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On the use of the method of images to investigate nearshore dynamical processes

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ABSTRACT

This note describes how the method of images may be used to determine the motion and evolution of two related kinds of phenomena within a wedge of inviscid fluid. The image field of a curved vortex within a wedge with vortex lines lying along sectors of circles around the apex of the wedge is that segment of a complete vortex ring which remains *outside* the wedge and of which the curved vortex forms a part. The image system can be used to describe the motion, interaction and stability of single or multiple vortices within the wedge. Axisymmetric jets form the image system for flow parallel to the edge of the wedge, akin to alongshore currents. Knowledge of the instability of jets provides information about the evolution of waves in the wedge domain. Existing results on the motion and instability of single or multiple co-axial ring vortices and of waves and instabilities in jets may be applied to describe the evolution of low Froude number eddies and waves in alongshore flow over a steadily shelving sea bed.

1. Introduction

The method of images is a well known and powerful mathematical technique for finding solutions of fluid motions, often those associated with vortices, in domains with boundaries (e.g. Acheson, 1990). The real flow domain is extended beyond its boundary and a flow field in the new 'virtual' or 'fictitious' region (the image of the flow in the real domain) is introduced which, together with the flow in the real domain, satisfies the conditions (e.g. zero normal flow) at the real flow boundary.

The purpose of this note is to draw attention to the images which can be found of vortices and flows with unstable modes of oscillation within the limited domain of a wedge, images for which many analytical and numerical results are already available. An application is made to the motion and evolution of flow in a nearshore coastal zone where water depth increases with distance from shore. This is known to be a region of alongshore flows induced by momentum lost from breaking waves, flows which are observed to become unstable (Oltman-Shay *et al.,* 1989; Bowen and Holman, 1989). Analytical progress in studying processes in the nearshore wedge domain has proven difficult because of the sloping lower boundary and problems associated with the vanishingly small water depth at the shore line, although progress aided by numerical methods has been made in the

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description of periodic disturbances known as 'vorticity waves' (see Bowen and Holman, 1989; Shrira *et al.,* 1997; Shrira and Voronovich, 1996). Numerical studies (Allen *et al.,* 1996; Slinn *et al.,* 1998; Özhan-Haller and Kirby, 1999) show that localized eddies or vortices are generated and are important in dispersing dissolved matter or suspended sediments. In the representation described below, the water depth increases uniformly with distance offshore and is independent of alongshore position. The upper surface will be taken as horizontal, rigid, but free slip, and the water is confined to a wedge between the surface and the sloping bottom, also free slip.

The image systems are useful in discovering how the motion field in the wedge evolves, providing predictions with which more sophisticated models may be compared, and in gaining prior insight into the stability of flow and eddy motion within the confined region. The theoretical ideas are described in Section 2 (the dynamics of vortices) and Section 3 (the dynamics of an alongshore current). The applications to the nearshore zone, and their limitations, are discussed in Section 4.

2. Axisymmetric vortex rings and vortices in a wedge

Images of line vortices lying parallel to plane boundaries are very useful in calculating the motion of the vortices and aspects of instability (e.g. Rosenhead, 1929, 1930; Robinson and Saffman, 1982; Thorpe, 1992). Other image systems have been known for a long time; for example, that of a vortex ring in a fluid bounded by a sphere when the axis of the ring passes through the center of the sphere (see Lamb, 1932).

Peregrine (1996, 1998) has shown that an image system exists which is useful in describing the motion of a vortex in a wedge of angle α. The motion of a vortex ring with no swirl and with axis z in an unbounded inviscid fluid is confined entirely to the z and radial, r, directions; there is no azimuthal, θ, component of motion around the z axis, nor, by symmetry, is there any pressure variation induced by the vortex ring in the θ direction. There is no flow normal to a plane, $\theta = 0$, say, containing the z-axis; the motion on the plane is parallel to its surface. A second plane, $\theta = \alpha$, also containing the z-axis is likewise parallel to the flow field (Fig. 1a). The flow in the sector $0 < \theta < \alpha$ satisfies the free-slip and zero normal velocity component boundary conditions on the planes $\theta = 0$ and $\theta = \alpha$, and the equations of motion. It, therefore, represents a possible steady motion field within the sector or wedge of fluid bounded by fixed, plane, free-slip boundaries at $\theta = 0$ and $\theta = \alpha$.

