FLUTTER SPEED LIMITS OF SUBSONIC WINGS

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BSTRACT

Flutter is a phenomenon resulting from the interaction between aerodynamic and structural dynamic forces and may lead to a destructive instability. The aerodynamic forces on an oscillating airfoil combination of two independent degrees of freedom have been determined. The problem resolves itself into the solution of certain definite integrals, which have been identified as Theodorsen functions. The theory, being based on potential flow and the Kutta condition, is fundamentally equivalent to the conventional wing-section theory relating to the steady case. The mechanism of aerodynamic instability has been analyzed in detail. An exact solution, involving potential flow and the adoption of the Kutta condition, has been analyzed in detail. The solution is of a simple form and is expressed by means of an auxiliary parameter K. The use of finite element modeling technique and unsteady aerodynamic modeling with the V-G method for flutter speed prediction was used on a fixed rectangular and tapered wing to determine the flutter speed boundaries. To build the wing the Ansys 5.4 program was used and the extract values were substituted in the Matlab program which is designed to determine the flutter speed and then predicted the various effects on flutter speed. The program gave us approximately identical results to the results of the referred researches. The following wing design parameters were investigated skin shell thickness, material properties, cross section area for beams, and changing altitude. Results of these calculations indicate that structural mode shape variation plays a significant role in the determination of wing flutter boundary.

الخلاصة

الرفرفة (الأرتجاج) هي الظاهرة التي تنتج من التداخل بين القوى الديناهوائية وديناميكية الهيكل مما يؤدي الى حالة من عدم الاستقرار وبالتالي تدمير وتحطم الجناح. تحسب القوى الديناهوائية لمقطع جناح مهتز له درجتان من الحرية باستخدام نظرية ثيودرسن(**Theodorsen function**)، حيث إن المسألة تحل باستخدام التكامل المحدد. ويعتمد أساس هذه دالة على التدفق الكامن (***potential flow***) وعلى شرط كوتا (**Kutta condition**) ، والتي تكون أساسا مكافئاً لنظرية مقاطع الأجنحة للحالة الثابتة .حيث يتم تحليل الآلية الديناهوائية الغير مستقرة بشكل مفصل. الحل الحقيقي يتضمن التدفق و تبني شرط كوتا وتحليله بشكل مفصل. يمكن تمثيل الحل باستخدام العامل المساعد (***K***) (**auxiliary parametric**). لحساب حدود سرعة الرفرفة على الجناح المستدق والجناح المستطيل حيث تم استخدام تقنية العناصر المحددة و النموذج الديناهوائى الغير مستقر مع طريقة السرعة-عامل التضاؤل (***V-g***) (***velocity - damping method***) للتنبؤ بسرعة الرفرفة باستخدام برنامج ***(*ANSYS 5.4***)*** تم بناء الجناح والقيم المستخرجة التي تستخدم في برنامج الـ***(*MATLAB***)*** الذي صمم لحساب سرعة الرفرفة. ومنه التنبؤ بالمتغيرات المؤثرة على سرعة الرفرفة.حيث إن البرنامج أعطى نتائج مطابقة تقريباً إلى نتائج البحوث المشار إليها. تم بحث المتغيرات التصميمية التالية للجناح سمك متغير للغلاف ومادة متغيرة ومساحة مقطع متغيرة لقطع التقوية وارتفاع متغير. اوضحت النتائج بان تغير النسق للهيكل يلعب دورا مهما في حساب سرعة الرفرفة.

KEY WORKS: **Flutter, V-g Method, Wings.**

INTRODUCTION

The problem of oscillating airfoils has been an important subject of unsteady aerodynamics because of its close link with flutter analysis. The sustained oscillation is a boundary between convergent and divergent motions. Hence, the speed thus obtained is the critical speed, above which flutter occurs.

(Sadeghi, 2003); developed a code for the computation of three-dimensional aeroelastic problems such as wing flutter. **(Bala Krishnan 2003)**; Investigated the initial mathematical theory of aeroelasticity centered on the canonical problem of the flutter boundary instability endemic to aircraft that limits attainable speed in the subsonic regime. **(Massimo Bianchin 2003)**; Studied a methodology to merge state-space time domain realizations of a complete numerical aeroservoelastic model with flight mechanics equations

UNSTEADY AERODYNAMIC FORCES OF THE TYPICAL SECTION MODEL:-

The unsteady aerodynamic forces are calculated based on the linearized thin - airfoil .In this section, Theodorsen's approach will be summarized and the flutter analysis will be conducted based on his approach **(Theodore Theodorsen 1935**).

In Theodorsen's approach, aerodynamic surfaces are modeled by flat plates. Theodorsen assumes that the flat airfoil is oscillating about the shear center (elastic axis) and unsteady flow is composed of two components, (a) non – circulatory flow which can be expressed through the sources and sinks and (b) circulatory flow related to the flat vorticity surface extending from trailing edge to infinity. For each flow component, he obtained the velocity potential and then calculated the pressure using Bernoulli's theory.

The Non-circulatory Flow:-

By using Joukoweski's conformal transformation **(Theodore Theodorsen 1935**), the airfoil can be mapped onto a circle. The velocity potential of a source (ε) on a circle (x_1, y_1) can be expressed as:

$$\varphi = \frac{\varepsilon}{4\pi} Ln\left[(x - x_1)^2 + (y - y_1)^2\right]$$

Similarly, the velocity potential is due to a source (2ε) at on circle (x_1, y_1) and a sink (-2ε) at on circle $(x_1, -y_1)$.

$$\varphi = \frac{\varepsilon}{2\pi} Ln\left[\frac{(x - x_1)^2 + (y - y_1)^2}{(x - x_1)^2 + (y + y_1)^2}\right] \quad (1)$$

Since ($y = \sqrt{1 - x^2}$), the velocity potential is a function of (x) only.
The downward displacement of the airfoil can be written as

$$z = h + \alpha(x - ab)$$

Then, up-wash will be

$$w_a(x,t) = -\left[\frac{\partial z}{\partial t} + V\frac{\partial Z}{\partial x}\right] =$$

$$-\left[\dot{h} + \dot{\alpha}(x - ab)\right] - V\alpha$$

Therefore, the velocity potential due to pitch angle (α) will be

$$\varphi_\alpha = b\int_{-1}^{1} \frac{-V\alpha}{2\pi} Ln\left[\frac{(x - x_1)^2 + (y - y_1)^2}{(x - x_1)^2 + (y + y_1)^2}\right] dx_1 = V\alpha b\sqrt{1 - x^2} \quad (2)$$

Similarly, velocity potentials due to plunge motion, ($\dot{h}$) and angular velocity, ($\dot{\alpha}$) are respectively expressed as:

$$\varphi_{\dot{h}} = \dot{h} b\sqrt{1 - x^2} \qquad \varphi_{\dot{\alpha}} = \dot{\alpha} b^2\left(\frac{x}{2} - a\right)\sqrt{1 - x^2}$$

The total velocity potential due to non-circulatory flow becomes:

$$\varphi_{NC} = \varphi_{\alpha} + \varphi_{h} + \varphi_{\dot{\alpha}}$$

$$= V\alpha b\sqrt{1-x^2} + \dot{h}b\sqrt{1-x^2} + \dot{\alpha}b^2(\frac{x}{2}-a)\sqrt{1-x^2} \qquad (3)$$

By Bernoulli theorem, the pressure is obtained as follows:

$$\Delta p = -2\rho\left(\frac{\partial\varphi}{\partial t} + V\frac{\partial\varphi}{\partial x}\right) = -2\rho\frac{\partial\varphi}{\partial t} = -2\rho\dot{\varphi} \qquad (4)$$

And the force (positive downward) and the pitching moment (positive nose-up) about the elastic axis will be expressed as:-

$$F_{NC} = b\int_{-1}^{1}\Delta p dx = -2\rho b\int_{-1}^{1}\dot{\varphi} dx$$

$$\equiv -\pi\rho b^2\left(\ddot{h} + V\dot{\alpha} - ba\ddot{\alpha}\right) \qquad (5)$$

$$M_{NC} = b\int_{-1}^{1}\Delta p(x-a)bdx = -2\rho b^2\int_{-1}^{1}\frac{\partial\varphi}{\partial t}(x-a)dx$$

$$\equiv \pi\rho b^2\left(V\dot{h} + ba\ddot{h} + V\dot{\alpha}^2 - b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right) \qquad (6)$$

The Circulatory Flow:-

To satisfy the Kutta condition, Theodorsen employs a bound vortex distribution over the airfoil and a vortex over the airfoil wake.

