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Queuing Theory in Theme Parks

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Abstract

The goal of this project is to find the most efficient way for theme parks to process their guests. There currently exist many types of lines such as standby lines, single rider lines, and fastpass or skip the line queues. We will simulate a single rides queue. This will be a popular roller coaster at the park. Hourly throughputs of rides will be calculated, and a simulation will be created for the ride to start processing guests through different line scenarios. Through this method, we will find the best method for a park to process as many guests as they can, therefore shortening wait times and improving guest experiences. We will assume a perfect operating day, meaning no breakdowns, rain or ride closures. The final step will be to look at the strength of the model and see how far it can be expanded to create better guest experiences at the park.

At some point in our lives, we will all stand in a line, whether it be a line at the bank, a line for food, or the line at the Department of Motor Vehicles. Lines are inevitable. Since so many businesses necessitate the use of a line, the next step is to make this experience as short and pleasant as possible. The goals of improving line experiences and efficiency led to the development of queuing theory. Queuing theory is the study of waiting in lines. Today, queuing theory is applied to many different situations such as grocery stores, airport runways, and theme parks. Theme parks have become a hot bed for queuing theory experts. Examples can be found in the innovative ‘ push’ system proposed by Iulian Beloiu and Gergely Szekely and the creation of the fastpass system for Disney by Bruce Laval. There are lines everywhere at a theme park and every customer wants the shortest wait for every ride. There are many psychological factors of waiting in a line that can affect a patron’s mood and perception of the length of a wait for a ride and turn that wait into a positive enjoyable experience. While this is part of the queuing process, the ultimate goal is to move as many people through the line in the shortest amount of time. A simulation to model a highly popular ride was creted. The simulation model can be used to observe different types of queues that may work better for the given ride. Statistics can be gathered on how many people ride per hour and how many empty seats are left on rides and why those seats are empty. Finally, different queuing management strategies can be added to rides such as single rider lines and fastpass (skip the line) queues. This will allow conclusions to be drawn on the most efficient queuing system for modern day theme parks to use with the goal of the most positive guest experience.

History of Queuing Theory

Anger Erlang led the way in with the formal study of queuing theory in the early 1900s. His original research was used to create a more efficient use of telephone lines in Denmark (Kalashnikov, 2013). One of the more important ideas he brought forward was the mathematical randomness of the arrivals of phone calls. “It is assumed that there is no greater probability of a call being attempted at one particular moment than at any other moment” (Erlang, 1909). He was able to show that the random arrivals could be expressed as a Poisson distribution which is still the standard arrival distribution for queues today (Kalashnikov, 2013). Other mathematicians were able to take these ideas and started applying them to models of traffic, hospital aid and computer systems. These studies became known as queuing theory.

The simulation model created for our simulation will follow multiple examples of queuing research that already exist. Queue models have been created to increase traffic flow and decrease accidents. The University of Merdeka Malang used a research-based simulation on a specific four-way intersection in Malang City in Indonesia. They collected data at the intersection to build a model based on the probabilities of whether a vehicle would turn left, go straight, or turn right. They applied the mean and variance of these choices to make the simulation as realistic as possible. They also differentiated between cars and motorcycles, and broke the traffic pattern into the morning, mid-day, and night (Yuniawan, 2018). They were able to suggest a change to the program used by one of the traffic lights to improve the overall flow of traffic.

A second study looked into improving the process of airplanes arriving at a single airport. The researchers understood that air traffic is expected to increase over the next 20 years. Therefore airports will need to operate more efficiently to accommodate for the increase in air travel. Interesting variables they had to account for were the variability of times it takes a plane to taxi to the runway as well as whether or not an airport had more than one runway that can be used. The model they made was applied to Tokyo International Airport, the fifth busiest airport at the time based on volume of customers. The model looked at the mean number of arrivals per hour and the interarrival time of planes (time between planes landing). There must be some separation between the planes for safety reasons. The study concluded with recommendations that the airport reorganize key transition points that they use for the airplanes to line up for landing. Certain transition points were leading to bottlenecks forming in the air and causing delays in landing procedures, and subsequently slowing down operations at the airport (Etoh, 2019).

