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The effect of East River on the barotropic motions in Long Island Sound

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ABSTRACT

A linearized, frequency-dependent analytical model is developed to examine the effect of East River on the barotropic motions in Long Island Sound. At tidal frequencies, East River creates a slightly imperfect reflecting wall at the western end of Long Island Sound, resulting in moderate reduction in the resonance of the M_2 tide. At subtidal frequencies the presence of East River permits a significant amount of volume exchange through the western end of Long Island Sound, causing large scale adjustments in both the amplitude and the phase of the barotropic flow well into the interior of the Sound. It appears that the impact of East River has to be considered for a proper assessment of low frequency motion in Long Island Sound.

1. Introduction

Cameron and Pritchard (1963) defined an estuary as "a semi-enclosed body of water which has a free connection with the open sea and within which sea water is measurably diluted with fresh water derived from land drainage." Implicit in this definition is the assumption that while an estuary is connected to the adjacent ocean at the mouth, the head of the estuary is completely surrounded by land. It is interesting to note that some of the major estuaries on the east coast of the United States do not fit this definition, as they are connected to either the ocean or other major bodies of water at both ends. Long Island Sound (Fig. 1) is one notable example of such estuaries. It is an elongated estuary roughly aligned with the east–west direction, bounded by Connecticut to the north and Long Island to the south. To the east at Montauk Point the Sound opens to Block Island Sound and the Atlantic Ocean, while to the west at Willets Point it communicates through East River with the lower Hudson estuary (better known as the New York Harbor). Since the Sound is not semi-enclosed, it can be forced by sea level fluctuations in the Atlantic Ocean and the lower Hudson estuary from both ends.

The presence of strong tidal motion in Long Island Sound has been reported by LeLacheur and Sammons as early as 1932. The most distinct feature of the dominant semi-diurnal tide in the Sound is its near fourfold increase in amplitude from Montauk Point to Willets Point due to resonance. Several attempts have been made

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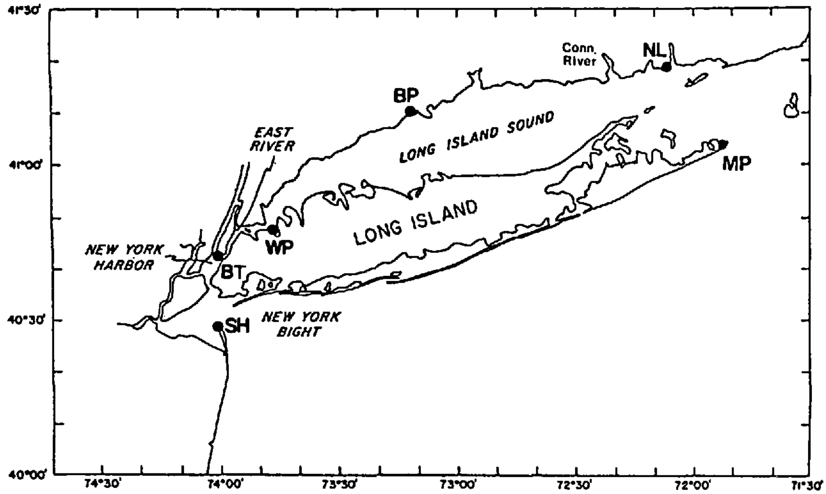

Figure 1. Location map of the East River—Long Island Sound system. The two ends of East River are located at Battery (BT) and Willets Point (WP). The eastern end of Long Island Sound is located at Montauk Point (MP). Additional locations of interest are Bridgeport (BP), Sandy Hook (SH), and New London (NL). Sea level records were available at Battery (η_{bt}), Willets Point (η_{wp}), Bridgeport (η_{bp}), and Montauk Point (η_{mp}).

to model the tidal variability in Long Island Sound. Redfield (1950) modeled the M_2 tidal height in the Sound by using two damped, progressive waves with equal amplitudes travelling in opposite directions. More recently, Redfield (1978) re-examined the system by taking into account the presence of tidal frequencies other than the M_2. Ianniello (1981) modeled the M_2 tidal motion and the tidally generated residual motion in the Sound by using a laterally averaged model which allowed for both depth and breadth variations. Murphy (1979) modeled the horizontal structure of M_2 and M_2-derived mean flow in the Sound. The barotropic M_2 tides and tidal currents in the Sound were also modeled by Kenefick (1985) using a vertically integrated finite difference model with realistic coastline and bathymetry.

In addition to the tidally-induced motion, past studies have also indicated the presence of low frequency subtidal variability within the Sound. These include the density-induced gravitational circulation (Riley, 1967; Gross and Bumpus, 1972; Wilson, 1976) and atmospherically-induced low frequency motion (Firstenberg, 1982; Ullman and Wilson, 1984; Wilson *et al.,* 1985; Wong, 1990a). It has been shown that the atmospherically-induced low frequency variability in the Sound is forced primarily by low frequency coastal sea level set-up or set-down, while local wind forcing over the surface of the Sound plays only a secondary role. The dominance of coastal sea level forcing on the subtidal variability in the Sound is consistent with the theoretical analysis of Garvine (1985). The low frequency currents are typically weaker than the M_2 current, but they are extremely important to the long term exchange process.

Historically, tidal variability along the East River has been examined in conjunction with studies looking into the tidal phenomena in the New York Harbor (e.g., Parsons, 1913; Marmer, 1935). Jay and Bowman (1975) and Bowman (1976) showed that to a first approximation, East River is a hydraulic tidal strait, with sea level and transport along it being governed by the hydraulic head between the two ends at Battery and Willets Point (Fig. 1). In addition to tidal variability, Filadelfo (1984) also showed that the barotropic subtidal current fluctuations along the river were highly coherent with subtidal hydraulic head between the two ends of the river.

