Journal of Marine Research, 60, 1–17, 2002

Journal of
MARINE RESEARCH

Volume 60, Number 1

Sverdrup-like theories of the Antarctic Circumpolar Current

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ABSTRACT

Two Sverdrup-like theories of the Antarctic Cirumpolar Current (ACC) have been proposed, due to Stommel and Webb, both of which assume an idealized geometry for the Southern Ocean in which all latitudes are blocked by either continents or "effective continents" consisting of major topographic features. However, the two models predict quite different dependencies of the ACC transport on wind stress: Stommel's model predicting transport proportional to wind stress curl, and Webb's model predicting transport proportional to the zonal wind stress. A generalization of Webb's model is shown to be equivalent to Godfrey's Island Rule, applied to an island straddling the South Pole. With realistic geometry, the strength of the ACC cannot be calculated, but a relationship can be derived which involves the transport in the western boundary current east of South America. It is shown that, if zonal wind stress is entirely balanced by form stress in Drake Passage, this transport is determined by wind stress curl as in Stommel's model. If there is no form stress supported by topography in Drake Passage, then a version of Webb's model applies, and the boundary current transport is determined by zonal wind stress. In the real Southern Ocean, neither of these extreme cases can be expected to apply, and the boundary current transport will, therefore, depend on both wind stress and distribution of form stress. The latter can only be calculated diagnostically from a more complete solution.

1. Introduction

The idea of Sverdrup balance in the ocean interior, with the circulation closed by a western boundary current, has proven to be a valuable simplification of the subtropical, wind-driven ocean circulation. Despite its simplifications, and the questionable evidence of its validity (e.g. Wunsch and Roemmich, 1985), it provides a helpful conceptual picture of

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the circulation, and a useful focus about which to ask more detailed questions about gyre dynamics. Concentrating as it does on the depth-integrated transport, the assumption of Sverdrup balance neatly separates questions of total transport from questions concerning the vertical structure of the flow.

For the Southern Ocean there is no comparable consensus theory. The Antarctic Circumpolar Current (ACC) is a complicated system in which wind and thermohaline forcing interact, and the role of eddies appears to be crucial; see Rintoul *et al.* (2001) for a recent review. Interactions with topography have long been known to occur along the length of the current (Sverdrup *et al.,* 1942, pp. 467–468 and 607), making a division into a Sverdrup interior and boundary currents hard to justify. Nonetheless, it might be hoped that an idealized division into such regions would contain the essential dynamics of the circulation, or at least provide a useful conceptual picture to build upon.

Unfortunately, although two attempts have been made to make this division, they produce completely different answers. The first (Stommel, 1957) predicts an ACC transport proportional to the wind stress curl at the latitude of Drake Passage, and the second (Webb, 1993) predicts a transport proportional to zonal wind stress. Furthermore, these two pictures both avoid the important issue of the range of latitudes which are unblocked by land, in the first case by extending the Antarctic Peninsula, and in the second by raising the Kerguelan Plateau to cover the Drake Passage range of latitudes. When unblocked latitudes occur, bottom pressure differences (form stresses) are required to balance zonal wind stress at these latitudes. As noted by Hughes (2000), these pressure differences must produce bottom pressure torques which upset Sverdrup balance, further complicating the Sverdrup interior/viscous boundary layer picture. The purpose of this paper is to reconcile these different pictures of the ACC, and to show how they relate to one another. The resulting generalized picture illustrates the interaction between wind stress forcing, the distribution of form stress, and the strength of the boundary current east of South America. The final relationship is diagnostic, in the sense that the strength of the boundary current can only be found once the form stress across a particular piece of topography is known. However, it reduces to a prognostic relationship in two extreme cases, corresponding to the different fundamental assumptions of Webb's and Stommel's models. Within the Drake Passage latitudes, the lack of any continental barriers means that the strength of the depth-integrated flow depends critically on the vertical structure of the flow, so little can be said here about the total ACC transport.

The real Southern Ocean is clearly much more complicated than either Stommel's or Webb's idealized theories allow, but there is a degree of truth in the idea of a small number of partial barriers. The topography in Figure 1 (a half-degree average of the Smith and Sandwell (1997) topography) shows, south of the southern tips of Africa and Australia, three significant barriers shallower than 2000 m. The most constraining is clearly the combination of South America, the Antarctic Peninsula, and the Scotia Island Arc, although at the latitudes where Drake Passage is deeper than 1000 m (about 57S to 61S), there are numerous deep gaps between the islands. The other main barriers are the

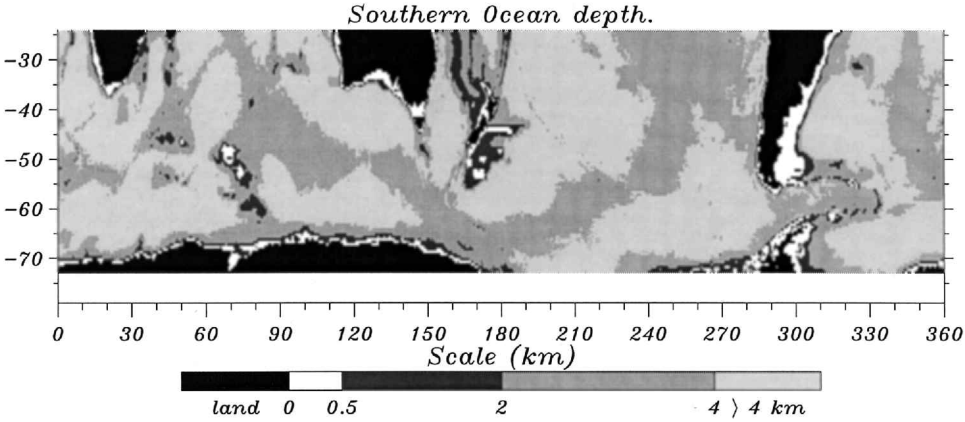

Figure 1. Southern Ocean bathymetry from a half degree average of the Smith and Sandwell (1997) topography.