The flow field outside the wedge (i.e. in $\alpha < \theta < 2\pi$) may be regarded as the image of the curved vortex confined within the wedge, shown hatched in Figure 1a. The curved vortex lines within $0 < \theta < \alpha$ intersect its boundaries at right angles. This avoids difficulties which may occur when a vortex filament meets a plane surface non-normally. As the position of contact of the vortex and its image at the surface is approached, the image of such a vortex filament in the plane (locally identical to its optical image) induces increasingly large motions parallel to the plane and steady conditions cannot be sustained at the location of the filament. This is a problem which occurs when a vortex with a vertical

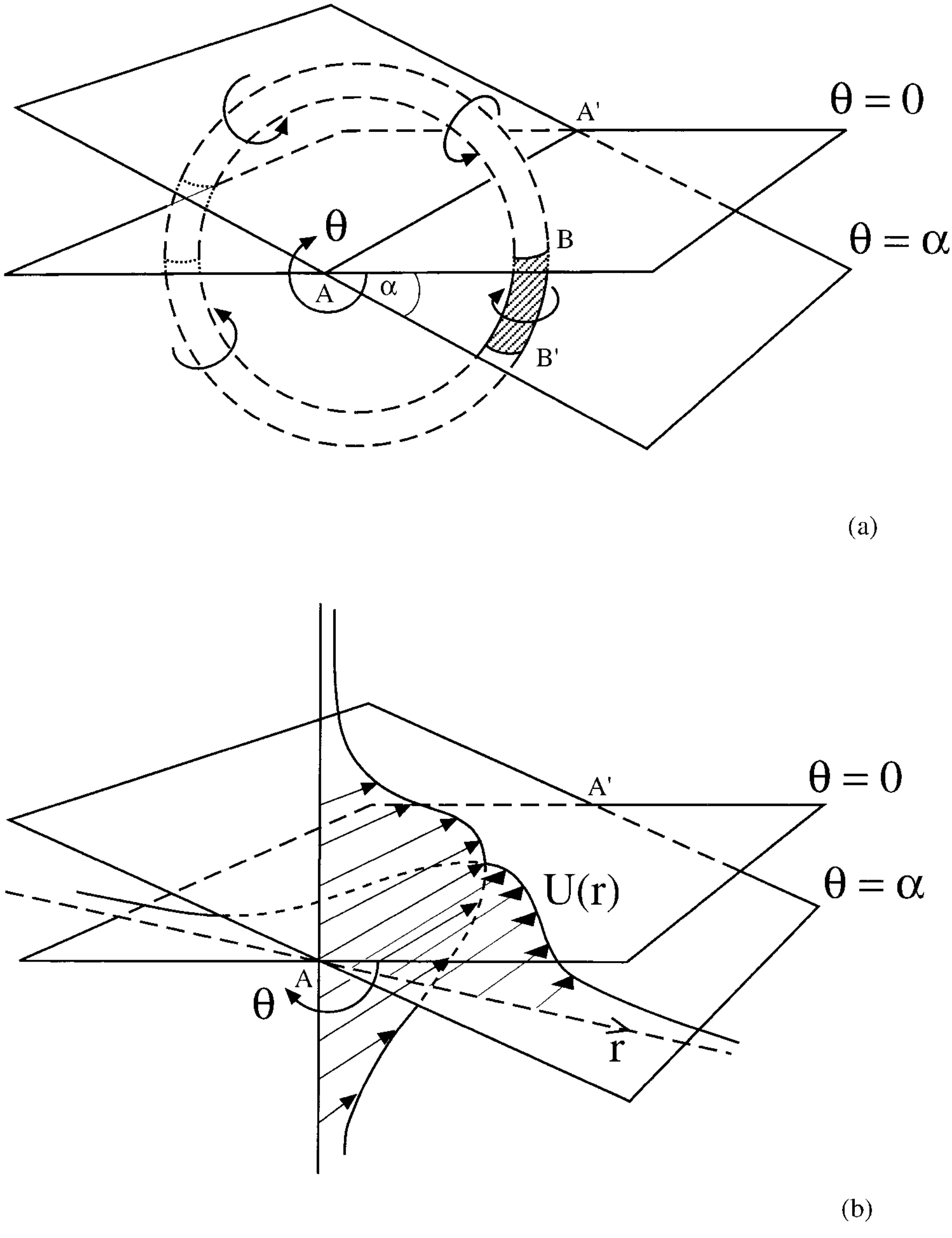

Figure 1. A perspective sketch showing two planes, $\theta = 0$ and $\theta = \alpha$, within the fluid domain which meet on AA′. When $\theta = 0$ is horizontal, it and $\theta = \alpha$ form the wedge. This represents a nearshore zone, with shore-line, AA′. (a) shows a vortex ring with axis AA′. The segment, BB′, of the vortex ring between the two planes (hatched) has an image consisting of the remainder of the vortex ring (dashed or dotted); (b) shows a jet with flow in direction AA′ and axisymmetric about AA′. The flow between the planes represents an alongshore drift current, U, in the nearshore zone. The distance of a point from the AA′ axis is r. The flow, U, is a function of r only. Axisymmetric disturbances (waves or growing instabilities) of the jet are similar to perturbations of the alongshore flow in the nearshore zone.

