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On the energetics of the Gulf Stream at 73W

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ABSTRACT

From September 1980 through May 1983 a series of nineteen sections of velocity profiles were obtained across the Gulf Stream 200 km northeast of Cape Hatteras. By decomposing the velocity and temperature observations into mean and fluctuating fields in two coordinate systems, geographic (or Eulerian) and 'stream' coordinates, it is shown that at least ⅔ of the eddy kinetic and potential energy is caused by the meandering of a well defined baroclinic front with a structure that is nearly independent of space and time. It is also shown that more than 95% of the kinetic energy of the front can be accounted for by a barotropic and a baroclinic mode with near equipartition between the two.

The cross-stream baroclinic, barotropic, and pressure-work terms in the eddy energy production equation are estimated to determine what processes contribute to the rapid growth of meandering after the current leaves the coast. In order of importance, the cross-stream average of the baroclinic conversion term is a factor three larger than the other two. The cross-stream averaged production of eddy energy is, however, clearly too large to be consistent with the observed rate of growth of the meander envelope since it would lead to a doubling of eddy kinetic and potential energy in only 2.1 days or 50 km following the mean flow. It is shown that in the case of the baroclinic conversion term the large cross-stream covariances $\langle u'T' \rangle$ have a simple geometric interpretation in terms of meander growth (and decay). They represent a down (or up) gradient heat flux that is not actually participating in the conversion processes suggesting that the baroclinic production terms are nearly horizontally nondivergent. Similarly, the pressure-work terms must be very nearly horizontally nondivergent (geostrophy). Thus, estimates of energy conversion rates are bound to be greatly exaggerated unless both horizontal components are included. Furthermore, conclusions about the relative importance of the cross-stream conversion terms to the production of eddy energy depend upon their horizontal divergence being in the same proportions, a very unsatisfactory assumption.

A simple kinematic model is used to show that the amount of energy needed to support meander growth is quite small. It is clear that to determine these rates experimentally puts great stress on conventional measurement procedures and suggests that alternative approaches such as paying more attention to boundary or flux conditions might be more rewarding in future studies.

1. Introduction

As the Gulf Stream flows north along the east coast of the United States and out to the east it deepens and increases its transport with the continual inflow of Sargasso Sea

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waters along its eastern edge. At the same time it wiggles and meanders about its mean path, but unlike the transport, the meandering does not increase monotonically; instead it seems to be coupled to the bathymetry along the course of the current. The meandering is at a minimum in the Florida Straits and at Cape Hatteras, in between which there is rapid growth and gradual decay off South and North Carolina (Bane and Brooks, 1979). East of Cape Hatteras the meandering grows rapidly at first, but levels off to a weak minimum near 70W. East of that meridian the meander envelope quickly expands to become uniformly wide (Cornillon, 1986). Thus, it appears that bathymetry exerts a varying degree of control and local stabilization on the path of the current. In Warren's (1963) study, the path of the current was dictated entirely by bathymetry together with upstream inflow or 'inlet' conditions.

In contradistinction to Warren's essentially deterministic model, Orlanski (1969) argued that the Gulf Stream, depending on the bottom topography, was baroclinically unstable, a view that has received recent observational support from Watts and Johns (1982) and Johns (1984), who using arrays of inverted echo sounders (IES), examined the space/time properties of meandering east of Cape Hatteras. From their measurements, an experimental dispersion relationship was determined in which the observed meander growth rates agreed well with Orlanski's (1969) model, but the observed phase velocities were faster than the model predicted. In the Watts and Johns' (1982) analysis of IES data it was assumed that the current had constant lateral width. Clearly, when we talk about an unstable current, we are referring to the path of the current, not the current itself, which, as we will emphasize, appears to be very robust.

An earlier and very remarkable study of the energetics of the Gulf Stream was that of Webster (1961), who noted that there was a conversion of kinetic energy from the fluctuating to the mean field. This work was remarkable for it suggested that the eddy field might be responsible for the maintenance of the mean flow. Subsequent to this study off Onslow Bay, N.C., where the meander envelope is decreasing toward Cape Hatteras, other studies by Webster (1965) at 30N and 25N, where the meander envelope width is rather uniform, gave similar if less striking results. These studies were all limited to the surface velocity field. More recently, however, Hood and Bane (1983) extended Webster's (1961) work off Onslow Bay to include the subsurface front of the Stream. Using arrays of current meters, they found conversion of not only kinetic, but also of potential energy from the fluctuating to the mean field in the cyclonic zone of the current, a result that not only supported Webster's original conclusions but also indicated that the entire cyclonic front was actively contributing to the conversion process, not just the surface waters.