Kerguelan Plateau at about 65E to 85E, split into a shallower northern plateau and a deeper southern region by a channel deeper than 2000 m, and the New Zealand/Campbell Plateau.

The extent of these three barriers is roughly shown in Figure 2, in which the shallowest topography in three bands (0–100E, 100–200E, and 270–340E) is plotted against latitude. While there are clearly three significant barriers north of Drake Passage, the Drake Passage latitudes are more ambiguous. The shallowest barriers at these latitudes are the Scotia islands and ridge but, being discontinuous, these features may not provide the greatest barrier to the flow. The southern part of Kerguelan Plateau provides a continuous barrier at depths greater than about 1700 m, and at 2000 to 2500 m topography south of Australia and New Zealand becomes significant. It is worth noting that the Kerguelan Plateau is isolated from Antarctica at depths shallower than about 3500 m.

The relatively deep passage between the northern and southern parts of Kerguelan Plateau, the discontinuous nature of the Scotia Islands, and clear evidence for interaction between the ACC and deep topography at a range of longitudes, makes the application of a simple Sverdrup-like model of the flow at Drake Passage latitudes seem very optimistic. On the other hand, the clear, continuous barrier presented by Kerguelan between about 45S and 55S, where South America also blocks the flow, leaves some hope for application of a Sverdrup-like model in this region, especially as this is on the northern side of the ACC where the current must join with the more strongly stratified subtropical gyres in which Sverdrup balance may be a good approximation. In any case, the concepts derived from an idealized geometry are valuable in interpreting the more complex geometry of the real ocean, and the final model presented here in Section 5 permits quite general geometries.

2. Stommel's model

This model, introduced with apologies for its incompleteness in a paper about the global ocean circulation (Stommel, 1957), appears at first to be the closest to the gyre circulation

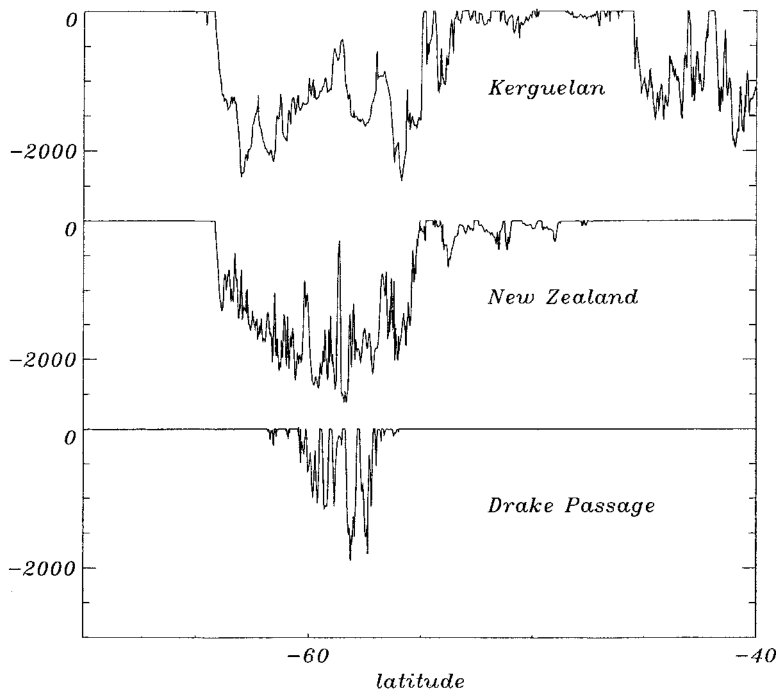

Figure 2. The shallowest bathymetry at each latitude from the Smith and Sandwell (1997) topography, in bands from 0–100E (top), 100–200E (center), and 270–340E (bottom).

model. The Southern Ocean is approximated (Fig. 3) by a flat bottomed ocean, with a meridional land barrier extending from the north to the "Drake Passage" latitude, representing South America. A second meridional land barrier a little farther to the east, representing the Antarctic Peninsula plus the Scotia Island Arc, extends from the south to the same latitude. The flow is considered to be in Sverdrup balance except in boundary currents close to the meridional barriers. (The term "Sverdrup balance" is taken to mean different things by different people. Throughout this paper it is taken to mean the relationship $\beta V = \nabla \times \tau_w$, where V is the depth integrated northward mass flux, τ_w is the surface wind stress, and β is the meridional gradient of the Coriolis parameter f.) The circulation is calculated north of Drake Passage by integrating Sverdrup balance to the west, from an eastern boundary (South America), and appending a western boundary current to supply the northward return flow. South of Drake Passage, Sverdrup balance is integrated to the east from the Antarctic Peninsula (assuming no western boundary current here), which implies some form of eastern boundary current close to the western side of the peninsula to provide the northward return flow. (Stommel was clearly uncomfortable with this idea, noting in a later paper (Stommel, 1962) that there are "difficulties in picturing the nature of the boundary conditions" in this region.) This eastern boundary current is then joined to the western boundary current to close the ACC. In Stommel's words "The flow