Psychology of Queuing

Disney parks represent 6 of the top 10 most attended theme parks in the world (worldwide-attendance-at-theme-and-amusement-parks, 2022). This means that they have some of the longest lines at theme parks, but it also allows them to be the best at queue design and queue management. Disney knows how popular each new ride is and designs their lines according to psychological laws and queueing theory models. One law of service to consider is $S = P - E$ (Maister, 1984). S is the satisfaction of a customer and it is equal to the perception of service quality (P) subtracted by the expected service quality (E). If a customer expects a high quality of service but perceives a quality lower than expected, their satisfaction is lowered. These are all psychological experiences and therefore can be influenced by non-concrete factors and variables. For example, mirrors were added to elevators and elevator waiting areas to allow people to check their appearance. No change was ever made to the efficiency of the elevators, but satisfaction of guests improved to their perception (Maister, 1984). This shows the power of perception in a queue.

Disney parks use these psychological factors in an attempt to influence customers minds to improve their overall satisfaction, regardless of actual waiting times. Disney knows that customers will have to wait, so they put a lot of money and design into the experience of a rides queue. David Maister lists the following crucial factors that affect the perception of wait time: occupied time feels shorter than unoccupied time, people want to get started, anxiety makes waits feel longer, uncertain waits are longer than known waits, unexplained waits are longer than explained waits, unfair waits are longer than equitable waits, the more valuable the service, the longer the customer will wait, and solo waits feel longer than group waits (Maister, 1984). Disney has had a new push for interactive queues. The queue itself is thought of as part of the experience or part of the attraction. The goal is for customers to have something to do to occupy their time. This could be a mobile game, and interactive game with physical objects in the queue, tactile objects in the queue that offer a reaction when engaged with, and simply an immersive queue that people can look at and find details in. All of these ideas occupy customers in the queue and will lead to a lower perceived waiting time. At restaurants, waiters are encouraged to give menus to a customer quickly and engage them in potential drink orders. This gives them the feeling of their meal getting started. It may be another 45 minutes before they are eating their dinner, but the process has begun. In a theme park, a ‘pre-show’ can be used to get the experience started while in line. A pre-show is usually just a movie that plays to explain the story of the ride or what customers are about to experience. It gives them something to occupy their time and gets them excited for the coming experience.

An important strategy that theme parks use to ease anxiety of guest is to post wait times. Maister says that uncertain wait times are longer than known wait times (Maister, 1984). When a person has no idea how long a wait time is supposed to be, they will feel that the wait is longer. They have no idea how much longer the wait may be or how close they are to being served. By posting a wait time, customers have a feeling of security and control since they can plan on how long they will be in line. This reduces the anxiety of the guests making their wait more comfortable. The more comfortable the wait, the shorter it will feel. Similar to the posting of wait times, theme parks now announce when a ride is experiencing down time. If a ride will be unavailable for a time, team members will make an announcement to those in line that there will be a delay in operation. This at least notifies guests that there is a reason their wait is longer. This also keeps guests more comfortable. They may be unhappy about the longer wait, but being notified significantly improves the way they feel towards a longer wait. By informing the guests, they feel included in the procedures that are affecting them as opposed to being left on the outside of the situation.

One of the biggest psychological factors seen at theme parks is that the value of service affects how long a customer is willing to wait. This means that the most popular rides will have the longest lines due to the willingness of the guests to wait longer for what they perceive will be a better experience. If they have a feeling that a ride will be really good, they will be willing to wait in a significantly longer line to make sure they get the chance to ride it. They may feel that the ride is ‘worth the wait’ (Maister, 1984). The last idea on the psychological impacts of waiting in lines is that waiting alone will always feel longer. When people are in groups, they have the ability to entertain, engage, and distract one another from the physical wait. Time moves quicker when engaged in an activity. Many groups will talk or play games while waiting in line to help pass the time. When a guest is on their own, they have less to engage with in line and are more likely to feel the full length of the line. While lines are bound to form in a theme park, understanding how a person is influenced by their wait time allows theme parks the chance to build queues that will minimize the feeling of waiting in line without actually changing the duration of the wait.