In order to consider wake, assume a bound vortex ($\Delta\Gamma = \gamma\, dx$) at ($\frac{1}{X_o}$), and a shed vortex ($-\Delta\Gamma$) at ($X_o$).

Then, the velocity potential due to vortex is

$$\Delta\varphi_{\Gamma} = \frac{\Delta\Gamma}{2\pi}\left[\tan^{-1}\frac{Y}{X-X_o} - \tan^{-1}\frac{Y}{X-1/X_o}\right]$$

$$= -\frac{\Delta\Gamma}{2\pi}\tan^{-1}\left[\frac{(X_o - 1/X_o)Y}{X^2 - X(X_o + 1/X_o) + Y^2 + 1}\right]$$

Define ($X_o + 1/X_o = 2x_o$), and ($X = x, y = \sqrt{1-x^2}$)

Then,

$$X_o = x_o + \sqrt{x_o^2 - 1}$$

$$1/X_o = \frac{1}{x_o + \sqrt{x_o^2-1}} = x_o - \sqrt{x_o^2-1}$$

The velocity potential can be expressed as

$$\Delta\varphi_{\Gamma} = -\frac{\Delta\Gamma}{2\pi}\tan^{-1}\left[\frac{\sqrt{1-x^2}(2\sqrt{x_o^2-1})}{x^2 - x(2x_o) + (1-x^2) + 1}\right]$$

$$= -\frac{\Delta\Gamma}{2\pi}\tan^{-1}\left[\frac{\sqrt{1-x^2}\sqrt{x_o^2-1}}{x^2 - xx_o}\right] \qquad (7)$$

Where

$$-1 \le x \le 1,\quad 1 \le x_o \le \infty$$

It is to be noted that the vortex is moving away from the airfoil with velocity of (V). Therefore, by Bernoulli theorem, the pressure due to the vortex is

$$\Delta p = -2\rho\left(\frac{\partial\Delta\varphi_{\Gamma}}{\partial t} + V\frac{\partial\Delta\varphi_{\Gamma}}{\partial x}\right)$$

Where:

$$\frac{2\pi}{\Delta\Gamma}\frac{\partial\Delta\varphi_{\Gamma}}{\partial x} = -\frac{\partial}{\partial x}\left[\tan^{-1}\left[\frac{\sqrt{1-x^2}\sqrt{x_o^2-1}}{1-xx_o}\right]\right]$$

$$= \frac{\sqrt{x_o^2-1}}{\sqrt{1-x^2}}\frac{1}{x_o - x}$$

$$\frac{2\pi}{\Delta\Gamma}\frac{\partial\Delta\varphi_\Gamma}{\partial x_o}=\frac{\sqrt{1-x^2}}{\sqrt{x_o^2-1}}\frac{1}{x_o-x}$$

The pressure at (X) due to the vortex at (x_o) is

$$\Delta p_\Gamma=-2\rho\frac{\Delta\Gamma}{2\pi}V\left[\frac{\sqrt{x_o^2-1}}{\sqrt{1-x^2}}+\frac{\sqrt{1-x^2}}{x_o^2-1}\right]\frac{1}{x_o-x}$$

$$=-\rho V\frac{\Delta\Gamma}{2\pi}\left[\frac{x_o^2-x^2}{\sqrt{1-x^2}\sqrt{x_o^2-1}}\right]\frac{1}{x_o-x} \quad (8)$$

$$=-\rho V\frac{\Delta\Gamma}{2\pi}\frac{x_o+x}{\sqrt{1-x^2}\sqrt{x_o^2-1}}$$

The force on the whole airfoil due to a vortex at (x_o) will be

$$\Delta F_\Gamma=b\int_{-1}^{1}\Delta p_\Gamma dx$$

$$=-\rho Vb\frac{x_o}{\sqrt{x_o^2-1}}\Delta\Gamma,1\le x_o\le\infty$$

The total force can be calculated by integrating with respect to (x_o)

$$F_\Gamma=\int\Delta F_\Gamma$$

$$=-\rho Vb\int_{-1}^{\infty}\frac{x_o}{\sqrt{x_o^2-1}}\gamma dx_o$$

Similarly,

$$\Delta M_\Gamma=b^2\int_{-1}^{1}\Delta p_\Gamma(x-a)dx,1\le x_o\le\infty$$

$$M_\Gamma=\int\Delta M_\Gamma$$

$$=-\rho Vb^2\int_1^\infty\left[\frac{1}{2}\sqrt{\frac{x_o+1}{x_o-1}}-\left(a+\frac{1}{2}\right)\frac{x_o}{\sqrt{x_o^2-1}}\right]\gamma dx \quad (10)$$

It has to be noted that the force and moment are functions of vortex strength (γ).By applying Kutta condition at trailing edge the vortex strength can be determined. The total velocity potential is:

$$\varphi_{total}=\varphi_\Gamma+\varphi_\alpha+\varphi_{\dot h}+\varphi_{\dot\alpha}$$

$$=\varphi_\Gamma+V\alpha b\sqrt{1-x^2}+\dot h b\sqrt{1-x}+\dot\alpha b^2(\frac{x}{2}-a)\sqrt{1-x^2}$$

By applying the Kutta condition, the following equation is obtained:

$$\frac{\partial\varphi_\Gamma}{\partial x}+\frac{V\alpha b(-x)}{\sqrt{1-x^2}}+\frac{\dot h b(-x)}{\sqrt{1-x^2}}+\frac{1}{2}\dot\alpha b^2\sqrt{1-x^2}+\frac{\dot\alpha b^2(\frac{x}{2}-a)(-x)}{\sqrt{1-x^2}}=$$

Finite. At (x=1) Therefore,

$$\left[\sqrt{1-x^2}\frac{\partial\varphi_\Gamma}{\partial x}\right]_{x=1}+\left\{-V\alpha b-\dot h b-\dot\alpha b^2(\frac{1}{2}-a)\right\}=0$$

Since

$$\frac{\partial\varphi_\Gamma}{\partial x}=\frac{\Delta\Gamma}{2\pi}\frac{\sqrt{x_0^2-1}}{\sqrt{1-x^2}}\frac{1}{x_o-x}$$

The following expression is obtained from the above equation.

$$\left[\sqrt{1-x^2}\frac{\partial\varphi_\Gamma}{\partial x}\right]_{x=1}=\int\frac{\Delta\Gamma}{2\pi}\frac{\sqrt{x_o-1}}{x_o-1}$$

$$=\frac{b}{2\pi}\int_1^\infty\frac{\sqrt{x_o^2-1}}{x_o-1}\gamma dx_o$$

$$=V\alpha b+\dot h b+\dot\alpha b^2\left(\frac{1}{2}-a\right)$$

Define

$$\frac{1}{2\pi}\int\frac{\sqrt{x_o^2-1}}{x_o-1}\gamma dx_o=V\alpha+\dot h+\dot\alpha b\left(\frac{1}{2}-a\right)\equiv Q$$

Then, the total force and moment on the airfoil will be as follows:

$$F_\Gamma=-\rho Vb\int\frac{x_o}{\sqrt{x_o^2-1}}\gamma dx_o$$

$$= -2\pi\rho VbQ \frac{\int_1^\infty \frac{x_o}{\sqrt{x_o^2-1}}\gamma\, dx_o}{\int_1^\infty \frac{\sqrt{x_o+1}}{\sqrt{x_o-1}}\gamma\, dx_o}$$

$$M_\Gamma = -\rho Vb^2 \int_1^\infty \left[\frac{1}{2}\sqrt{\frac{x_o+1}{x_o-1}} - \left(a+\frac{1}{2}\right)\frac{x_o}{\sqrt{x_o^2-1}}\right]\gamma dx_0$$

$$= -2\pi\rho VbCQ$$

$$= -2\pi\rho Vb^2 Q\left[\frac{1}{2} - C\left(a+\frac{1}{2}\right)\right] \qquad (11)$$

Where (c) is the Theodorsen function, and is defined as

$$C = \frac{\int_1^\infty \frac{x_o}{\sqrt{x_o^2-1}}\gamma dx_o}{\int_1^\infty \frac{\sqrt{x_o+1}}{\sqrt{x_o-1}}\gamma dx_o}$$