The importance of East River on the density-induced gravitational circulation in Long Island Sound has been well documented (Hardy, 1972; Wilson, 1976). Despite the fact that a major river, the Connecticut River, enters the Sound near its eastern end, salinities in the Sound vary from *30 ‰* in the eastern section to *24 ‰* in the western section (Hardy, 1972). This salinity gradient is supported by fresh water input from the lower Hudson estuary via East River. Wilson (1976) indicated that such longitudinal salinity and associated density gradients existed in the Sound throughout the year and maintained a well-developed two-layer gravitational circulation. In contrast, the effect of the opening at East River on the barotropic response of the Sound has received little attention in the past. Previous modeling studies on motions derived from barotropic tides (Murphy, 1979; Ianniello, 1981; Kenefick, 1985) have all assumed the western end of the Sound to be closed. The common argument is that the cross-sectional area of the river is much smaller than that at the eastern end of the Sound. The fact remains that virtually nothing is known about the impact of East River on the barotropic tides in Long Island Sound. More importantly, the effect of East River on the barotropic response of the Sound may be most pronounced for low frequency subtidal motion forced by atmospherically driven subtidal sea level fluctuations imposed at the two ends of the East River-Long Island Sound system. The present study examines the barotropic motion in Long Island Sound under sea level forcing from Battery and Montauk Point, with the objective of gaining physical insight into the importance of East River on Long Island Sound at both tidal and subtidal frequencies. A linearized, frequency-dependent analytical model is developed so that the importance of East River can be examined in some detail.

2. Volume flux comparison

One way to assess the potential impact of East River on the barotropic motion in Long Island Sound is to examine the magnitude of the volume flux through the western end of Long Island Sound (or the eastern end of East River) versus that through the eastern end of the Sound at different frequencies. Based on an array of current meters near the eastern end of East River, Bowman (1976) estimated the amplitude of the M_2 tidal flux there to be $Q_r = 7.1 \times 10^3$ m^3/s. The amplitude of the M_2

flux through the eastern end of Long Island Sound was estimated by Wilson (1976) to be about $Q_s = 3.5 \times 10^5$ m^3/s. At the semi-diurnal frequency, $Q_r/Q_s \cong 2\%$.

At subtidal frequencies the variability in the East River-Long Island Sound system is driven by nondeterministic atmospheric events. As a result, the ratio of Q_r/Q_s may be strongly time-dependent. Since no long-term current observations are available to us, we will estimate the volume flux with readily available long-term sea level data in the system. For our particular application, we obtained year-long (1975) sea level records at Battery (η_{bt}), Willets Point (η_{wp}), Bridgeport (η_{bp}), and Montauk Point (η_{mp}) (see Fig. 1 for station locations). Subtidal signals in these records are separated from the raw records by passing the time series through a low-pass Lanczos filter (Bloomfield, 1976) with a cut-off period of 34 hr.

The estimation of Q_r through the eastern end of East River is straightforward. Since East River is basically a hydraulic strait, to first order of approximation we have

$$u_1 \approx \frac{-gh_1}{r_1}\frac{(\eta_{wp} - \eta_{bt})}{L_1} = \frac{gh_1}{r_1}\left(\frac{\Delta\eta}{L_1}\right). \tag{1}$$

Here u_1 is sectionally averaged current along the river, r_1 is the linearized resistance coefficient, h_1 is the mean depth, L_1 is the length of East River, and g is the acceleration of gravity. Here we define $\Delta\eta = (\eta_{bt} - \eta_{wp})$ so a positive hydraulic head corresponds to a positive current to the east into the Sound. The subtidal volume flux through East River is thus $Q_r = u_1 w_1 h_1$ where w_1 is the mean width of the river. For East River, it can be shown (see the next section) that the appropriate resistance coefficient is $r_1 = 6.0 \times 10^{-3}$ m/s. The physical dimensions of East River give $h_1 = 9.3$ m, $w_1 = 1.2 \times 10^3$ m, and $L_1 = 2.8 \times 10^4$ m. Based on Eq. (1), we can come up with an estimate of the transfer function representing the magnitude of the volume flux generated per unit change in $\Delta\eta$ to be about 6.0×10^3 (m^3/s)/m. The subtidal volume flux through East River into the western end of Long Island Sound can thus be estimated from differences between the observed η_{bt} and η_{wp} (Fig. 2, top panel).

The estimation of the total subtidal volume flux into and out of Long Island Sound, hereafter termed Q_f, is a more difficult task. Following Goodrich (1988), the change in volume δV_{AB} between two sea level stations A and B over the time interval δt can be estimated as the surface area between A and B times the time rate of change of sea level between these two stations. Based on observed subtidal sea level fluctuations at Willets Point, Bridgeport and Montauk Point, an estimate of Q_f was obtained (Fig. 2, lower panel). Ideally, Q_f should equal the sum of Q_s and Q_r, as the total volume flux should equal the sum of the volume flux from the two ends of the Sound. Because of the crude way in which Q_f is estimated, it is perhaps more appropriate to measure the relative importance of the subtidal volume flux through East River simply by comparing some summary statistics of Q_r and Q_f. Table 1 shows a comparison of the standard deviations of Q_r and Q_f. On an annual basis $\sigma_r/\sigma_f = 4.57\%$. This ratio shows strong seasonal variation, with a minimum of $\sigma_r/\sigma_f = 2.98\%$

