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On the transition between different dynamical regimes of the Antarctic Circumpolar Current

by V. M. Kamenkovich[1]

ABSTRACT

Two different dynamical regimes of the Antarctic Circumpolar Current are considered: the Sverdrup regime and the frictionally controlled one. In the former the intensity of the current does not depend on friction, while in the latter it is inversely proportional to the coefficient of friction. The transition between these two regimes is studied. It is shown that the frictionally controlled regime is generated not only in the case of closed isolines of ambient potential vorticity q. The regime is formed in the case of blocked (or partially blocked) q isolines as well, if the slope of the q isolines in the zonal direction is sufficiently small.

The Antarctic Circumpolar Current (ACC) is the only large-scale current that has no apparent meridional barriers in its way. It is well known that a specified external forcing can generate in the ACC region two completely different dynamical regimes. Depending on the geometry of the isolines of ambient potential vorticity q, the nondimensional transport streamfunction ψ can be of $O(1)$ or $O(1/\epsilon)$ where ϵ is a small nondimensional coefficient of friction. In the former the dynamics are controlled by the Sverdrup relation which is why the intensity of the current does not depend on friction. In the latter, the friction controls the intensity of the current, although the position of the streamlines is determined by the position of q isolines. Despite numerous publications on the subject it is still not very clear what dynamical regime governs the real ACC (see, for example, Warren *et al.* (1996) and the subsequent discussion in Hughes (1997) and Warren *et al.* (1997)).

One usually believes that the Sverdrup regime is formed when all q isolines cross the side boundaries of the region (the blocked case) while the frictionally controlled regime is formed when all q isolines are closed contours encircling Antarctica (the closed case). It is useful to notice that this is not necessarily the case. The frictionally controlled regime can exist in the blocked (or partially blocked) case as well, if the slope of the q isolines in the zonal direction is sufficiently small.

To explain the basic arguments in the simplest way, consider the barotropic model in a zonal channel D: $-L \leq x \leq L, 0 \leq y \leq 1$ with periodic conditions in the x-direction. All the flow characteristics are supposed to be nondimensional. The mathematical formulation of

1. Institute of Marine Sciences, The University of Southern Mississippi, Building 1103, Room 249, Stennis Space Center, Mississippi, 39529, U.S.A.

the problem is as follows. The transport streamfunction ψ is the solution of the vorticity equation

$$\epsilon\Delta\psi + J(\psi, q) = \mathrm{curl}_z\,(\boldsymbol{\tau}/H) \tag{1}$$

satisfying the impermeability and periodic boundary conditions

$$\psi = Q \qquad \text{at} \qquad y = 0 \tag{2}$$

$$\psi = 0 \qquad \text{at} \qquad y = 1 \tag{3}$$

$$\psi(-L, y) = \psi(L, y) \tag{4}$$

where Q, the total transport of the ACC, is to be determined.

Here $\partial\psi/\partial x = vH$, $\partial\psi/\partial y = -uH$; u, v are the zonal and meridional velocities respectively; q is the ambient potential vorticity; H is the depth of the ocean; $\boldsymbol{\tau} = (\tau_x, \tau_y)$ is the wind stress; ϵ is the coefficient of bottom friction, $\epsilon \ll 1$; other notations are traditional. The functions $\boldsymbol{\tau}$, H and q are supposed to be periodic in the x-direction.

For the barotropic model $q = f/H$, f is the Coriolis parameter. Eq. (1) holds for the equivalent barotropic model as well (with some slight changes in the coefficients at the friction term and the right-hand side). In this case q remains a specified function of x and y, however, the formula $q = f/H$ should be modified to take into account the vertical structure of the currents (for a discussion of the equivalent barotropic model see Krupitsky *et al.* (1996) and the references therein). The purpose of the note is to consider the influence of the geometry of q isolines on the Antarctic circulation.