In order to construct a more complete picture of the energetics of the current, Brooks and Niiler (1977) addressed this question in a very thorough study of energy conversion processes in the Florida current using the dropsonde technique (Richardson and Schmitz, 1965). Their study, which continued earlier efforts by Schmitz and Niiler (1969), included measurements throughout the water column and across the entire current. Thus, they could determine energy conversion due to both kinetic and potential energy fields across the entire Florida Current. Their results substantiated the earlier work by Schmitz and Niiler (1969) and Webster (1965) in that there was a conversion of kinetic energy to the mean flow in the cyclonic shear zone, but they also found that this was offset by an opposite flux elsewhere in the Stream such that the net transfer to the mean field was insignificant. Similar conclusions applied to the potential energy field as well. In summary, there is on the one hand evidence for topographic control of a potentially unstable baroclinic current drawing its energy from the mean density field, and on the other hand suggestive evidence that the mean current is in part maintained by the fluctuating velocity field.

In this study we shall look at the energetics of the meandering Gulf Stream in the area of the Watts and Johns' (1982) study near 73W, about 200 km east of Cape Hatteras where the current is an unbounded jet, both laterally and vertically. We attempt to estimate the rate at which energy is converted to the eddy field from the mean field (or vice versa), and to assess the relative importance of the conversion processes that can be observed. Of these, we will find the conversion of mean to fluctuating potential energy to be the largest. The rate is much too large, however, and we will show that the reason for this is the neglect of the downstream component of energy conversion, which is of the opposite sign. As a consequence, any assessment of the relative importance of these processes can only be done on the assumption that the net production rates are proportional to the observed cross-stream terms themselves.

Our approach is virtually the same as that of Brooks and Niiler's (1977) study except that the profile data we will be working with was obtained with the instrument 'Pegasus', an acoustically tracked continuous profiler of currents and temperature from the surface to close to the bottom (Spain *et al.*, 1981). This data set consists of nineteen bimonthly sections across the Gulf Stream between September 1980 and May 1983. Each section is made up of up to nine equidistant velocity and temperature profiles about 24 km apart. (Four of the sections were taken a few days to a week later on the same cruise.) Figure 1 shows the area of study. The location of the center of the stream and the direction of flow is indicated for most of the sections. The shaded area shows the envelope of meandering of the surface thermal (IR) front according to Cornillon (1986). The reader is referred to the paper by Halkin and Rossby (1985) for a report on the results of the observational program including detailed estimates of the volume transport. A striking property of these transects is the similarity of the current from section to section. Much of what might be called 'eddy' variability is clearly associated with the meandering of the current, and not changes to the current itself.

We begin with a brief restatement of the energy transformation equations and a summary of the data that will be used here (Sections 2 and 3). A conventional modal analysis of the velocity field is included in Section 4 to show the vertical structure of the current and how it changes across the current. Reconstruction of the pressure field is discussed in Section 5. In Section 6 the estimates of energy conversion between the

