the incandescence of a carbon filament, the displacement of the needle of a galvanometer, etc.). No one of these actions alone is sufficient to reveal it. But no action indicates anything which is behind itself; it indicates only itself and the total series. The obvious conclusion is that the dualism of being and appearance is no longer entitled to any legal status within philosophy. The appearance refers to the total series of appearances and not to a hidden reality which would drain to itself all the being of the existent. And the appearance for its part is not an inconsistent manifestation of this being. To the extent that men had believed in noumenal realities, they have presented appearance as a pure negative. It was "that which is not being"; it had no other being than that of illusion and error.* (Sartre, 1966, Introduction)

in its more general form corresponds to the dialectic "universal-particular" or the relation between the one and the multiple.

Incidentally, the Meno paradox has been considered in relation to abduction in (Paavola and Hakkarainen, 2005). The paradox is:

> *If you know what you're looking for, inquiry is unnecessary.*
>
> *If you don't know what you're looking for, inquiry is impossible.*
>
> *Therefore, inquiry is either unnecessary or impossible.*
> *(consulted December 14th 2022)*

The dialectical opening means that we create the form (the idea) because in a pragmaticist sense it serves the purpose of organising the observable. The idea is born out of the inquiry itself. The duality does not pre-exist, it is created in a single unitary act and it is justified because of its appropriateness. Dialectical openings dissolve the possibility of applying the paradox since the paradox presuposses the existence of one term before the other.

Abstraction does not explain, it only organises, then abstraction is not abduction despite it being pragmatic as well. At the same time, theories cannot be made with respect to a unique event, the singular. Before deserving further study each experiment must be reproduced, and by reproduction it is understood not an impossible new production in exactly the same conditions (including time, space, personal, apparatus, ...) but rather the production of a new experiment related to the original by an abstract form. It is the abstract form what is reproduced[8].

Establishing this relation represents the essence of abstraction, the abstract form into which other forms of putting in relation can be mapped, as for example *cognitive surpass* (Piaget and García, 1982) (see Section 4).

Abstraction is not performed without guidance, when abstracting we have in consideration some matters of concern that lead our quest. For example, we all know that tigers are cats, but if what is in consideration is the ability of purring, then tigers are not that kind of cats as they do not have this ability because of physiological reasons. Thus, any particular being relates to a multitude of abstract forms.

The dialectic particular-universal pervades science. Every time we "discover" a "regularity" of nature we can say with identical precision that we have established a new abstract form for organising nature.

[8]It is not unusual for experiments to be irreproducible. In such cases what often happens is that the abstract experiment does not completely describe the determining circumstances. Some determining conditions have not been controlled, much less communicated, as they are outside what deserves control in terms of the theoretical background underlying the design and development of the experiment. Then, the theoretical background must be rejected, being considered at least incomplete.

By the name *experience* we usually designate the construction of an abstract form that relates to a finite sequence of events. By the way of abstraction, events known in isolation become particular realisations of the same experience. Thus, experience is in part abstraction. Abstraction enacts the identification of the different. When the abstract form has been established we can ask whether new events correspond to this form or not. In case they do, we can translate to them, caring for their particularities, the results of theories that correspond to the abstract form.

Theories actually relate ideas, abstract forms, and it is this character of abstract what allows us to relate them to new particular events. An explanatory hypothesis introduced to explain only a singular event does not constitute a theory. When we propose hypotheses to explain a unique event but do not establish a belief that rules our future actions, we are not constructing theories, and we are not in the process where scientific retroduction belongs. The ground for advancing hypotheses is prepared by abstraction or must be performed alongside of, or before, abstraction.

But if abstraction is not abduction, and abstraction produces a new idea in our consciousness, how is it possible that abduction is the only creative moment of science? Since abduction and abstraction are synthetic judgments, the right to be called the only creative moment of science corresponds to the synthetic judgment. Have we come back to Kant?

Consider the equation of state of ideal gases,

$$PV = n(kT)$$

where P stands for pressure, V for volume, n number of moles, T is the absolute temperature and k is a constant such that (kT) represents energies. The equation has a clear abstraction since it applies to all diluted gases irrespectively of their chemical nature and the form of the container (at least). All diluted gases map into the same expression, but only in the limit of infinite dilution the expression matches experiments, hence it is an idealisation in the sense of Galileo (Galilei, 1914). Finally, it is a synthetic relation. Each one of the elements introduce ideas into our consciousness and concurrently produce an interesting relation that explains observations. But idealisation is not a synthetic judgment.

Idealisation and abstraction produce indeed new ideas. However, they enter into explanation only through abduction. Thus, thinking in terms of the production of theories (explanations) the abductive step can be synthesised with abstraction and idealisation as they are complementary when the goal is understanding nature.

3.1 Abstraction and analogy

> *Analogy is the inference that a not very large collection of objects which agree in various respects may very likely agree in another re-*

> *spect. For instance, the earth and Mars agree in so many respects that it seems not unlikely they may agree in being inhabited. [CP 1.69]*

Between two similar problems, analogy makes a direct connection. In contrast, abstraction makes indirect connections conditioned to the possibility of producing an universal form which can be particularised in each of the different problems originally perceived as potentially related. In so doing, abstraction opens the possibility of a manifold of connections other than those initially considered, largely enlarging the possibility of performing empirical contrastive comparisons. In the quoted example, if instead of producing the inference we propose a (tentative) abstract form consisting of the class of objects that have the identified set in common (call it *habitable planets*), the route just started leads us into attempting to prove that the characteristics selected to define the class can, by themselves, determine the possibility of life on the planet. If this is the case, the analogy is correct, if not, it is incorrect. The analogy lacks the element of rationality which is present in "clear thinking", this is, abstract thinking, thinking using abstractions.

3.2 Abstraction and phantasy

The English spelling phantasy (Greek: $\phi\alpha\nu\tau\alpha\sigma\acute{\imath}\alpha$, Latin: *imaginatio*) is predominantly associated with "imagination, visionary notion" (Oxford). The Latin etymology of *imaginatio* relates it to *imitare* (to copy). According to Hume (Hume, 2011, p. 7), we think in terms of elements pertaining to imagination which, according to Aristotle (Aristotle, 1907, p. 123, see 3.3–3.15), lies close to both perception (Impressions in Hume) and belief, but not to clear thinking. We further learn from Husserl (1983, Ch. One) that the production of ideas by intuition (eidetic seeing) can be triggered by both the real (observable) world as well as by phantasy. Very much like Aristotle, Husserl reminds us that whatever predicate having to do with "matters of fact" must be grounded on experience ("And thus not even the most insignificant matter-of-fact truth can be deduced from pure eidetic truths *alone*." Husserl (1983, p. 11)).

In short, concerning the study of nature (physics) there are two possible approaches at least since Aristotle: "clear thinking" and "phantasy". The former struggles for being correct while the latter may be correct or not, and its correctness cannot be established without the participation of clear thinking[9].

[9]Notice that the current use of fantasy (modern spelling) does not correspond well with Aristotle's use. Phantasy here does not mean fantasy as in day-dreaming for example but rather imagination as it is praised in [CP 1.46].

