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Evaluating eddy mixing coefficients from eddy-resolving ocean models: A case study

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ABSTRACT

A numerical model of a baroclinically unstable jet in a zonally periodic channel is used to analyze mesoscale eddy fluxes and their relationship with the gradients of the mean flow. The quasi-geostrophic approximation proves the best way to calculate potential vorticity fluxes in the primitive equation model. Away from the surface layers, eddy fluxes of potential density are consistent with advection by eddy-induced velocities v^* and w^* as suggested by Gent *et al.* (1995). Eddies mix potential vorticity along isopycnals, so that v^* is related to the gradients of potential *vorticity* rather than potential *density* as implicitly assumed by Gent *et al.* The mixing coefficient for potential vorticity, associated with the advective component of the eddy fluxes, is found to be similar to the mixing coefficient of tracer anomalies on isopycnals. Both show a maximum at mid-depth below the jet core. The present calculations support the analysis of Treguier, Held and Larichev (1997) and encourage further attempts to derive a parameterization based on true mixing of potential vorticity.

1. Introduction

Mesoscale eddies are very energetic in the ocean, and they contribute to the mixing of tracers and potential vorticity on isopycnal surfaces. Mesoscale mixing processes certainly play an important part in the dynamics of the large-scale ocean circulation and of the coupled ocean-atmosphere system. However, the representation of those processes in numerical climate models is not very satisfactory at the present time. Mesoscale eddies do not develop in those models because of their coarse spatial resolution, dictated by the limited computer power available.

The search for better parameterizations of mesoscale mixing in climate models has been very active in the past few years. Perhaps the most significant progress is the proposition by Gent and McWilliams (1990) of a parameterization based on downgradient diffusion of tracer anomalies along isopycnals and a mixing of isopycnal thickness. The latter can be viewed as a representation of eddy-induced transport (Gent *et al.,* 1995), and therefore the parameterization has both a diffusive and an advective component, each being associated with one mixing coefficient.

There are two ways of validating such a parameterization of mesoscale eddies. One

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approach is to implement it in a large-scale ocean model to see if the results improve. Danabasoglu and McWilliams (1995), England (1995) and Robitaille and Weaver (1995) among others, have introduced the parameterization in global ocean models and found more realistic circulations and tracer distributions. The parameterization has also been implemented in idealized coarse resolution models by Visbeck *et al.* (1997) and compared with eddy resolving experiments. Another approach is to analyze the eddy fluxes in eddy resolving models to validate the theory underlying the parameterization. A recent example is the study by Lee *et al.* (1997), who showed the importance of eddy-induced advection in a high resolution isopycnal model.

We follow this approach with the specific goal of calculating mixing coefficients for tracers and isopycnal thickness, using eddy fluxes diagnosed from a high resolution model. Our purpose is to show that the flux-gradient relationships that exist in an unstable baroclinic jet are not compatible with some of Gent *et al.*'s assumptions. This has been discussed recently by Treguier *et al.* (1997). The present paper provides an illustration and numerical confirmation of their analysis.

Before calculating mixing coefficients, we compare different estimations of the eddy-induced velocities, and show the relevance of quasi-geostrophic scaling to the analysis of eddy fluxes in z-coordinate primitive equation models.

2. Eddy-induced velocities in an unstable zonal jet

a. Numerical experiment

We consider a baroclinically unstable zonal jet, periodic in the x direction. The only eddy generation mechanism is baroclinic instability of a forced mean shear. Eddies extract their energy from the available potential energy of the mean flow and eddy fluxes work to reduce the slope of the isopycnals. This is the dynamical regime that mesoscale eddy parameterizations like Gent and McWilliams (1990) try to represent. Pavan and Held (1996) have shown that diffusive closures can be a reasonable approximation for the eddy fluxes in such flows; therefore it should be possible to calculate eddy-induced velocities and diffusion coefficients.

The jet is forced by linear relaxation to a mean shear. Such a forcing has been used more often to represent the atmosphere than the ocean, because it is viewed as a representation of radiative forcing which has no counterpart in the ocean. With this forcing the turbulent flow reaches a statistical equilibrium easily in a limited domain, which is our aim. Note that the forcing is weak, in the sense that the relaxation time scale (578 days) is long compared with turbulent time scales.

The numerical code is SPEM5, a finite-difference version of the primitive equation model of Haidvogel *et al.* (1991). The initial condition is a meridional potential density gradient over the central zone of the domain, 350 km wide (Fig. 1b). This gradient is associated with a geostrophic zonal current intensified in the upper 1000 m. It is representative of mid-latitude, mid-ocean eastward jets like the Azores current or the North Atlantic

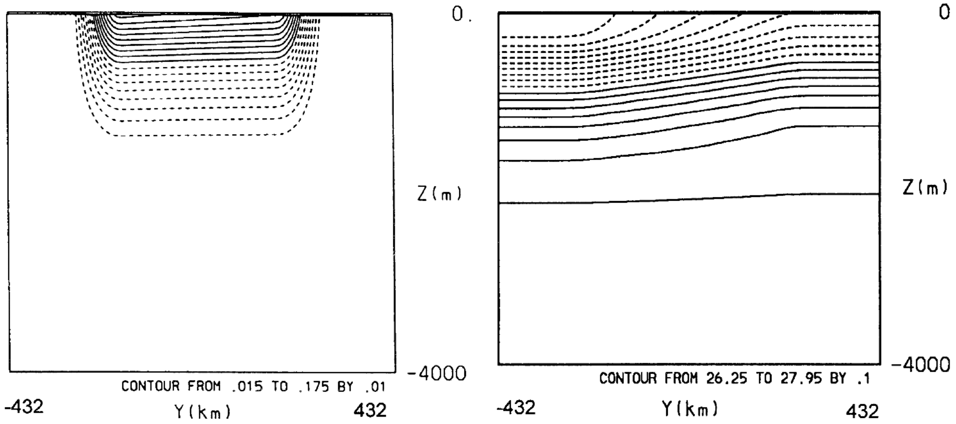

Figure 1. (a) Initial geostrophic zonal velocity (m s^{-1}); (b) Initial potential density profile (σ_0). The lowest contours are dashed.

current. We assume the initial bottom velocity is zero. A small random perturbation is added to the initial field to start the instabilities. The density field is relaxed to the initial zonal-mean value, and there is no direct forcing on the velocities excepted in narrow (50 km) sponge layers near the walls where a Rayleigh damping is used to prevent an eventual build up of boundary currents.