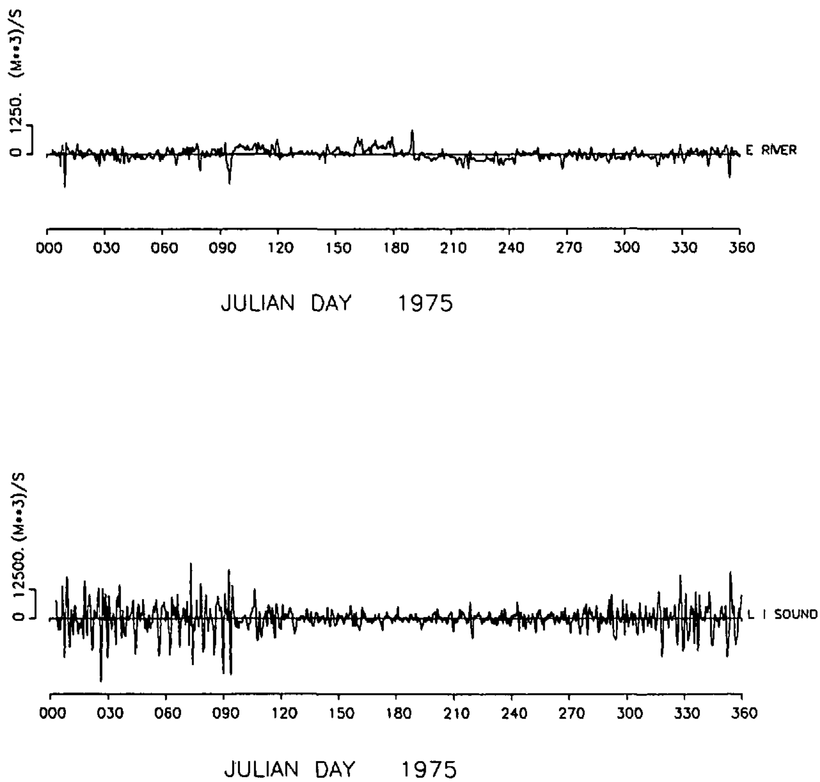

Figure 2. (A) Subtidal volume flux through the East River into or out of Long Island Sound (see text). (B) Total subtidal volume flux into or out of Long Island Sound (see text). Note the change in scale for the magnitude of volume flux.

in the winter months and a maximum of σ_r/σ_f = 9.23% in the summer. It is apparent that while σ_f goes through strong seasonal variation, σ_r shows only slight variation with season. The variation of σ_f with season is easily understood, as stronger atmospheric forcing in the winter induces larger subtidal sea level and current fluctuations within the Sound. The subtidal volume transport through the East River, however, depends not on the absolute magnitude of the sea level but differences in

Table 1. Some statistics of the subtidal volume flux through the eastern end of the East River (Q_r) and the flux into or out of the Long Island Sound (Q_f). Here σ_f represents the standard deviation of Q_f and σ_r represents the standard deviation of Q_r. σ_r and σ_f are in unit of (m^3/s).

	Jan–Dec	Jan–Mar	Apr–Jun	Jul–Sep	Oct–Dec
σ_r	2.39×10^2	2.26×10^2	2.55×10^2	2.04×10^2	1.93×10^2
σ_f	5.23×10^3	7.58×10^3	3.96×10^3	2.21×10^3	5.75×10^3
$\frac{\sigma_r}{\sigma_f}$	4.57%	2.98%	6.44%	9.23%	3.36%

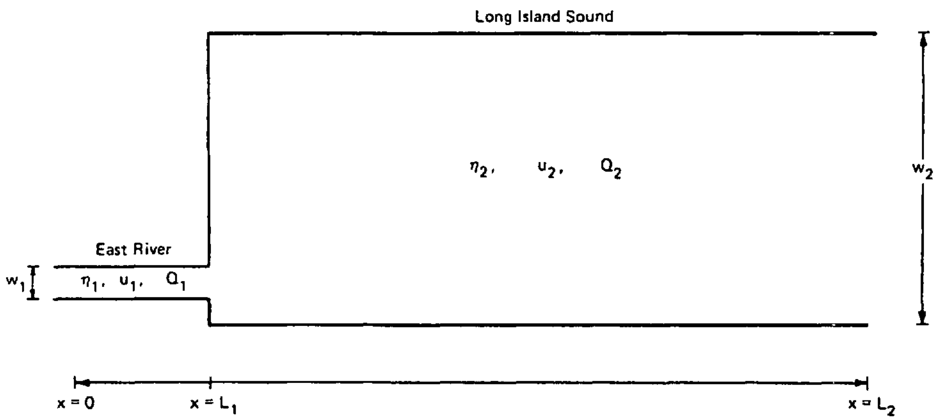

Figure 3. A schematic diagram of the interconnected channels representing the East River-Long Island Sound system.

sea level across the two ends of the river. Apparently the hydraulic head does not change appreciably with season. The result is that the subtidal volume flux through East River may become a significant fraction of the total volume flux into or out of the Sound. There is apparently a need to further assess the importance of the River to the Sound, particularly at subtidal frequencies.

3. A simple conceptual model

The impact of East River on the barotropic motion in Long Island Sound will be considered by simplifying the two waterways into two interconnected rectangular channels with constant depths and breadths. Figure 3 shows a schematic diagram of the simplified model domain. The channel representing East River has a mean depth h_1, mean width w_1 and mean length L_1. The point $x = L_1$ corresponds to the position of Willets Point where East River enters Long Island Sound. The symbols η_1, u_1, and Q_1 represent the surface elevation, sectionally averaged current, and volume flux in East River, respectively. The volume flux is defined as $Q_1 = u_1 h_1 w_1$. The channel representing Long Island Sound has a mean depth h_2, width w_2, and length $(L_2 - L_1)$. Again the symbols η_2, u_2, and Q_2 represent the surface elevation, sectionally averaged current, and volume flux in the Sound. A linearized, analytical model is constructed to examine the first-order barotropic response of the coupled East River-Long Island Sound system under sea level forcing from the two ends of the system at Battery ($x = 0$) and Montauk Point ($x = L_2$). The imposition of sea level as forcing mechanism at the open boundary is a common practice for modeling flows in tidal channels (van de Kreeke and Dean, 1975). Wong (1990a) has shown that sea level fluctuations at Battery were essentially the same as those at Sandy Hook. Due to the relatively short distance between Battery and Sandy Hook, the lower Hudson estuary simply co-oscillated with the coastal sea level at Sandy Hook. With a two-input linear model, the intent is to represent all dependent variables in the interior of the system (η_1, u_1, Q_1, η_2, u_2, and Q_2) in terms of linear combinations of the two forcing inputs (η_{bt} and η_{mp}). Once that is done, the effect and importance of East River on the barotropic motion in Long Island Sound can be readily assessed.