3.3 When abstraction was left behind

As the XIX century advanced a new social type irrupted in the scene. By 1833, the words physicist and scientist were coined by W. Whewell, a philosopher of nature (Yeo, 1993). Science became a matter of interest for the State and then, the conditions that according to Kant (Kant, 1798) favoured the development of reason in the minor faculty disappeared for science. The philosopher of nature faded out.

During the second half of the XIX century, two forms of thinking had evolved without a neat distinction, differing in the relation among clear thinking and phantasy. Both of them using the word *idea* but meaning different things with it. The requisites for achieving belief, or the cessation of doubt are expected to be unlike as well. The subtleties of the different meanings may emerge when we push our reasoning to its limits. We call *imaginative thinking* to the form originating in the *Bild* concept (D'Agostino, 2004), where images (often called ideas) were the central tool in developing knowledge, and *abstract thinking* to the form supported by abstraction (cognitive surpass (Piaget and García, 1982, Introduction)).

Abstract thinking requires the development of abilities from within, in the form in which Peirce indicated that our current logic must approve its own improvement. Abstract thinking is then linked to W. von Humboldt's *bildung* (self formation) (Sorkin, 1983). As it comes from within, self formation does not lend itself for massive education. In contrast, always according to Kant, the formation of professionals requires them to master the use of established ideas, to think with their master's ideas. The emphasis at the Prussian universities first, and later in the rest of Europe, changed from bildung into instruction (Helmholtz, 1908, VI On academic freedom in German universities). The focus of higher education becomes not as much clear thinking but rather *certified beliefs*.

The investigation of the influence brought about by the second industrial revolution into science deserves an independent inquire. We will only provide an example here.

An example of the different meanings associated to understanding, coupled with the introduction of auxiliary concepts, comes from mechanics. Newton's mechanics can be introduced to students in several forms. Some will rest upon "absolute space". Einstein (1924) and Boltzmann (1974, p. 102), for example, believed that absolute space was essential to Newton's mechanics. In his treatise of mechanics, Mach (1919) rejected absolute space as nonsense and introduced as reference the fixed stars (whatever they are [10]). In both cases, a first reference for motion allowed the introduction of inertial systems in which Newton's laws were claimed to hold. In abstract terms, "absolute space" refers to any belief that indicates that there is a primordial reference for motion. In this abstract form,

[10] Actually, Newton had considered the proposition and rejected it.

Mach believes in absolute space[11] . Perhaps more importantly, the notion of space connects directly with the intuition of the child who organises relative positions of objects in space using as reference her/himself. Yet, Newton's mechanics is based upon the notion of "absolute motion" which is *not* motion in absolute space and it is rather close to relative motion (Newton, 1687; Thomson, 1884; Solari and Natiello, 2021). Thus, while for a relevant group of physicists the auxiliary notion of space and a first reference for motion are necessary elements for the understanding of classical mechanics, yet, others can understand without those elements or with a suppressible version of space.

Absolute space was defended by Bertrand Russell in his early period as well. Russell (1901, p. 274) writes:

> *En ce qui concerne les nécessités de la pensée, la théorie kantienne semble amener ce résultat curieux, que tout ce qu'on ne peut s'empêcher de croire est faux. Dans le cas actuel, ce qu'on ne peut s'empêcher de croire, c'est quelque chose qui se rapporte à la nature de l'espace, non pas à celle de notre esprit.*

Despite Russell's obfuscation, it is clear that all that we can observe are spatial relations, and from them, we can construct a form of description of these spatial relations in terms of the relations between the intervening bodies and a main body and its surroundings. We owe this representation (in abstract form) to Descartes: the *space*. In the words of Kant:

> *By means of the external sense (a property of the mind), we represent to ourselves objects as without us, and these all in space.(Kant, 1787, p. 51)*

Thus, space is a phantasy: a representation of the real observable in the mind of the observer. By erasing the own body from the perceived scene, the observer idealises (creates) the space out of the observed spatial relations with the intervening bodies (Solari and Natiello, 2018). Space exists only in our minds but relates to the perceived as Aristotle dictates for all phantasies. Space is not real, and it is not entirely imagined either, it contains both nature and some invention as well.

Poincaré (1913, p. 185-186) summarised the situation at the beginning of the XX century:

[11] This is another example on how by considering how a specific idea (or observation) enters in a theory, we can reach insight into its abstract form. Moreover, in this case forms that were perceived to be in opposition become actually two particular realisations of the same abstract form. Such situation is quite common, it can be used for self-criticism: when exposing a criticism of something perceived as belonging to a different perspective than ours, we can ask under which abstract form both perspectives are just different realisations of this form. The abstraction quite often produces a symmetrisation of the opponents, thus removing arbitrariness.

> *Most theorists have a constant predilection for explanations borrowed from physics, mechanics, or dynamics. Some would be satisfied if they could account for all phenomena by the motion of molecules attracting one another according to certain laws. Others are more exact: they would suppress attractions acting at a distance; their molecules would follow rectilinear paths, from which they would only be deviated by impacts. Others again, such as Hertz, suppress the forces as well, but suppose their molecules subjected to geometrical connections analogous, for instance, to those of articulated systems; thus, they wish to reduce dynamics to a kind of kinematics. In a word,* they all wish to bend nature into a certain form, and unless they can do this they cannot be satisfied. *Is Nature flexible enough for this? [Emphasis added]*

Concluding this section, it must be asserted that the desires of Peirce in [CP 5.384] for science to be a method upon which "the ultimate conclusion of every man should be the same" cannot be fully carried out in current scientific practice since the requirements for the cessation of doubt are possibly different for different persons. The fade out of reason (Horkheimer, 1947; Feyerabend, 1987) during the XX century has completely changed the scene.

3.4 Glossary

Before moving on we summarise our use of polysemous words:

- Abstract (verb): The mental activity that produces relations between the multiple particulars and the universal form. It creates the association and produces relations between the particulars.
- Analogy: The inference that objects which agree in various respects agree in another respects. It produces a direct relation between the analogous elements not mediated by an universal form.
- Arbitrary: Not produced by reason. In conflict with reason.
- Critical thinking: The reasoning directed towards the fundaments (as in (Hegel, 2001, § 101)). The discovery of the hidden assumptions that frame our thoughts (epistemic frames, habits, simple intuitions, phantasies, ...) and their subsequent removal or rationalisation, making our thoughts clearer.
- Phantasy/imagination: An idea not firmly grounded in observations but necessary for our thinking. A posteriori they may be grounded, rejected or

remain as phantasies. The negative sense usually associated to them corresponds to: something created by our mind to avoid questioning our theories or fundamental beliefs.

- Reason: The mental faculty which is used in adapting thought or action to some end; the guiding principle of the mind in the process of thinking. Frequently contrasted with will, imagination, passion, etc. (Adapted from the Oxford dictionary).