The model parameters are summarized in Table 1. Time series of kinetic energy over nine years are plotted in Figure 2. Initially, all the kinetic energy is in the zonally symmetric component of the flow and there is little barotropic energy. During the first year, baroclinic instability sets in and produces a large increase in the total kinetic energy, due to the appearance of meanders and eddies. The perturbations have a large barotropic component, and this allows bottom friction to act as an efficient energy sink. A statistical equilibrium

Table 1. Physical and numerical parameters.

Parameters of the experiment			
Horizontal resolution	Channel length	L_x	1080 km
	Channel width	L_y	864 km
	Gridpoints		120×96
First Rossby Radius		λ_1	32.8 km
Ocean depth		H	4000 m
Vertical resolution	28 gridpoints	δz	50 to 230 m
Coriolis parameter		f_0	$0.78\ 10^{-4}\ s^{-1}$
		β	$1.94\ 10^{-11}\ m^{-1}\ s^{-1}$
Density forcing	linear relaxation	ε_ρ	$2\ 10^{-8}\ s^{-1}$
Friction	Bottom friction	ε	$4.6\ 10^{-4}\ m\ s^{-1}$
		$\tau_\varepsilon = \varepsilon/H$	$1.15\ 10^{-7}\ s^{-1}$
	Biharmonic friction	ν	$4\ 10^{9}\ m^4\ s^{-1}$

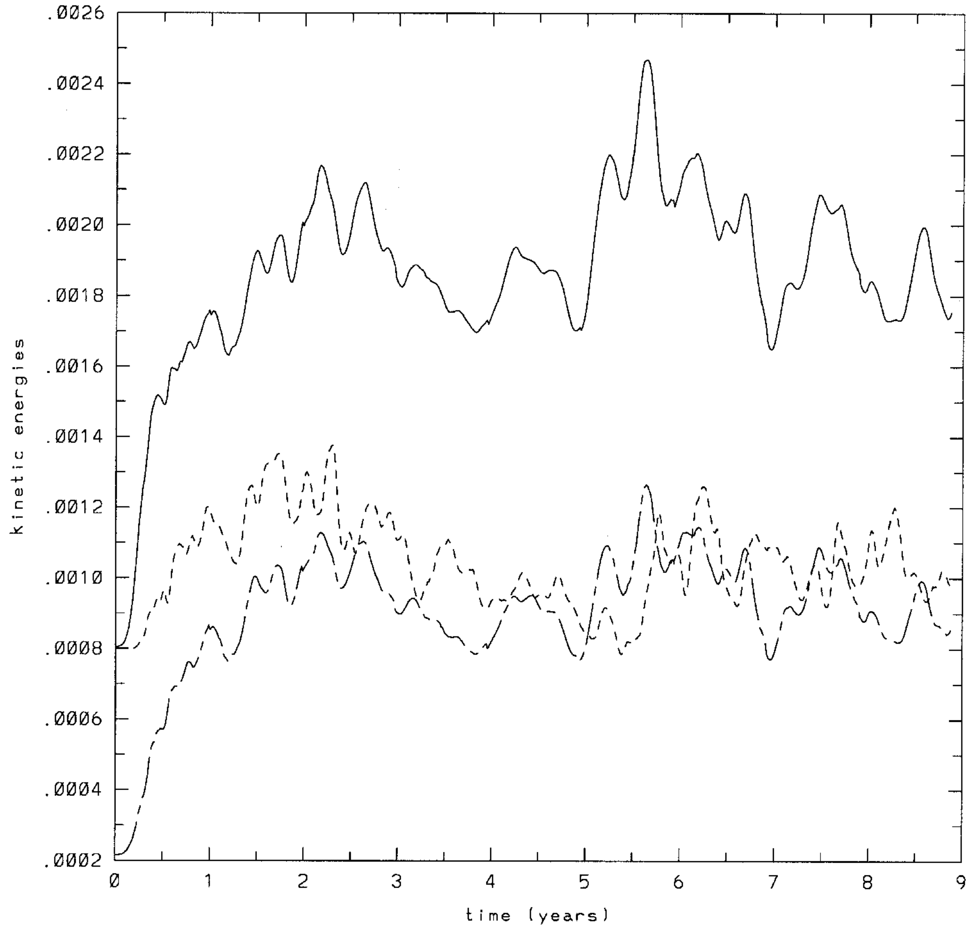

Figure 2. Time series of kinetic energy averaged over the domain ($m^2 s^{-2}$). Solid line: total kinetic energy. Dashed line: kinetic energy of the zonally symmetric component of the flow. Dashed-dotted line: kinetic energy of the barotropic (depth-averaged) flow.

between potential energy input by the forcing and kinetic energy dissipation is reached after about two years.

A "mean" component of the flow is defined as the average over the the last five years of the experiment (using snapshots every 10 days) and over the length of the channel. The eddy statistics are similar when averaging over two years only. The time and zonal average of temperature and zonal velocity in the equilibrated state are shown in Figure 3. They differ from the forcing field (similar to the initial condition, Fig. 1) because of the strong nonlinearities. Rather than the forced broad jet there is a narrower main jet and a secondary jet to the north. The concentration of momentum into the jets is due to the divergence of Reynolds stresses $\partial \overline{u'v'}/\partial y$. It is a familiar feature of eastward currents on a β-plane which

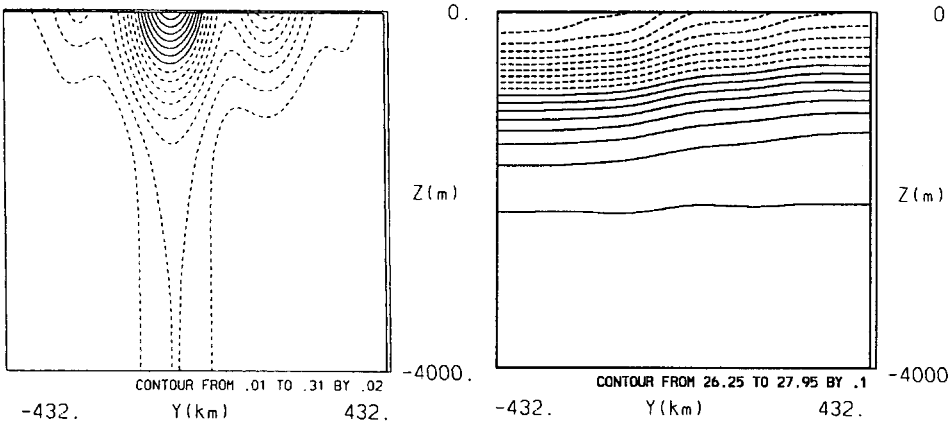

Figure 3. Time and zonal averages as a function of latitude and depth. (a) zonal velocity; (b) potential density (σ_0). The lowest contours are dashed.