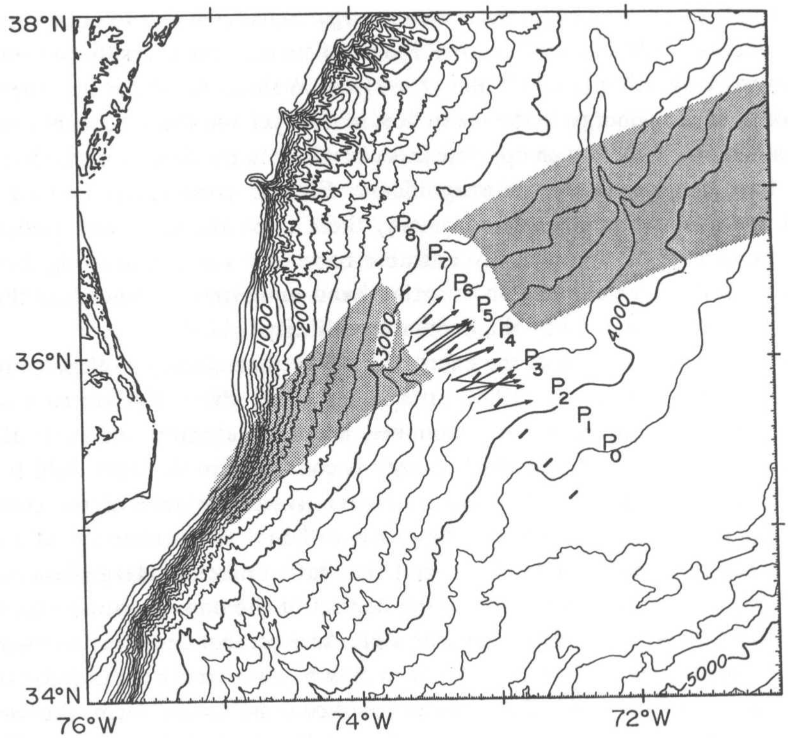

Figure 1. Location of the nine Pegasus sites where the bimonthly velocity profiles used in this study were obtained. The arrows indicate the center of the Gulf Stream and direction of flow at the time of each section. The shaded region delineates the standard deviation envelope of meandering according to Cornillon (1986).

mean and fluctuating fields are presented. In Section 7 a kinematic model is employed to estimate the energy requirements for simple meander growth. Further discussion about these observations and their implications is given in the last section.

2. Energy transformations

An expression for growth or decay of eddy kinetic and eddy potential energy (EKE and EPE) following the mean flow has been derived by many authors. For simplicity and continuity with past work we follow the formulation used by Brooks and Niiler (1977). This is shown in Eq. 1.

$$\frac{d}{dt}\left\{\frac{1}{2}(\overline{u'^2}+\overline{v'^2})+\frac{1}{2}g\overline{\rho'^2}\Big/\left|\frac{\partial\bar{\rho}}{\partial z}\right|\rho_o\right\}= \tag{1a}$$

$$-\left\{\frac{\partial}{\partial x}(\overline{u'p'}/\rho_o)+\frac{\partial}{\partial y}(\overline{v'p'}/\rho_o)+\frac{\partial}{\partial z}(\overline{\omega'p'}/\rho_o)\right\} \tag{1b}$$

$$+$$

$$-\left\{\overline{u'^2}\frac{\partial \bar{u}}{\partial x} + \overline{v'^2}\frac{\partial \bar{v}}{\partial y} + \overline{u'v'}\left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y}\right)\right\} \tag{1c}$$

$$\qquad * \qquad\qquad\qquad *$$

$$-\left\{g\overline{u'\rho'}\frac{\partial \bar{\rho}}{\partial x}\Big/\left|\frac{\partial \bar{\rho}}{\partial z}\right|\rho_o + g\overline{v'\rho'}\frac{\partial \bar{\rho}}{\partial y}\Big/\left|\frac{\partial \bar{\rho}}{\partial z}\right|\rho_o\right\} \tag{1d}$$

$$*$$

$$-\left\{\overline{u'w'}\frac{\partial \bar{u}}{\partial z} + \overline{v'w'}\frac{\partial \bar{v}}{\partial z}\right\} \tag{1e}$$

$$-\left\{\frac{1}{2}u'\left(u'^2 + v'^2 + g\rho'^2\Big/\left|\frac{\partial \bar{\rho}}{\partial z}\right|\rho_o\right)_x + \frac{1}{2}v'\left(u'^2 + v'^2 + g\rho'^2\Big/\left|\frac{\partial \bar{\rho}}{\partial z}\right|\rho_o\right)_y\right\}. \tag{1f}$$

The first terms on the right-hand side (1b) represent the pressure-work terms, the second terms (1c) the conversion of fluctuating kinetic energy, the third terms (1d) the conversion of fluctuating potential energy from mean kinetic and potential energy, respectively. The fourth terms (1e) represent transformations due to small-scale shear instabilities (these will not be considered in this study, Brooks and Niiler, 1977), and finally the triple correlations in the last line (1f) the conversion of fluctuation kinetic and potential energy by the fluctuating field. In this paper we examine these transformation processes in the area just east of Cape Hatteras where the envelope of meandering is rapidly growing in the downstream direction. As with most earlier studies, we are limited to those terms associated with cross-stream gradients. We will show that they are all large and positive, but the omission of downstream gradients (in the y-direction) makes energy budget calculations incomplete, which raises some very interesting questions of interpretation that will be discussed in Section 6.