- Turbid thinking. The construction of ideas by not completely conscious and rational methods such as: simple intuition, habit (usually unconscious), imagination/phantasy, analogy and the dogmatic use of epistemological frames and pre-existing theories.

- Understand: To have a rational theory that explains the matter in question.

4 Science, reality and the rules of rational retroduction

We have illustrated the existence of different approaches to the cessation of doubt, and consequently different *sciences*. They cannot be considered equivalent since one is based in clear thinking while the other resorts to non-rational ingredients. In order to develop a rational retroduction it is mandatory that the very notion of rationality becomes objective and intersubjective. We attempt here a proposal for the objective determination of rationality.

We further notice that for Peirce explanation can be equated to rationalisation as it is evident from the following quotation:

> *I think I have now said enough to show that my theory – that that which makes the need, in science, of an explanation, or in general of any rationalization of any fact, is that without such rationalization the contrary of the fact would be anticipated, so that reason and experience would be at variance, contrary to the purpose of science – [that this theory] is correct, or as nearly so as we can make any theory of the matter at present. (Peirce, 1994, CP 7.201)*

4.1 Science and reality

The task of understanding involved in scientific theories requires some precision on what we mean by reason and the requisites for inference.

4.1.1 The principle of reality

In the first place, we must indicate that the attempt of constructing a cosmos out of sensorial input implies the assumption that there is something real that reaches us through the senses, this is to say, that there are subject and object. While the truth of this statement is debatable, we can consider the dangers involved in accepting or rejecting it. Little damage is done if accepting reality were an error and it turns to be that everything is part of a unique encompassing being. On the contrary, if we were in error when rejecting reality, we would become completely dysfunctional and miss one of the greatest opportunities in life. The principle is addressed in:

> *Such is the method of science. Its fundamental hypothesis, restated in more familiar language, is this: There are Real things, whose characters are entirely independent of our opinions about them; those Reals affect our senses according to regular laws, and, though our sensations are as different as are our relations to the objects, yet, by taking advantage of the laws of perception, we can ascertain by reasoning how things really and truly are; and any man, if he have sufficient experience and he reason enough about it, will be led to the one True conclusion. The new conception here involved is that of Reality.(Peirce, 1955, p. 18)*

Reality in Peirce can be seen as a duality. In front of us we have what is perceivable (observable but not yet "observed") and what is elaborated by ourselves from this input: the ideated or ideal (*facts* in (Piaget and García, 1982)) [12].

The progression of knowledge

A schema for the progression of knowledge is depicted in Figure 1.

According to this schema, to grasp reality we need the sensible world (SW) that we perceive "out there", its ideated or intuited forms or ideal world (IW) which rest within us and a form of correspondence from one world with the other: the phenomenological map, Π, Γ (see Figure 1). We also need to include in reality as a primary element the consciousness of our mental operations as they are not ideas produced by elaboration of input from the senses –they are neither in SW nor in IW– and are not part of the phenomenological map, but rather its producer [13]. In natural science we restrict the study to the development of the relation between IW and SW, something that is not possible if we want to study cognition or

[12] This traditional view goes back at least to Goethe's aphorisms: "Experience is only half of experience" and "Everything factual is already theory".

[13] Thus, reason does not belong to the real world (SW, IW) but rather to an internal world which acquires other sort of reality in self-consciousness.

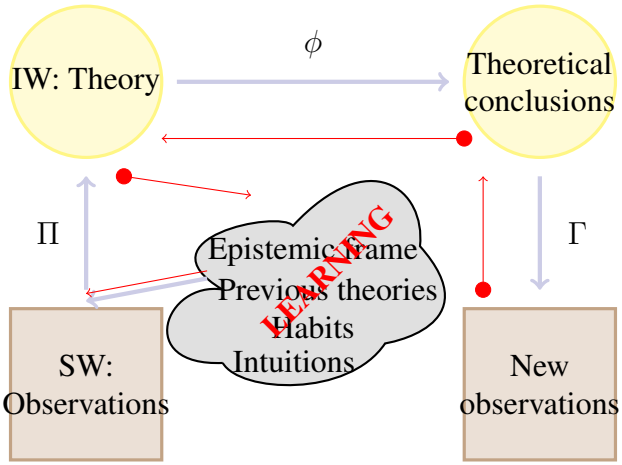

Figure 1: Schema for the progression of knowledge. Π is a projection that produces the real out of the observed, ϕ stands for a theoretical elaboration (which eventually can be empty, in such a case ϕ is the identity Id) and Γ is the interpretation that produces an expected observation. The red circuit indicates the effect of refutations, eventually leading to an improved version of Π .

psychology ignoring self-consciousness. Notice that whatever is in IW, it is not what is sensed, it is clearly not in SW. Hence, we can think of IW as a negation of SW, and the dialogue through the phenomenological map between the two forms of the real constitutes what science is. We have included a cloud of "turbid thinking" in Figure 1. The turbid thinking provides suggestions on how to produce facts and theories from observations. Sooner or later these precursors of the final ideas must be brought to our conscience to be examined and depurated by reason. The situation becomes mandatory when the theory is refuted. The observable events cannot be changed; in contrast, the facts we associated to the observations must be changed. The rational option is not to doubt about reason but rather doubting the pre-rational elements. Out of the process more observations are sought and, hopefully a renewed (more rational) theory emerges. We usually say that we have learned from our mistakes. Finally, ϕ represent theoretical elaborations, basically ruled by mathematical logic.

4.1.2 Scientific reasoning

The casting of science into the forms of dialectics, performed in Section 3, is leaving out an important fact. The ideal world shares a fundamental characteristic with living things: it is self-reproducing, it entails creative production, *poiesis*. We call this reproductive act reasoning, an activity that produces new ideas out

of previous ideas or observations. It is then the poiesis of our conscience what opens up the duality IW/SW. In other words, IW is a result of the dialectic corresponding to the opening Ego-SW, where Ego is the knowing and SW is the knowable. The destiny of IW is then to change permanently, ideas generating experiences and experiences generating ideas. Our beliefs are not stable, and every time we try to see whether they come from the reasoning ego or from SW we end up finding the opponent [14]. This ever changing IW is then part of the dialogue (*becoming* (Hegel, 2001, § 134)) between Ego and SW, accounting in this form for the mixed character of experience and facts. Then, if reasoning is the activity that institutes new ideas in IW, the sensorimotor cognitive activity of children in their earliest times in full contact with SW must be considered as reasoning, since it institutes the idea of self (ego), and alter (not ego), the idea of the permanent (the identity, what remains unchanged through perceived changes) and the transition between states of permanency (change), at the same time they conceive space and time (Piaget, 1999)[15]. These ideas frame all further knowledge. The development of early cognition just presented certainly belongs to IW and was developed by Piaget from hints obtained by the observation of SW. We can then say that it is reasoning what institutes the duality we call reality, and it does so through the sensorimotor activity of the child.[16]

In this sense, abduction is the kind of inference entering the construction of theories in science and not any kind of inference as discussed in Sections 2 and 3. It corresponds to say that, according to the abstract mind, the cognitive activity we call science aims at the production of cognitive surpasses (see a detailed discussion in Section 4.2.3). Consequently we can call *scientific knowledge* to the

[14] Reason casts doubts on observations and observations cast doubts on reasoning. They continually "negate" each other and strive to explain themselves to the other (to agree) to preserve our unity. In this "becoming" they construct reality. Beliefs are held when this becoming rests for a while.