often leads to the formation of multiple jets, as observed here (Panetta, 1993; Treguier and Panetta, 1994).

b. Calculation of the eddy-induced velocities

In this first experiment potential density is the only tracer. The mean, zonally-averaged tendency due to the divergence of eddy density fluxes

$$D = \frac{\partial \overline{v'\rho'}}{\partial y} + \frac{\partial \overline{w'\rho'}}{\partial z}$$

is represented in Figure 4a. The eddies decrease (increase) the density on the heavy (light) side of the front: they are a sink of available potential energy for the mean flow and tend to flatten the isopycnals. It is this divergence D of the eddy fluxes that one wishes to represent in low resolution climate models, where baroclinic instability is not present. Note that this structure of the eddy fluxes is not directly related to the structure of the forcing field, but rather results from a balance between mean and eddy advective fluxes. A similar balance, and a similar structure of the eddy fluxes is found in free decaying experiments (not shown), which suggests that our results do not depend qualitatively on the forcing prescription.

Gent *et al.* (1995), following earlier studies of atmospheric tracer mixing (e.g., Plumb and Mahlman, 1987), argue that in nearly adiabatic conditions the parameterization of eddy density fluxes should be purely advective. This is justified by considering the equations in isopycnal coordinates. The continuity and potential density equations are combined to form an equation for thickness, $\sigma = \partial z/\partial \rho$, where $z(\rho)$ is the depth of an isopycnal surface.

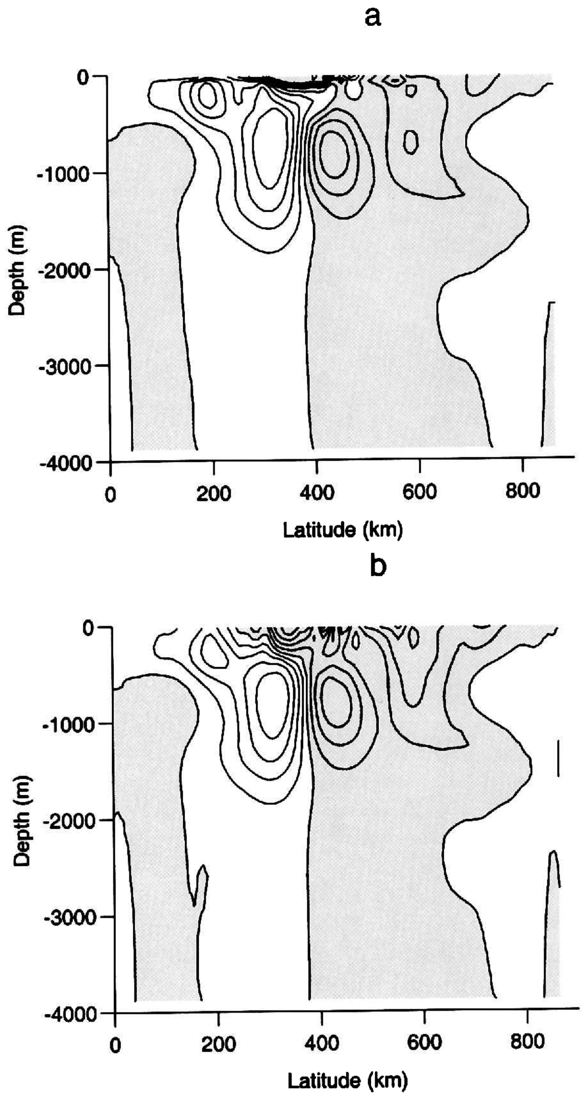

Figure 4. (a) Eddy flux divergence D. (b) Advection by eddy-induced velocities $v^*\overline{\rho}_y + w^*\overline{\rho}_z$. Contour intervals are the same in both figures; positive values are shaded.

Assuming a separation between mean and eddy flow, the thickness equation is:

$$\frac{\partial \overline{\sigma}^{\rho}}{\partial t} + \nabla_{\rho}(\overline{V}^{\rho}\overline{\sigma}^{\rho}) + \nabla_{\rho}(\overline{V'\sigma'}^{\rho}) + \frac{\partial}{\partial \rho}\left(\sigma \overline{\frac{d\rho}{dt}}^{\rho}\right) = 0, \tag{1}$$

where $V = (u, v)$ is the horizontal velocity vector, ∇_{ρ} a derivative along isopycnals and the overbar represents an average at constant ρ. When the flow is adiabatic ($d\rho/dt = 0$), the only eddy effect is the divergence of the eddy isopycnal thickness flux $\overline{V'\sigma'}^{\rho}$ which can be written as an advection by the eddy-induced velocity

$$V^{**} = \frac{\overline{V'\sigma'}^{\rho}}{\overline{\sigma}^{\rho}}. \tag{2}$$

The adiabatic thickness equation becomes

$$\frac{\partial \overline{\sigma}^{\rho}}{\partial t} + \nabla_{\rho}(\overline{V}^{\rho}\overline{\sigma}^{\rho}) + \nabla_{\rho}(V^{**}\overline{\sigma}^{\rho}) = 0. \tag{3}$$

Coming back to the equations in Cartesian coordinates, Gent *et al.* (1995) show that the last term in (3) is equivalent to advection by a three-dimensional nondivergent velocity field V^*, w^*. According to Treguier *et al.* (1997, hereafter THL) V^*, w^* can be defined approximately by

$$\begin{aligned} V^* &= -\frac{\partial}{\partial z}\left(\frac{\overline{V'\rho'}}{\overline{\rho}_z}\right) \\ w^* &= \nabla\left(\frac{\overline{V'\rho'}}{\overline{\rho}_z}\right) \end{aligned} \tag{4}$$

where the overbar is an average at a constant z level, in the (quasi-geostrophic) limit of small isopycnal displacements. We show that this limit applies to the interior of our model by comparing the meridional components v^{**} and v^* (Fig. 5). v^* has been calculated as a function of depth and is plotted as a function of the time- and horizontal-mean density coordinate $\rho(z)$. v^{**} has been calculated by linear interpolation of the model results onto instantaneous isopycnals, and then performing the time and zonal average on those isopycnals. It is costly and the results are somewhat noisy in the deep ocean where the interval between model levels is large (250 m).

Except for the surface layers, the agreement between v^{**} and v^* is quite striking, since isopycnal excursions over the domain are not negligible (up to 500 m from south to north). This suggests that the quasi-geostrophic approximation should be used to calculate eddy fluxes in z-coordinate models, since it is much easier to calculate averages at a constant level rather than interpolate instantaneous values onto isopycnals. This has been noted by Rix and Willebrand (1996).