Assume that the airfoil has a simple harmonic motion

$$\gamma = \gamma_o e^{i\left[k\left(\frac{s}{b}-x_o\right)+\varphi\right]} = \gamma_o e^{i[\omega t - kx_o + \varphi]}$$

$$k = \frac{\omega b}{V} = \omega t$$

Where $s = Vt$

Then, Theodorsen function is expressed as

$$C = \frac{\int_1^\infty \frac{x_o}{\sqrt{x_o^2-1}}\gamma_o e^{i\omega t}e^{-ikx_o}e^{i\varphi}dx_o}{\int \frac{\sqrt{x_o+1}}{\sqrt{x_o-1}}e^{-ikx_o}dx_o}$$

$$= \frac{\int_1^\infty \frac{x_o}{\sqrt{x_o^2-1}}e^{-ikx_o}dx_o}{\int_1^\infty \frac{\sqrt{x_o+1}}{\sqrt{x_o-1}}e^{-ikx_o}dx_o} \qquad (12)$$

The Theodorsen function is frequently replaced by simple algebraic approximation as follows:-

$$C(k) = 0.5 + \frac{0.0075}{0.0455+ik} + \frac{0.10055}{0.3+ik} \qquad (13)$$

The total force and moment resulting from the noncirculatory and circulatory flows are expressed as:

$$F_\Gamma = -\pi\rho b^2\left[\ddot{h} + V\dot{\alpha} - ba\ddot{\alpha}\right] - 2\pi\rho VbQC(k)$$

$$M = \pi\rho b^2\left[ba\ddot{h} - Vb\left(\frac{1}{2}-a\right)\dot{\alpha} - b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right] + 2\pi\rho Vb^2\left(a+\frac{1}{2}\right)QC(k)$$

Where

$$Q = V\alpha + \dot{h} + \dot{\alpha} b\left(\frac{1}{2}-a\right)$$

If a quasi-steady aerodynamic is assumed (The aerodynamic characteristics of an airfoil whose motion consists of variable linear and angular motions are equal, at any instant of time, to the characteristics of the same airfoil moving with constant linear and angular velocities equal to actual instantaneous values.), then C (k) becomes (1), and the force and moment will be

$$F_{QS} = -\pi\rho b^2\left[\ddot{h} + V\dot{\alpha} - ba\ddot{\alpha}\right] - 2\pi\rho Vb\left[V\alpha + \dot{h} + \dot{\alpha} b\left(\frac{1}{2}-a\right)\right]$$

$$M_{QS} = \pi\rho b^2\left[ba\ddot{h} - Vb\left(\frac{1}{2}-a\right)\dot{\alpha} - b^2\left(\frac{1}{8}+a\right)\ddot{\alpha}\right] + 2\pi\rho Vb^2\left(a+\frac{1}{2}\right)\left[V\alpha + \dot{h} + \dot{\alpha} b\left(\frac{1}{2}-a\right)\right] \qquad (14)$$

3. Flutter Equation Of The Typical Section Model.

Equation of Motion:-

Consider the typical section shown in **Fig. (3).**

The model has a translation spring with stiffness (k_h) and torsion spring, with stiffness (k_T). These springs are attached to the airfoil at the shear center. Therefore, it is two degrees of freedom model (h,α). And (h) is measured at the shear center (elastic axis).

The downward displacement of any other point on the airfoil is

$$z = h + x\alpha$$

Where (x) is a distance measured from the shear center.

The strain energy and the kinetic energy are respectively given by

$$U = \frac{1}{2}K_T\alpha^2 + \frac{1}{2}K_h h^2$$

$$T = \frac{1}{2}\int \rho \dot{z}^2 dx$$

where (ρ) is the mass per unite length of the airfoil.

$$T = \frac{1}{2}\left(\dot{h}^2 \int \rho dx + 2\dot{h}\dot{\alpha}\int \rho x dx + \dot{\alpha}^2 \int \rho x^2 dx \right)$$

Define the following.

Mass $(m = \int \rho dx)$

The second moment of inertia of the airfoil about shear center,

$$I_\alpha = \int \rho x^2 dx = mr_\alpha^2$$

The first moment of inertia of the airfoil about shear center,

$$S_\alpha = \int \rho x dx = mx_\alpha$$

Where (r_α) is the radius of gyration and (x_α) is a distance from the coordinate to the mass center.
Then, the kinetic energy can be written as follows:

$$T = \frac{1}{2}m\dot{h}^2 + mx_\alpha \dot{h}\dot{\alpha} + \frac{1}{2}I_\alpha \dot{\alpha}^2$$

The virtual work due to the unsteady aerodynamic forces is

$$\delta W_\alpha = \int \Delta p \delta z dx = \int \Delta p\{\delta h + x\delta\alpha\}dx = Q_h\delta h + Q_\alpha \delta\alpha$$

Where the force (Q_h) is positive downward and moment (Q_α) is positive nose-up.
Lagrange's equations provide the equation of motion of the airfoil.

$$\frac{d}{dt}\left(\frac{\partial(T-U)}{\partial \dot{q}}\right) - \frac{\partial(T-U)}{\partial q} = Q_q$$

$$\begin{bmatrix} m & mx_\alpha \\ mx_\alpha & mr_\alpha^2 \end{bmatrix}\begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_T \end{bmatrix}\begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} Q_h \\ Q_\alpha \end{Bmatrix} = \begin{Bmatrix} f \\ m \end{Bmatrix}$$

where $q = h, \alpha$

$$\begin{bmatrix} 1 & \bar{x}_\alpha \\ \bar{x}_\alpha & \bar{r}_\alpha \end{bmatrix}\begin{Bmatrix} \ddot{h}/b \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} \omega_h^2 & 0 \\ 0 & \omega_\alpha^2 \bar{r}_\alpha^2 \end{bmatrix}\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F/mb \\ M/mb^2 \end{Bmatrix}$$

where

$$\omega_h^2 = K_h/m, \omega_\alpha^2 = K_T/I_\alpha, \bar{x}_\theta = x_\alpha/b, \bar{r}_\alpha = r_\alpha/b.$$

The harmonic motions ($h = h_o e^{i\omega t}$) and ($\alpha = \alpha_o e^{i\omega t}$) assumed the equations of motion will be:-

$$-\omega^2\begin{bmatrix} 1 & \bar{x}_\alpha \\ \bar{x}_\alpha & \bar{x}_\alpha^2 \end{bmatrix}\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} + \begin{bmatrix} \omega_h^2 & 0 \\ 0 & \omega_\alpha^2 \bar{r}_\alpha^2 \end{bmatrix}\begin{Bmatrix} h/b \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F/mb \\ M/mb^2 \end{Bmatrix}$$

The unsteady aerodynamic force and moment are:

$$F = -\pi\rho b^2\left[\ddot{h} + V\dot{\alpha} - ba\ddot{\alpha}\right] -$$

$$2\pi\rho VbC(k)\left[V\alpha + \dot{h} + b\left(\frac{1}{2} - a\right)\dot{\alpha}\right]$$

$$= \pi\rho b^3\omega^2\left[\frac{h}{b}\left(1 - i2C\frac{1}{k}\right) + \alpha\left(-a - i\frac{1}{k} - 2C\frac{1}{k^2} - 2i\left(\frac{1}{2} - a\right)\frac{C}{k}\right)\right]$$

$$= \pi\rho b^3\omega^2\left[\frac{h}{b}L_h + \alpha\left(L_\alpha - \left(\frac{1}{2} + a\right)L_h\right)\right]$$

Where the reduce frequency is ($k = \frac{\omega b}{V}$), and

$$L_h = 1 - i2C\frac{1}{k},\ L_\alpha = \frac{1}{2} - i\frac{1+2C}{k} - \frac{2C}{k^2}$$

Similarly,

$$M = \pi\rho b^4\omega^2\left[\left\{M_h - \left(\frac{1}{2} + a\right)L_h\right\}\frac{h}{b} + \left\{M_\alpha - \left(\frac{1}{2} + a\right)(L_\alpha + M_h) + \left(\frac{1}{2+a}\right)^2 L_h\right\}\alpha\right]$$

Where

$$M_h = \frac{1}{2}, M_\alpha = \frac{3}{8} - i\frac{1}{k}$$

Then, the equation of motion can be rewritten as:

$$-\omega^2\begin{bmatrix}1 & \bar{x}_\alpha \\ \bar{x}_\alpha & \bar{r}_\alpha^2\end{bmatrix}\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}+\begin{bmatrix}\omega_h^2 & 0 \\ 0 & \omega_\alpha^2 \bar{r}_\alpha^2\end{bmatrix}\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}$$

$$=\frac{\omega^2}{\mu}\begin{bmatrix}L_h & L_\alpha-\left(\frac{1}{2}+a\right)L_h \\ M_h-\left(\frac{1}{2}+a\right)L_h & M_\alpha-\left(\frac{1}{2}+a\right)(L_\alpha+M_h)+\left(\frac{1}{2}+a\right)^2 L_h\end{bmatrix}\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix} \quad (15)$$

where the mass ratio is defined as: ($\mu=\frac{m}{\pi\rho b^2}$), (m) is the airfoil mass per unit length.