To formulate an additional constraint, necessary to determine Q, recall that initially we have a system of momentum and continuity equations for velocity vector (u, v) and the sea surface elevation ζ. It is convenient to write the momentum equations as

$$-q\frac{\partial\psi}{\partial x} = -\frac{\partial\zeta}{\partial x} + \frac{\tau_x}{H} + \epsilon\frac{\partial\psi}{\partial y} \tag{5}$$

$$-q\frac{\partial\psi}{\partial y} = -\frac{\partial\zeta}{\partial y} + \frac{\tau_y}{H} - \epsilon\frac{\partial\psi}{\partial x} \tag{6}$$

Let's assume that ψ is already known. Then to find ζ from (5) and (6) one solves the problem of determining a function from its known first derivatives. A necessary and sufficient condition of single-valuedness of ζ in the channel is

$$\oint_\Gamma d\zeta = 0 \tag{7}$$

where Γ is a closed contour encircling Antarctica (Kamenkovich, 1961). If one takes $y =$ const as such a contour, then using (5) gives

$$\int_{-L}^{L}\left(q\frac{\partial\psi}{\partial x} + \epsilon\frac{\partial\psi}{\partial y} + \frac{\tau_x}{H}\right)dx = 0. \tag{8}$$

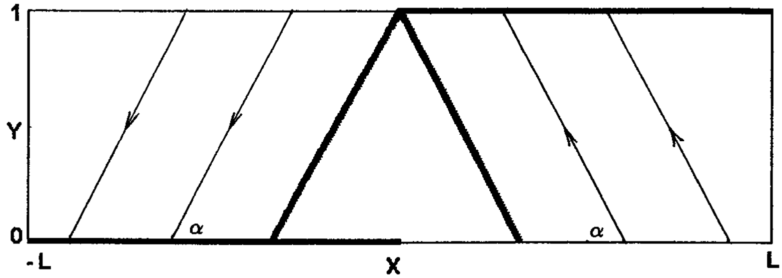

Figure 1. A simple pattern of closed q isolines when $\partial q/\partial x = O(1)$, $\partial q/\partial y = O(1)$ ($\alpha = O(1)$). The Sverdrup regime is valid outside the boundary layers located in the vicinity of the bold face lines (see text). Arrows show the direction of integration of the Sverdrup relation (10).

This is the sought additional constraint on the transport streamfunction ψ.

To determine the transport streamfunction ψ, two auxiliary functions are usually introduced, ψ_0 and ψ_1. The function ψ_0 is the solution to (1) with the boundary conditions $\psi_0(x, 0) = 0$, $\psi_0(x, 1) = 0$ and the periodic condition in the x-direction; the function ψ_1 is the solution to the homogeneous form of (1) ($\boldsymbol{\tau} = 0$) with the boundary conditions $\psi_1(x, 0) = 1$, $\psi_1(x, 1) = 0$ and the periodic condition in the x-direction. After determining ψ_0 and ψ_1 the function ψ is represented as

$$\psi = \psi_0 + Q\psi_1. \tag{9}$$

Substituting (9) into (8) yields an algebraic equation for calculating the total transport Q of the ACC.

Suppose that all q isolines within the channel are blocked by the side boundaries, and $\partial q/\partial x = O(1)$ and $\partial q/\partial y = O(1)$. In this case the Sverdrup relation

$$J(\psi, q) = \mathrm{curl}_z\,(\boldsymbol{\tau}/H) \tag{10}$$

is valid everywhere in the channel except the boundary layers. For a simple configuration shown in Figure 1 these boundary layers are the Stommel boundary layers along bold face parts of the side boundaries, internal boundary layers along slanted bold face lines, and some transitional (or corner) regions in the vicinity of the end points of these lines. Eq. (10) gives the variation of the transport streamfunction ψ along the q isolines caused by the wind-stress action. Using (8)–(10) gives $\psi_0 = O(1)$, $\psi_1 = O(1)$, $Q = O(1)$ and $\psi = O(1)$.

The general criterion for determining the position of the Stommel boundary layers was derived by Pierre Welander (Welander, 1966, 1968). Pierre initiated a series of studies dealing with matching together the Sverdrup-relation regions, the Stommel boundary layers, and the transitional boundary layers (or corner regions) for different configuration of q isolines and the boundary walls (see, for example, Kamenkovich and Mitrofanov, 1971; Kamenkovich and Reznik, 1972). Traditionally this approach was applied to the mid-latitudes regions. Stommel (1957) assumed the application of the Sverdrup regime to the ACC region by incorporating some meridional barriers; his idea was developed in a