As discussed in THL, the relationship between v^* and v^{**} is similar to the equivalence that exists between the *horizontal* flux of *quasi-geostrophic* potential vorticity on one hand, and the *isopycnal* flux of *Ertel* potential vorticity on the other hand. This equivalence was first pointed out by Charney and Stern (1962). These two fluxes are similar in our model, while the eddy flux of Ertel potential vorticity averaged at constant z is very different and even of opposite sign at some depths. The quasi-geostrophic form of potential vorticity must be preferred for an analysis in z coordinates, even in a primitive equation model.

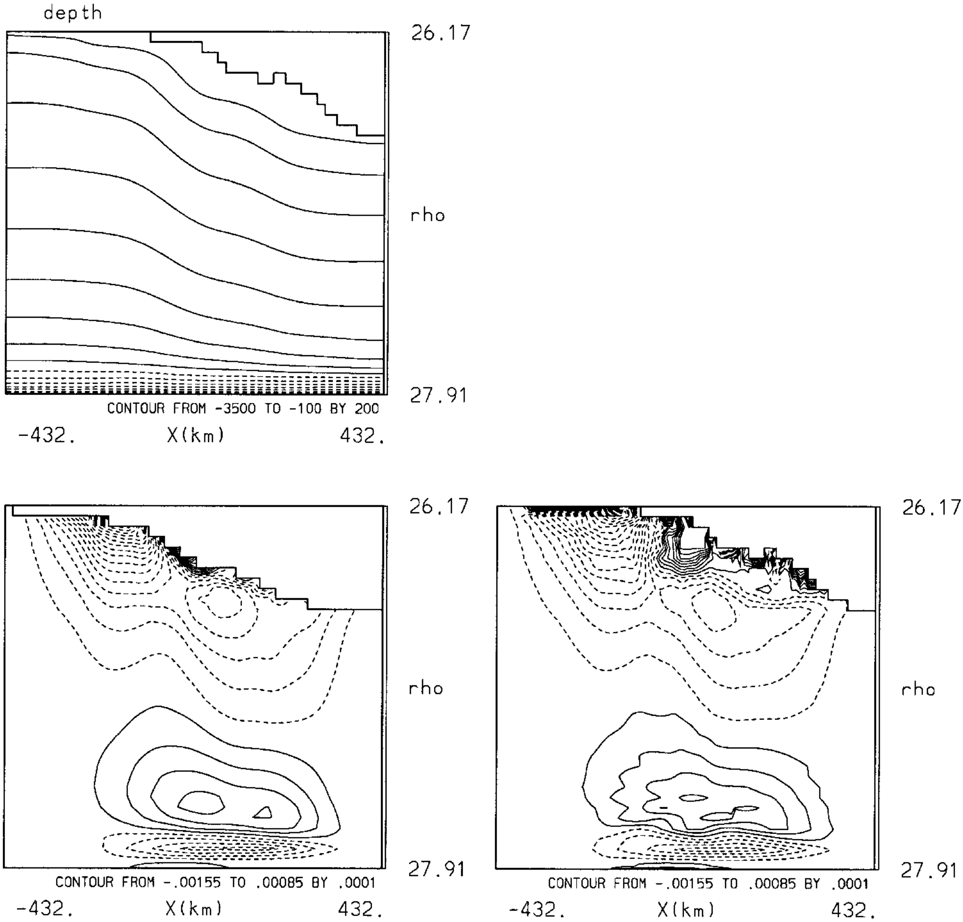

Figure 5. (a) (top): depth of the mean isopycnals as a function of potential density ρ. (b) (bottom left): v^* calculated using the z-coordinate form (4). (c) (bottom right): v^{**} calculated using the isopycnal form (2).

v^{**} and v^* differ near the surface, because the isopycnal calculation (2) takes into account the mass flux in isopycnals that are not present in the mean state. Therefore, in isopycnal coordinates the domain of v^{**} is larger than the domain of v^* (Fig. 5). THL show that the mass flux in those "transient" isopycnals, when converted back to z coordinates, is related to the vertical density flux divergence $(\overline{(w'\rho')})_z$. This term is small in the interior but becomes important near the surface because of the rigid lid condition ($w' = 0$).

c. Spatial structure of the eddy-induced velocities

Having calculated eddy-induced velocities in our model, we now ask whether advection by v^* and w^* is a good representation of the total eddy density flux divergence pictured in

Figure 4a. The approximate eddy advection

$$A = v^* \frac{\partial \overline{\rho}}{\partial y} + w^* \frac{\partial \overline{\rho}}{\partial z}$$

with the velocities calculated by (4) is shown for comparison in Figure 4b.

Even though our model is not adiabatic (there is a forcing in the interior and biharmonic friction), we find that the agreement is good in the interior, probably because the forcing is small enough. The main discrepancies occur near the surface, above 500 m (it is the depth above which isopycnals may outcrop). The advective representation of eddy fluxes is expected to fail at the surface because (as argued by THL) the surface flow is never close to adiabatic conditions. Even in the absence of buoyancy forcing, surface density tends to cascade toward small scales and be smoothed by biharmonic subgrid-scale mixing. Surface density is not constrained by the secondary circulation that maintains geostrophy in the interior. Small-scale filaments appear in the instantaneous density fields at 25 m, but they are absent at 550 m (Fig. 6). This illustrates the point made by THL that diffusion of surface density cannot be neglected, even when there are no air-sea fluxes. Eddy effects are both advective and diffusive near the surface.

Let us consider now the eddy-induced velocities v^*, w^* plotted as a function of latitude and depth (Fig. 7). THL underline the fact that in a quasi-geostrophic regime, vertical advection by w^* is the dominant eddy term. Indeed the divergence of density fluxes (Fig. 4a) is dominated by the *horizontal* divergence $\partial \overline{v'\rho'}/\partial y$, which is represented by the *vertical* advection by w^*. The spatial structure of w^* is easy to interpret: w^* is positive on the south side of the front, and negative on the north, which tends to flatten the isopycnals.

The structure of v^* is more complex than in the simple picture of Gent *et al.* (1995). Their Figure 3 shows a single cell with positive v in the top layers and negative in the bottom layers. In our case v^* changes sign many times with depth. This is related to the structure of the mean potential vorticity gradient as shown below.