3. Data preparation

The data base consists of 145 velocity profiles. Most of these are of good quality, but there are instances of gaps as long as 200 m in the vertical (due to acoustic tracking difficulties), especially in and above the main thermocline. To prepare the data for uniform analysis the profiles were linearly interpolated and resampled every 25 m from the surface to the bottom. For simplicity the velocity profiles were extrapolated at constant velocity from the deepest point of observation to the bottom. While irrelevant to these analyses since we will limit ourselves to the upper 2000 m, the extrapolation is necessary for a modal analysis of kinetic energy, which we will summarize in the next section.

Two coordinate systems are used. The first is simply the line of nine 'Pegasus' stations across the stream. They can be thought of as nine moorings with instrumentation throughout the water column. Up to 19 profiles were obtained at the central sites.

This yields nearly as many degrees of freedom since they are, on average, taken two months apart. We refer to this as the geographical system. It could have been called the Eulerian coordinate system, but Eulerian usually carries the connotation of time series analysis which the bimonthly sampling hardly qualifies for. The other system is called the 'stream' coordinate system, and is defined by the direction of transport (y) and the cross current position (x) derived from the temperature field, specifically the point halfway between where 12°C is at 400 and 600 m (Halkin and Rossby, 1985). The stream coordinate system provides a simple yet powerful means of examining the structure and stability of the current itself (Halkin and Rossby, 1985).

4. Modal structure

Each velocity profile is an instantaneous observation to which fluctuations on all length and time scales contribute. In order to gain some sense of the vertical scales of horizontal motion and their cross-stream distribution, we began this study with a modal analysis of kinetic energy. It was motivated by the thought that the energetics analyses below would be improved if we were to first remove the high vertical wavenumber structure, and presumably therefore the high frequency inertial component of motion by using only the lowest five modes. This concern proved to be unwarranted, because the amount of energy present at high wavenumbers is so small. The results of the modal analysis are nonetheless of interest and are summarized here.

The procedure is to decompose each profile using a set of linear, flat bottom orthogonal modes defined by the local density field. The methodology is identical to that employed by Rossby (1974). Specifically, we define a set of modes that satisfy the w-equation:

$$\frac{\partial^2 w}{\partial z^2} + N^2(z)\, f(\mathbf{x}, \sigma, f)\, w = 0$$

where w is the vertical velocity and N is the local Brunt-Vaisala frequency. Solutions to this equation have the same vertical structure for both low frequency internal waves and baroclinic planetary waves. Density was determined from temperature using an empirical T/S relationship (Armi and Bray, 1983). It is satisfactory for temperatures less than 18°C south of the north wall, and less than about 12°C north of the wall, which are at a depth of 300 and 170 m, respectively. At shallower depths the T/S relationship breaks down, but the modal functions are by their nature insensitive to local irregularities in density, so we assume little error arises therefrom. (Stable stratification is of course ensured.)

The shape of the first baroclinic mode as a function of cross-stream position is shown in Figure 2. These orthonormal velocity profiles (vertical average = 1 erg gm^{-1}) were computed to full ocean depth, but only the top 2000 m are shown. Going from left to right (north to south) the zero crossing deepens from about 600 to 1150 m while the

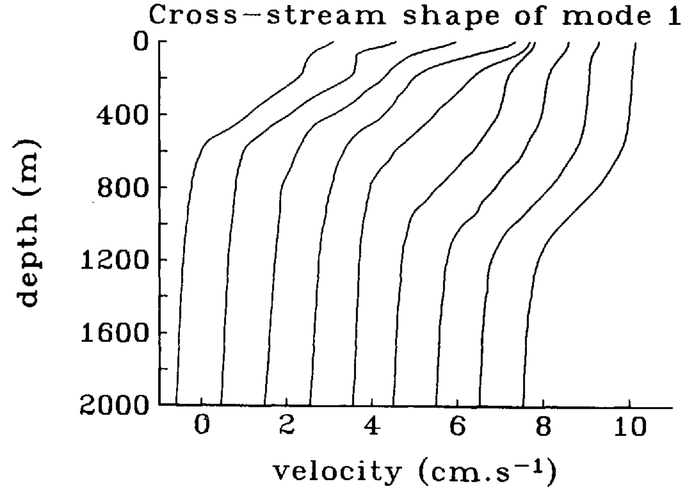

Figure 2. Shape of the first baroclinic mode across the Gulf Stream in July 1982. The profiles are based on the local density profile at each site. The normalization used sets the vertically integrated kinetic energy equal to 1 erg gm^{-1}.