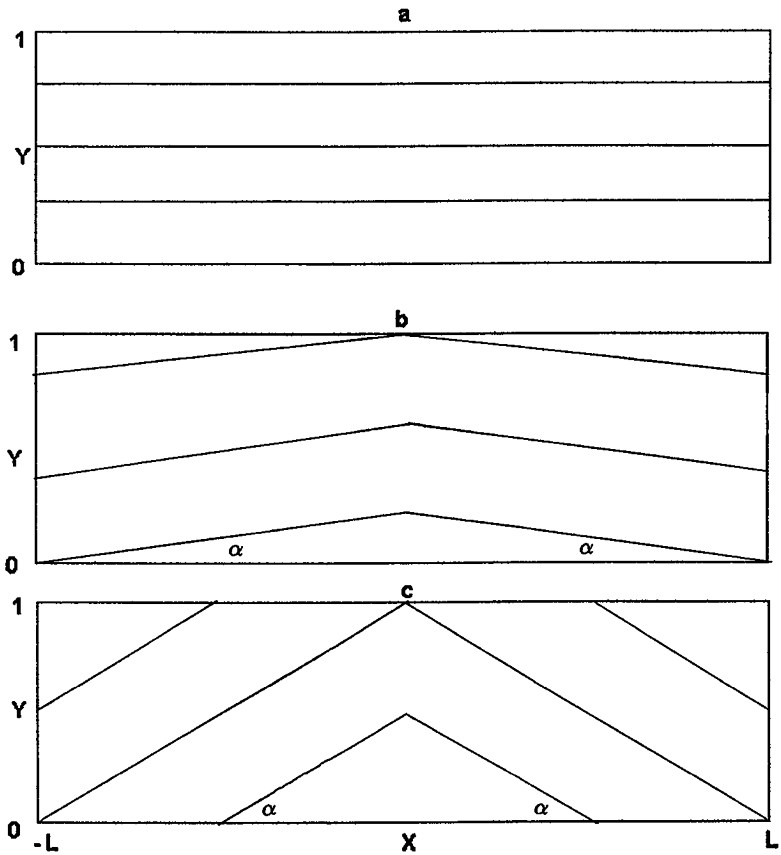

Figure 2. Three different cases of q isoline patterns for an elongated channel ($L \gg 1$): (a) closed q isolines; (b) partially blocked q isolines; (c) blocked q isolines. The slope angle α of q isolines is assumed to be small.

series of papers (see, for example, Baker, 1982). Recently, Krupitsky and Cane (1994) and Wang and Huang (1995) (see also Wang and Huang, 1994) modeled the ACC region as a zonal channel and showed that under certain bottom topographies the Sverdrup relation remains applicable.

The specific regime for the case of closed q isolines was considered by Kamenkovich (1962). In the case of $q = y$ (Fig. 2a) the asymptotics (the leading behavior) of the transport streamfunction ψ depends only on y,

$$\psi = \psi(y), \tag{11}$$

and satisfies the following equation

$$\epsilon \frac{\partial^2 \psi}{\partial y^2} = \overline{\mathrm{curl}_z \left(\frac{\tau}{H}\right)} \tag{12}$$

where the overbar means the averaging along y = const over the length of the channel. In contrast to the Sverdrup case, we see that the diffusion of vorticity (or the bottom friction) plays a leading role in determining the values of ψ, however the same diffusion does not influence the position of the ψ isolines at all. Using (8), (9), and (12) gives $\psi_0 = O(1/\epsilon)$, $\psi_1 = O(1)$, $Q = O(1/\epsilon)$, and $\psi = O(1/\epsilon)$.

The formulas (10)–(12) are asymptotic. Thus, under the same forcing $\psi = O(1)$ for the case of blocked q isolines (with $\partial q/\partial x = O(1)$, $\partial q/\partial y = O(1)$) while for the case of closed q isolines encircling Antarctica $\psi = O(1/\epsilon)$. This disparity is due to the different dynamical balances for these two regimes: the Sverdrup balance and the balance between vorticity diffusion and external source of vorticity caused by the action of wind.

Assume now that $\partial q/\partial x = O(\epsilon)$ while $\partial q/\partial y = O(1)$. Then it is not difficult to show (see, for example, Krupitsky *et al.*, 1996; Appendix A) that for small ϵ the asymptotics of the transport streamfunction ψ depends only on y and satisfies the following equation

$$\epsilon \overline{\left(\frac{\partial q}{\partial y}\right)^{-1}} \frac{\partial^2 \psi}{\partial y^2} + \overline{\left(\frac{dy}{dx}\right)_q} \frac{\partial \psi}{\partial y} = \overline{\left(\frac{\partial q}{\partial y}\right)^{-1} \operatorname{curl}_z \left(\frac{\tau}{H}\right)} \tag{13}$$

where

$$\left(\frac{dy}{dx}\right)_q = -\frac{\partial q}{\partial x}\left(\frac{\partial q}{\partial y}\right)^{-1} \tag{14}$$

is the slope of q isolines in the zonal direction.