3. Mixing coefficient for eddy-induced velocities

a. *Potential vorticity mixing*

The eddy-induced velocities are closely related to the quasi-geostrophic potential vorticity flux. In fact, $v^* \approx -\overline{v'q'}/f_0$ with

$$q = \zeta + \beta y - f_0 \frac{\partial}{\partial z} \frac{\rho}{\overline{\rho}_z},$$

ζ being the vertical component of the relative vorticity. $-\overline{v'q'}/f_0$ is shown in Figure 8 and can be compared with v^* (Fig. 7). They are similar because the contribution from vortex stretching dominates the potential vorticity flux, even though relative vorticity fluxes are not negligible in the model (they are responsible for the concentration of momentum into jets).

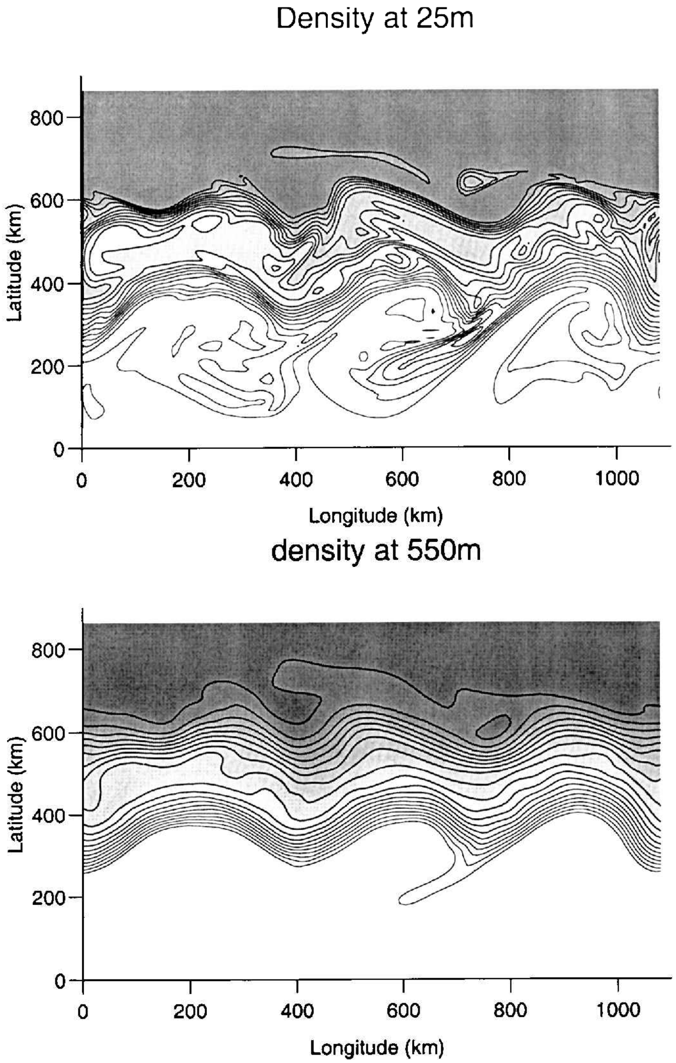

Figure 6. Instantaneous potential density fields. (a) at 25 m depth; (b) at 550 m depth. Contour interval is 0.02 kg m^{-3}.

Many studies like that of Pavan and Held (1996) suggest that eddies perform a downgradient diffusion of potential vorticity in baroclinically unstable jets. This is true here, and indeed the sign changes of v^* in the vertical correspond to sign changes of the meridional potential vorticity gradient. Note that v^* is positive in the bottom layers, even though there is no density gradient in the mean state at those depths: this is related to the presence of a meridional gradient of potential vorticity equal to β. A similar feature has been noted recently by Lee *et al.* (1997) in an isopycnic model.

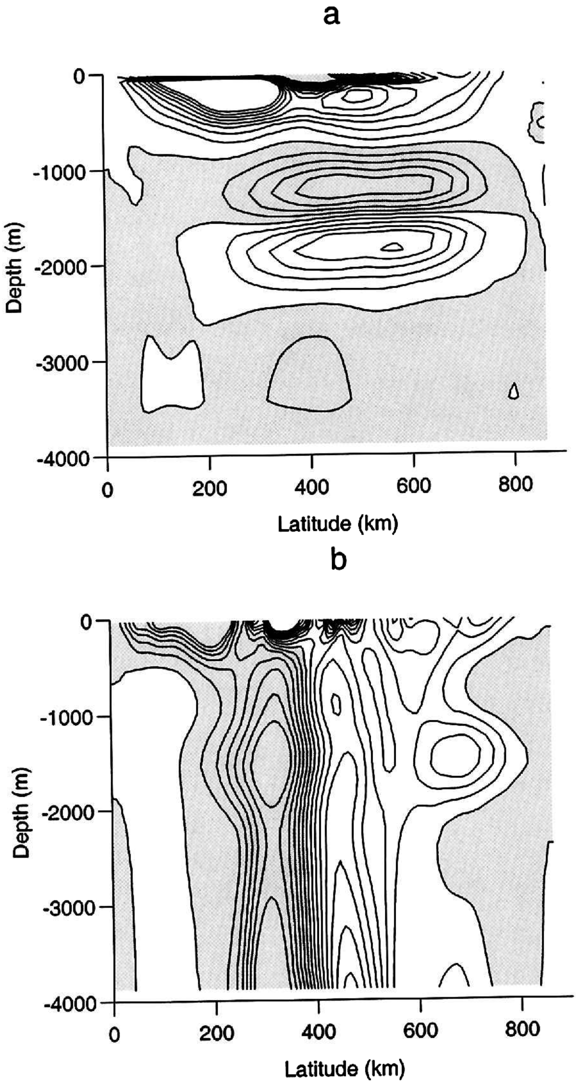

Figure 7. Eddy induced velocities calculated by (4) as a function of depth. (a) v^*; contour interval is 0.8 10^{-4} m s^{-1}. (b) w^*; contour interval is 3. 10^{-7} m s^{-1}. Positive values are shaded.