Define ($\Omega^2=\frac{\omega^2}{\omega_\alpha^2}$) and ($R^2=\frac{\omega_h^2}{\omega_\alpha^2}$), then

$$-\Omega^2\begin{bmatrix}1 & \bar{x}_\alpha \\ \bar{x}_\alpha & \bar{r}_\alpha^2\end{bmatrix}\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}+\begin{bmatrix}R^2 & 0 \\ 0 & \bar{r}_\alpha^2\end{bmatrix}\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}$$

$$=\frac{\Omega^2}{\mu}\begin{bmatrix}L_h & L_\alpha-\left(\frac{1}{2}+a\right)L_h \\ M_h-\left(\frac{1}{2}+a\right)L_h & M_\alpha-\left(\frac{1}{2}+a\right)(L_\alpha+M_h)+\left(\frac{1}{2}+a\right)^2 L_h\end{bmatrix}\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}$$

V-G METHOD FOR FLUTTER ANALYSIS:-

The above flutter equation is expressed in the following matrix form.

$$[K_{ij}]\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}=\Omega^2[A_{ij}+M_{ij}]\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}$$

Where (K_{ij}) is the stiffness matrix, (M_{ij}) mass matrix, and (A_{ij}) is the aerodynamic matrix. Note that the aerodynamic is function of the reduced frequency, (k).

V-g method assumes first the artificial structure damping, (g).

$$\lfloor K_{ij}\rfloor=(1+ig)\lfloor K_{ij}\rfloor$$

This artificial damping indicates the required damping for the harmonic motion. The eigenvalue of the equation of motion represents a point on the flutter boundary if the corresponding value of (g) equals the assumed value of (g).

For a given reduced frequency, ($k=\frac{\omega b}{V}$) will be a complex eigenvalue problem.

$$\frac{(1+ig)}{\Omega^2}[K_{ij}]\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix}=[A_{ij}+M_{ij}]\begin{Bmatrix}h/b \\ \alpha\end{Bmatrix} \quad (16)$$

The Eigen value is:-

$$\lambda=\frac{1+ig}{\Omega^2}$$

From this eigenvalue:

$$\frac{1}{\lambda_{Re}}=\frac{\omega_i^2}{\omega_\alpha^2}$$

$$g=\frac{\lambda_{Im}}{\lambda_{Re}}$$

ANSYS ANALYSIS OF WING MODEL:-

The wing model analysis in the Ansys program is by using the suitable element for the work. The (Shell 93) may be used for skin and the spar web and the (Beam 4) (3D elastic beam) is used for the stiffeners in the isotropic case

FLUTTER PROGRAM: -

The combination between the (ANSYS 5.4) and the (MATLAB 7.0) is employed. The program is solved by using the Theodorsen's theory with velocity damping (V-g) method.

The inputs of program for the wings model are:

1. From (ANSYS 5.4) the natural frequencies are taken.
2. The static unbalance, frequency ratio, mass ratio, radius of gyration and non-dimensional location of airfoil elastic axis.
3. Density of air at any altitude.

And the outputs of program are:

1- The bending and torsional mode shapes for both rectangular and tapered wings as shown in Figures a to d.

2- The relation between the non-dimensional parameter (1/k) with structural damping.
3- Calculation of the flutter speed.

RESULTS AND DISCUSSIONS:-

RESULTS OF COMPARISON: -

By using analytical and numerical solution for the case where (μ=60, r_α = 0.4, x_α = 0.2, a = -0.3) it is found that the results in the work are approximately equal to the results in references as shown table 1.The following parameter are to be investigated

Effect of the Changing Wing Skin Thickness

The shell thickness is one of the main important variables in the wing design; therefore the effect of variation thickness from (0.001m) to (0.0035m) was studied in the reduced frequency, flutter speed and mass for two types of wing (rectangular wing and straight-tapered wing).

Rectangular Wing

Table (2) shows the shell thickness effects on the vibration modes. For the configuration (3x5) with area (A=44mm^2) and thickness (0.001m) the first two natural frequencies are equal to (20.016 HZ, 101.15 HZ), with mass (8.231kg), **Figs. (5)** and **(6)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\,\omega_\alpha}$ =1.66) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.351). For these values (ω_f =35.503HZ, V_f =474.751m/sec). But when the thickness increases to (0.0035m) the first two natural frequencies are equal to (19.489HZ, 108.7HZ) with mass (20.704kg), **Figs. (15)** and **(16)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\,\omega_\alpha}$ =2.66) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.322) and (ω_f =35.0014HZ, V_f =817.115m/s). From **Figs (7), (8) (9), (10) (11), (12) (13)** and **(14)** it is found that the reduced frequency is increased while the frequency ratio decreases therefore; the flutter speed is increased with thickness i.e. mass increases and the effect of thickness on the flutter speed is under investigation.

Tapered Wing

Table (3) shows that the effects of thickness are high on the flutter speed. For thickness (0.001m) the first two natural frequencies are equal to (31.807HZ, 128.22 HZ), with mass (7.4752kg) and **Figs. (17)** and **(18)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\,\omega_\alpha}$ =2.97) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.416) and (ω_f =53.33HZ, V_f =765.671m/s). And for thickness (0.0035m) the first two natural frequencies are equal to (32.446HZ, 138.75HZ), with mass (18.721kg), **Figs. (27)** and **(28)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\,\omega_\alpha}$ =4.956) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.396) and (ω_f =54.945HZ, V_f =1381.891m/s). **Figs (19), (20), (21), (22), (23), (24), (25)** and **(26)** show the thickness effect on flutter speed for taper wing type.

From the static solution **(Mechanical and Electrical Systems, Operation Manual, Boeing Commercial Airplane Company 1984)**. the optimum thickness is taken for the wings (0.001m) for rectangular wing and (0.003m) for tapered wing.

EFFECT OF THE USED MATERIAL

One of the important and necessary factors in the wing design is the material that is used. The material chosen gives the high resistance with little weight (high resistance to weight ratio). Therefore, three types of materials to build the wing structure are tested in this work.

Rectangular Wing:-

From **Table (4)** the (adv. Aluminum) is used in the wing design. For configuration (3x5) with thickness equal to (0.001m) the first two natural frequencies are equal to (21.20HZ, 107.49HZ) with mass (8.5437kg) and **Figs. (29)** and **(30)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\omega_\alpha}$ =1.688) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.337) and (ω_f =36.22HZ, V_f =513.018m/s). And for the same thickness for (7075-T6) the first two natural frequencies are equal to (20.016 HZ, 101.15 HZ), with mass (8.231kg) and **Figs. (5)** and **(6)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\omega_\alpha}$ =1.66) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.351). For these values (ω_f =35.503HZ, V_f =474.751m/sec). From **Table (5)** and for the same thickness when using the (Ti6A14V) the first two natural frequencies are equal to (19.825HZ, 101.21HZ) with mass (13.02 kg). **Figs. (33)** and **(34)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\omega_\alpha}$ =2.044) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.327) and (ω_f =33.095HZ, V_f =584.624m/s). From above it is clear that the effects of the materials (7075-T6) and (Adv.Aluminum) are approximately equal in angular flutter frequency but the difference in the flutter speed is equal to (7.45%) using the same mass. But when using (Ti6A14V) the angular flutter frequency is less than the (7075-T6) and (Adv.Aluminum) while the flutter speed is greater with high value of mass. The percentage between (Ti6A14V) and (7075-T6) is equal to (18.7%), and (Ti6A14V), (Adv.Aluminum) is equal to (12.2%). The percentages differ because of the wing mass difference.