Finally, consider the transition from the frictionally controlled regime to the Sverdrup regime by perturbing the q isolines. The simplest type of such a perturbation is shown in Figure 2. Starting with the closed q isolines ($\alpha = 0$) we gradually increase the slope angle α to provide a smooth transition to the case presented in Figure 1 ($\alpha = O(1)$). Recall that in the case of Figure 1 we have the Sverdrup regime. It is evident that the transition can be reversed, from the Sverdrup regime to the frictionally controlled one. It is important to stress that as long as the slope angle α is small, the function $\psi_0 = O(1/\epsilon)$, $\psi_1 = O(1)$, as is seen from (13). Therefore $Q = O(1/\epsilon)$ and the transport streamfunction $\psi = O(1/\epsilon)$ or, in other words, the regime remains frictionally controlled whether q isolines are blocked, or partially blocked, or closed. Figure 2 illustrates this statement for a very elongated channel ($L \gg 1$). In Figure 2a all q isolines are closed; in Figure 2b only part of q isolines is closed while in Figure 2c all q isolines are blocked. Nevertheless, in all three cases $\psi = O(1/\epsilon)$ since in all these cases $\tan \alpha = O(\epsilon)$. Thus, the dynamical regime in a zonal channel is substantially influenced by the slope of q isolines in the zonal direction. Whether the q isolines in the channel are blocked or closed, this is of secondary importance.

Based on the Sverdrup solution it is easy to understand why ψ_0 (and therefore ψ and Q) has large values when α is small (as compared to ψ_0 for $\alpha = O(1)$). To find $\psi(A)$ in the case of $\alpha = O(1)$ one integrates $\operatorname{curl}_z(\tau/H)$ along a q isoline from B to A, see Figure 3. For small α the integration goes also along a q isoline but from D through C to A. For an elongated

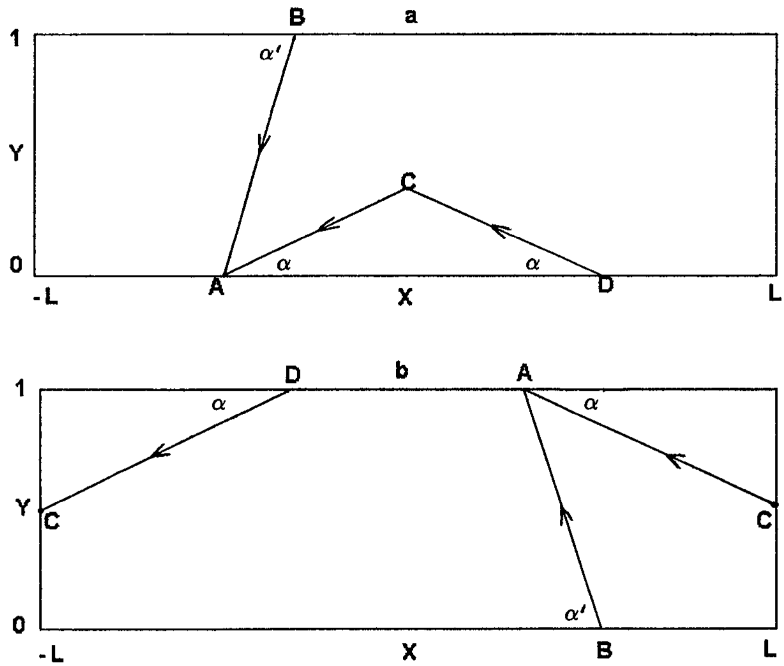

Figure 3. The comparison of paths BA and DCA of integration of the Sverdrup relation (10) for different slope angles of q isolines α and α' for an elongated channel ($L \gg 1$). The integration in different parts of the channel is shown. Arrows give the direction of integration.

channel the path DCA is clearly longer than the path BA. It seems that it is the Sverdrup relation that dictates large values of ψ_0 for small α.

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