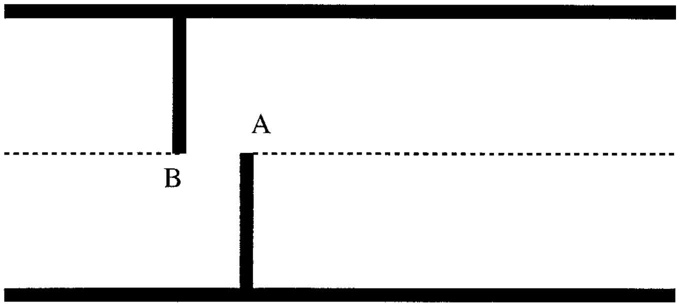

Figure 3. Schematic of Stommel's model of the ACC.

through the passage itself is something of a mystery, but doubtless models can be devised to describe it."

Applying Sverdrup balance along the section AB (eastward from A to B) in Figure 3, gives the total transport of the ACC as

$$\beta_0 T = \int_A^B \nabla \times \tau_w dx, \tag{1}$$

where β_0 is β at the Drake Passage latitude, and T is the mass transport of the ACC. The strength of the ACC in this picture is thus determined entirely by the wind stress curl.

This seems like a gyre model, in that the flow depends only on wind stress curl and is independent of zonal wind stress. However, it differs from the gyre model in permitting a form of eastern boundary current, and no western boundary current south of Drake Passage. If only western boundary currents were permitted, the analogous flow would simply be a gyre circulation, with the two sections of western boundary current connected by a zonal flow in the "Drake Passage" region, again assuming that such a connection of the two boundary currents can be dynamically sustained.

The transport prediction of this model has been tested by a number of authors (Baker, 1982; Chelton *et al.,* 1990), by comparing measured transport (approximately $130 \times 10^6\ m^3\ s^{-1}$) with that predicted from various wind stress climatologies. Agreement is generally good within the errors which are large (of order 50%). This apparent agreement led Warren *et al.* (1996) to suggest that future work on the dynamics of the ACC should be concentrated on Sverdrup dynamics, citing Stommel's model as a promising example, while labeling as "obscurantist" work related to the question of balance between wind stress and form stress first noted by Munk and Palmén (1951). This contention was challenged by Hughes (1997) and Olbers (1998) (see also replies: Warren *et al.* (1997, 1998)).

3. Webb's model

The model proposed by Webb (1993) (also independently derived by Ishida (1994), and generalized to a barotropic source-sink flow with topography by Wang and Huang (1994)),

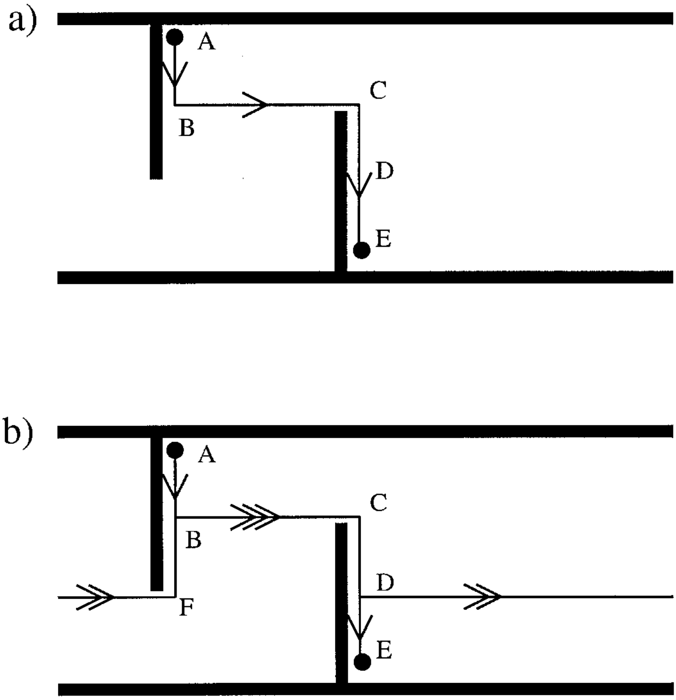

Figure 4. Schematic of Webb's model of the ACC, showing (a) the simplest configuration for the source-sink flow, which turns out to be impossible, and (b) the actual configuration in which a zonal recirculation arises.

initially looks less like a Sverdrup model since it does not explicitly consider vorticity. However, the model does obey Sverdrup dynamics in the ocean interior and is closed by boundary currents, this time confined to western ocean boundaries. The model, illustrated in Figure 4, is a simple source-sink flow in a flat-bottomed barotropic ocean. The geometry is similar to that of Stommel's model, except that the meridional land barrier which extends to the north from Antarctica extends farther north than the southern tip of South America, producing a region (latitude range) of overlap. The southern barrier is here taken to represent the Kerguelan Plateau. A more complicated version was also considered in which the Kerguelan Plateau did not connect to Antarctica and the range of latitudes south of the plateau was blocked by the Antarctic Peninsula, but the simpler geometry of Figure 4 serves to illustrate the main point.