TAPERED WING

From **Table (6)**,and when using the (adv. Aluminum) in the wing design with changing thickness, it is seen that for configuration (3x5) with thickness equal to (0.003m) the first two natural frequencies are equal to (34.306HZ, 146.07HZ) with mass (17.098kg), **Figs. (31)** and **(32)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\omega_\alpha}$ =4.769) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.429) and (ω_f =62.66HZ, V_f =1399.903m/s). And for the same thickness for (7075-T6) the first two natural frequencies are equal to (32.387HZ, 137.75HZ), with mass (16.472kg), **Figs. (13)** and **(14)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\omega_\alpha}$ =4.675) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.421) and (ω_f =57.992HZ, V_f =1294.144m/sec). From **Table (7)** and for the same thickness when using the (Ti6A14V) the first two natural frequencies are equal to (32.07HZ, 136.88HZ) with mass (26.055kg), **Figs. (35)** and **(36)** show the reduced frequency ($\frac{1}{k}=\frac{V_f}{b\omega_\alpha}$ =5.79) and frequency ratio ($\frac{\omega_f}{\omega_\alpha}$ =0.405) and (ω_f =55.436HZ, V_f =1592.679m/s). From above, it is found that the effects of the materials (7075-T6) and (Adv.Aluminum) are different in angular flutter frequency but the flutter speed difference percentage is equal to (7.5%) with the same mass approximately. But when using (Ti6A14V) it is found that the angular flutter frequency is less than the (7075-T6) and (Adv.Aluminum) while the flutter speed is greater with high value for mass. The percentage difference between (Ti6A14V) and (7075-T6) is equal to (18.7%) and (Ti6A14V) (Adv.Aluminum) is equal to (12.2%). These differences are because of the wing mass difference.

From above, the material (7075-T6) is recommended for both wings rectangular and taper because it gives good results for flutter speed and angular flutter frequency with little mass.

Figs. (1) and **(2)** show the bending and torsion mode shapes and the corresponding deformations of the material, type (7075-T6) in configuration (3x5) and shell thickness (0.001m) with beam cross section area of

($44x10^{-6}m^2$). These deformations are due to free vibration in the rectangular wing. **Figs. (3)** and **(4)** show the bending and torsion mode shapes and the corresponding deformations of the material, type (7075-T6) in configuration (3x5) and shell thickness (0.003m) with beam cross section area of ($44x10^{-6}m^2$). These deformations are due to free vibration of the tapered wing.

CONCLUSIONS

From the results achieved in this work the following points may be concluded.
Increase of radius of gyration (r_α) tends to increase the flutter speed especially for higher mass ratio. The static unbalance (x_α) increases (the distance between the rotation center and center of gravity) the flutter speed decreases because of the strong coupling between heaving and pitching motion.

- The flutter speed is sensitive to the ratio of uncoupling natural frequencies, where the increasing of the frequency ratio increases the flutter speed and the flutter speed has a minimum near ($\frac{\omega_h}{\omega_\alpha} = 1$).With structure damping omitted the typical section model is neutrally stable until ($V = V_f$) for ($V = V_f$) the bending and torsion frequencies merge and for ($V > V_f$) the system is unstable.
- With including structure damping (g) for small (V) all values of structure damping (g) are stable and flutter speed is sufficiently large (V) where structure damping (g) changes its sign from negative to positive.
- The higher wing aspect ratio decreases the flutter speed, while the increasing of the taper ratio increases the flutter speed. The flutter speed changes linearly with the altitude and it is increased with increasing the altitude. Flutter prevention can be summarized by adding mass or redistribute mass so that ($x_\alpha < 0$) mass balance, increases torsional stiffness i.e. increase (ω_α), Increasing or decreasing ($\frac{\omega_h}{\omega_\alpha}$) if it is near one (for fixed ω_α), adding damping to the structure and require the aircraft to be flown below its critical mach number.

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Symbols:-

Symbol	Definition	Unit
Aij	Aerodynamic matrix	-
A	Area of element	m2
a	Non-dimensional location of airfoil elastic axis	-
b	Semi chord of the wing	m
C	Theodorsen function	-
C_r	Wing root chord	m
C_t	Wing tip chord	m
[D]	Elasticity matrix	-
E	Modulus of elasticity	Gpa
F	Total force	N
F(k)	Real part of Theodorsen function	-
FNC	Force due to non-circularity flow	N
FΓ	Force due to vortex	N

Symbol	Definition	Unit
Lα	Aerodynamic coefficient	-
M	Total moment	N.m
MNC	moment due to non-circularity flow	N.m
Mh	Aerodynamic coefficient	-
Mij	Mass matrix	-
Mα	Aerodynamic coefficient	-
MΓ	moment due to vortex	N.m
m	Mass of wing	Kg
[N]	Shape function matrix	-
p	Pressure	N/m^2
ΔpΓ	Pressure due to vortex	N/m2
R	Frequency ratio	-
rα	Radius of gyration	m
Sα	First moment of inertia	kg.m
s	The distance from the vortex element to the airfoil	m
T	Kinetic energy	N.m

Symbol	Definition	Unit
t	time	sec
U	Strain energy	N.m
V	Velocity of air	m/s
Wα	Up-wash velocity of the airfoil	m/s
δWα	Virtual work due to unsteady aerodynamic forces	N.m
x	The distance measured from the shear center	m
xα	The center of gravity distance of the wing	m
α	Angle of attack	Degree
$\dot{\alpha}$	Angular velocity	rad/s
$\ddot{\alpha}$	Angular acceleration	rad/s2
ε	Source intensity	m2/s
γ	Function of the distance from the first vortex element	-
φ	Velocity potential	m/s

Symbol	Definition	Unit
φα	Velocity potential due to pitch angle	m/s
$\varphi_{\dot{h}}$	Velocity potential due to plunger motion	m/s
$\varphi_{\dot{\alpha}}$	Velocity potential due to angular velocity	m/s
φNC	Velocity potential due to non circularity	m/s
φΓ	Velocity potential due to vortex	m/s
φtotal	Total Velocity potential	m/s
Γ	Bound vortex and shed vortex	
ρ	Air and material density	kg/m3
ωh	Bending frequency	Hz
ωα	Torsion frequency	Hz
Ω	Frequency ratio	-
μ	Mass ratio	-
λ	Eigen value	-

Symbol	Definition	Unit
G(k)	Imaginary part of Theodorsen function	-
g	Structural damping	-
H	Altitude	km
h	Te Vertical coordinate of the axis of rotation	m
$\dot{h}$	Plunger motion	m/s
$\ddot{h}$	Plunger acceleration	m/s2
Iα	Second moment of inertia about shear center	kg.m2
i	$\sqrt{-1}$	-
$[K^e]$	Element stiffness matrix	-
Kij	stiffness matrix	-
Kh	Stiffness for translation spring	N/m
Kα	Stiffness for tensional spring	N/m
k	Reduced frequency	-
Lh	Aerodynamic coefficient	-

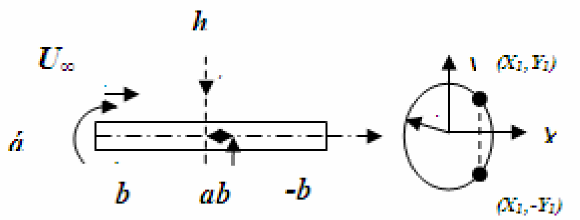

Fig. a. Flat airfoil and the transformed circle [4]

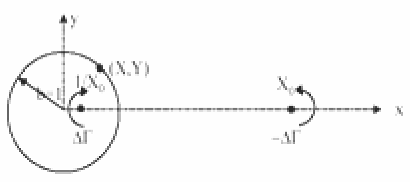

Fig. b. Circle in circulatory flow (**Theodore Theodorsen 1935**).

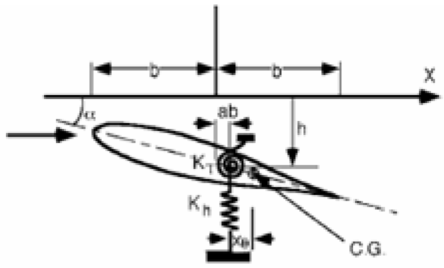

Fig. C. Typical section model

Table. 1 Results of the flutter speed parameter comparison.

Results of present work for flutter	Results of ref.[5],[6]
$\frac{V_f}{b\,\omega_\alpha} = 3.327$	$\frac{V_f}{b\,\omega_\alpha} = 3.69$ [5] $\frac{V_f}{b\,\omega_\alpha} = 3.89$ [6]

RECTANGULAR WING

Material: - cast-aluminum (7075-T6).