In scatter plots of potential vorticity flux vs. potential vorticity gradient (not shown), the mixing coefficient is positive everywhere. However, the mixing coefficients vary widely. In the horizontal, they tend to be maximum ($>$1000 m^2 s^{-1}) in the region of the strong mean flow and small elsewhere. This information is not relevant to the parameterization of eddies in a low resolution model since the latitudinal structure of the jet would not be resolved by the coarse horizontal grid (typically 1 to 4 degrees). On the other hand, information about the vertical structure of the mixing coefficients may be relevant because climate models

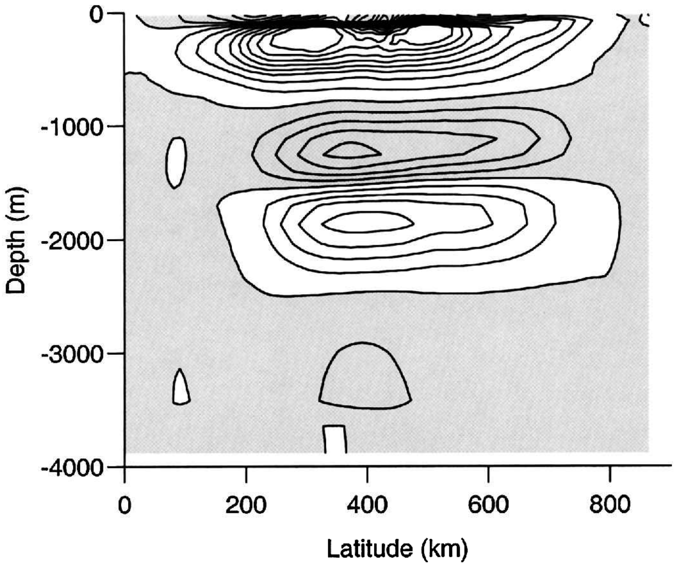

Figure 8. Quasi-geostrophic potential vorticity flux $-\overline{v'q'}/f_0$ (m s^{-1}) Contours are the same as in Figure 7a.

usually have a large number of levels on the vertical (20 or more). It is not uncommon for modelers to use horizontal mixing coefficients that are large in the upper layers and smaller at depth, based on the observation that eddies are more energetic near the surface.

We have plotted vertical profiles of the mixing coefficient for potential vorticity (Fig. 9), calculated from latitudinal averages over the central portion of the channel. Estimates based on the isopycnal flux of Ertel potential vorticity and on the horizontal flux of quasi-geostrophic potential vorticity agree well. Note that there is no estimate at a few model levels, which correspond to a sign change of both the flux and the gradient: where fluxes and gradients are very small, numerical discretization errors may lead to very large and unphysical values of the mixing coefficients.

Both isopycnal and quasi-geostrophic estimates show a mixing coefficient that is moderate (200 m^2 s^{-1}) in the top and bottom layers, with a large maximum between 1000 m and 1500 m depth. This structure is completely different from the structure of the eddy kinetic energy, which is maximum at the surface and minimum at the bottom. Maximum mixing below the core of an eastward current is observed in the atmosphere (in the case of the jet stream) as well as in the Gulf Stream (Bower, 1991). In the linear theory it can be interpreted in terms of the existence of a critical layer below the core of the jet, allowing large meridional excursions of particles (Green, 1970). On the basis of that theory Killworth (1997) has recently proposed an analytical form of the mixing coefficient κ with a vertical structure similar to the one found here (his Fig. 3). Linear theory may be partly relevant to our model because the eddy field is clearly dominated by a wavenumber 3 wave (Fig. 6) which is associated with a time scale of 160 days in the wavenumber-frequency

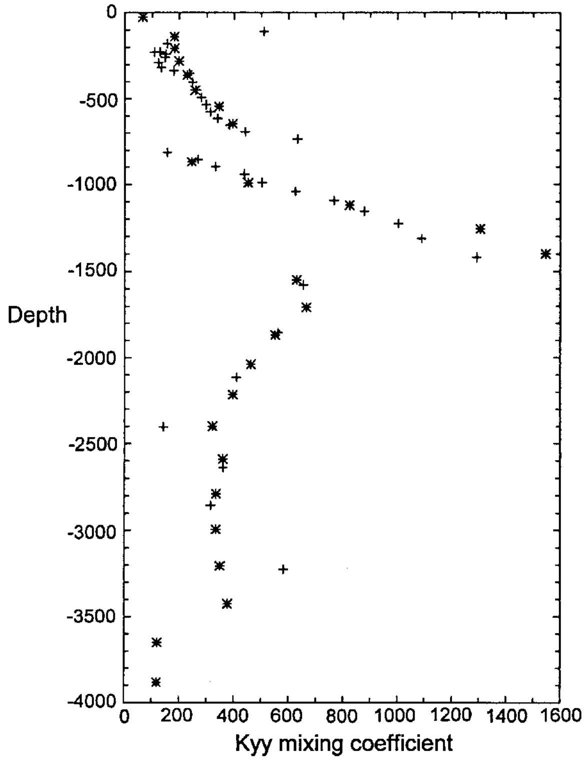

Figure 9. Vertical profile of mixing coefficients, calculated from the ratio of the fluxes and gradients averaged over the central portion of the domain. Crosses represent estimates based on the isopycnal fluxes and gradients of Ertel potential vorticity, and stars are estimates based on horizontal fluxes and gradients of quasi-geostrophic potential vorticity. Units are $m^2\ s^{-1}$.

spectrum (not shown). The corresponding phase speed ($c \approx 0.026$ m s^{-1}) is independent of depth and matches the mean zonal velocity $\overline{u}$ averaged over the central portion of the channel at a depth of 1200 m, close to the observed maximum of κ. This structure of mixing (with a maximum at mid-depth) may be valid only in the case of eastward baroclinic jets. The main point is that mixing by mesoscale eddies is everywhere depth-dependent, and the depth where mixing is strongest does not generally coincide with the maximum of eddy kinetic energy.

b. Parameterization of the eddy-induced velocities

Gent *et al.* (1995) propose a parameterization for the eddy-induced velocities (hereafter GM parameterization). It is based on the hypothesis of a ‘‘nearly downgradient Fickian diffusion of thickness in isopycnal coordinates.’’ It is an exact downgradient of thickness when the mixing coefficient is constant in the vertical.

Our estimate of the eddy fluxes supports the GM assumption of neglecting relative vorticity fluxes, compared to the thickness (or vortex stretching) fluxes for an unstable baroclinic flow. On the other hand, we have seen that where the thickness gradients are small, the thickness flux does not vanish but is related to the meridional gradient of potential vorticity (β effect). This produces the northward eddy-induced velocity near the bottom in Figure 7. The fact that GM parameterization is based on thickness gradients (and not potential vorticity gradients) is certainly a weakness. THL discuss the possibility of incorporating the planetary vorticity gradient β into an eddy parameterization. If one wishes to preserve the conservation properties of eddy fluxes this can be done only by defining a suitable horizontal and vertical structure of the eddy mixing coefficient, as attempted by Killworth (1997).