[15] We return to the issue of pairs of opposite concepts in Subsection 4.2.2.

[16] Peirce (1994, CP 8.41) discusses Kant's a-prioris of Space and Time concluding that something of the kind of Will, for which he finds no better word than volition, enters into these sensations. A sense of collision or clash between an inward and outward motion, a "consciousness of duality" or a "duality of consciousness". This duality leads to a critic of Hegel: "The capital error of Hegel which permeates his whole system in every part of it is that he almost altogether ignores the Outward Clash". Peirce writes: "Feeling is simple consciousness" which can be linked to the "simple intuition" in (Husserl, 1983) and the consciousness of the duality to "philosophical intuition" (the "lower and higher consciousness" in Pierce). He recognises as well that "conceptions which are proved to be indispensable in Formal logic, must have been already rooted in the nature of the mind when reasoning first began". A thought in the same direction as Piaget's ideas and well aligned with Hegel (2001, § 4) where "how to think" is put at the same autonomous level than digestion and moving. Thus, as an innate activity with no need for education in logic. All these philosophers appear to present particular expressions of an abstract idea not completely apprehended.

outcome of this activity.

The notion of scientific understanding we have coined contrasts with the notions of assimilation and accommodation. Both these forms of cognition are present in the child's development. The idea of assimilation and accommodation belongs to the family of imitation (Gruber and Vonèche, 1995, Introduction) which is within the same realm as analogical thinking. It is present since early times in life and is in use by the time of the development of abstract thinking, likely a pre-requisite and precursor. Abstract thinking is characterised by the overpassing increasingly present in the adolescent formal thinking[17], since the universal is reached by form of abstraction which is to put the actual as a case of the possible, being then the possible the universal form of the actual.

4.2 On the relation between subjective, intersubjective and objective

4.2.1 The no arbitrariness principle

If we introduce some arbitrary decisions in the scientific discourse (be it for the sake of the argument or with the aim of facilitating an explanation), the set of possible arbitrary elements must have the internal structure of a group[18], being then the set of all possible presentations of the argument a representation of the group and as such all of them equivalent. Further, it is shown that the facilitation of the relational concept of space due to Leibniz produced by the introduction of a privileged observer introduces a (useful) subjective element, the *subjective space* (the space in all elementary physics texts) along with a series of properties of this space as well as conditions that the statements regarding physical laws must satisfy if they are going to remain rational.

[17] *Formal thinking is both thinking about thought (propositional logic is a second-order operational system which operates on propositions whose truth, in turn, depend on class, relational, and numerical operations) and a reversal of relations between what is real and what is possible (the empirically given comes to be inserted as a particular sector of the total set of possible combinations). These are the two characteristics—which up to this point we have tried to describe in the abstract language appropriate to the analysis of reasoning— which are the source of the living responses, always so full of emotion, which the adolescent uses to build his ideals in adapting to society.*(Gruber and Vonèche, 1995, p. 438)

[18] For example: We can say that the relations in the invariant relational space are lifted into relations in the subjective spaces by arbitrary decisions, but since the subjective statements must remain equivalent, there must be a group of transformations, T, that allows us to move from one presentation to the other. If we conceive now a theory as a space of statements, E, relating different concepts belonging to our subjective presentation, what is real in them is only the core that remains when we remove (mod out) the arbitrariness, $D = E/T$, which is the result of identifying statements that only differ by the introduced arbitrariness. Thus, D is invariant while E is equivariant with respect to T.(Solari and Natiello, 2018)

When constructing a theory we have to make an early decision: are we going to introduce arbitrariness or not? The decision has not much relevance if we keep track of the introduced arbitrariness, and acknowledge the necessity of (and the methods for) removing it. However, if we lose consciousness of our constructive effort, we might inadvertently enter into the realm of arbitrariness. No amount of mathematics will take us ever out of the subjectivist cage, since the necessary step is not an analytic/deductive judgment but rather a synthetic/critical one. This is, we need to understand not what the consequences of our beliefs are, but rather which is the foundation of our beliefs.

The rejection of arbitrariness is a condition put on every rational construction (Solari and Natiello, 2018).

Principle 1. [No Arbitrariness Principle (NAP)] *No knowledge of Nature depends on arbitrary decisions.*

A criticism of empiricism A property is a quality proper of something. Whenever there is a property, there is something to which it belongs. If s stands for something and p for property, the basic enunciation is: "s is p". The set of properties, $\mathcal{P}$ is the set of all possible values of p irrespective of the s. It is true that the enunciation "s is" (produced after elimination of all the properties) is meaningless, an argument that is found in Carnap (1959, The significance of a sentence), but it only indicates that the search for the essence by depriving the object of its attributes is the wrong path. In the same form in which we admit a set of properties, we are forced to admit the set of objects constituted by all those things, $\mathcal{S}$, pointed by s in statements of the type "s is p", regardless of the property p. Doing otherwise is an instance of arbitrariness, since –as already discussed– the universal of something is nothing but the set of all particular forms of the matter/object under consideration[19]. It is important to notice that here as well as in Sartre, the metaphysical (and Kantian) "thing-in-itself", the noumenon, is eliminated as in Mach and Carnap, but the abstract in Sartre survives.

The elements in S bear all of them in common an undeniable property which cannot be suppressed, a essential property in the words of Mach: all members of S can be used in statements of the form "s is p" for some well selected $p \in \mathcal{P}$. The set of well selected properties regarding an object s, call them $\mathcal{P}_s \subseteq \mathcal{P}$ is the bundle of properties associated to the object in Mach. Reciprocally, the intersection of all $s \in \mathcal{S}$ having the property p is the only possible indication of the property. It then entails the same risks to admit the existence of $\mathcal{P}$ than the admission of the existence of $\mathcal{S}$, we must admit both or none of them, for otherwise we incur in arbitrariness. Mach restricts these considerations to regard

[19] A similar discussion is found in (Sartre, 1966, p. 3), see footnote 7.

observable objects, which is mere willingness and not a logical demand, for if it were true, we could never speak of the properties of mathematical objects such as vector spaces or numbers. In conclusion, the fact that the search for the essence in Mach's method fails does not mean that there is no essence, it only means what it shows: the search cannot reach the target, it is an inadequate searching method. Having failed to grasp the universal, Mach is then forced to pick one of the particulars (the "simplest one" in the words of Hertz) and to think in terms of it extending the results obtained to the whole class by invocation of analogy[20]. There is then no abolishment of metaphysics but rather what is abolished are higher levels of abstraction at the cost of introducing subjectiveness.