The source and sink are meant to represent the effect of a northward Ekman transport due to an eastward wind stress, which must be returned to the south below the Ekman layer. Thus, this is a model of the flow outside the Ekman layer, for a situation with no Ekman pumping except in the source and sink regions (this form of wind stress was recently used by Hallberg and Gnanadesikan (2001) to investigate the role of transient eddies in an idealized ACC). Sverdrup balance then allows only zonal flows except in boundary currents to the east of the land barriers, where meridional flow is also permitted.

The problem then consists of constructing a series of zonal jets and western boundary currents to carry a flux from source to sink. The twist comes from the realization that the simple path *ABCDE* illustrated in Figure 4a is not allowed. The current along *BC*, being geostrophic, implies a pressure drop from north to south. However, the region south of *BC* is connected to the region to the north by a westward path which crosses no currents in this picture. Since it crosses no currents, there can be no pressure drop and therefore no current along *BC*.

The solution to this conundrum may be found by introducing a second zonal jet DF, and an additional western boundary current region FB, as in Figure 4b. Any path from south of BC to the north must now either cross BC or must cross the current along DF. The only constraint now is that the pressure drop across DF must be the same as that across BC. However, since BC and DF are at different latitudes, the same pressure drop will result in a different transport in the two currents because transport is given by $H\delta p/f$ where H is the (constant) water depth, δp is the pressure drop, and f is the (latitude dependent) Coriolis parameter. Since $|f|$ is smaller nearer to the equator, the current along BC is greater than that along DF, leaving a residual current to "leak out" to the sink at E (and requiring a matching supply along AB).

In this model then, the zonal wind stress (represented by the source and sink) results in a zonal recirculation which is proportional to the wind stress, and inversely proportional to the difference in f at the tips of the blocking meridional barriers. The transport, therefore, tends to infinity as the overlap between meridional barriers tends to zero (viscous effects apart). The current is seen to result from the constraint that the pressure gradient must integrate to zero for closed circuits around Antarctica, a point which was glossed over in Stommel's model which did not specify how to link the two boundary currents. New dynamics are required as a result of the fact that Antarctica is isolated from South America, even if all latitudes are blocked by continents.

This model highlights an aspect of ACC dynamics which was dismissed by Warren *et al.* (1996): the possibility of a link between the meridional overturning circulation and the strength of the ACC. The wind-driven Ekman flux across the ACC is returned in a geostrophic current at depth. That geostrophic current is supported by topography, but the need (in this idealized geometry) for the associated pressure difference to switch from one piece of topography to another in order for the return flow to jump the Drake Passage gap, results in the need for a strong zonal current.

Webb also compared the model prediction with wind stress climatology, and obtained a value for the ACC transport as good as that obtained using Stommel's formula, although agreement was much worse in the more complex model with a gap to the south of the Kerguelan Plateau.

4. Application of the Island Rule

Although Webb's model seems highly idealized and un-Sverdrup-like, it is in fact easily generalized to the case of a baroclinic flow in an ocean with bottom topography, with a general wind stress distribution as long as the interaction with bottom topography is confined to western boundary currents. Take the steady, linear momentum equation

$$\rho f\,\mathbf{k} \times \mathbf{u} = -\nabla p + \tau_z + \mathbf{a}, \tag{2}$$

where $\mathbf{a}$ is the force due to lateral viscous stresses, and τ_z is the vertical divergence of viscous stresses acting on horizontal planes. An integral over depth, from the bottom at $z = -H$ to the surface at $z = \eta$ gives

$$-f\nabla\Psi = -\nabla P + p_b \nabla H + \tau_0 + \mathbf{A}, \tag{3}$$

where $(P, \mathbf{A}) = \int_{-H}^{\eta} (p, \mathbf{a})dz$, p_b is pressure at the ocean bottom, and $\tau_0 = \tau_w - \tau_b$, the difference between wind stress and the (usually negligible) bottom stress. The depth-integrated mass transport has been written in terms of a streamfunction, $\mathbf{k} \times \nabla\Psi = \int_{-H}^{\eta} \rho\mathbf{u}dz$, on the assumption that there are no mass sources. Taking the curl of (3) gives the barotropic vorticity equation:

$$\beta\Psi_x = \nabla p_b \times \nabla H + \nabla \times (\tau_0 + \mathbf{A}). \tag{4}$$

This reduces to Sverdrup balance if $\tau_b = \mathbf{A} = 0$ and $p_b = p_b(H)$. If we now separate the ocean into "interior" regions, where Sverdrup balance holds, $\mathbf{A}$ and τ_b are negligible, and $p_b = p_b(H)$, and "boundary layer" regions where these other terms are important, we can integrate (3) to give

$$-\oint_{\text{interior}} f\nabla\Psi \cdot \mathbf{ds} = \oint_{\text{interior}} \Psi\beta dy = \oint_{\text{interior}} \tau_w \cdot \mathbf{ds}, \tag{5}$$

for any closed path which lies entirely within the interior, since the viscous terms are zero and ∇P and $p_b\nabla H$ integrate to zero (the criterion that pressure must be single-valued, from Webb's simple model, translates here to the criterion that ∇P must integrate to zero along the chosen path). One such path for a generalization of Webb's model is shown in Figure 5a. If we write $\Psi = 0$ on land to the north, and $\Psi = T$ on land to the south, where T is the ACC transport, (5) becomes

$$T(f_N - f_S) = \oint_{abcda} \tau_w \cdot \mathbf{ds}, \tag{6}$$

where f_N is f along cd and f_S is f along ab. The transport T is then proportional to the wind stress along $abcda$, and inversely proportional to the difference in Coriolis parameter at the ends of the overlap region, just as in Webb's simple model. Unlike Webb's model, this model includes the Ekman layer, so there are no sources or sinks (representing Ekman pumping), and the wind stress is included explicitly instead.