Table. 2 .Shell thickness variation for rectangular wing.

case	Thickness (m).	Mass (Kg).	Mode 1 (Hz).	Mode2 (Hz).	$R = \frac{\omega_h}{\omega_\alpha}$	I_α
1	0.001	8.231	20.016	101.15	0.197	0.1157
2	0.0015	10.726	19.82	104.75	0.189	0.1576
3	0.002	13.22	19.69	106.54	0.184	0.1995
4	0.0025	15.715	19.6	107.57	0.182	0.2414
5	0.003	18.21	19.536	108.24	0.180	0.2832
6	0.0035	20.704	19.489	108.70	0.179	0.3251

TAPERED WING

Material: - cast-aluminum (7075-T6).

Table. 3. Shell thickness variation for tapered wing.

case	Thickness (m).	Mass (Kg).	Mode 1 (Hz).	Mode 2 (Hz).	$R = \frac{\omega_h}{\omega_\alpha}$	I_α
1	0.001	7.4752	31.807	128.22	0.248	0.1111
2	0.0015	9.7244	32.066	132.33	0.242	0.1546
3	0.002	11.974	32.214	134.72	0.239	0.1981
4	0.0025	14.223	32.313	136.45	0.237	0.2416
5	0.003	16.472	32.387	137.75	0.235	0.285
6	0.0035	18.721	32.446	138.75	0.233	0.328

RECTANGULAR WING

Material: - cast-aluminum (Adv.Aluminum).

Table 4 .Effect of (Adv.Alum) shell thickness variation for rectangular wing.

case	Thickness (m).	Mass (Kg).	Mode 1 (Hz).	Mode 2 (Hz).	$R = \frac{\omega_h}{\omega_\alpha}$	I_α
7	0.001	8.5437	21.204	107.49	0.197	0.1201
8	0.0015	11.133	20.994	111.25	0.188	0.1636
9	0.002	13.723	20.856	113.11	0.184	0.2071
10	0.0025	16.312	20.760	114.19	0.182	0.2505
11	0.003	18.902	20.691	114.87	0.180	0.2940
12	0.0035	21.491	20.641	115.34	0.178	0.3375

RECTANGULAR WING

Material: - cast-aluminum (Ti6A14V).

Table 5. Effect of (Ti6A14V) Shell thickness variation for rectangular wing.

case	Thickness (m)	Mass (Kg).	Mode 1 (Hz).	Mode2 (Hz).	$R = \frac{\omega_h}{\omega_\alpha}$	I_α
13	0.001	13.02	19.825	101.21	0.195	0.1831
14	0.0015	16.966	19.625	104.63	0.187	0.2493
15	0.002	20.912	19.494	106.29	0.183	0.3155
16	0.0025	24.858	19.403	107.24	0.181	0.3818
17	0.003	28.804	19.338	107.84	0.179	0.4480
18	0.0035	32.75	19.29	108.25	0.178	0.5143

TAPERED WING

Material: - cast-aluminum (Adv.Aluminum).

Table. 6 .Effect of (Adv.Alum) shell thickness variation for tapered wing.

Case	Thickness (m).	Mass (Kg).	Mode 1 (Hz).	Mode2 (Hz).	$R=\frac{\omega_h}{\omega_\alpha}$	I_α
7	0.001	7.7593	33.679	136.01	0.247	0.1153
8	0.0015	10.094	33.969	140.35	0.242	0.1605
9	0.002	12.429	34.124	142.87	0.238	0.2057
10	0.0025	14.763	34.229	144.69	0.236	0.2508
11	0.003	17.098	34.306	146.07	0.234	0.2960
12	0.0035	19.433	34.368	147.12	0.233	0.3412

TAPERED WING

Material:- cast-aluminum (Ti6A14V).

Table. 7. Effect of (Ti6A14V) Shell thickness variation for tapered wing.

case	Thickness (m).	Mass (Kg).	Mode 1 (Hz).	Mode2 (Hz).	$R=\frac{\omega_h}{\omega_\alpha}$	I_α
13	0.001	11.824	31.512	127.54	0.247	0.1757
14	0.0015	15.382	31.761	131.57	0.241	0.2446
15	0.002	18.940	31.903	133.91	0.238	0.3134
16	0.0025	22.497	31.999	135.60	0.235	0.3822
17	0.003	26.055	32.07	136.88	0.234	0.4511
18	0.0035	29.613	32.126	137.86	0.233	0.5199

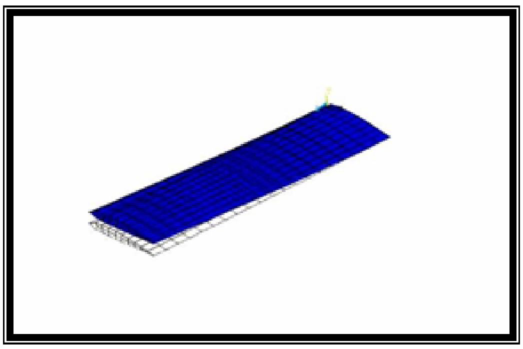

Fig. 1 Bending mode shape for rectangular wing

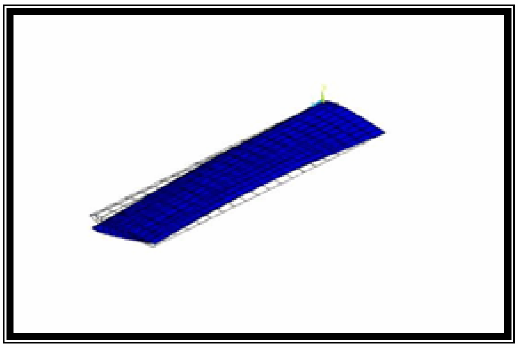

Fig. 2 Torsional mode shape for rectangular wing

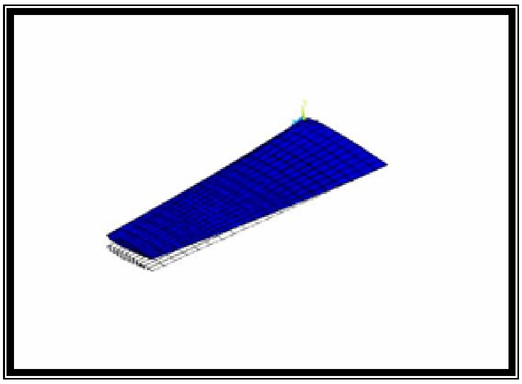

Fig. 3 Bending mode shape for tapered wing

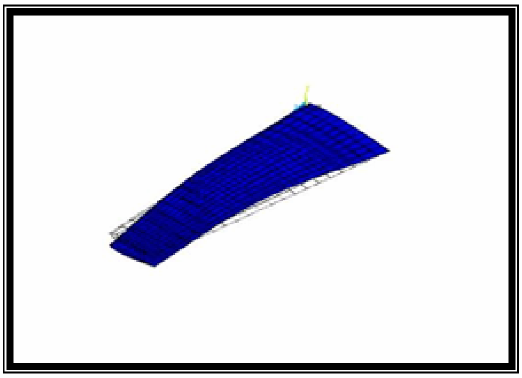

Fig. 4 Torsional mode shape for tapered wing

Rectangular Wing: - Figures below show the shell thickness variation.

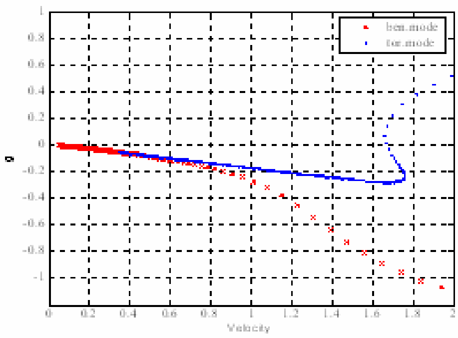

Fig. 5. The relation between non-dimensional parameter 1/k and damping ratio.

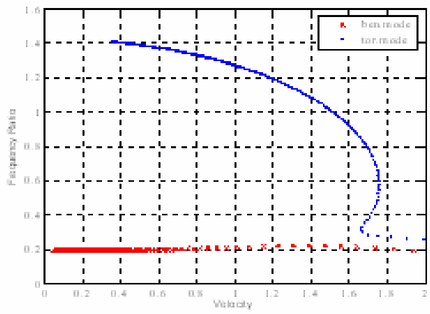

Fig. 6. The relation between the non-dimensional parameter 1/k and frequency

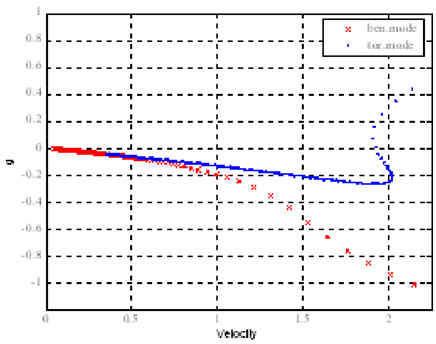

Fig. 7. The relation between non-dimensional parameter 1/k and damping ratio

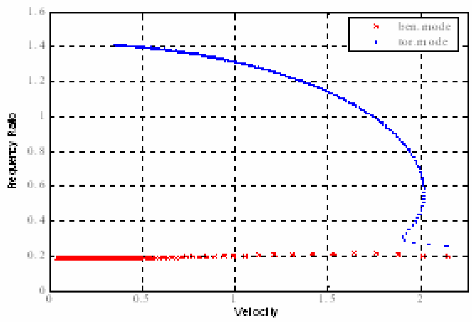

Fig. 8. The relation between the non-dimensional parameter 1/k and frequency ratio.