We have tried without success to recover the observed v^* of Figure 7 using GM parameterization with a constant mixing coefficient. When the mixing coefficient has a vertical structure, the advective parameterization in the form proposed by GM no longer implies downgradient mixing of thickness. As discussed by THL, in that case the GM parameterization is based on the isopycnal gradient of z (the height of an isopycnal surface), rather than $\partial z/\partial\rho$ (the thickness). The corresponding definition of V^* in z-coordinates is

$$V^*_{\mathrm{GM}} = \partial_z \left[\kappa_{\mathrm{GM}} \frac{\nabla\overline{\rho}}{\overline{\rho}_z} \right] \tag{5}$$

rather than the relevant form that would be based on the gradients of the quasi-geostrophic potential vorticity q

$$V^* = \frac{\kappa_q}{f_0} \nabla\overline{q}. \tag{6}$$

In the quasi-geostrophic limit, it is easy to show that (5) is equivalent to downgradient horizontal mixing of density. In our zonally symmetric channel, where we have demonstrated that the quasi-geostrophic approximation is appropriate in the interior, κ_{GM} can be estimated by

$$\kappa_{\mathrm{GM}}(y, z) \approx -\overline{v'\rho'} \Big/ \frac{\partial\overline{\rho}}{\partial y}\,. \tag{7}$$

κ_{GM} and κ_q are plotted in Figure 10 as a function of depth. Note that the vertical structure of κ_q is similar to the structure of the mixing coefficient for a passive tracer (dashed line). Indeed the similarity between the dynamics of potential vorticity and a tracer on isopycnal surfaces has often been pointed out.

On the other hand, the vertical structure of κ_{GM} is clearly different. The first discrepancy occurs in the bottom layers, and is due to the absence of β in GM's formulation as noted above (the thickness flux, which is very close to the potential vorticity flux, does not vanish

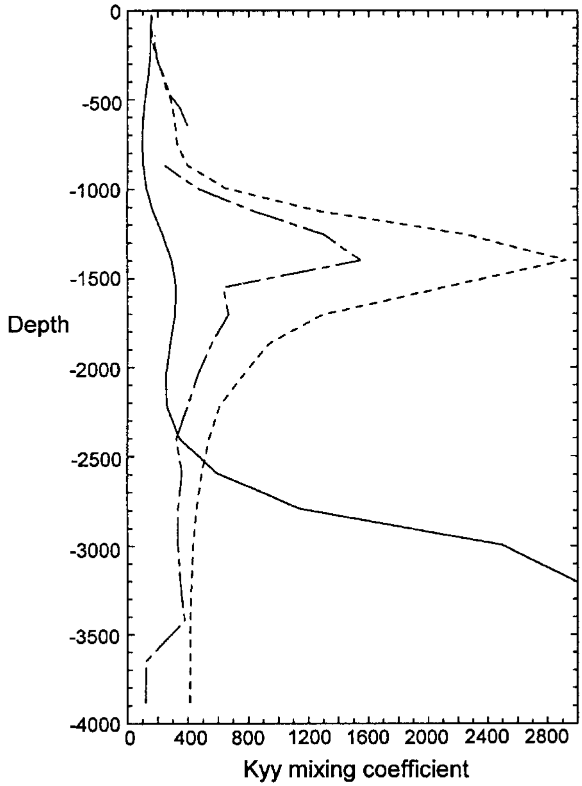

Figure 10. Vertical profiles of mixing coefficients, calculated from the ratio of the fluxes and gradients averaged over the central portion of the domain. Dashed-dotted line: estimate based on fluxes and gradients of quasi-geostrophicpotential vorticity (same as Fig. 9). Dashed line: estimate based on fluxes and gradients of a passive tracer. Solid line: estimate (5) compatible with GM parameterization.

because there is a potential vorticity gradient, even though the thickness gradient itself is negligible). The relationship between eddy PV fluxes and the planetary vorticity gradient β is also shown by Lee *et al.* (1997). It may explain why Rix and Willebrand (1996) failed to find a statistically reliable structure for κ in their North Atlantic eddy-resolving model. Maybe their difficulty was due to the fact that they tried to find a relationship between the potential vorticity flux and the gradient of vortex stretching, rather than the gradient of PV, thereby neglecting β.

A second discrepancy occurs near 1200 m where κ_q has a maximum but not κ_{GM}. We have argued above that this maximum mixing is to be expected in eastward jets and is observed in the ocean; therefore, we believe that the estimate κ_{GM} is not compatible with the physics of our model. Finally, κ_{GM} tends to have much lower values than κ_q except near the bottom where it is ill-defined. Its value is around 200 $m^2\ s^{-1}$, which is lower than the values used in most low-resolution numerical models to represent eddy mixing.

GM parameterization attempts to represent the sink of potential energy due to baroclinic instability. This is presented as a justification for the special form chosen by GM (namely, (5) instead of (6)), because in that case the parameterization is a net sink of potential energy for the large-scale flow, whatever the shape of the mixing coefficient. THL argue that eddies satisfy the constraint of a global potential energy sink in the ocean not by departing from true mixing of potential vorticity, but rather by producing an adequate horizontal and vertical structure of the mixing coefficients. Especially, the mixing coefficient must of course be zero in regions where the large-scale flow is not baroclinically unstable. This adjustment of the spatial structure of the mixing coefficient is what we observe in the eddy-resolving model: the mixing coefficient κ_q does have a vertical structure, and it is not compatible with GM's definition κ_{GM}.

This importance of the spatial structure means that one has to be very careful when using integrated balances to infer the order of magnitude of the mixing. For example, a discrepancy appears when one tries to relate the mixing coefficient κ_{GM} to the domain-integrated sink of potential energy per unit mass

$$B' = \frac{1}{\mathscr{V}} \iiint -\frac{g}{\rho_0} \overline{w'\rho'}\, dx\, dy\, dz \tag{8}$$

where $\mathscr{V}$ is the volume of the domain. In a model using GM parameterization B' is parameterized as a function of κ_{GM} by

$$B' = \frac{1}{\mathscr{V}} \iiint \kappa_{GM} \frac{g}{\rho_0} \frac{|\nabla\rho|^2}{\rho_z}\, dx\, dy\, dz. \tag{9}$$

Tandon and Garrett (1996) use (9) with a constant κ_{GM} to infer the magnitude of B' in the Antarctic Circumpolar Current. With $\kappa = 1000\ \mathrm{m^2\ s^{-1}}$ and rough estimates of the large-scale density gradients in the region, they find $B' = 8\ 10^{-9}\ \mathrm{m^2\ s^{-3}}$. This is much larger than the values calculated directly from (8) by Ivchenko *et al.* (1997) in FRAM (Fine Resolution Antarctic Model): $B' = 4 10^{-10}\ \mathrm{m^2\ s^{-3}}$. The very high value of B' estimated by Tandon and Garrett led them to the conclusion that a large diapycnal mixing was necessary to dissipate the kinetic energy generated by baroclinic instability. The smaller value that is actually found in eddy resolving models is compatible with dissipation of the kinetic energy by bottom friction corresponding to a linear decay time of 100 days over the ocean depth. Note that the order of magnitude of B' in FRAM agrees with the value found in the North Atlantic CME model by Treguier (1992).