The assumptions made here are precisely the same as those in the Sverdrup Gyre model:

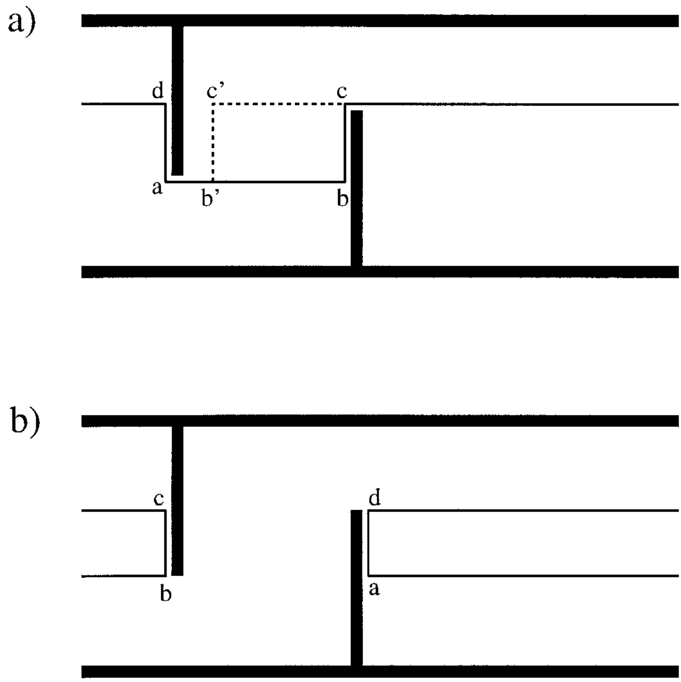

Figure 5. Illustration of contours used for integrals in the text.

Sverdrup balance in the ocean interior, and the circulation closed by western boundary currents only. The generalization of Webb's model is thus the most natural extension of Sverdrup dynamics to a Southern Ocean with all latitudes blocked. In fact, this solution can be seen to be simply the Island Rule of Godfrey (1989), adapted to the geometry of an island which straddles the pole. Note, though, that any form of boundary current is permitted, including one resulting from topographic interactions; Godfrey assumed an ocean with vertical sidewalls, but the result generalizes to sloping walls as long as the nonSverdrup region does not extend far in latitude from the northern and southern tips of the continents. A crucial assumption is that the form stress $p_b \nabla H$ integrates to zero along the chosen contour.

Stommel's model is rather more complicated seen in these terms, and it is useful to start by considering the same geometry as in the Webb model, with overlapping partial barriers. In this case, with only western boundary layers supported by the northern continent, and only eastern boundary layers supported by the southern continent, the contour analogous to that in Figure 5a is the one shown in Figure 5b, which again leads to (6), but with a different contour. In this case though, the contour doesn't contain the island of Antarctica, so the line integral of wind stress can be replaced by an area integral of wind stress curl. A problem then arises when we realize that the sections ab and cd in Figure 5b can be at any latitude within the range of overlap of the two partial barriers. In fact, in the limit as the two sections approach each other, the integral reduces to

$$\beta T = \int_a^b \nabla \times \tau_w dx. \tag{7}$$

This is Stommel's result for the ACC transport, but with overlapping barriers it becomes ambiguous: at which latitude is the wind stress curl to be evaluated? The answer is clear with hindsight. If there is no eastern boundary current along bc, and no western boundary current along da, then the Sverdrup transports across ab and dc in Figure 5b must be equal. The constraint on position of boundary currents imposes a constraint on the wind stress curl distribution. If that constraint is broken, then Stommel's model is inadmissable, and a western boundary current must be supported by the southern barrier. Stommel avoids this problem by allowing no overlap between the partial barriers, so there is only a single latitude at which the wind stress curl specifies the transport. However, this produces a second problem.

Since the contour in Figure 5b does not enclose Antarctica, the constraint that P must be single-valued on integrating around Antarctica has not been used yet. The effect of this constraint becomes clear upon integrating around the contour $ab'c'd$ in Figure 5a. The stretch $b'c'$ is chosen to be just outside the western boundary current adjacent to South America so, instead of determining the total transport T, (5) now imposes a constraint on the strength of the boundary current as measured by the streamfunction just outside it:

$$\int_{b'}^{c'} \Psi \beta dy = \oint_{ab'c'da} \tau_w \cdot \mathbf{ds}, \tag{8}$$

showing that the streamfunction must reach a value along $b'c'$ proportional to a (nearly zonal) wind stress integral, and inversely proportional to the length of $b'c'$. The boundary current transport tends to infinity in the limit as the length of overlap tends to zero. With these assumptions then, Stommel's model predicts a western boundary current strength determined by wind stress, but an ACC transport determined by wind stress curl. Note that precisely the same analysis can be applied to Webb's model, giving the same value for the western boundary current which, in that case, is closely related to the ACC transport.