[20]It is interesting to compare the present view with Mach's position in (Mach, 2012, The economical nature of physical inquiry). We read:

> *Nature exists once only. Our schematic mental imitation alone produces like events. Only in the mind, therefore, does the mutual dependence of certain features exist.*
>
> *Let us endeavor now to summarise the results of our survey. In the economical schematism of science lie both its strength and its weakness. Facts are always represented at a sacrifice of completeness and never with greater precision than fits the needs of the moment. The incongruence between thought and experience, therefore, will continue to subsist as long as the two pursue their course side by side; but it will be continually diminished. In reality, the point involved is always the completion of some partial experience; the derivation of one portion of a phenomenon from some other. In this act our ideas must be based directly upon sensations. We call this measuring.*

In Mach "schematic mental imitation" corresponds to the role of abstraction, now exercised by blurred images. Rather than decorative detail or particularities, what is sacrificed –according to Mach– is completeness. This action in Mach parallels the action of Π. The restitution of the particulars that corresponds to Γ becomes in Mach "the derivation of one portion of a phenomenon from some other", which requires analogy and imagination. Thus, the central difference is Mach's relying on "imitation" as opposed to our relying on abstraction. Mach (2012, The principle of comparison in physics) explains:

> *The adoption of a theory, however, always involves a danger. For a theory puts in the place of a fact A in thought, always a different, but simpler and more familiar fact B, which in some relations can mentally represent A, but for the very reason that it is different, in other relations cannot represent it. If now, as may readily happen, sufficient care is not exercised, the most fruitful theory may, in special circumstances, become a downright obstacle to inquiry.*

Mach will soon name the substitution described as "analogy".

4.2.2 The mediation principle and the dialectical openings to understanding

Dialectical openings Our discussion of the construction of science is based upon the repeated action of synthetic cognition (Hegel, 2001, §1720).

This consideration takes as given the multiplicity of determinations, the observable. It then underplays the fact that in order to perceive as multiple what can be clearly argued that is different, we need to disregard some part of what is determined in each element of the multiple. The creation of concepts that do not refer to previous understanding but rather are inter-defined (usually, just two concepts indicating the ideal and opposing extremes of a perceived difference), is called a *dialectical opening* (Solari and Natiello, 2018, Note 4). Abstraction, the grasping of something as a particular case of an universal ideal and the synthesis of the particulars in universals is a dialectical opening. However, not all openings are abstractions. The opening that recognises an Ego and an external world (SW) is more complex. When we recognise ourselves and at the same time that "there is something out there" whose action reaches us through the external senses, we become conscious of SW and create a dual of it, IW, our ideas of the world which lie in our consciousness alongside of the idea of self, this is, self-consciousness. At this point our road begins to depart from Hegel[21], as he addressed not IW but rather self-consciousness. The basic dialectical opening of natural science corresponds to the creation of the duality (SW, IW). However, the most striking form of dialectical opening is the one performed by the child conceiving space and time (see Section 4.1.2). This operation requires that there is a something (name it relative position with respect to ego, or position in space) that is not permanent, that is a non-permanent property of the object. Further, if there are non-permanent matters, the idea of non-permanent becomes the idea of change, later *time*. Any attempt to explain one element in the dualities ego-alter, permanence-change calls for the other term, any attempt to explain position requires an idea of time, and an idea of object. Thus, the elements of the descriptions are interdependent, they have been constructed by idealising (taking to extremes) perceived differences and all of them together open the possibility of organising what reaches our senses. A dialectical opening institutes the terms that make possible to organise the idealised world and with it the sensorial world, it makes understanding possible.

Notice that dialectical openings operate on the basis of perceived differences which are ideated into complementary options within their universe of application. Being complementary, both of them are the Universe minus its complement: the negation of each other. We emphasise that the differences are perceived, they belong to SW. Notice further, that the frequently encountered duality essence-appearance is not a dialectic opening when the essence is thought of as the *noumen*. In such a case, it leaves all the perceived as appearance, as opposed

[21] Hegel addresses self-consciousness in his first book, The phenomenology of spirit.

to a metaphysical entity: the essence, the "thing-in-itself". In this sense, dialectical openings realise the elimination of metaphysics as the Vienna Circle sought, but preserve the abstract with a necessary nexus to the observable. But if we say: "the essence of a key is to open doors" we are constructing a valid duality where essence corresponds to "function" (relation with other objects) and we are leaving its material form in the appearance.

Mediation principle We do not usually accept as reasonable that which appears out of nothingness as self-evident assertions. We normally request a new rational belief to be derived (mediated) by acceptable argumentation from accepted beliefs. This recurrent form of reasoning cannot be pursued indefinitely. It comes to an end when we reach a point in which beliefs can no longer be derived from other accepted beliefs. At this point there seems to be only one option: Either we make explicit a layer of arbitrary assumptions (axioms) which is the opaque end that reason lets us see, or we find a set of opposing concepts and ideas that in their interplay constitute the foundation of our discussion; the dialectical openings.

Axiomatisation turns natural science into exact science, physics into mathematics, by removing the links between IW and SW. However, a purely abstract science is void. Instead of pushing physics into the exact sciences we must consider mathematics as a natural science, being the fundamental elements of mathematics the idealisation of quantitative relations in the observable world, which are always in relation to qualities (Usó-Doménech et al., 2022). Thus, projecting out the quality in SW we obtain the quantity. The operation requests us to conceive the dialectical opening quality-quantity along which we make the projection.

4.2.3 Cognitive surpass

The introduction of explanatory hypothesis, the process of abduction, is subject to the control of rationality and to the condition that the newly introduced hypothesis explains a class of problems larger than the one that motivated it, this is, that the hypothesis bears some of the main ingredients of *cognitive surpass* (Piaget and García, 1982) and offers itself more openly to refutation. However, the requisites for the acceptance of explanatory hypotheses (i.e., to be able to stand in front of refutation attempts) say little about the method of production.

4.2.4 The continuity principle (reduction to the obvious/evident)

Argumentations are constructed in such a way that they rest upon small units we consider evident or obvious. Yet, what is obvious or evident for some, may not be so for others. One of the forms in which we usually identify potentially irrational arguments is by detecting hiatus or *lacunæ* in the argumentation. The

request "please fill in the gap" quite often reveals a belief that cannot be supported, while being necessary for the argument. On the contrary, the rational argumentation proceeds to fill the gaps by explaining how they consist of the concatenation of smaller pieces, iterating the process until the pieces are accepted as evident or obvious. This self-similar form corresponds to what in mathematics is called *continuity.*

4.2.5 Logical action in front of contradictions

Whenever a chain of deductive reasoning arrives to a contradiction, the whole chain is rejected. When the contradiction results from comparing theoretical prediction and experimental reality we speak of experimental refutation. The logical scheme can be depicted as $A \Rightarrow [\text{consequences}] \Rightarrow B$ and B evaluates to $False$. No matter how pleasant the intermediate consequences are, there is no support for them. The most evident example is the hypothesis of the ether which is fundamental for the proposition of Maxwell's displacement field. Discarding the existence of the ether (following empirical evidence) under the present principle would mean the refutation of the hypothesis as well as its consequences. Yet, in general $A = a_1 \& \ldots \& a_n$, i.e., A may be a composite statement consisting of different parts. Only the hypothesis and consequences involved in the deduction of B are necessarily affected by the falsity of B. Thus, part of the theory survives and only part needs to be constructed under new hypotheses.