Basics of Queuing Theory

To explore the efficiency of a theme park queue, a basic introduction to queuing theory is required. A queue is made up of two initial ideas. There is an arrival rate of customers (the speed at which customers are getting into the line) and a service rate of customers (the speed at which customers are given service). If the arrival rate is exceeding the service rate for the entire simulation, the line will grow to an infinite length (Willig, 1989). There are three main types of queues that are being used in daily life: structured queues, unstructured queues, and virtual queues. A structured queue is a queue with a physical organization to it. There are some markings to show the order of the line. Examples of a structured queue would be supermarket or airport security lines. Airport security usually uses a host to organize the line and feed people into the correct spaces. Unstructured lines form out of necessity but are not organized. These lines may form at ATM machines or in some retail stores that don't plan on having a longer line. The final type that has gained popularity is a virtual queue. These are popular in restaurants where customers are given a pager or text message to inform them when their table is ready. They are allowed to roam around and engage in other activities instead of having to save their spot in a physical line. Once they have checked in, their spot is saved virtually in a queue and they are informed when it is there turn. Virtual queues are being seen more and more in theme parks. Guests may be eating lunch while they are virtually waiting for their chance to ride the next roller coaster.

Once a type of queue is decided on, a method of service is the next procedure to work through. A single server, single phase queue will have one person serving people and they only have one action required to serve them. An example of this service model would be a McDonalds or an ATM. A single server, multi-phase service model will have a single person serving customers, but that server has multiple actions to get through to provide service. The easiest way to visualize this model is a Chipotle restaurant. Chipotle has to move customers down the line with stations adding either meat or toppings. A multi-server, single phase line will have multiple servers, but only one action needed to serve the customers. This is similar to a Chick-Fil-A restaurant where they will have multiple servers available instead of one. The final service model seen is a multiple server, multi-phase line. This is similar to a laundromat. In a laundromat, people are waiting for any washing machine to open. Once they successfully wash their clothes, they then need to wait again for any of the dryers to open.

When working with different queue models, the accepted notation is known as Kendall's Notation (Willig, 1989). Kendall's notation is a sequence of letters and numbers that represent different distributions and numbers related to the queuing system. Given the notation *A/B/m/N-S*, *A* represents the interarrival time distribution. The interarrival time is the time between arrival of customers. If one customer shows up at minute 2 and the next customer arrives at minute 5, the interarrival time for that single case would be 3 minutes. *B* represents the distribution of service times. *m* represents the number of servers in the queuing system. *N* represents the maximum number of people that can wait in the line. There are situations where *N* can be unbounded. If *N* is unbounded, it will be omitted in Kendall's notation. The final letter *S* is an optional notation that specifies the service discipline being used. The common two options are referred to as FIFO and LIFO. FIFO stands for first in, first out. This means that customers are served in the same order that they arrive in. LIFO stand for last in first out. This means that the last person to enter the line is the first person to be served (Willig, 1989). An example of this is usually found in inventory management. The last items that have been placed onto a shelf will be the first items used in selling.

The most common arrival distribution is a Poisson distribution. The average rate of arrival is given by the equation $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ with the average arrival rate of λ and x number of customers (Gosavi). This arrival distribution takes the variance of interarrival rate into account. This means the interarrival rate of customers will vary according to the distribution instead of cu arriving at a constant rate. This model reflects the fact that people to not arrive according to a strict every 2 minute schedule in the real world. Many people may arrive in a minute or nobody may arrive for 5 minutes. The most common service distribution is an exponential distribution. The exponential distribution is given by the equation $P(t) = e^{-\mu t}$ where μ is the average service time, t is the service time and $\frac{1}{\mu}$ is the mean. When combining a Poisson arrival distribution with an exponential service distribution, the queuing model is known as Markovian.

For this project, a simulation will be created for four different scenarios. Each scenario models a different type of line for the same marquee ride at a theme park. For this simulation, we will be using the Raging Bull rollercoaster at Six Flags Great America as our model. This roller coaster runs trains that can hold 32 riders. The ride can safely dispatch a train every 60 seconds, meaning it can send 60 trains every hour. After multiplying, the theoretical capacity of the ride is 1920 riders per hour. Our line simulation has the goal of getting as close to this number as possible. There are always going to be people in line, but the way they are managed will change the efficiency of the ride.