The way in which western boundary current transport and ACC transport are decoupled in this version of Stommel's model is nicely illustrated by Webb's source-sink model. When only eastern boundary currents are permitted to the south, this results in the circulation illustrated in Figure 6. The western boundary current is exactly the same as in Webb's model, but is now associated with a local recirculation closed by an eastern boundary current rather than with a circulation around Antarctica.

Again, though, this conclusion depends crucially on the assumption that $p_b \nabla H + \tau_b + \mathbf{A}$ integrates to zero along the chosen contour. Even if viscosity plays a significant role in

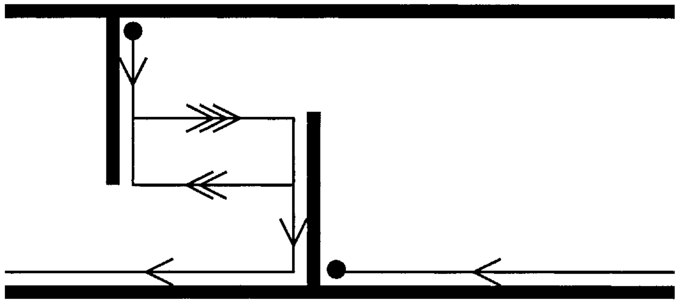

Figure 6. The flow in a version of Webb's model which permits only eastern boundary currents on the southern barrier, and only western boundary currents on the northern boundary. A recirculation is required to make the pressure single valued.

the jets in Figure 6, the contour crosses these jets at right angles making the probable viscous contribution to the line integral rather small. However, if there is a submarine ridge connecting South America and Antarctica, then it is quite possible for this to support a form stress so that there would be no contour encircling Antarctica along which $p_b \nabla H$ integrates to zero. In that case, although it is hard to see how the northward flow east of the Antarctic Peninsula crosses the ridge to become a northward flow to the west of South America, there is no need for a local recirculation to develop in Stommel's model.

In short, where Stommel says "doubtless models can be derived to describe" the flow through Drake Passage, such models must involve form stress across topography which connects South America and Antarctica, if a large (unobserved) local recirculation is not to be generated.

5. A general model

Both Webb's and Stommel's models assume a highly idealized and unrealistic geometry, particularly in the assumption that all latitudes are blocked by continents. Given the assumed geometry, Webb's model (or its extension to Godfrey's Island Rule) is more complete, and represents the natural extension of Sverdrup dynamics to the Southern Ocean, requiring as it does only western boundary currents to close the circulation. However, a fundamental assumption of Webb's model is that a contour encircling Antarctica can be found along which the form stress integrates to zero. If (as in reality) South America is connected to Antarctica by a submarine ridge capable of supporting form stress, that assumption may not be valid.

Stommel's model is more problematic, with its eastern boundary current and unspecified Drake Passage dynamics. However, the ACC is a strong current which flows to the east faster than the westward intrinsic propagation speed of Rossby waves (Killworth *et al.,* 1997; Hughes *et al.,* 1998), and it is the westward proprogation of Rossby waves which establishes the Sverdrup interior and western boundary current of conventional gyre

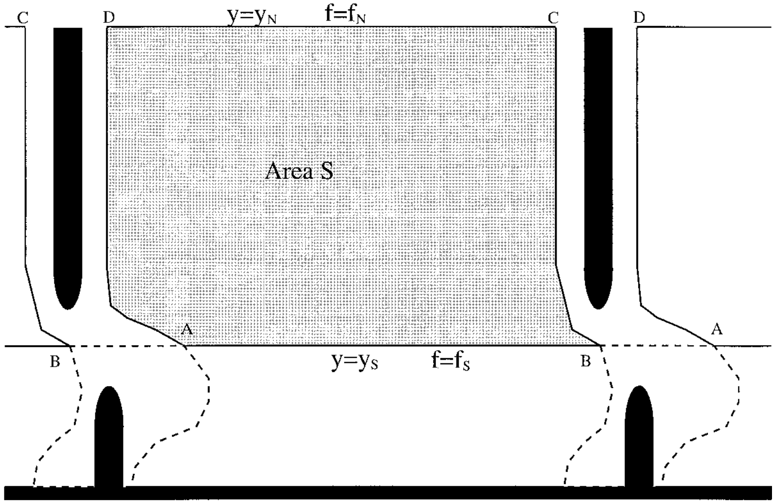

Figure 7. The geometry of the generalized model.

models, so we should perhaps expect something unusual in this region. The unspecified dynamics in the Drake Passage region leads to a strong western boundary current, the same as that in the extension of Webb's model, and a local recirculation (which is not observed) unless Antarctica and South America are connected by a submarine ridge which supports form stress.

In the real Southern Ocean, there are no continents blocking the Drake Passage range of latitudes. We must, therefore, abandon hope of a theory which predicts the transport of the ACC without addressing the vertical structure of the flow. This can be seen by noting that a purely zonal flow of any magnitude is an exact solution to the inviscid, unforced equations of motion as long as the flow does not intersect topography. Bearing this in mind, we will turn attention to prediction of the strength of the western boundary current adjacent to South America, which is a measure of that part of the ACC which is deflected to the north at this longitude. A constraint on this current has already been constructed in the case of idealized geometry and no submarine ridge connecting South America and Antarctica. In this section, we will show how this constraint generalizes to the real geometry of the Southern Ocean, in which a connecting ridge exists as well as other complex topography, on the assumption that there is a region to the north of the ACC in which Sverdrup balance is a good approximation.