There is another instance of the same logical scheme which is not usually considered, namely when the contradiction stems from the logical structure of the theory (e.g., inconsistent postulates). Assume A is True, then $-A$ is False. The construction $A \Rightarrow [\text{consequences}] \Rightarrow (-A)$ discloses an internal contradiction of the theory and, as above, it forces us to reject the full chain.

Again, it is worth to realise that a refuted theory may require only some minor repair since usually A is a composite statement. It is enough for one of its terms to be wrong for the theory to be refuted. Refutation does not mean "throw away all your thoughts".

4.2.6 Example: demarcation of a non-scientific belief

Let us show how the requisites proposed in this Section change our perception of what is acceptable as scientific and what is not. Suppose we have a belief, T such as "All swans are white", and we have a form of determining what a swan is without considering its colour, call it A; we have that $T \& A \Rightarrow A \& W$, meaning that if we believe the theory it "explains" that the swan I am observing is white. The most immediate reason why such a belief is to be called non scientific is

that has not been explicitly linked to observations, thus, we cannot recognise the construction of a theory, which is, at the end, what is to be subject to appraisal.

Let us now give reasons for our belief: we have been observing fowl during ten years at a lake nearby our home. In these years of observations we have recognised N swans (mostly by their neck, beak and swimming, say) and the totality of them were white. Call these observations O and we have conjectured T out of O, this is our basic theory. Would now our belief be scientific (given the fundaments)? Our observation consists of a triple: a statistics, a place and a time frame. The theory T is produced by projecting out place and time frame. In order to confront the theory with new data we have to produce statistics in other places and time frames. Let us check the requirements of rationality. The initial violation of the mediation principle has been repaired in our second attempt. Is there continuity? What allows us to go from the observed into the idealised theory? As we shall presently see, the answer is: a rudimentary version of probability theory.

Actually, we have no support to disregard the alternative explanation that the observed fact occurs only in the lake we observe or in some lakes or period of time, all doubts that tell us that our theory is not under a firm ground. We may restrict the theory to our lake and the observed period of time. During that period other bird-watchers might have collected data as well. In this case, statistics would allow no more than establishing a bound, p^*, on the probability, p, of detecting an individual not being white in the experimental situation we are exploring. Let $P(p; N)$ be the probability of observing non-white swans in N trials. Hence, for a null observation record, given P we can estimate p^* as the solution of the equation $P(p^*, N) = \frac{1}{2}$. For all $p < p^*$ it is more likely not to observe a non-white swan in N trials than observing it. Hence, the observations only allow us to say that having *not* observed a non-white swan is not just bad luck but rather what it is expected for such a probability p. But then, why have we chosen $p = 0$ in our theory T? We meet arbitrariness once again. With the sole support of statistics we cannot make such a bold theory. We would have to make then a third version of the theory which changes our belief for something in terms of probabilities, which eventually will cast doubts about the quality of our statistics: Do we have bias in our sampling?

The example shows how we proceed from an initial belief, a hunch, critically searching for its fundaments using some of the rules for reasoning that are not (at least in this presentation) formal logic. The lesson gained from the observation of white swans in some place and period of time is: If things do not change too much since I made my observations, I am likely to encounter only white swans at that place next time. This suggests ways to improve the theory through further enquiry: I might suspect genetic similarities among the swans and also its counterpart: That there may be more genetic diversity in swans at other locations (or different epochs of time, but changing location is feasible while moving to "another time" is not).

The "hunch" and its criticism trigger a research programme by putting together other knowledge and the awareness of the limitations of my observations. This generates an ampliative hypothesis to investigate.

4.2.7 Example: the principle of relativity

Most physicists, including us, accept the principle of relativity as correct. Why is it so? While Einstein does not offer any argument (Einstein, 1905), Poincaré does (Poincaré, 1913, Ch. VII), when he states:

> *The movement of any system whatever ought to obey the same laws, whether it is referred to fixed axes or to the movable axes which are implied in uniform motion in a straight line. This is the principle of relative motion; it is imposed upon us for two reasons: the commonest experiment confirms it; the consideration of the contrary hypothesis is singularly repugnant to the mind.*

Poincaré comes to no better argument after several pages dwelling in Newton's mechanics as it was taught at his time (based upon absolute space). No search in the realm of mechanics will serve the purpose of finding the foundations of the principle after Newton's axioms. The foundations are to be found behind them, a matter Newton did not discuss at large. Moreover, the little he did discuss was misunderstood, such as the notion of "true motion" which was shadowed and finally replaced by the notion of "motion in absolute space" which is not in Newton (Solari and Natiello, 2021).

The idea of the relativity principle as proposed by Poincaré and others under various names (such as "symmetry principle" (Mach, 1919)) is a belief coming from the habit instructed to physicists by the teaching of Newton's mechanics. Observational inferences such as "All swans are white" can be put to experimental test (and in this case proved wrong by e.g., displaying black swans from Australia or black-necked swans from South America). On the contrary, the relativity principle is not observational. To assess its truth value we must seek its foundation in the demands of reasoning. The principle can be regarded as a special case of the no arbitrariness principle (NAP) since it states that any choice of reference frame within the class of "admissible systems" is as good as any other. To pick a system is an arbitrary decision, then arbitrary decisions should be transparent for the laws of physics. For Newton's laws of dynamics this amounts to the Galilean group of transformations that relate positions and velocities as recorded by any (inertial) system without altering the physical laws (Newton's equations). The transformations must form a group since the composition of valid transformations is required to be another valid transformation.

At those times, the issue was how to merge electrodynamics with the principle of relativity. Maxwell's equations were known to obey a symmetry with respect to Lorentz transformations. Hence, it appeared as Lorentz transformations were the needed connection between admissible systems. It is interesting to examine the argument in (Einstein, 1940):

> *The so-called special or restricted relativity theory is based on the fact that Maxwell equations (and thus the law of propagation of light in empty space) are converted in equations of the same form when they undergo Lorentz transformations. This formal property of Maxwell's equations is supplemented by our fairly secure empirical knowledge that the laws of physics are the same with respect to all inertial systems. This leads to the result that the Lorentz' transformations – applied to space and time coordinates– must govern the transition from one inertial system to any other.*

In the first place, the conclusion is not deducible from the premises. Secondly, the Lorentz transformations (LT) do not form a group, since the associative law fails [22]. The Lie algebra associated to the group of Poincaré-Lorentz symmetries has dimension six and what is needed has dimension three. [23]

Third, there is nothing such as "...empirical knowledge that the laws of physics are the same with respect to all inertial systems". Inertial systems, as well as the laws of physics, are abstract concepts. The invariance of natural laws with respect to the choice of inertial system is a matter of reason and of habitus. Einstein refers to the habitus developed when studying Newtonian mechanics where the concept of "inertial system" was originated. What belongs to the habitus and what to the empirical knowledge was in any case clearer for Poincaré than for Einstein.

