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Comments on "The non-wavelike response of a continental shelf to wind" by G. T. Csanady

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1. Introduction

Csanady (1998) presents solutions for time-dependent wind-driven flow in a barotropic coastal ocean. We disagree with two of his three boundary condition options and wish to clarify the origin of the non-wavelike aspect of the flow.

2. Offshore boundary condition

There is nothing arbitrary about the appropriate choice of boundary condition at $x = l$ (where the topography meets the flat-bottom deep ocean). This point was dealt with clearly by Buchwald and Adams (1968), Gill and Schumann (1974), and Allen (1976b), for example. The cross-shelf transport and the pressure must be continuous at $x = l$. This implies that cross-shelf velocity must be continuous if depth h and wind stress G are continuous. Hence, from Csanady's equation (1), both components of the pressure gradient, ζ_x and ζ_y must be continuous at $x = l$. This in turn implies that pressure and its derivative normal to the boundary, ζ and ζ_x must be continuous at $x = l$. Csanady's condition 1 (ζ defined as zero offshore) allows ζ to be continuous, but not ζ_x. His second (channel) condition causes pressure to be discontinuous at $x = l$, another unsatisfactory property. His third condition is the traditional $\zeta_x = 0$ at $x = l$, which allows both ζ and ζ_x to be continuous.

Csanady (1998) advocates that pressure approach zero far from shore in the "boundary layer" or "long wave" approximation. This is equivalent to taking limits—far offshore vs. long wave—in the wrong order, as we now demonstrate. Once the long-wave approximation is made, Csanady's (4) reduces to

$$\zeta_{xx} = 0 \tag{1}$$

when the bottom is flat, far offshore. Requiring that ζ be bounded far from shore yields

$$\left.\begin{aligned}\zeta &= \zeta(y, t)\\ \zeta_x &= 0\end{aligned}\right\}\text{for } x > l. \tag{2}$$

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Choosing $\zeta = 0$ for all $x > l$ at this point makes it impossible, in general, for ζ_x to be continuous at $x = l$.

On the other hand, if the long-wave approximation is *not* made first, then the equivalent of Csanady's (4), for $h_x = 0$, becomes (in the absence of a wind stress curl)

$$\zeta_{xx} + \zeta_{yy} = 0 \tag{3}$$

or, for harmonic alongshore behavior with wavenumber k,

$$\zeta_{xx} - k^2\zeta = 0. \tag{4}$$

In this case, the bounded solution indeed decays as $x \rightarrow \infty$, but at a rate k^{-1}. Matching to a shelf solution allows both ζ and ζ_x to be continuous. The long-wave limit (1) is equivalent to letting k become infinitesimally small, so the offshore decay scale for pressure becomes infinitely large. This, then, requires that ζ offshore be independent of x in the long-wave limit, so ζ_x is zero at $x = l$, and ζ is generally nonzero at this location. Thus, a careful consideration of behavior far from shore leads to a nonzero solution offshore in the long wave limit.

The outer shelf boundary condition for pressure can be derived quite systematically for the wind-forced problem (by a derivation analogous to that in Allen (1976b)). Curiously, Csanady (1998) does not present a mathematical argument for defining pressure to be zero offshore. Instead, he argues on intuitive grounds that, in a realistic ocean, coastal influences should not be felt far from the shelf-slope topography. Thus, he comes to choose solutions that do not conserve interior mass transport because of imposed discontinuities in ζ or ζ_x. Of course, in the real ocean, coastal-trapped waves are not appreciably felt far offshore, but the reason is that the inclusion of a realistic, continuous stratification leads to decay of the dominant baroclinic component (offshore of the shelf-slope topography) on the relatively short scale of the internal Rossby radius of deformation.

Stated succinctly, Csanady (1998) advocates the use of a non-mass-conserving boundary condition as a means to compensate for incomplete model physics.

As Csanady (1998) states, "the best boundary condition [at $x = l$] is no boundary condition." We agree. It is straightforward to solve Csanady's shelf wave problem numerically, without using a condition at $x = l$, or assuming *a priori* that $\zeta_x = 0$ offshore. If the long-wave approximation is not made, then the inviscid governing equation (equivalent to Csanady's (4)) becomes

$$(h\zeta_{xt})_x + h\zeta_{yyt} + fh_x\zeta_y = 0. \tag{5}$$

Wave solutions are sought in the form

$$\zeta = \overline{\zeta}(x) \exp [i(\omega t + ky)] \tag{6}$$

so

$$\omega(h\overline{\zeta}_x)_x - \omega hk^2\overline{\zeta} + fh_x\overline{\zeta}k = 0. \tag{7}$$

This is solved subject to

$$h(\omega\bar{\zeta}_x + k\bar{\zeta}) = 0 \qquad \text{at } x = 0, \tag{8}$$

and

$$\bar{\zeta}_x + |k|\bar{\zeta} = 0 \qquad \text{at } x = L, \tag{9}$$

where $L > l$. This offshore condition derives from (3), and in fact allows $\zeta \to 0$ as $x \to \infty$.