The realistic geometry is schematically illustrated in Figure 7. The area S is enclosed by a contour $ABCDA$, which has a southern part AB at $y = y_S$ corresponding to Coriolis parameter $f = f_S$. This southern part can be at any latitude, including those blocked by land. The northern section CD, at $y = y_N$ and $f = f_N$ is presumed to lie entirely within a Sverdrup balance region (i.e. a region in which the flow does not penetrate to the bottom, so that friction is negligible and $p_b = p_b(H)$). Sections BC and DA lie along a depth contour

$H = H_0$ (4000 m is a reasonable assumption). It is also assumed that the western boundary current lies to the west of DA, so that viscous terms τ_b and $\mathbf{A}$ are confined to the boundary current region and negligible along DA and BC. In fact, Hughes and de Cuevas (2001) show that bottom pressure torques dominate the vorticity balance even of western boundary currents, so this should be a safe assumption.

Within the area S, and along AB, all terms are permitted to be important; Sverdrup balance is not assumed in this region, and interactions with topography (particularly Kerguelan Plateau) are expected.

Integrating (3) around $ABCDA$ gives

$$\int_{ys}^{yN} (\Psi_E - \Psi_W)\beta dy = \oint_{ABCDA} (p_b \nabla H + \tau_0 + \mathbf{A}) \cdot \mathbf{ds}, \tag{9}$$

where $\Psi_E(y)$ is Ψ along BC, and $\Psi_W(y)$ is Ψ along DA. The difference $\Psi_E - \Psi_W$ is equal to the net northward flow across latitude lines within area S, or to the net southward flow across the short zonal section joining 4000 m depth contours either side of South America, passing across the continent. It is, therefore, a measure of the western boundary current strength west of DA, assuming there is a negligible eastern boundary current adjacent to South America. In any case, it measures the meridional mass flux over a narrow region around South America.

Substituting $\tau_b = \mathbf{A} = 0$ along $BCDA$, and $p_b = p_b(H)$ along CD, this becomes

$$\int_{ys}^{yN} (\Psi_E - \Psi_W)\beta dy = \int_{AB} (p_b \nabla H - \tau_b + \mathbf{A}) \cdot \mathbf{ds} + \int_S \nabla \times \tau_w dA. \tag{10}$$

The boundary current flowing between DA and CB is, therefore, that which would be expected from Sverdrup balance (as in Stommel's model), plus an extra contribution given by the nonSverdrup terms $p_b \nabla H + \tau_b + \mathbf{A}$ along AB.

It is well known (Hughes and de Cuevas, 2001) that the zonal wind stress in the Southern Ocean (and in fact at all latitudes) is balanced by form stress in a zonal integral, with friction and nonlinear terms a small residual. The integral of (3) around ABA gives

$$\oint_{ABA} (p_b \nabla H - \tau_b + \mathbf{A}) \cdot \mathbf{ds} = -\oint_{ABA} \tau_w \mathbf{ds}, \tag{11}$$

and the left-hand side is dominated by the form stress term $p_b \nabla H$, so there is little error in referring to $\int_{AB} (p_b \nabla H - \tau_b + \mathbf{A}) \cdot \mathbf{ds}$ as the form stress along AB. Eq. (10) can then be written as

$$\int_{ys}^{yN} (\Psi_E - \Psi_W)\beta dy = FS_{\text{Other}} + \int_S \nabla \times \tau_w dA, \tag{12}$$

and (11) as

$$FS_{\text{Drake}} + FS_{\text{Other}} = -\oint_{ABA} \tau_w \mathbf{ds}, \tag{13}$$

where $FS_{\text{Drake}} = \int_{BA} (p_b \nabla H - \tau_b + \mathbf{A}) \cdot \mathbf{ds}$, and $FS_{\text{Other}} = \int_{AB} (p_b \nabla H - \tau_b + \mathbf{A}) \cdot \mathbf{ds}$ (both integrals taken along an eastward direction).

We can now see the Webb and Stommel models as special cases of this balance. If the only active topography is that in the Drake Passage region so that the zonal wind stress along ABA is all balanced by FS_{Drake}, the form stress across the connecting submarine ridge, then $FS_{\text{Other}} = 0$ and the western boundary current is the same as that determined by Sverdrup balance, as in Stommel's model. Seen in terms of the meridional overturning, this is the situation of the northward Ekman flux being returned as a deep geostrophic flow which is entirely supported by Drake Passage topography. If there is no submarine ridge in Drake Passage, then $FS_{\text{Other}} = -\oint_{ABA} \tau_w \mathbf{ds}$, and there is an additional western boundary current flow due to the zonal wind stress (the FS_{Other} term in (12)), as in Webb's model, resulting from the fact that a pressure field cannot be found which will support a direct deep geostrophic flow from north to south.