Assumptions

We make the following assumptions for the simulation. The ride may have delays, but no full breakdowns throughout the day. Every customer in line will ride the ride. The line itself has no physical length (once you walk into line, you are effectively at the front of it and do not have to walk for 2 minutes to get to the station). A train cannot be dispatched in less than 60 seconds to avoid colliding with another train on the course. Each train is 8 rows long with 4 across seating for a maximum capacity of 32 riders per train. Finally, this simulation is limited to a single run. Each simulation has only been run once when in practice it should be run hundreds of times. Theses numbers will give a good starting point, but further simulations would need to be done to give more weight to the final conclusions we draw.

Building the Simulation Models

The first simulation run is the closest to an ideal scenario as possible. In this situation, there is an attendant assigning rows to each person. They will assign 4 people to row one, 4 people to row 2, and so on. The key to this system is that all groups can be broken up. The first 4 people go to row one, regardless of whether they are in a group together. This simulation should hit the theoretical capacity. This was a useful first simulation since it allowed a set up for arrival times and service times.

For the first simulation of all individual riders, the line was constructed around two main parameters. The first is the interarrival time between riders in the line. This is amount of time between customers getting in line. A rider may get in line, and another may get in line 10 seconds later. This would yield an interarrival time of 10 seconds. These times were modeled by a Poisson distribution. Each rider was assigned two random numbers between zero and one to be used in the calculation of interarrival and service times(decimals included). The interarrival time is one divided by the arrival rate. For this simulation, we assume an arrival rate of 35 customers every 60 seconds. This gives an interarrival time of roughly 1.7 seconds. To have a Poisson distribution of arrivals with rate λ, it is equivalent to have the interarrival time be exponentially distributed with a mean of $\frac{1}{\lambda}$. The random numbers where then sent through the logarithmic formula (Willig, 1989):

$$T_{\mathrm{a}} = -\mu_{\mathrm{a}} * (\ln(1 - r_1))$$

Where T_a is the interarrival time for that customer, μ is the average interarrival time (1.7 seconds), and r represents the random number that was assigned to that customer. This creates an interarrival time for each customer. By adding these together, we can get a physical arrival time for each guest. For example, if guest one has an interarrival time of 6 seconds, they get in line 6 seconds into the simulation or at $t = 6$. Guest two has an interarrival time of 2 seconds, they get in line 8 seconds after the start of the simulation or at $t = 8$.

There is now a line forming for the ride, the next step is to get people on the coaster so that they can ride it. In queuing theory, this will be known as the service time. For our simulations, the service time will be the time it takes to get riders onto the coaster, check all the restraints and dispatch the train. The service time will not include the riders time on the ride. Since we are assuming no breakdowns, the ride time should not be a variable. The simulation is focusing on the queue itself. The service time will be modeled using an exponential distribution.

For this calculation, an average service time of 60 seconds will be used, along with the second random number. These will then be sent through the logarithmic formula (Willig, 1989):

$$T_s = -\mu_s * \ln{(1 - r_2)}$$

where T_s is the service time for that customer, μ_s is the average service time of 60 seconds, and r_2 is the second random number assigned to that customer. The service time will be the same for 32 consecutive passengers since they will all be on the same train. This completes the two factors that impact the wait time of the queue. To get an idea of queue length, a few more calculations are made. The time service begins for each customer is linked to when the 32nd customer arrives. Once all 32 customers are ready to board the train, the service time begins. Then, each customer's time waiting in the queue can be calculated by subtracting their original arrival time in the line from the time their service begins. The final calculation will find out how many customers are currently in the line. The customer will be counted as 'in line' if their time that service begins is later than the current time. Once the simulation is built, some key analytics will be taken and measured against other line models.

The second simulation built was less ideal for efficiency, but better represented what happens in the real world at theme parks. The second simulation will allow groups of people to get in line and ride together. Once again, a few assumptions will be made to put this into practice. Assume the size of groups will range from 1-6 people. The following look-up table will be used for probabilities assigned to each group size.