axis is introduced within a wedge since, although vortex lines meet the upper horizontal surface normally, they intersect the sloping boundary at angle $(\pi/2 - \alpha)$.

At first sight there appears to be a fundamental difference between the nature of the image systems of a line vortex parallel to a plane surface and that of the vortex in a wedge. A single line vortex in an infinite fluid induces a circular motion around its core which must be modified to satisfy the condition of no-normal-flow when the plane boundary is introduced. A second vortex, the image located at the location of the optical image of the first in the plane, makes the appropriate adjustment to the flow field. In contrast, vortex lines in the wedge lying along segments of circles intersect both boundaries at right angles. The vortex elements close to each boundary do *not* induce any flow normal to the nearby surface. Those more remote elements of the vortex in the wedge *are* however inclined non-normally to the boundary and *do* induce a component of normal flow, and it is this which has to be balanced by the presence of the image composed of the remaining segment of the circular vortex outside the wedge region.

As Peregrine (l.c.) has pointed out, there is much information immediately available to describe the motion of the vortex in the wedge from well-established results found for vortex rings. These are discussed in many standard texts, e.g. Lamb (1932), Batchelor (1967) and Saffman (1992). Steady vortices propagate parallel to the edge or apex of the wedge. For example, when the core of the vortex is of uniform vorticity and has a circular cross-section of radius, a, vortex strength, κ, and its center lies at distance $L \gg a$ from the edge or apex of the wedge, the speed of the ring speed parallel to the apex line (i.e. along the direction, AA′ in Fig. 1a) is approximately

$$V = (\kappa/4\pi L)[\log (8L/a) - 1/4]. \tag{1}$$

Lamb (1932) describes how vortices carry with them a body of irrotational fluid. The existence of a family of steady vortex rings extending from Hill's spherical vortex to rings with very small cross-section is discussed by Norbury (1973). He extends earlier results, notably those of Fraenkel (1972), describes the shape of the boundary of steady vortex cores, and estimates the speed of the rings.

The representation of the image system of a single vortex ring can be extended immediately to multiple vortices in a wedge by observing that they are segments of a corresponding array of co-axial vortex rings. Calculations and experiments made, for example of the movement through one-another of pairs of vortex rings with vorticity of the same sign, or the increasing radius, L, of approaching equal strength vortices of opposite sign (Weidman and Riley, 1993) can be applied to the evolution of the location of vortices within a wedge provided that viscous effects on the boundaries are negligible (see Section 4). The instability of regular arrays of coaxial vortex rings, and, therefore, of the corresponding vortices in a wedge, can be predicted using the results developed by Levy and Forsdyke (1927).

Peregrine's image system can be extended to study the instability of single vortices within a wedge. It is known that vortex rings become unstable to an azimuthal instability in which the vortex ring develops stationary growing waves around its perimeter. These are well illustrated by Van Dyke (1982; see plates 112–115) and have been studied in laboratory experiments by Widnall and Sullivan (1973) and Maxworthy (1977). The instability is sometimes known as Widnall instability (e.g. see Van Dyke, 1982; plate 113), after the theory and experiments by Widnall and colleagues. Widnall and Tsai (1977) consider the instability of a ring of radius, R. The core is of uniform vortex strength, κ, and radius, a, and $a/R \ll 1$. Instability is found in the form of stationary waves having no self-induced rotational motion around the axis of the ring. Their wavenumber, k, is such that $ka = C$, where C takes a set of prescribed values, 2.50, 4.35, etc. found by Widnall and Tsai. The expression for the instability contains terms proportional to $\exp(in\theta)$. The number of growing waves, n, around the perimeter of a ring is given by

$$n = CR/a, \tag{2}$$

and their growth rate is a function of the constant, C. For example, when $C = 2.5$, the growth rate is approximately