In the real Southern Ocean we must in general expect some of the form stress to fall across the Drake Passage topography, and some across the remaining topography, so that neither Webb's nor Stommel's model will apply in its pure form. In this case, neither a transport proportional to zonal wind stress nor a transport proportional to wind stress curl can be predicted, since a change in the wind stress (or other forcing) could also result in a change in the proportion of form stress which is supported by Drake Passage topography. We are left only with a diagnostic relationship (12) which tells us how the boundary current transport depends on wind stress curl and on the part of the zonal wind stress which is not balanced by form stress in Drake Passage.

The relationship (12) represents an integrated measure of the boundary current strength, but it was noted earlier that the southern latitude $y = y_S$ can be anywhere in this derivation, so more local information about the boundary current can be obtained (diagnostically) by applying (12) with two different southern latitudes, and differencing the resulting equations to obtain a measure of boundary current strength between the two chosen latitudes.

6. Summary

Two Sverdrup-like models of the ACC have been investigated, due to Stommel (1957) and Webb (1993). Webb's model has been extended to include effects of stratification and localized topography. In this form, it is seen to be the natural extension of Sverdrup dynamics to a Southern Ocean in which all latitudes are blocked by (effective) continents, and continents are the only active topography. The resulting ACC transport estimate is equivalent to the application of Godfrey's (1989) Island Rule to the geometry of an island which straddles the South Pole.

Stommel's model, which requires the presence of eastern boundary currents, was also shown to result in a large western boundary current proportional to zonal wind stress, unless South America and Antarctica are connected by a submarine ridge capable of supporting a form stress.

A more general model was then considered, in which the Drake Passage latitudes are blocked not by a continent, but by submerged topography, and quite general topography is permitted. In this case, the ACC transport cannot be calculated, but a diagnostic relationship (12) was derived. This states that the strength of the western boundary current adjacent to South America is that which would be predicted from Sverdrup balance plus a form stress related component. This extra component is determined by the part of the zonal wind stress which is not balanced by form stress in the Drake Passage region.

If there were no topography connecting South America and Antarctica, this extra component would be proportional to the zonal wind stress at Drake Passage latitudes, as in Webb's model. In the other extreme, if a ridge connecting South America and Antarctica were the only topography capable of supporting form stress, there would be no extra component to the western boundary current, and Stommel's model would work for latitudes blocked by South America. In reality, we should expect part of the form stress to be balanced in Drake Passage, and part elsewhere, reducing (12) to a purely diagnostic relationship, which illustrates how the distribution of form stress is relevant to the strength of the western boundary current flow. Ironically, given the suggestion of Warren *et al.* (1996) that "obscurantist" form stress considerations be abandoned in favor of Sverdrup dynamics, it turns out that the reconciliation of two Sverdrup-based models of the ACC requires an understanding of the differing roles of form stress in the two cases.

It would be very lucky if an integral balance could be found which determines ACC transport without finding a complete solution for the flow, including its vertical structure. Stommel's model (with a submerged ridge connecting South America and Antarctica to support a form stress), and Webb's model (or it's extension to the Island Rule) are examples of particular, idealized geometries in which the bottom topography is very localized, and assumptions about the position of boundary currents have been made, resulting in such an integral balance. However, simply combining a submerged ridge in Drake Passage and a submerged Kerguelan Plateau is sufficient to turn the transport prediction into a diagnostic relationship between forcing, form stress distribution, and the strength of a western boundary current flow which may not represent the total strength of the ACC. We thus see how a realistic distribution of interactions with topography releases the ACC transport from the apparent constraint imposed by these idealized models.

Other analytical predictions of ACC transport suffer similarly, for example Gill (1968) provides a complete analytical solution (which is virtually independent of vertical structure) for a flat-bottomed Southern Ocean, but the solution cannot be extended to include topography and implies an unrealistic role for friction. Kamenkovich (1997) discusses a class of solutions for which the asymptotic dependence of transport on friction can be calculated, but these require the assumption that the flow is barotropic or equivalent barotropic, leaving the transport dependent on the assumed vertical structure. Johnson and Bryden (1989) make a prediction of the ACC transport based on an eddy parameterization, on the assumption that it is essentially a zonal current with negligible flow at the bottom, that wind stress is transmitted down through the water column by eddy interfacial form stress, and ignoring any thermohaline flow (variations on this model are discussed by Rintoul *et al.* (2001)). These models give different dependences of ACC transport on wind stress, according to their different assumptions about topography and the vertical structure of the flow, and all ignore any influence of the thermohaline circulation. However, numerical studies such as Gent *et al.* (2001) indicate that both winds and thermohaline forcing are important, and the thermohaline forcing is crucial in determining the vertical structure of the flow.

The essential assumption which leads to Sverdrup balance is that the flow does not interact with the ocean bottom topography. When boundary currents which do not obey Sverdrup dynamics are confined to the western side of ocean basins, this leads to a complete solution for the depth-integrated flow, independent of the details of the vertical distribution, at all latitudes blocked by land. This picture breaks down in two ways in the ACC: there are latitudes which are not blocked by land, and interaction with bottom topography occurs in many places. In these circumstances, it seems that an understanding of the vertical structure of the flow is a requirement for prediction of the transport.

Acknowledgments. Thanks to David Marshall, Ric Williams, and two anonymous referees, whose comments have substantially improved this paper. This work was supported by the U.K. Natural Environment Research Council.

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Received: *1 March, 2001;* revised: *2 October, 2001.*