Table 1

Group size	Outcome
0	1
0.05	2
0.15	3
0.35	4
0.65	5
0.9	6

Probabilities of a specific sized group entering the line

This distributes the group sizes so that 5% of groups are one person groups, 10% are groups of two people, 20% are groups of three people, 30% are groups of four, 25% are groups of five, and 10% are groups of six. Since this ride can fit four people across, most larger group sizes over six people will split themselves into smaller groups that are listed above. Groups of people were then randomly generated and added together to get a train as full as possible. Based on the group sizes above, a train could have anywhere from 26 to 32 passengers on it. There are now empty seats on most trains due to the randomness of group sizes and the unpredictability of how guests will sit on the train. Train sizes were then calculated by a similar look-up table to the one above. The percentages of each train size were observed and used to generate a line where people are grouped by their train.

Table 2

Customer	Random #1	Interarrival Time (seconds)	Arrival Time (seconds)	Random #2	Service Time (seconds)	Time Service Begins	Waiting Time in Queue (seconds)	Time Service Ends	Time Customer Spends in System (seconds)	People on Train	Trains of People Waiting	People in line
1	0.28492564	10	10	0.515947	44	10	0	54	44	31	0	0
2	0.585752197	26	36	0.993495	302	54	18	356	320	31	0	0
3	0.808281664	50	86	0.506434	42	356	270	398	312	28	1	28
4	0.39007247	15	101	0.976374	225	398	297	623	522	30	2	58

Time service begins and ends measured in seconds. Table is looking at full groups of people getting on the roller coaster together

In table 2, customer 1 represents a train of 31 people instead of just one person. Since all riders on the same train are served together, a line can still be formed by how many trains of people are waiting in line. Once the line is formed, throughput and maximum waiting times can be calculated and compared to that of other line simulations.

The third simulation built combined both prior simulations together. It was a line of groups of people with the same distribution as in simulation number two. This is meant to simulate the main line for the ride. This line is then combined with a single rider line of individual people. This allows a realistic way to maximize the efficiency of the roller coaster. It's unlikely that patrons group themselves into perfect groups of 32 customers to perfectly fit on the train. With the addition of the single rider line, the total number of customers from the group line is calculated first. For example, if the number of customers on the train is 29, three customers from the single rider line will be brought onto the ride to fill the empty seats.

Results

For all three simulations, data was collected with the purpose of comparing the efficiency of each line system. Key data points are the slowest dispatch, average dispatch, dispatches per hour and theoretical capacity of the ride based on those operational limitations. From there, a 10-hour operating day was simulated. The throughput for each hour was calculated and then totaled to give the throughput for the entire 10-hour day. Then, the average hourly throughput is calculated, and we can see how close it is to the theoretical capacity and how it compares to the other line models.

These simulations are looking at the efficiency of how to process the line and not the efficiency of wait time. The time it takes to dispatch a train was randomly generated in the same way for each of the simulations. This means there should not be a noticeable difference between wait times. Next, to incorporate some realism into the simulations, a waiting limit was assigned to each customer or group of customers. In simulation one, no customer wants to wait more than 90 minutes. The same is true for simulation two. For simulation three, the groups of people will not wait for more than 90 minutes, but the single riders do not want to wait more than 20 minutes. The reason for the lower threshold is twofold. Since their party is being split up, they are less willing to wait as much time as the standby line. The other reason is that the line moves less frequently or in smaller amounts. There are trains that will already be full and will not use any single riders and others my use up to six. This inconsistency can be frustrating and leads to a lower threshold for waiting times in the single rider line.

Simulation 1

Table 3

Slowest Dispatch	Average Dispatch	Dispatches per Hour	Theoretical Capacity
438	105.2748728	34.19619424	1094.278216
7.3	1.754581214		

Dispatch times for the first simulation

First row of slowest dispatch and average dispatch are in seconds. Second row is in minutes. Theoretical capacity is riders per hour

The table shows the dispatch information for the individual person simulation or simulation one. For slowest dispatch and average dispatch, the top number is measured in seconds and the bottom number is in minutes. The way the simulation is set up, the average dispatch time is set at 60 seconds. There is variability in the service time due to the exponential distribution and random number input that leads to the service time. With this in mind, the average ending up at 99 seconds per dispatch is realistic and expected. In the world of theme parks, unexpected events happen. Dispatch times can be slowed down if people do not remove loose articles, don’t follow ride operator instructions, or do not fit in the rides restraint system.