The student in physics and the learned physicist know about Galilean transformations in the context of Newton's laws. They can legitimately ask: why this symmetry must be the symmetry of space instead of some other set of symmetries? What textbooks fail to indicate in this respect is that Newton aimed at describing relative motion, an aim that the facilitation of Newtonian physics forgot. Relative

[22] This observation already indicates that LT's cannot connect different inertial systems, since such connections are automorphisms of the inertial systems and as such must form a group (this is today an elementary theorem in Category theory).

[23] A defender of relativity would argue that the three group generators not considered correspond to rotations. This is true in abstract terms, but the necessary correspondence with rotations in the sensible world is not present. The rotations correspond to $L(-(u \oplus v)) \circ (L(u) \circ L(v))$ (Gilmore, 1974, p. 503), where $\oplus$ stands for Einstein's addition of velocities and $L(u)$ is a Lorentz transformation based on the velocity u. To imagine in our minds the consecutive application of three Lorentz transformations does not enact the rotation of physical objects. There is no correspondence between actually rotating a given object and the rotation group regarded as a subgroup of the Poincaré-Lorentz group.

motion relates to the SW in a direct form, it is real in the terms presented here. If, in order to facilitate mathematical expressions, we introduce a Cartesian frame (space), the reference point of the frame can be changed almost arbitrarily with time (provided we satisfy proper continuity conditions) and this is done without altering the relative motion (let us call this group the *inertial group*). It is this set of transformations what matters to the representation of the observable-real. A subgroup of this group corresponds to Galilean transformations, that preserve as well the form of Newtons equations. Thus, any transformation aiming at replacing Galilean transformations must be a subgroup of the inertial group, or otherwise it must present the form in which the new space relates to the observable making room for the replacement transformations proposed. Educated habits play a fundamental role in this matter. We teach in physics to represent motion by a graph, this is, we create a pedagogical space (not a movie) called space-time: $\mathcal{R}^{1+1}$. This opens for mistaking time with a second spatial coordinate. Indeed, if the student is asked to represent a trajectory of constant speed going from (x_0, t_0) to (x_1, t_1) they will efficiently draw a straight line. However, if we ask: which was the distance travelled by the represented body? they might answer $d = |x_1 - x_0|$ more likely than coming to realise that there is not enough information to answer the question. Notice that in any other representation (y, t) with $y = x + z(t)$ the answer would become $d' = |x_1 - x_0 + z(t_1) - z(t_0)|$ depending on the arbitrary election of frame used for the representation (d actually corresponds to the change in the relative position between body and origin of coordinates, it is not a property of the body). The information in space-time is not enough to reconstruct distances and relative positions. Space-time is subjective, it depends on the observer, it is not directly linked to SW but rather it is the result of intuitions cemented by the teaching of physics.

Being such the case, there is a possibility for other rules of reasoning being broken by Special Relativity. The exploration points to the meaning of the velocity in a Lorentz transformation. First, the meaning of velocities in the context of Electromagnetic theory changed from "relative velocities" in experiments, to velocities relative to the ether later and finally to velocities with respect to a reference frame. Such changes are a signal of trouble in itself (Assis, 1994). It is not possible to connect any observable, objective, velocity to provide meaning to the velocity involved in a Lorentz transformation (Solari and Natiello, 2022). This matter puts us in front of the dilemma: either Special Relativity is imperfect or we have to abandon the hopes for science to be based only in observations and reasoning. Historically, the second alternative was taken.

Indeed, Einstein advocates that there is no abstraction (or any other relation) between observations and the theories at the time of production (Einstein, 1936), they arise by free invention and are validated by their results. The failure to recognise this problem originates in the suppression of abstraction:

> *An adherent to the theory of abstraction or induction might call our layers "degrees of abstraction"; but, I do not consider it justifiable to veil the logical independence of the concept from the sense experiences. The relation is not analogous to that of soup to beef but rather of wardrobe number to overcoat. (Einstein, 1936)*

Einstein gives "logical independence" to the concept from its conception originated in our sense-experiences. Adopting "free invention" implies to break the connections with nature, to have purely abstract concepts, detached from their conceptualisation. Science would then no longer be a matter of understanding nature, but rather a sort of game. In Einstein (1936) words:

> *It is an outcome of faith that nature – as she is perceptible to our five senses– takes the character of such a well formulated puzzle.*

4.3 The "marvellous self-correcting property of Reason"

Laudan (1981, p. 188) has stated that

> *No one was able to suggest plausible rules for modifying earlier theories in the face of new evidence so as to produce demonstrably superior replacements*

in support of his general idea that Peirce's "Self-corrective thesis" (Laudan, 1981, Chapter 14) cannot be demonstrated. He claims further that (Laudan, 1981, p. 239):

> *Peirce, in short, gives no persuasive arguments to establish that qualitative induction is either strongly or weakly self-corrective. [...] What the facts do not show, of course, is how the hypothesis is to be altered so as to bring it closer to the truth.*

Laudan's criticism to Peirce is rather a criticism to Laudan's own interpretation of Peirce. Indeed, Peirce writes

> *So it appears that this marvellous self-correcting property of Reason, which Hegel made so much of, belongs to every sort of science, although it appears as essential, intrinsic, and inevitable only in the highest type of reasoning, which is induction. (Peirce, 1994, CP 5.579) [the part quoted by Laudan is highlighted, and Laudan's sentence finishes with: 'and every branch of scientific inquiry exhibits "the vital power of self-correction".(Peirce, 1994, CP 5.582)']*

Laudan's selective quotation left behind a number of fundamental ingredients, criticising thereafter a weaker version of self-correction rather than Peirce's developed concept. It is not clear whether Laudan restricts science to the science recognised as such by Peirce [CP 1.43–1.45] or not.

The connection with *all* reason, with logic [CP 2.191] and with Hegel are missing, the latter being important to grasp abstraction, cognitive surpass and the grounding of hypotheses (see 2). Further, Laudan focuses his criticism in a version of inductive reasoning isolated from other reasoning processes, hence disregarding the "vital power" of reason as a unity. In all, the possibility of learning from our mistakes and improving on them with the aid of reason is degraded, if not neglected.

The self-correcting property belongs to reason and it is inherited by science only as much as science inherits reason. In this Section we highlight the existence of "rules of reasoning" restricting science in the same form in which Peirce did. Along this work, we have shown how the concept of abstraction can be made a little bit more abstract (Section 3), i.e., offering a closer focus on the essence/nature of the concept, and how this abstraction immediately opens the possibility of showing e.g., that Mach's relationism is actually a secular form of absolute space. We have improved the "Relativity principle" by showing that in terms of clear thinking it is not a separate principle but a particular application of the, more demanding, No-arbitrariness-principle. We have further shown how one classical example of Inference to the best explanation can be polished by making it rational according to the given rules (Section 4.2.6).

In short, when the production of true beliefs as a process, science, and logic are regarded –inspired in Peirce's ideas– as particular expressions of reason following a few identified rules, the "self-correcting property of Reason" can hardly be denied.