In the Fourier-transformed frequency domain, the sectionally averaged linearized momentum equation for $0 \le x \le L_1$ (channel 1), and the corresponding continuity equation are

$$i\omega\hat{u}_1 = -g\frac{\partial\hat{\eta}_1}{\partial x} - \left(\frac{r_1}{h_1}\right)\hat{u}_1 \tag{2a}$$

$$i\omega\hat{\eta}_1 = -h_1\frac{\partial\hat{u}_1}{\partial x} \tag{2b}$$

Similarly, for $L_1 \le x \le L_2$ (channel 2), we have

$$i\omega\hat{u}_2 = -g\frac{\partial\hat{\eta}_2}{\partial x} - \left(\frac{r_2}{h_2}\right)\hat{u}_2 \tag{3a}$$

$$i\omega\hat{\eta}_2 = -h_2\frac{\partial\hat{u}_2}{\partial x}. \tag{3b}$$

Here g is the acceleration of gravity, and r_1 and r_2 are the linearized resistance coefficients for East River and Long Island Sound, respectively. The dependent variable with the circumflex indicates complex-valued transformed variable in the frequency domain, $i = \sqrt{-1}$, and ω represents angular frequency. The transformed variable is complex-valued so phase information is preserved. Throughout the system, the typical tidal amplitude is about $0\,(10^{-1})$ as compared to the mean depth of the channel, so we feel the linearized approach represents a reasonable approximation (Ianniello, 1981). The simplified geometry allows us to develop analytical solutions so the essence of the system response can be examined in some detail.

Eqs. (2a)–(3b) are ordinary differential equations in x with the frequency ω being simply a parameter free to be specified. Four boundary conditions are required to solve the four dependent variables (η_1, η_2, $Q_1 = u_1w_1h_1$, and $Q_2 = u_2w_2h_2$) in the interior of the model domain. Two such conditions are readily available from open boundary sea level forcing as

$$\hat{\eta}_1(x = 0, \omega) = \hat{\eta}_{bt}(\omega) \tag{4a}$$

$$\hat{\eta}_2(x = L_2, \omega) = \hat{\eta}_{mp}(\omega). \tag{4b}$$

The other two boundary conditions can be defined at the internal boundary $x = L_1$ where East River connects with Long Island Sound. These conditions are

$$\hat{\eta}_1(x = L_1, \omega) = \hat{\eta}_2(x = L_1, \omega) \tag{5a}$$

$$\hat{Q}_1(x = L_1, \omega) = \hat{Q}_2(x = L_1, \omega) \tag{5b}$$

Eqs. (5a) and (5b) simply match the sea surface elevation and volume flux at the point where the two channels meet.

With Eqs. (2a)–(3b) and the four boundary conditions, the sea surface elevations and volume fluxes within the system can be expressed in terms of linear combinations of $\hat{\eta}_{bt}$, $\hat{\eta}_{mp}$, and a set of complex-valued transfer functions as

$$\hat{\eta}_1(x, \omega) = \hat{\eta}_{bt}(\omega)H_{1bt}(x, \omega) + \hat{\eta}_{mp}(\omega)H_{1mp}(x, \omega) = \hat{\eta}_{1bt}(x, \omega) + \hat{\eta}_{1mp}(x, \omega) \quad (6a)$$

$$\hat{Q}_1(x, \omega) = \hat{\eta}_{bt}(\omega)U_{1bt}(x, \omega) + \hat{\eta}_{mp}(\omega)U_{1mp}(x, \omega) = \hat{Q}_{1bt}(x, \omega) + \hat{Q}_{1mp}(x, \omega) \quad (6b)$$

$$\hat{\eta}_2(x, \omega) = \hat{\eta}_{bt}(\omega)H_{2bt}(x, \omega) + \hat{\eta}_{mp}(\omega)H_{2mp}(x, \omega) = \hat{\eta}_{2bt}(x, \omega) + \hat{\eta}_{2mp}(x, \omega) \quad (6c)$$

$$\hat{Q}_2(x, \omega) = \hat{\eta}_{bt}(\omega)U_{2bt}(x, \omega) + \hat{\eta}_{mp}(\omega)U_{2mp}(x, \omega) = \hat{Q}_{2bt}(x, \omega) + \hat{Q}_{2mp}(x, \omega). \quad (6d)$$

The eight transfer functions ($H_{1bt}, H_{1mp}, U_{1bt}, U_{1mp}, H_{2bt}, H_{2mp}, U_{2bt}, U_{2mp}$) fully describe the response of the system to boundary sea level forcing. Because of the significant difference in the cross-sectional area between the two waterways, it is more meaningful to examine the volume fluxes in the two channels rather than the sectionally-averaged currents. The term $\hat{\eta}_{2bt}$ in Eq. (6c) represents the part of the sea level within Long Island Sound which is induced by boundary sea level at Battery ($\hat{\eta}_{bt}$). Because $\hat{\eta}_{2bt} = \hat{\eta}_{bt}H_{2bt}$, the amplitude of the transfer function $|H_{2bt}|$ gives the normalized response amplitude to a unit of forcing from $\hat{\eta}_{bt}$, while $\phi(H_{2bt}) = \tan^{-1}[Imag(H_{2bt})/Real(H_{2bt})]$ gives the phase relationship between $\hat{\eta}_{bt}$ and the interior sea level. The subscript *2bt* indicates that the transfer function relates the boundary forcing $\hat{\eta}_{bt}$ to the dependent variable in channel 2, the Sound. Similarly, $\hat{\eta}_{2mp}$ in Eq. (6c) represents the part of the sea level in the Sound that is induced by boundary sea level at Montauk Point ($\hat{\eta}_{mp}$). Again, $|H_{2mp}|$ and $\phi(H_{2mp})$ provide the relative response amplitude and phase information, and the subscript *2mp* indicates that the transfer function relates the boundary forcing $\hat{\eta}_{mp}$ to the dependent variable within channel 2. The transfer functions H_{2bt} and H_{2mp} thus provide a measure of the effectiveness of the two open boundary sea levels on forcing the sea level in the interior of the Sound. The transfer functions also provide the spatial structure of the interior sea level response. Eq. (6d) describes the way in which volume flux in the Sound is forced by the two open boundary sea levels, and the role of the transfer functions U_{2bt} and U_{2mp} can be interpreted in a similar way. It is important to note that while H_{2bt} and H_{2mp} are dimensionless, U_{2bt} and U_{2mp} have the unit (m^3/s)/m.