$$q = (\kappa/4\pi R^2)[0.856 \ln(8R/a) - 0.9102]. \tag{3}$$

The number of growing waves depends, however, on the distribution of vorticity inside the core of a ring and can vary considerably from the uniform case (Saffman, 1978). Instability waves with n up to about 28 have been observed in experiments (e.g. see Maxworthy, 1977). If the wedge angle, α, is an integer fraction of π (i.e. when there exists an integer m such that $m = \pi/\alpha$), the position of the wedge boundaries can be chosen to be located at the crest or trough of the growing wave where there is only motion in the r-z plane, the remainder of the 'wavy' vortex ring now forming the image. The instability 'fits' into the wedge with azimuthal wavenumber, n, which is an integer multiple of m. The value $n = m$ gives a half wave length perturbation within the wedge, $n = 2m$ gives a full wave etc., and each wave form has associated with it a vortex core of particular radius, a, which will be given by (2) when α is small (as it is in the applications in mind). The growth rates are as given by (3). This implies that vortices in wedges with core radii much larger than $CR\alpha/\pi$ do not contain unstable wave modes. They will be stable to perturbations along their axes.

3. Axisymmetric jets and alongslope flows in a wedge

There is another class of well-studied dynamical systems in a wedge which have simple images. An inviscid axisymmetric jet, and stable or unstable waves on the jet, may similarly be notionally divided by free-slip surfaces intersecting along its axis of symmetry, so as to represent the motion within a wedge bounded by free-slip planes, and its image field (Fig. 1b).

Jets are known to become unstable through the growth of axisymmetric ripples forming in the shear layer around the core of the jet. These rapidly develop into circular, azimuthally-coherent, vortex rings which subsequently merge and become turbulent (Crow and Champagne, 1971; Zaman and Hussain, 1980; see also Van Dyke, 1982, plates 102, 118–120). The stability of inviscid jets of circular section which are spreading very slowly have been studied by Batchelor and Gill (1962) and Mattingly and Chang (1974). Useful results are also given by Morris (1976, quoting inviscid cases) and Michalke and Hermann (1982, e.g. when their external flow, U_∞ is zero.)

Results are available for a variety of velocity profiles. Batchelor and Gill provide estimates of the conditions for instability, the growth rate and wavelength of the most unstable disturbances, for mean axial flows, U, with the 'top hat' form where U is equal to U_0 for $r < a$ and is zero for $r > a$, where r is the distance from the axis, AA$'$ in Figure 1b, U_0 is a constant reference speed and a is a constant radius. In addition to proving a semicircle theorem akin to that in plane shear flows, Batchelor and Gill also consider the stability of $U = U_0/[1 + (r^2/a^2)]$, showing that it is only unstable when the disturbances are non-axisymmetric and proportional to $\exp(in\theta)$, where θ is the azimuth angle, in the particular case when $n = 1$. In this case the inviscid flow, U, in a wedge appears to be stable. Mattingly and Chang compute eigenfunctions, phase speeds and growth rates for $n = 0$, 1 and 2 disturbances of the jet profile $U = U_0(r < a)$ and $U = U_0 \exp[-\sigma(r - a)^2/b^2]$ for $r \geq a$, with the constant $\sigma = 0.693$, chosen so that $U = U_0/2$ when $r = (a + b)$, a measure of the half width of the jet including the core of radius a and half the shear zone thickness, b. The fastest growing $n = 0$ axisymmetric disturbances have maximum spatial growth rates, $0.35(a + b)^{-1}$, $0.255(a + b)^{-1}$ and $0.115(a + b)^{-1}$, wavelengths of $12.6(a + b)$, $8.6(a + b)$ and $5.8(a + b)$, and phase speeds $0.73U_0$, $0.699U_0$ and $0.763U_0$, at $(a + b)/a =$ 1.21, 1.36 and 1.83, respectively. (The $n = 1$ and 2 disturbances correspond to what Mattingly and Chang describe as helical and double helical modes. These are suppressed by the introduction of the wedge's rigid boundaries and do not appear to be useful as images or in the applications, except when α is large, i.e. $\pi/4$ when $n = 2$; see Section 2. We are not aware of solutions with large n in jets, similar to those in vortex rings, which would be useful in providing solutions in a wedge with small α.) The theoretical results agree well with the observations when the disturbance amplitudes are small. Morris (1976) provides estimates of the phase speeds and growth rates of the mode with $n = 0$ for three jet speed profiles:—(i) $U = U_0(1 + r^2/a^2)^{-2}$, (ii) $U = U_0$ for $0 \leq r/a < 1 - \delta/2$, $U = U_0\{1 + \tanh[2(1 - r/a)/\varepsilon]\}$ for $r/a > 1 - \delta/2$, with δ chosen so that $\tanh(\delta/\varepsilon)$ is close to unity and $\varepsilon = 4U_0^{-2} \int U(U_0 - U)\, dr$, the momentum boundary layer thickness of the jet shear layer, and (iii) $U = (U_0/2)\{1 - \tanh[(r/a - a/r)/\varepsilon]\}$. The velocity gradient at the center of the shear layer in (iii), where $U = U_0/2$ and $r = a$, is U_0/ε. Michalke and Hermann (1982) also give solutions for profile (iii), showing, for example, maximum spatial growth rates of $0.364\varepsilon^{-1}$, $0.172\varepsilon^{-1}$ and $0.048\varepsilon^{-1}$, disturbance wavelengths of 7.3ε, 6.3ε and 4.8ε, and phase speeds of $0.65U_0$, $0.76U_0$ and $0.91U_0$, when $4a/\varepsilon = 10$, 5 and 2.5, respectively; the growth rates and phase speeds of the disturbances depend on the velocity profile.