4.4 Multiple abstract projections and the case of science

The idealisation or abstraction of an observed phenomena is performed with the aim of organising our view of it, linking the new facts to pre-existing matters in our understanding. Such operation is motivated by the need of answering questions regarding the phenomena. Thus, both the questions being posed and the pre-existing knowledge suggest which features of the phenomena carry the potential for *explanation* (providing an answer to the questions) and which do not. This process is followed by logical elaboration and interpretation that provides the opportunity for contrasting the ideal with the observed, and, in case of refutation it triggers a new attempt at producing understanding. The process is directed by the posed questions, and as such, different idealisations are possible for different sets of questions. The observable real is then crossed by several idealisations,

each one can be said to correspond to a *dimension* of the phenomena. The associated projections on IW characterising these aspects are in principle considered independent of each other, something that will later be modified by synthetic judgment, that confronts the alleged independence. Let us illustrate this process with the idea of "science as it is practised".

The notions of science so far discussed correspond to an ideal, flawless functioning, science. The practice of science develops in a society which is part of a civilizational movement, thus there is a science idealised in terms of its relation to the society at large. Science is practised by human beings that constitute a particular field of symbolic production (Bourdieu, 1999). Thus, we easily find three different dimensions in the consideration of science.

From the point of view of society in general the goals of science are often related to the production of goods and practices that enhance well being. Central to well being is techno-science, geared towards the production of new goods, enhancing comfort and capabilities. Techno-science frequently adopts the criteria proper of technology and focuses on predictive success. The quality of this science is hence rooted in prediction. If something *works*, this is taken as support for it being *correct*. The foundation of scientific theories is subordinated to their success capabilities. In the schema of Figure 1, the focus is on Γ.

In addition, science is requested to guide some important decisions. For example, decisions in matters of global warming, epidemics, nuclear energy safety, human environmental impact and the extinction of species. In such endeavours, the contrastive comparison of the predictions is not possible. This aspect has been called "science in the post-normal age" (Funtowicz and Ravetz, 1993; Waltner-Toews et al., 2020). Such practice is forced to root its quality in the elements Π, ϕ of Figure 1, since Γ is not available.

The most traditional perspective is that of science as the search for harmony and understanding. All three elements Π, ϕ, Γ are then equally important and they cooperate (e.g., via auto-correction) to enhance understanding.

Thus, differences in epistemology are to be expected in correspondence with the demand put on science by society at large.

This situation is a constant source of misunderstanding. It may be argued that the "correct" relation between theory and observation is the one reflecting the current practices of the community of scientists, as if the practices of scientists were not conditioned by the necessity of justifying science in front of the supporting society (society at large, governments, granting institutions, etc.) or were not conditioned by the need to conform to established practices of their scientific social field. There have been attempts at explaining science as a practice directed towards the acquisition of knowledge in terms of features of the social structure of the field, such as competition for resources and social respect. A third source of misunderstanding is to believe that what has been observed for science in some

age (say after World War II) can be used to explain the development of science in another age (e.g., before the second industrial revolution). To consider science as "that which is analogous to what is currently observable" is to operate against the process of abstraction, which, as Piaget taught us, is geared toward the discovery of the possible as opposite to the given. It should then be considered a political act of conservatism that deprive us of ideals and the exercising thoughts directed towards the search of the foundations: the critical thinking.

5 Final thoughts

The idea in Peirce that it is possible to agree about the real depends on the criteria for the cessation of doubt and the admissibility criteria for hypotheses. We have shown that there are different criteria in use for both actions and as a consequence we are not reaching truth but rather opinion. If (the pragmaticist's) Truth is to be achieved, clear thinking must take the ultimate word above the turbid thinking that leads to opinion. The restoration of critical thinking requires the rejection of arbitrariness and the recovering of a rationality that goes beyond the instrumental reason. The arc of the construction leads then to investigate reason in its objective form rescuing the role of abstraction, indicating it as a pre-requisite for rational abduction.

In Section 3.3 we discuss how spatial relations link to a deeper abstraction level than the traditional notion of space, by discussing Ernst Mach as a representative of empiricism. In several respects, Mach is simply unable to understand Newton and/or the relational space; he needs a first reference of position and motion, and he boldly sustains the "fixed stars" as such reference when confronted with more abstract ideas such as those of Lange. However, space is only a production of the child, made out of spatial relations and the suppression of her/himself from the total picture. In an early stage of development we fail to perceive us as one arbitrary particular of a universal, we fail to produce abstraction. Spatial relations are real, their observable form corresponds to relative positions of objects, space is only a convenient form of representing them. The idea that space does not exist as such, which is clear in Kant and later in Piaget, simply obfuscates a great mind such as a young (30 years old) Bertrand Russell. Actually, the idea that the form in which we ordered the observable in our early infancy might not be the universal form for understanding appears as inconceivable to some minds. To sustain physics on the phantasy of a space that is no longer linked to completely observable events, rather than on the observable spatial relations, "simply" requires more phantasy: things that are unobservable except for their alleged consequences, the observable facts. We have already been warned by Faraday that what began by being a conjecture too often becomes a belief just by habituation:

> *But it is always safe and philosophic to distinguish, as much as is in our power, fact from theory; the experience of past ages is sufficient to show us the wisdom of such a course; and considering the constant tendency of the mind to rest on an assumption, and, when it answers every present purpose, to forget that it is an assumption, we ought to remember that it, in such cases, becomes a prejudice, and inevitably interferes, more or less, with a clear-sighted judgment. (Faraday, 1844, p. 285)*

Is it possible for a science based upon reason and experience to recovered the lost track? How is rational retroduction different from other forms of inference? We discussed some characteristics of rational abduction/retroduction in Section 4. We have made an effort to produce a few rules of rational thinking, and showed how they help to construct more solid beliefs. We hope it is just the beginning of a collective task long overdue, and that other scientists and philosophers will contribute their own rules. In front of us rises the most formidable task: to rethink the possibilities of humanity and the life on Earth. To believe that the same science that gave us the menace of nuclear destruction, global warming and an accelerated extinction of species will give us the means to avoid catastrophe is only the characteristic insistence of the dogmatic. Critical thinking, a critical philosophical attitude and critical science are urgently needed alongside techno-science.

If we drop the requirement for explanations to be rational, we can then include elements of turbid thinking as explanations, for example, we may accept to stop doubting when we have an analogy. It is not the formal part of abduction – the logic in abduction– what changes but rather its quality. Once rationality has been removed from hypothesis making and from theory testing, we preclude clear thinking from taking us back into reason.

What we call rational retroduction is not merely a reasoning step, but rather the integrated process of generating abductive hypotheses grounded in reason and in previous knowledge while accepting them tentatively only after all efforts to refute them have failed. The rules of reasoning summarise the guidance given by reason when attempting to improve theories. We present a dialectical view of abstraction, contesting Mach's criticism. If making abstractions is a necessity for understanding, abstraction cannot be rejected.

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