To further demonstrate that using (9) with a spatially uniform mixing coefficient is not justified, we have calculated a constant mixing coefficient from (9) in our model:

$$\kappa_{GM} = B' \Big/ \frac{1}{\mathscr{V}} \iiint \frac{g}{\rho_0} \frac{|\nabla\rho|^2}{\rho_z}\, dx\, dy\, dz = 40\ \mathrm{m^2\ s^{-1}}. \tag{10}$$

It is one order of magnitude smaller than the spatially averaged mixing coefficient κ_q based on the local flux/gradient relationship for potential vorticity (about 400 $m^2 s^{-1}$). The fact that (9) with constant κ_{GM} leads to a wrong estimate of B', or that (10) leads to a wrong estimate of κ_{GM} is due to two factors: (1) the eddy mixing coefficient is highly variable in the horizontal and the vertical and (2) the flux-gradient relationship implied by GM is not correct.

4. Isopycnal diffusion of tracers

GM parameterization has two components: one component is advective, and the other diffusive. Potential density is affected by the advective component only, and the associated mixing coefficient κ_q should ideally be a mixing coefficient for potential vorticity along isopycnals. Tracers are advected by the eddy-induced velocity and diffused along isopycnals with a coefficient κ_T that does not need to be identical to κ_q.

We introduced passive tracers in the jet model to calculate κ_T from the tracer fluxes and gradients. A mixing coefficient can be estimated only when gradients are non-zero. Either an initial value problem must be considered, where the analysis of tracer fields is done for a short time (before initial gradients are destroyed by turbulent mixing) or a restoring force must be added in the tracer equation to maintain the gradients. Both procedures have been used for the present study. Different restoring times (100 to 600 days) and different initial conditions have been considered for the tracers. The aim of those sensitivity studies (which do not need to be described in detail) was to assess the robustness of the results.

For a tracer, eddy fluxes have a dual role: mixing the tracer anomaly along isopycnals, but also advecting it with the eddy-induced velocity. As pointed out by Lee *et al.* (1997) the diffusive effect is likely to dominate for short times and the advective effect for long times.

Over a period of one year, we expect diffusion to be the dominant effect. We have derived a mixing coefficient along isopycnal surfaces from the ratio of fluxes and gradients averaged over one year. The coefficient is similar in shape to the mixing coefficient for potential vorticity (Fig. 10). The existence of a mid-depth maximum and the order of magnitude of the mixing coefficient are very robust, independent of the initial tracer gradients and of the tracer dynamics (pure advection/diffusion or presence of a restoring force). On the other hand, the precise value of the maximum does depend on the tracer considered. Typically, it may vary by up to a factor of three according to the tracer gradient used in the calculation. Estimated diffusivities are high when the tracer gradient is low (for experiments with no restoring force). The isopycnal mixing coefficient for a passive tracer is found to be larger than the mixing coefficient for potential vorticity, but because of the sensitivity mentioned above this result may not be significant.

In Lee *et al.*'s model (1997) the effect of advection is demonstrated by considering the evolution of a tracer front over 18 years: after a rapid smoothing of the front, the dominant effect is advection which moves the front northward or southward according to the layer considered. In our model, horizontal advection by v^* is apparent when considering the deformation of a tracer distribution with initial meridional gradient. In the middle layers the zonally averaged time-mean velocity $\overline{v}$ is very small but the tracer contours show northward and southward excursions which correspond to the sign of the eddy-induced velocity v^* at a given depth (not shown).

5. Conclusion

Eddy fluxes in an unstable baroclinic jet model have been analyzed taking as a working hypothesis the existence of a local relationship between fluxes and gradients. The hypothesis seems completely appropriate for our simple, zonally symmetric model. Eddy potential density fluxes are well represented by advection by eddy-induced velocities; tracer fluxes are both diffusive and advective.

The action of eddies on the density field can be understood as a mixing of potential vorticity along isopycnals, but with a coefficient highly variable both meridionally and vertically. The mixing coefficient is maximum at mid-depth, at the latitude of the jet core. In our simple model, eddies clearly do not work in the way implied by the GM parameterization, which is based on an assumption of "near" (but not quite) Fickian diffusion of thickness. The numerical results support the conclusion of THL, that parameterization of mesoscale eddies should be as close as possible to a mixing of potential vorticity. If one wishes to remain within the framework of the GM parameterization, the mixing coefficient κ should be *constant* on the vertical since the GM parameterization is farther from a true mixing of potential vorticity when κ depends on z.

We evaluated the mixing coefficient for the isopycnal spreading of tracer anomalies, and found that it has the same vertical structure as for potential vorticity mixing. Potential vorticity is not exactly a passive tracer but at small scales it is expected to behave in the same way. Pierrehumbert (1991) argues that "small-scale vorticity concentrations can resist being sheared out by the large-scale flow field." This argument would be consistent with a lower mixing coefficient for PV than for the passive tracer, which is what we find in the model; however the difference (a factor of two) may not be significant since mixing coefficients derived from flux-gradient relationships are sensitive to the way gradients are averaged spatially. We consider that the present study provides no strong basis to choose different mixing coefficients for the advective and the diffusive part of an eddy parameterization.

The analysis of the present simplified model can be viewed as a first step toward a calculation of eddy mixing coefficients over the global ocean using a realistic eddy-resolving ocean model. Our results give some hints as to how such estimates may be performed.

Mesoscale eddies are quasi-geostrophic, and the quasi-geostrophic form of potential vorticity can be safely used to compute fluxes and gradients, even in a primitive equation model. Using the QG approximation allows the averaging to be performed at constant z which is much cheaper to compute in a z-coordinate model.

The vertical eddy-induced velocity w^* is the most important component (in agreement with the QG limit) and has a simple structure in the vertical and horizontal. The horizontal eddy-induced velocity v^* reflects the horizontal potential vorticity fluxes $\overline{v'q'}$, and its structure may be complicated. Moreover, in a three-dimensional basin geometry u^* and v^* are likely to have a large nondivergent component which does not contribute to the mean flow forcing (THL). w^* being the divergence of the horizontal flux is not affected by the rotational component and may be easier to interpret in eddy-resolving models.

In the near future, global eddy-resolving experiments should allow direct estimates of the spatial structure of mixing coefficients in different baroclinically unstable regions of the ocean, and show whether that structure is indeed related to the local growth rate of baroclinic instability.

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