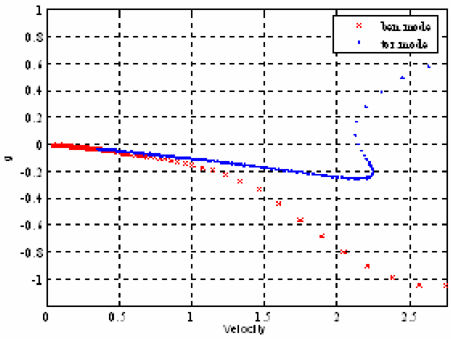

Fig. 9. The relation between non-dimensional parameter 1/k and damping ratio.

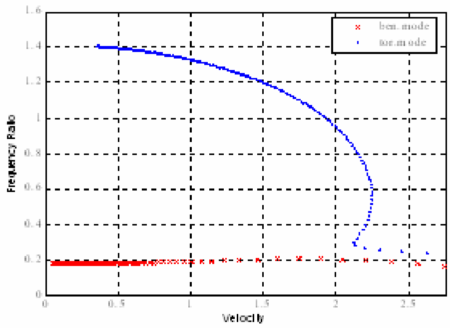

Fig. 10. The relation between the non-dimensional parameter 1/k and frequency ratio.

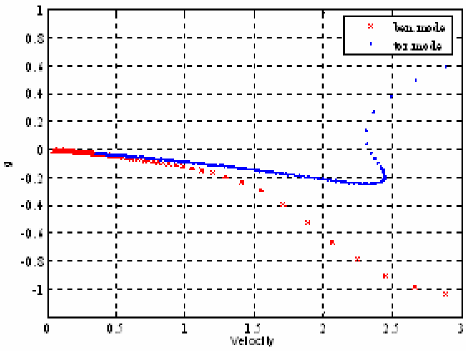

Fig. 11 .The relation between non-dimensional parameter 1/k and damping ratio.

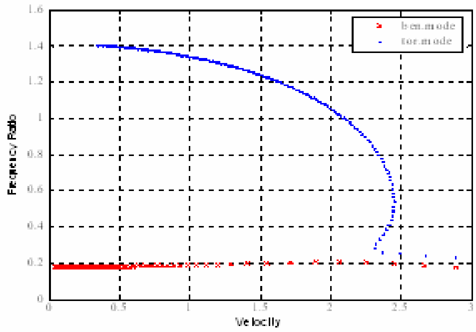

Fig. 12. The relation between the non-dimensional parameter 1/k and frequency ratio.

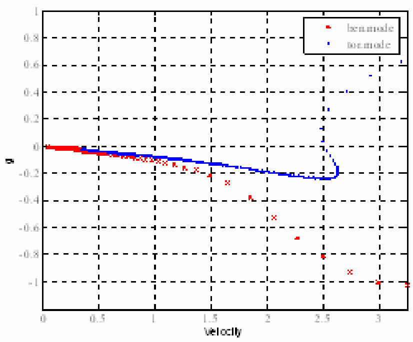

Fig. 13 .The relation between non-dimensional parameter 1/k and damping ratio.

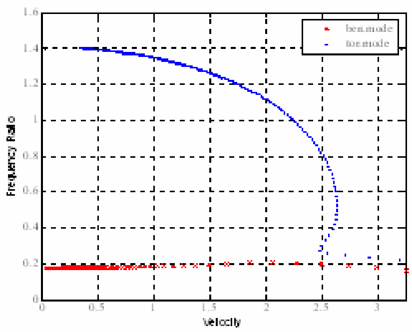

Fig. 14. The relation between the non-dimensional parameter 1/k and frequency ratio.

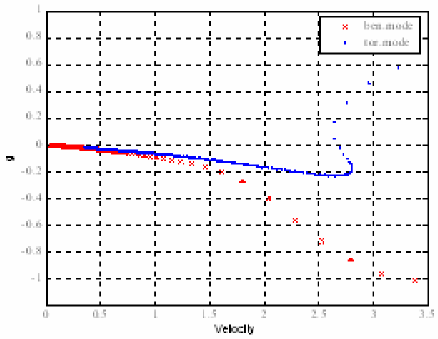

Fig. 15 .The relation between non-dimensional parameter 1/k and damping ratio.

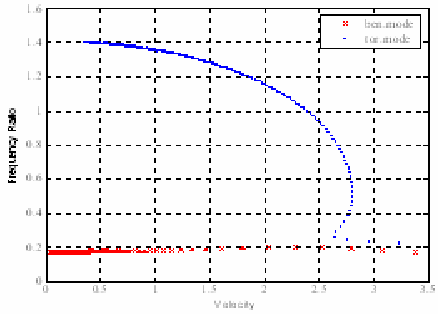

Fig. 16 . The relation between the non-dimensional parameter 1/k and frequency ratio.

Tapered wing:- Figures below show the shell thickness variation.

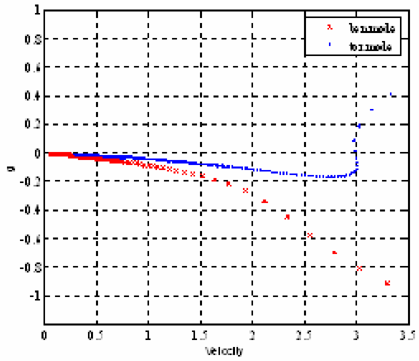

Fig. 17. The relation between non-dimensional parameter 1/k and damping ratio.

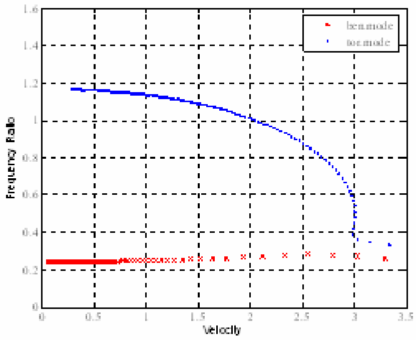

Fig. 18. The relation between the non-dimensional parameter 1/k and frequency ratio.

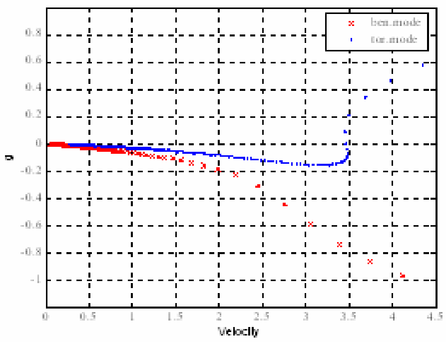

Fig. 19. The relation between non-dimensional parameter 1/k and damping ratio.

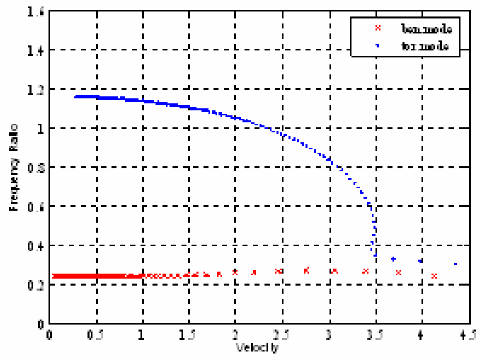

Fig. 20. The relation between the non-dimensional parameter 1/k and frequency ratio.

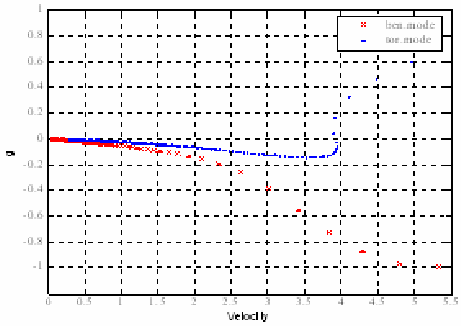

Fig. 21. The relation between non-dimensional parameter 1/k and damping ratio.