4. Application to the nearshore zone

The image results can be applied to predict the development of instability in an alongshore current over a uniformly shelving sea bed in which the current speed depends only on the distance, r, from a straight shore line, provided viscous or small-scale turbulent effects can be neglected. The image system is an axisymmetric jet. Since the instability of jets is known to include the formation of axisymmetric vortex rings, it is, therefore, of no surprise that vortices are significant features in the development of disturbances to alongshore flows.

Whilst results from studies of jets may provide insight into the instability of an alongshore flow, several limitations must be taken into consideration before using the results in a quantitative way. Jets become unstable at relatively low Reynolds numbers of order 100 in laboratory studies reviewed by Morris (1976), although the studies of Crow and Champagne (1971) and Zaman and Hussain (1980) show development of ring vortices around the jet at Reynolds numbers of order 10^4 to 10^5. Mattingly and Chang's (1974) experiments are at a diametrical Reynolds number of 300 which appears to be 'sufficiently large to correspond to the inviscid theory.' Alongshore currents with a width of 50 m and maximum speeds of 0.5 m s^{-1} at Leadbetter Beach or about 150 m and 1 m s^{-1}, respectively, in the SUPERDUCK data set, are reported by Dodd *et al.* (1992). Oltman-Shay *et al.* (1989) report growing waves with wavelengths of 45 m to 300 m and periods of 45 s to 1000 s in SUPERDUCK. Alongshore currents of 0.5 m s^{-1} and a width of order 50 m have Reynolds numbers based on molecular viscosity and width (the appropriate length scale for the equivalent jets) of about 25×10^6, apparently sufficiently large for inviscid theory to apply.

The presence of the turbulent bottom boundary in the nearshore zone and the breaking waves of the surf zone introduce stresses not represented in the inviscid image system and imposes a more severe limitation to the use of the image theory. Where the radial component of viscous stresses are dominant, the flow evolving in a jet at high Reynolds number may be similar to that in comparative structures in a nearshore zone in which transfer of momentum in the direction radial from the shoreline (or approximately horizontal when α is small) by turbulent eddies is far greater than the vertical. In general, however, the effect of the momentum transfer and dissipation by turbulent motions leading to the loss of energy of growing waves, factors considered by Dodd *et al.* (1992) and Dodd (1994), will limit the direct application of the simple model described here. A further limitation is that the jets for which solutions are available as described above all have maximum speeds on the $r = 0$ axis, at the center of the jet. A jet with maximum flow speed at some nonzero distance from the shoreline, r, would be more appropriate in representing the alongshore flows calculated by Thornton and Guza (1986) and reported by Dodd *et al.* (1992). Solution of the stability equation in cylindrical coordinates (see for example Batchelor and Gill, 1962) for these jet-like flows is now computationally easy. Dodd (1994) infers, however, that control of the instability is in fact dominated by shear in the deeper water, offshore shear layer (where dissipation may be much less important), which *is* represented by a jet with a single maximum. Other effects, such as stratification, wind stress, and Coriolis effects are not represented in the image system. (Shrira and Voronovich (1996) find that the Earth's rotation does not affect the linear dynamics of vorticity waves.) In spite of these reservations, the analogy with jets does appear to allow qualitative insight to be gained into the evolution of disturbances to an alongshore flow.