The transfer functions are determined by the physical dimensions of the system and the resistance coefficients. The coupled East River-Long Island Sound system gives $h_1 = 9.3$ m, $w_1 = 1.2 \times 10^3$ m, $L_1 = 2.8 \times 10^4$ m, $h_2 = 21.5$ m, $w_2 = 2.0 \times 10^4$ m and $L_2 = 1.83 \times 10^5$ m. The model is then forced with known amplitudes and phases of M_2 tides at Battery and Montauk Point (Wong, 1990a), and the resistance coefficients are chosen to be $r_1 = 6.0 \times 10^{-3}$ m/s and $r_2 = 0.9 \times 10^{-3}$ m/s to provide a best fit of known M_2 variability in the interior of the system. Figure 4 shows the computed M_2 tide and M_2 tidal flux in the Sound. To simplify the presentation, a normalized

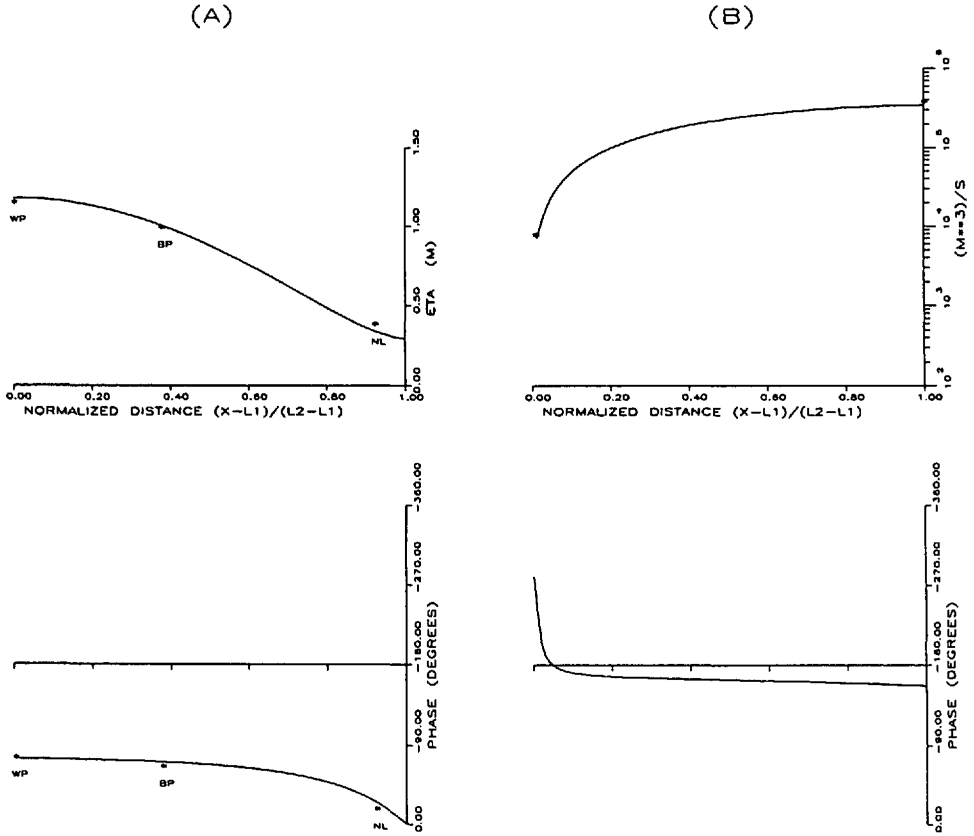

Figure 4. (A) The amplitude and phase distributions of the M_2 tide as given by the analytical model. Asterisks indicate known amplitude and phase of M_2 tide at Willets Point, Bridgeport, and New London (Wong, 1990a). (B) The amplitude and phase distributions of the M_2 tidal flux as given by the analytical model. Asterisks indicate known estimates of the amplitude of M_2 flux at the western end of the Sound (Bowman, 1976) and at the eastern end of the Sound (Wilson, 1976).

longitudinal position along the Sound is chosen to be $x' = (x - L_1)/(L_2 - L_1)$, with $x' = 0$ corresponds to Willets Point and $x' = 1$ corresponds to Montauk Point. The known amplitude and phase of the M_2 tide at Willets Point, Bridgeport, and New London (Wong, 1990a), as well as known amplitude of the M_2 tidal flux in East River (Bowman, 1976) and in the Sound (Wilson, 1976) are used to obtain the appropriate resistance coefficients. The same resistance coefficients are then used for motions at other frequencies without further adjustments.

Previous studies concerning the barotropic response of Long Island Sound to ocean forcing have assumed the western end of the Sound to be closed, so the system can only be forced by open boundary sea level at Montauk Point. With the opening of East River in our conceptual model, the Sound can now be forced by sea level from

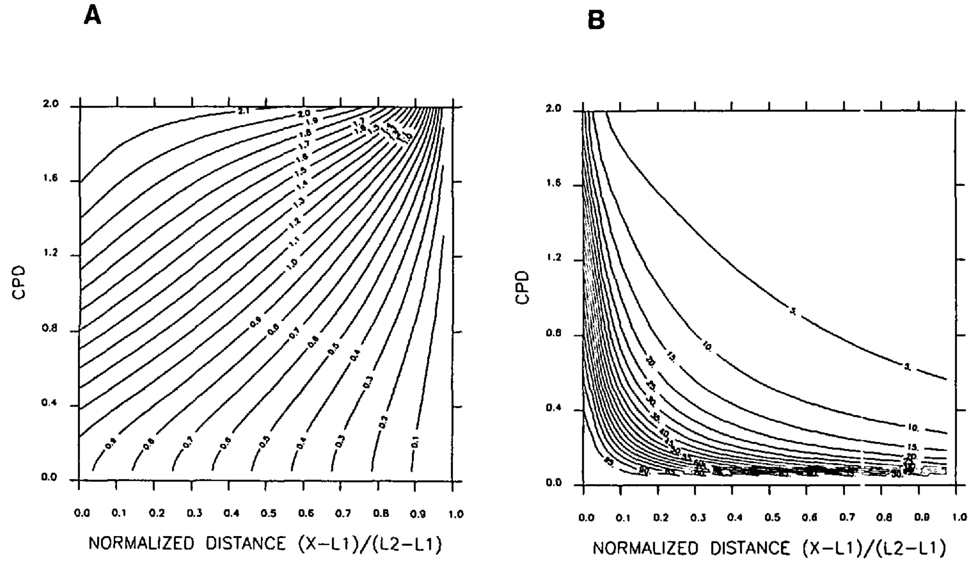

Figure 5. (A) The distribution of $(|H_{2bt}|/|H_{2mp}|) \times 100\%$ with normalized distance along the Sound and frequency (in cycles per day). This ratio measures the relative effectiveness of $\hat{\eta}_{bt}$ and $\hat{\eta}_{mp}$ in forcing the sea level response within the Sound. (B) The distribution of $(|U_{2bt}|/|U_{2mp}|) \times 100\%$ with distance along the Sound and frequency. This ratio measures the relative effectiveness of $\hat{\eta}_{bt}$ and $\hat{\eta}_{mp}$ in forcing the volume exchange within the Sound.