The same could be said for the slowest dispatch nearing eight minutes. This would be a long time to wait, but there are occasions where customers get into arguments with staff members and security can be called. There could be a small malfunction in the ride operations. Any of these could lead to a longer dispatch time. The dispatches per hour is calculated by dividing 60 minutes by the average dispatch time in minutes. This number is multiplied by 32 customers per train (potentially) to give the theoretical hourly capacity of 1159 riders per hour. Again, the ride can operate faster at a rate of 60 second dispatches, but the goal is to look at realistic simulation with numbers we may observe at a regional theme park.

Table 4

	Longest Line	Throughput HR 1	Throughput HR 2	Hr 3	Hr 4	Hr5
	2204	1248	1184	960	1216	1216
Avg Wait Time	Longest Wait (min)	Running Total Throuughput				
80.89567129	98.72136644	1248	2432	3392	4608	5824

Hr 6	Hr 7	Hr 8	Hr 9	Hr 10	Avg Per Hr
1120	1184	1376	1152	1184	1184
					Total Thrpt
6944	8128	9504	10656	11840	11840

Average wait time in minutes and throughput measured in number of riders

Table 4 also shows results from simulation one, the individual simulation where every seat should be occupied on each train. These results show the statistics for how the ride and line operate over a 10-hour day. The throughput is broken down by hour and the total for the day is shown at the end. The longest the line ever gets is 2,204 people long with the average customer waiting 80 minutes. The wait time and length of line are a direct result of setting the maximum wait limit of the customers at 90 minutes, so we shouldn't expect a lot of variation in that number, nor take it as a sign of a more or less efficient line system. If this is the best-case scenario, the other lines are looking to arrive at a 10-hour day throughput of 11,840 riders with an average of 1184 riders each hour.

Simulation 2

Table 5

Slowest Dispatch	Average Dispatch	Dispatches per Hour	Theoretical Capacity
511	97.3205	36.99117863	1183.717716
8.516666667	1.622008333		

First row is measured in seconds for slowest and average dispatch. Second row is measured in minutes. Theoretical capacity is riders per hour

Table 5 above shows the dispatch speed for simulation two. We do want to look at it to see that it doesn't show a significant difference from simulation one. Any change we see in the throughput of the ride should be from the management of the ride and not that simulation having an extra 5 dispatches for hour. In this case, the numbers are similar enough to look at the throughput of this simulation over 10 hours.

Table 6

Longest Line	Throughput HR 1	Throughput HR 2	HR 3	Hr4	Hr 5
1960	979	1013	1007	1282	1034
avg wait time	Running Total				
84.69595104	979	1992	2999	4281	5315

Hr 6	HR7	Hr 8	Hr 9	Hr 10	Avg per hr
1008	1098	1009	909	1148	1048.7
					Total Thrpt
6323	7421	8430	9339	10487	10487

Average wait time measure in minutes. Throughputs all measured in number of riders

Looking at the results from simulation two (table 6), the simulation in which groups of people ride together, we see some similarities and differences from simulation one. The average wait time and longest line are very similar to simulation one. This is to be expected since the lines are built using the same formula for service time. The wait time is once again capped at 90 minutes so we see the average wait time stay below this. The interesting conclusions can be drawn from the average throughput per hour and the total throughput for a 10-hour day. We see that by not having every seat on the ride filled, the efficiency of the ride decreases by over 1000 customers per day, and over 100 riders per hour. These are significant numbers in terms of how efficiently a theme park can be run. 100 riders per hour is the equivalent of sending over 3 empty trains every hour with no one riding them. For the 1000 customers that do not ride each day, this ride experience could make or break their day at the park. This is considered to be the most popular ride at the park, and whether or not someone gets the opportunity to ride it could impact whether or not they return to the park again.

Simulation 3

Table 7

Slowest Dispatch	Average Dispatch	Dispatches per Hour	Theoretical Capacity
519	99.88196886	36.04254142	1153.361326
8.65	1.664699481		

First row measured in seconds for slowest and average dispatch. Second row measured in minutes. Theoretical capacity measured in riders per hour

We start simulation 3 (the group simulation with a single rider line added) by showing that the speed of the line remains the same (table 7). The average dispatch, dispatches per hour, and theoretical capacity remain very similar to both simulation one and two. We expect these numbers to be similar since they are all created using the same exponential distribution for the service time of the ride. The service time in each case being the speed at which the trains are dispatched.