The growth and phase speed estimates of Mattingly and Chang imply that an alongshore flow of 0.5 m s^{-1} between the shore and $r = 30$ m and a current which then falls to

0.25 m s^{-1} over the next 11 m will support fastest growing waves of wavelength 350 m which propagate in the current direction at about 0.35 m s^{-1} and grow by a factor e in a distance 160 m. Michalke and Hermann's calculations with jet profiles (iii) and with current $U_0 = 0.5$ m s^{-1} in a 20 m wide shear zone centered 50 m from shore ($4a/\varepsilon = 10$) have fastest growing waves of wavelength 146 m and phase speed 0.32 m s^{-1} growing by a factor e in 55 m. A 48 m wide shear zone centered 120 m from shore ($4a/\varepsilon = 10$) with $U_0 = 1$ m s^{-1} will have fastest growing waves of length 356 m moving at 0.65 m s^{-1} and growing by a factor e in 131 m. These estimates are independent of the slope of the sea bed and assume that the shear zone itself is not changing rapidly.

The evolution of ring vortices, the finite amplitude stage of transition to turbulence following the growth of unstable waves in the jet, provides a link between the two image systems described above. Provided their Froude number is small, the results imply that single eddies in the nearshore zone will propagate parallel to shore, driven by their image system through a background mean alongshore flow, at speeds which do not depend on the bottom slope. Not represented is the effect on the orbits of eddies of variation of the alongshore current with r, although when their vorticity greatly exceeds that of the mean flow, the effect of the latter may be small. Eddies of opposite sign escaping the nearshore shear flow or generated, for example by rip-currents (Smith and Largier, 1995), will continue to move offshore into deeper water. Like-signed eddies may be expected to rotate around one another. Eddies with core radii much larger than $2R\alpha/\pi$ (i.e. with radii much greater than the water depth) should be stable to perturbations along their axes. Much smaller eddies appear likely to be unstable to such perturbations.

Viscosity will affect the eddies. Whilst vertical vortices approaching a plane vertical boundary in a viscous fluid are known to move along orbits qualitatively as described by the inviscid image system, the effect of viscosity is to generate secondary vortices near the wall which subsequently cause the primary vortices to move away from the wall (e.g. see Doligalski *et al.,* 1994). Laboratory experiments on the approach of vertical vortices to a uniformly shelving beach (to be reported elsewhere by LC) show a general agreement with the predictions of the wedge image theory when the slope angle is moderate, but secondaries are also formed as for a vertical boundary. Moreover, the interaction of the rotating vortex with the sloping rigid bottom and with the free surface promotes a vertical circulation within the vortex, apparently as a consequence of viscosity.

The results obtained in these ways to the nearshore zone are applicable only when the vertical displacement of the free water surface induced by the circulation of naturally-occurring eddies can be disregarded. The radii of a pair of eddies observed in the nearshore zone by Dr J. A. Smith (Scripps Institution of Oceanography; private communication, 2000) are of order 20 m in water depth of 5 m and their vortex strength, κ, is of order 10 m^2 s^{-1}. The Froude number of the eddies, $Fr = \kappa^2/gHR^2$, is about 5×10^{-3}, very small, and their Reynolds number, $Re = \kappa H/\nu R$, is of order 10^6. The surface slope, dh/dr, derived from the θ equation of motion in cylindrical co-ordinates assuming uniform pressure on the water surface and neglecting viscosity, is about κ^2/gR^3, giving a depression at the center of an eddy of about κ^2/gR^2. This displacement will usually be very small, typically 0.025 m, if Smith's eddies are representative of those in the nearshore zone, so that the rigid upper surface is likely to be a good approximation.

A further factor limiting application is the extent to which alongshore shear flows or eddies may, in reality, be produced with a structure symmetrical about the shore line, depending on r but not on θ. Near-vertical structures seem more likely. For moderate values of α when the θ variation is small and the vortices or vorticity in the mean flow are nearly vertical, the theory may, however, provide good approximations to the flow and useful predictions. In the extreme case when $\alpha = 90°$, the axisymmetric theory fails to have realistic application. The only realistic steady solution for a vortex which intersects the upper horizontal boundary of a domain of depth, h, with a vertical (shore) boundary, appears to be a vertical two-dimensional vortex. The ninety degree circular vortex arcs suggested by the present theory have sets of circular images in the upper, lower and shore boundaries which do not offer steady solutions. (A horizontal vortex with multiple images in the upper and lower boundaries (e.g. Rosenhead, 1929), is a further solution in the domain, but does not intersect the upper boundary.)

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