Battery to the west as well as Montauk Point to the east. One way to assess the importance of East River on Long Island Sound is to examine the relative effectiveness of $\hat{\eta}_{bt}$ and $\hat{\eta}_{mp}$ in producing barotropic motion in the Sound. This relative effectiveness can be readily determined by the ratio $|H_{2bt}|/|H_{2mp}|$ and $|U_{2bt}|/|U_{2mp}|$ (Fig. 5). The computations are conducted from $x' = 0$ to $x' = 1.0$ in steps of $\Delta x' = 0.025$ over a frequency range of $\omega = 0.05$ cpd (cycles per day) to $\omega = 2.0$ cpd in steps of $\Delta\omega = 0.05$ cpd.

Figure 5A shows the distributions of $|H_{2bt}|/|H_{2mp}|$ with x' and ω. The ratio reaches its maximum value at the western end of Long Island Sound for the semi-diurnal tidal motion. The general tendency is for the ratio to decrease with distance away from the western end of the Sound, and to decrease with decreasing frequency. Since $|H_{2bt}|/|H_{2mp}|$ never exceeds 3% anywhere in the model domain, it is apparent that the sea level response in the Sound is forced primarily by $\hat{\eta}_{mp}$ to the east.

Figure 5B shows the distributions of $|U_{2bt}|/|U_{2mp}|$ with x' and ω. It can be seen that the ratio generally increases with distances away from the eastern end of the Sound, and it also increases significantly with decreasing frequency. Except in a rather narrow band near the western end of Long Island Sound, $|U_{2bt}|/|U_{2mp}|$ is smaller than 5% over much of Long Island Sound at the semi-diurnal frequency. This indicates that the semi-diurnal tidal volume flux in the Sound is primarily forced by $\hat{\eta}_{mp}$. The relative importance of $\hat{\eta}_{bt}$ does increase substantially, however, as one approaches the western end of Sound. At subtidal frequencies, say $\omega = 0.4$ cpd, $|U_{2bt}|$ is very important in forcing the subtidal volume flux near the western end of the Long Island Sound. Furthermore, $|U_{2bt}|$ remains a significant fraction of $|U_{2mp}|$ well into the interior of the Sound, indicating the increased importance of forcing from $\hat{\eta}_{bt}$ at subtidal frequencies. This tendency becomes even more apparent with decreasing subtidal frequencies. At subtidal frequencies the volume flux in the Sound can be effectively forced by $\hat{\eta}_{mp}$ to the east as well as $\hat{\eta}_{bt}$ to the west. The opening of East River allows the effect of $\hat{\eta}_{bt}$ to be transmitted into the Sound. The significant response in volume flux, however, is not accompanied by a significant response in sea level. At such low frequencies (or long time scales), the volume flux in the Sound has time to make large scale adjustments. An increase in the volume flux response near the western end of the Sound as a result of forcing from $\hat{\eta}_{bt}$ produces higher response in the remaining part of the Sound. Since the volume flux adjustment takes the form of a flow-through situation, no significant sea level adjustment is necessary.

In addition to permitting the effect of $\hat{\eta}_{bt}$ to be transmitted into the Sound, the presence of East River also changes the way in which the Sound responds to sea level forcing from Montauk Point to the east. Without East River, the western end of the South is a perfect reflector to $\hat{\eta}_{mp}$. With East River, the western end of the Sound becomes an imperfect reflector, and the barotropic response of the Sound to forcing from $\hat{\eta}_{mp}$ may be significantly modified as a result of this opening.

This effect cannot be determined by simply setting $\hat{\eta}_{bt} = 0$ in the model. Instead, one has to physically decouple Long Island Sound from East River by imposing a no-flux condition at Willets Point. The governing equations in the Sound are the same as Eqs. (3a) and (3b). We have

$$i\omega\hat{u}_{2d} = -g\frac{\partial\hat{\eta}_{2d}}{\partial x} - \left(\frac{r_2}{h_2}\right)\hat{u}_{2d} \tag{7a}$$

$$i\omega\hat{\eta}_{2d} = -h_2\frac{\partial\hat{u}_{2d}}{\partial x} \tag{7b}$$

with the boundary conditions

$$\hat{\eta}_{2d}(x = L_2, \omega) = \hat{\eta}_{mp}(\omega) \tag{8a}$$

$$\hat{Q}_{2d}(x = L_1, \omega) = 0. \tag{8b}$$

The additional subscript *d* indicates that the solutions are for the decoupled Long Island Sound (channel 2) with the western end closed. The solutions can be expressed as

$$\hat{\eta}_{2d}(x, \omega) = \hat{\eta}_{mp}(\omega)H_{2dmp}(x, \omega) \tag{9a}$$

$$\hat{Q}_{2d}(x, \omega) = \hat{\eta}_{mp}(\omega)U_{2dmp}(x, \omega). \tag{9b}$$