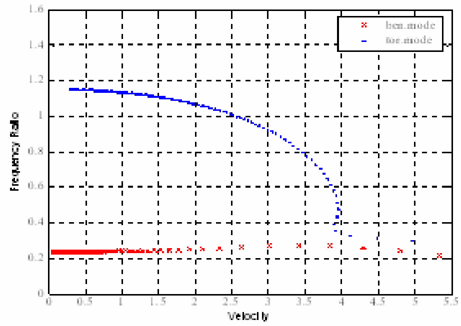

Fig. 22. The relation between the non-dimensional parameter 1/k and frequency ratio.

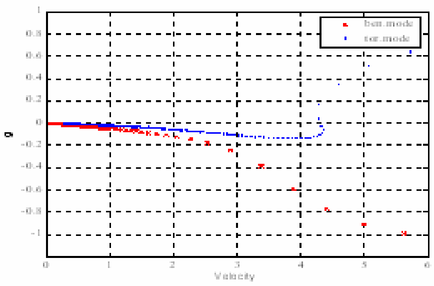

Fig. 23. The relation between non-imensional parameter 1/k and damping ratio.

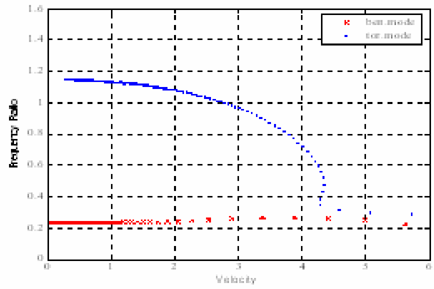

Fig. 24. The relation between the non-dimensional parameter 1/k and frequency ratio.

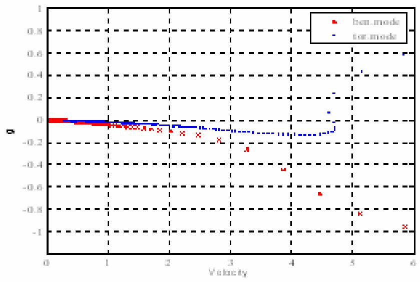

Fig. 25. The relation between non-dimensional parameter 1/k and damping ratio.

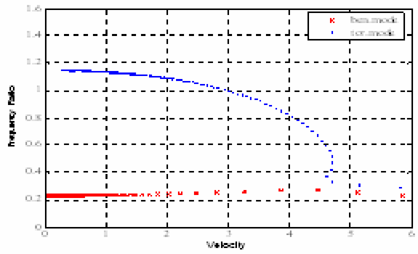

Fig. 26 . The relation between the non-dimensional parameter 1/k and frequency ratio.

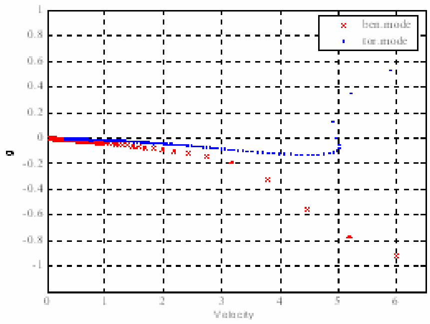

Fig. 27. The relation between non-dimensional parameter 1/k and damping ratio.

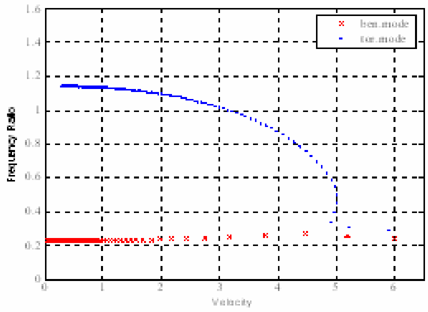

Fig. 28. The relation between the non-dimensional parameter 1/k and frequency ratio.

Rectangular wing:- The (Adv.Aluminum) effect with thickness=(0.001m).

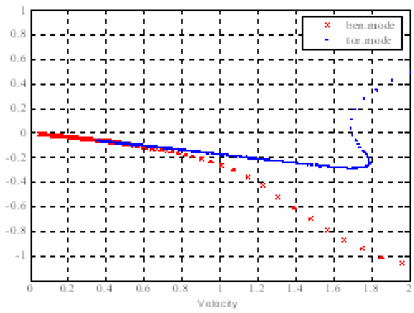

Fig. 29. The relation between non-dimensional parameter 1/k and damping ratio.

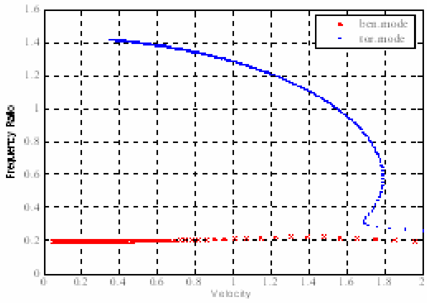

Fig. 30. The relation between the non-dimensional parameter 1/k and frequency ratio.

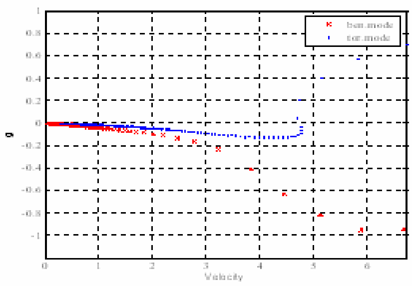

Fig. 31. The relation between non-dimensional parameter 1/k and damping ratio.

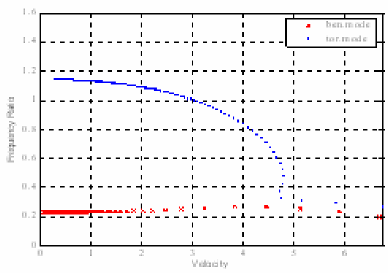

Fig. 32. The relation between the non-dimensional parameter 1/k and frequency ratio.

Rectangular Wing: - The (Ti6A14V) effect with thickness= (0.001m).

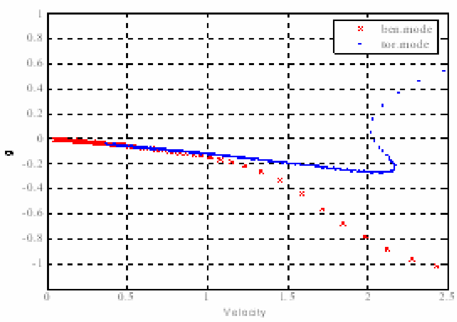

Fig. 33. The relation between non-dimensional parameter 1/k and damping ratio.

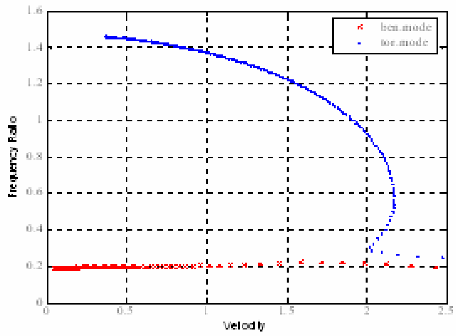

Fig. 34. The relation between the non-dimensional parameter 1/k and frequency ratio

Tapered Wing: - The (Ti6A14V) effect with thickness= (0.003m).

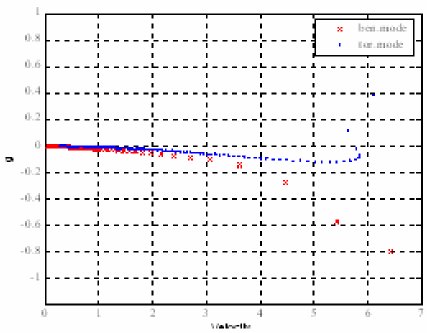

Fig. 35. The relation between non-dimensional parameter 1/k and damping ratio.

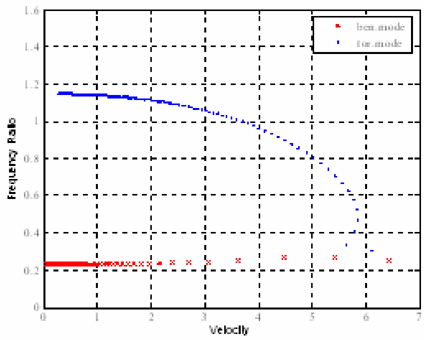

Fig. 36. The relation between the non-dimensional parameter 1/k and frequency ratio.