Table 8

Longest Line	Throughput HR 1	Throughput HR 2	Hr 3	HR 4	HR 5
1926	1089	1323	1065	1283	1174
Avg Wait time	Single rider HR 1				
90.13926	63	53	55	61	74
	Total Group				
	1089	2412	3477	4760	5934
Avg Single Rider wait	Total Single Rider				
20.80673333	63	116	171	232	306
	Totals				
	1152	1376	1120	1344	1248

HR 6	HR 7	HR 8	Hr 9	HR 10		
1153	1001	791	1273	1151		
63	55	54	61	62		
7087	8088	8879	10152	11303	11303	
369	424	478	539	601	601	
					Total Overall	Avg per hr
1216	1056	845	1334	1213	11904	1190.4

Average wait time measured in minutes. Throughputs measured in number of riders

Table 8 shows the total throughput over a 10-hour day of park operations. The main difference in the presentation of the data is that it shows the breakdown of how many people are getting on the ride as a large group (coming from the standby line) and how many single riders are being added from the single rider line to fill the train. As predicted, this model that utilizes a single rider line exceeds the throughput of the random groups and avoids sending trains with empty seats on them. In this case, the average throughput per hour actually exceeded the theoretical throughput. This is due to this case having a lower overall dispatch time than the listed 99 second dispatch times. The ride team had good luck getting trains out faster than expected and therefore exceeded the expected rider throughput. The chart also shows that the overall throughput for the day gets back the 1000 riders that simulation 2 did not allow to ride.

<u>Simulation 4</u>

The final simulation to discuss is the simulation that involves a fastpass system. A fastpass system can be a free or paid system that allows guests a separate line at each attraction. Guests use the fastpass to get into a different line that should be shorter and move quicker. At the station of the ride, operators give priority to the fastpass line to ensure that it moves quickly. A well run fastpass line should never be longer that 20 minutes. This speed is achieved by only allowing a certain number of fastpasses for a given time frame. The downside of this line is that it slows the speed of the standard line significantly A full simulation was not created for this scenario for a few reasons. The first is that a full simulation of this system would be identical to either simulation 2 or simulation 3 depending on whether or not a single rider line is still added. If there are three lines, which would be most efficient, there would be the standby line of people in large groups, the fastpass line which would also have people in groups, and finally the single rider line that would fill the empty spaces on each train. The second reason is based on what a fastpass linecannot change the efficiency of a ride system, it changes the satisfaction rate of guests. This goes beyond the scope of this paper. A fastpass system allows certain guests to experience shorter wait times while others remain in the longer standby or single rider line. A well run fastpass system should improve the experience for all guests who have a choice between three lines depending on their priorities while in the park. The only downside is if the fastpasses are paid for. Once these are an upcharge, one of the options for guests is now unavailable depending on how much money they are willing to spend on their day at the park. While it decreases the potential to improve guest experiences, parks generally used a paid fastpass system to improve their profits.

Conclusion

After working with these simulations and observing the results, an obvious conclusion can be drawn. The idea of a single rider line may appear obvious, but is still rarely utilized in the theme park industry. What gets lost at theme parks is the importance of guest experience. Paid fastpass become the priority to increase profits and actually become a better value if the ride operations are slower and less efficient. This creates a backwards model for how to best run a park. By allowing 3 free options: a standby line, a single rider line, and a fastpass line, the ride is most importantly optimized for efficiency. The inclusion of the single rider line guarantees that there will be as few empty seats as possible on every train. The inclusion of a fastpass line gives guests an opportunity to wait less for certain rides and improve their day. The best situation is if this is free for all guests since they would all then have the same opportunity to improve their guest experience.

It needs to be noted that these simulations where only run once. The simulations examined here prove these ideas to be true. Given more time, each simulation would be run hundreds of times to get a more precise vision of what is happening each day in a theme park.

Lines are inevitable, and people are going to have to wait in them. There are a lot of variables in theme park operations. Delays in operations and break downs will happen, but the simple design of the queue can have a significant impact for close to 10% of the guest looking to ride a certain ride. Improving guest experience should always be important in the minds of executives and these queue techniques should be implemented in far more rides across the theme parks of the world.

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