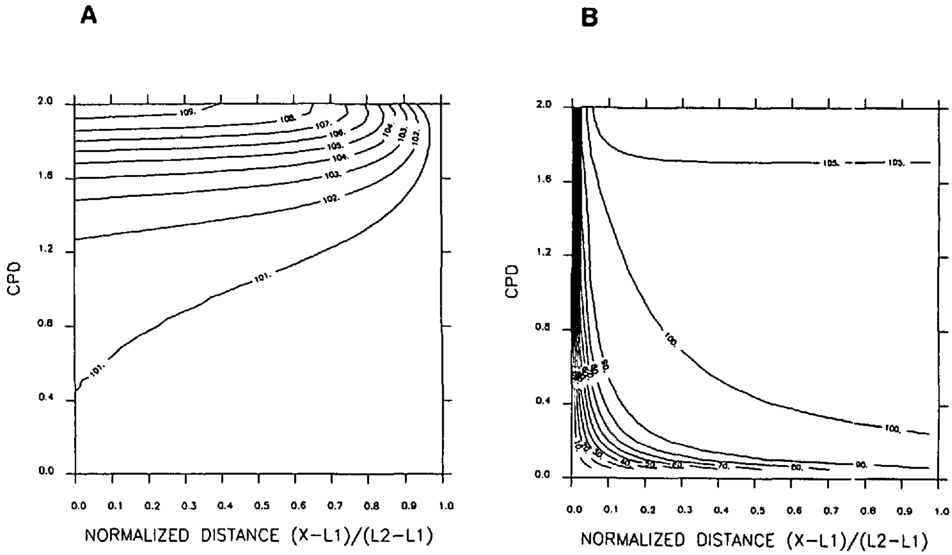

Figure 6. (A) The distribution of $(|H_{2dmp}|/|H_{2mp}|) \times 100\%$ with distance along the Sound and frequency. This ratio measures the impact of the presence or the absence of East River on the sea level response of the Sound to forcing from $\hat{\eta}_{mp}$. The subscript *2mp* indicates the response with the presence of East River, and the subscript *2dmp* indicates the response of the decoupled Long Island Sound without East River. (B) Same as in (A), except for the distribution of $(|U_{2dmp}|/|U_{2mp}|) \times 100\%$, so the ratio measures the impact of East River on the volume exchange response of the Sound to forcing from $\hat{\eta}_{mp}$.

The subscript *2dmp* indicates that the transfer function now relates the boundary forcing $\hat{\eta}_{mp}$ to the dependent variables in channel 2 which is decoupled from channel 1. The differences between H_{2mp} and U_{2mp} versus H_{2dmp} and U_{2dmp} thus reflect the different response of the Sound to sea level forcing from Montauk Point with or without the opening of East River at the western end of the Sound.

Figure 6A shows the distributions of $|H_{2dmp}|/|H_{2mp}|$ with distance and frequency, and Figure 6B shows similar distributions for $|U_{2dmp}|/|U_{2mp}|$. At the semi-diurnal tidal frequency, $|H_{2dmp}|/|H_{2mp}|$ increases with distance away from Montauk Point. Since this ratio is always larger than 100%, it is apparent that the opening of East River (the imperfect reflector) reduces the resonance of semi-diurnal tide in the western end of the Sound. The ratio $|U_{2dmp}|/|U_{2mp}|$ is also larger than 100% over much of the Sound, indicating that a smaller volume flux is required to support the slightly reduced resonance of M_2 tide. $|U_{2dmp}|$ remains larger than $|U_{2mp}|$ until very near the western end of the Sound where $|U_{2dmp}|$ begins to feel the no-flux boundary condition at Willets Point. At $x' = 0$, $|U_{2dmp}| = 0$.

At the subtidal frequency of $\omega = 0.4$ cpd, $|H_{2dmp}|/|H_{2mp}| \cong 100\%$. This indicates that at such low frequencies the amplitude of the sea level response in the Sound is insensitive to the presence or the absence of East River. In contrast, Figure 6B shows

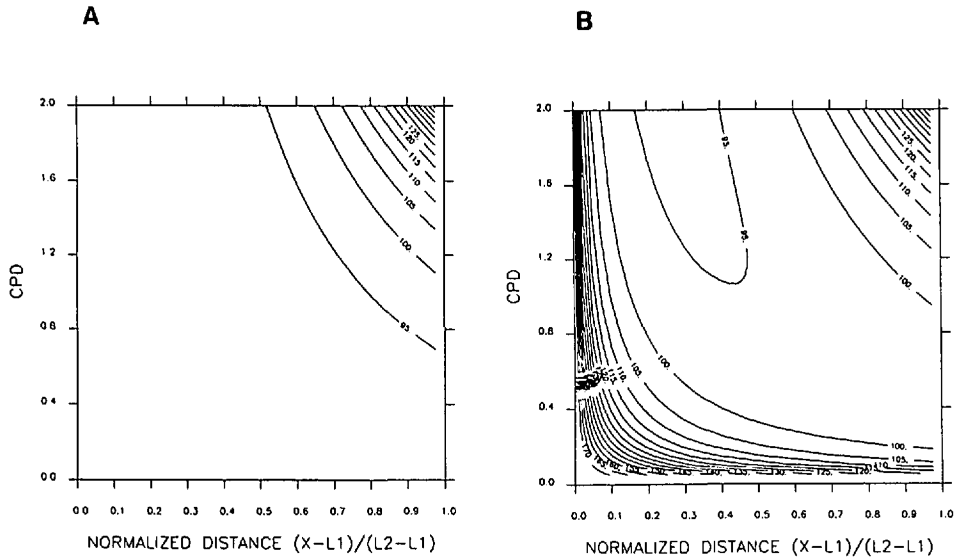

Figure 7. (A) The distribution of $[\phi(U_{2dmp}) - \phi(H_{2dmp})]$ with distance along the Sound and frequency. The western end of Long Island Sound is closed off at Willets Point. (B) The distribution of $[\phi(U_{2mp}) - \phi(H_{2mp})]$ with distance along the Sound and frequency. East River is present in this case.

that the presence of East River causes very significant differences between $|U_{2dmp}|$ and $|U_{2mp}|$, indicating that the response of subtidal volume exchange in Long Island Sound to sea level forcing from Montauk Point is very sensitive to the presence of East River. At low frequencies, $|U_{2dmp}|/|U_{2mp}|$ can be much less than 100% over a significant portion of the Sound, suggesting that the volume exchange is substantially increased as a result of the opening of East River. This is due to the fact that over such long time scales barotropic flow in the western end of the Sound has time to adjust to the narrow opening of East River. Since subtidal flow can easily go through the western end of the Sound, the system responds by simply drawing in more water from the east. The contour lines in Figure 6B indicate that low frequency motion can feel the presence of East River well into the interior of the Sound. Again the flow adjustment is achieved without significant adjustment to the sea level response.