This problem (7–9) can readily be solved using resonance iteration on a finite-difference grid, using the code of Brink and Chapman (1987). We use the geometry:

$$h = h_0 + \alpha x \qquad x \le l \tag{10a}$$

$$h = h_0 + \alpha l \qquad x \ge l \tag{10b}$$

where $h_0 \ll \alpha l$ since the numerical code does not allow $h = 0$ anywhere. We chose $h_0 =$ 0.5 m, $\alpha l = 4000$ m, $f = 10^{-4}$ s^{-1}, $l = 100$ km and $L = 2l$. We use 300 grid points. The long wave limit corresponds to letting $kl \to 0$, so we solve the problem with $k = 10^{-6}$, 10^{-7}, and 10^{-8} m^{-1}. The results, respectively, for the first mode are $c_1 = 2.65$, 2.72, and 2.72 m/s. Results do not change, to this accuracy, when h_0 is halved (0.25 m). Clearly, in the "long wave" or "boundary layer" limit, $kl \to 0$, $c_1 \to 2.72$ m/s or $c_1 = 0.272fl$, thus vindicating Csanady's third (correct) $x = l$ boundary condition and invalidating the other two.

Given that there is only one correct choice of $x = l$ boundary condition, Csanady's (1998) Eq. (6) also needs correction to

$$g\int_0^\infty h\zeta_x dx = \int_y^\infty G dy - \int_y^\infty gh\zeta_y dy|_{x=\infty} \tag{11a}$$

although part of his (7),

$$\psi_\infty = \int_y^\infty (G/f) dy \tag{11b}$$

is correct.

3. Interpretation

The "non-wavelike" aspect of Csanady (1998) was found in the solutions obtained by Allen (1976a) by expanding the long-wave wind-forced problem in terms of its free modes. This is, of course, a valid procedure since Huthnance (1975) demonstrated that the long-wave modes represent a complete set.

The governing wind-forced, undamped first order wave equations are

$$c_n^{-1} F_{nt} - F_{ny} = b_n G, \tag{12}$$

where $F_n(y, t)$ is a modal amplitude function and b_n is a coupling coefficient computed knowing across-shelf modes $\phi_n(x)$ and wind structure. If there is no alongshore variation in the wind, then

$$F_{nt} = c_n b_n G \tag{13}$$

for all modes. Following Allen (1976a) or by analogy with Csanady (1998), we now choose

$$G = G_0 H(-y) H(t). \tag{14}$$

The total solution for pressure is then

$$p = G_0 \sum_{n=0}^{\infty} b_n \phi_n(x) [H(t) H(-y - c_n t) c_n t - H(-y) H(y + c_n t) y]. \tag{15}$$

At $y = -Y$, the response for a given mode is governed by (13), a two-dimensional non-wavelike behavior, until time $t = c_n^{-1} Y$, when the information that an edge to the forcing exists reaches the site. More complicated geometries lead to the same result: that "knowledge" of alongshore variations reach a site after a wave propagation time. Before that time, there is no way to tell that the system is not two dimensional. Further, in the rigid lid limit that is relevant here, the mode equivalent to the barotropic Kelvin wave has $c_0 = \infty$ and $\phi_0 =$ constant. Thus, a spatially uniform, steady alongshore pressure gradient is set up instantly. In both senses (short time or fast waves), the wave Eq. (12) yields solutions with non-wavelike aspects.

This physical behavior is difficult to observe in Csanady's (1998) unbounded wedge problem because all coastal-trapped waves propagate with infinite celerity in that case: the knowledge of the edge is immediate and the adjustment is continuous.

REFERENCES

Allen, J. S. 1976a. Some aspects of the forced wave response of stratified coastal regions. J. Phys. Oceanogr., *6,* 113–119.

—— 1976b. On forced, long continental shelf waves on an *f*-plane. J. Phys. Oceanogr., *6,* 426–431.

Brink, K. H. and D. C. Chapman. 1987. Programs for computing properties of coastal-trapped waves and wind-driven motions over the continental shelf and slope—2nd ed., Woods Hole Oceanographic Institution Technical Report WHOI-87-24, 119 + iii pp.

Buchwald, V. T. and J. K. Adams. 1968. The propagation of continental shelf waves. Proc. Roy. Soc. London, A *305,* 235–250.

Csanady, G. T. 1998. The non-wavelike response of a continental shelf to wind. J. Mar. Res., *56,* 773–788, (this issue).

Gill, A. E. and E. H. Schumann. 1974. The generation of long shelf waves by the wind. J. Phys. Oceanogr., *4,* 83–90.

Huthnance, J. M. 1975. On trapped waves over a continental shelf. J. Fluid Mech., *69,* 689–704.

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