At this point it is informative to examine the difference in phase angle between $\phi(U_{2dmp})$ and $\phi(H_{2dmp})$, as it is the same as the phase difference between volume flux $\hat{Q}_{2d}$ and surface elevation $\hat{\eta}_{2d}$. Figure 7A gives this phase relationship as a function of distance along the Sound and frequency. At the semi-diurnal tidal frequency, $\hat{Q}_{2d}$ and $\hat{\eta}_{2d}$ are nearly 180° out of phase at the eastern end of the Sound. The wave is thus near progressive, and a rise in sea level at Montauk Point corresponds to a westward (negative) flow into the Sound. As one travels away from Montauk Point, the phase between $\hat{Q}_{2d}$ and $\hat{\eta}_{2d}$ changes until the two are nearly in quadrature phase in the central portion of the Sound. This indicates that the wave changes from near progressive to near standing in the central Sound. With the western end of the Sound being a perfect reflector, the wave remains near standing from central Sound to western Sound. At subtidal frequencies $\hat{Q}_{2d}$ and $\hat{\eta}_{2d}$ are near quadrature phase throughout the Sound, indicating standing wave characteristics for the entire system.

Figure 7B indicates the distribution of $\phi(U_{2mp}) - \phi(H_{2mp})$ for the system where East River is present. Again the distribution indicates the phase relationship between volume flux $\hat{Q}_{2mp}$ and surface elevation $\hat{\eta}_{2mp}$ for the open-ended Long Island Sound system. At semi-diurnal frequency, $\hat{Q}_{2mp}$ and $\hat{\eta}_{2mp}$ are nearly 180° out of phase at the eastern end of the Sound, indicating a near progressive wave. $\hat{Q}_{2mp}$ and $\hat{\eta}_{2mp}$ then approach quadrature phase toward the central portion of Long Island Sound, indicating a switch from near progressive to near standing wave. The wave remains near standing in western Sound until very near the western end of the Sound where the wave changes to near progressive again. At the western end of the Sound a rise in sea level corresponds to a westward (negative) flow out of the Sound. We can thus say that the semi-diurnal tidal wave begins to feel the presence of East River when the wave starts to switch from standing to progressive. At subtidal frequencies the phase between $\hat{Q}_{2mp}$ and $\hat{\eta}_{2mp}$ is once again near progressive at the western end of the Sound. In addition, the phase between $\hat{Q}_{2mp}$ and $\hat{\eta}_{2mp}$ deviates substantially from that of a standing wave well into the interior of the Sound. The contrast between $\phi(U_{2dmp}) - \phi(H_{2dmp})$ in Figure 7A and $\phi(U_{2mp}) - \phi(H_{2mp})$ in Figure 7B clearly indicates that the presence of East River not only changes the amplitude of the subtidal volume exchange but also the subtidal wave characteristics over a substantial portion of Long Island Sound.

4. Discussion

Our analyses suggest that the presence of East River at the western end of Long Island Sound has significant implications for the barotropic motion within the Sound. The influence of East River on sea level and volume exchange in the Sound is highly frequency-dependent. At the semi-diurnal tidal frequency, the presence of East River creates a slightly imperfect reflecting wall at the western end of the Sound which causes moderate reduction in the resonance of M_2 tide. At the subtidal frequencies the presence of East River permits significant amount of volume exchange through the western end of the Sound, causing large scale adjustment in both the amplitude and phase of the barotropic flow well into the interior of the Sound.

Within the context of the simple analytical model, channel 1 represents a constrictive waterway which is connected to the head of a main channel (channel 2). The degree in which the presence of channel 1 affects the barotropic flow in channel 2 depends strongly on whether volume flux can be effectively conveyed through channel 1. In terms of volume exchange, channel 1 behaves as a low-pass filter which allows effective exchange only at low frequencies. The cut-off frequency of this low-pass filter depends on the relative cross-sectional areas of the two channels and friction. The cut-off frequency decreases with decreasing cross-sectional area and increasing friction in channel 1, and vice versa.

It is not uncommon for an estuary to be connected with either the ocean or other major bodies of water through relatively narrow and constrictive channels. Our analysis suggests that while such channels may pose a significant restriction for the transmission of tidal variance, subtidal variability can often be transmitted through these channels effectively. Despite the narrowness of these channels, the interior of the estuary can be strongly influenced by the adjacent water bodies at subtidal frequencies. In the past, such a phenomenon has been documented primarily for relatively small estuaries and coastal lagoons. For example, Lake Pontchartrain, Louisiana, has two relatively narrow passes connecting it to the Mississippi Sound and thence to the Gulf of Mexico. Swenson and Chuang (1983) showed vigorous subtidal volume exchange through the narrow passes, and Chuang and Swenson (1981) showed that the subtidal variability in the lake and the passes were forced by a strong coupled ocean-lake response. Such coupled ocean-estuary response at subtidal frequencies has also been observed across the narrow Aransas Pass which connects Corpus Christi Bay with the northwestern Gulf of Mexico (Smith, 1977, 1978). Wong and Wilson (1984) found that the coastal lagoons on the south shore of Long Island, New York, were strongly coupled at subtidal frequencies. They showed that the atmospherically induced subtidal disturbances on the adjacent continental shelf could propagate into these interconnected bays through long, narrow, and shallow waterways without significant attenuation, while higher frequency tidal motions were substantially damped. Such frequency-dependent response has also been observed in other coastal lagoons with restrictive communication to the ocean (Wong, 1987; DiLorenzo, 1988; Kjerfve and Knoppers, 1991).

The importance of the subtidal motion through narrow connections between major estuaries has received only limited attention in the past, even though its potential oceanographic and ecological implications may be immense. Recently, Wong (1990b) showed that the subtidal motion in the upper reaches of Delaware Bay could be strongly influenced by the adjacent Chesapeake Bay via subtidal volume exchange through the C&D Canal. Our result indicates that the narrow opening at the western end of Long Island Sound may substantially affect the response characteristics of the Sound at subtidal frequencies. It appears that more research is needed to examine the coupled response between major estuaries which are connected by either man-made or natural waterways.

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