vertical shear in the main thermocline weakens by about a third. In the earlier study (Rossby, 1974) it was shown that more than 90% of the vertically averaged kinetic energy was accounted for by modes 0 and 1. The same is true across the Gulf Stream. In fact, 95 to 99% of the total kinetic energy can be accounted for with modes m_0, m_1, and m_2. In Figure 3 we show the distribution by mode across the Stream in stream coordinates. These modes may not be the 'correct' ones to employ in an energetic current over a sloping bottom, but, evidently, they are efficient. Note the approximate equipartition of energy between m_0 and m_1, and that m_2 and m_3 are negative and somewhat larger on the anticyclonic side of the current. This is to accommodate the lack of and even reversal of vertical shear that the first mode expects to find in the top 100 to 300 m, Figure 2.

Figure 3 also shows the vertically averaged residual kinetic energy (RKE) after modes 0–5 have been removed. In the center and particularly along the cyclonic edge the RKE is a factor 2–3 larger than outside the current. This may indicate increased high vertical wave number internal (inertial?) wave energy since there is no indication (from modes 2–5) that mode 1 cannot accommodate the shear of the basic velocity profile. The RKE outside the current is less than 10 ergs gm^{-1}. This is comparable to the vertically averaged inertial and high frequency energy levels (~6 ergs gm^{-1}) observed in the LOTUS study at 34N, 70W (Briscoe and Weller, 1984).

5. The pressure field

The traditional analyses of energy transformations have been limited to terms checked with an asterisk in Eq. 1. In this study we will examine those terms, but, in

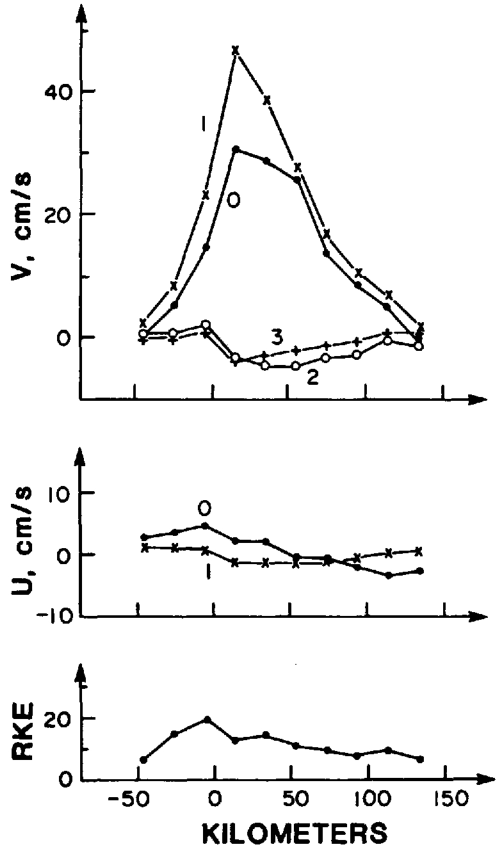

Figure 3. Amplitude of the barotropic and lowest three baroclinic modes as a function of cross-stream position in the stream coordinate system, averaged over all nineteen sections. Modes 2 and 3, although small, are important in correcting for the lack of expected downstream shear on the anticyclonic side of the current. Mode 0 in the middle panel shows inflow on both sides of the current. (The negative dip in m_1 is caused by leakage from the v-field.) The bottom panel shows the vertically averaged residual eddy kinetic energy (ergs gm^{-1}) after modes 0–5 have been removed. The origin is where the 15°C isotherm crosses 200 m (the North Wall).

addition, we also estimate the contributions from the pressure-work term marked with a + by computing the hydrostatic pressure everywhere using the observed temperature and estimated salinity fields (Armi and Bray, 1983) relative to 2000 m. The fluctuations at 2000 m were estimated using geostrophy:

$$p'(x) = -\rho f \int_0^x v(x)dx + p'(0)$$

where v is the observed cross-section velocity field at 2000 m. For lack of any information, the pressure at the northwestern end is assumed to be constant, i.e. $p'(0) = 0$. This is reasonable; the transverse correlation scale is less than 50 km in the deep waters (Johns, 1984), so the influence of this assumption on the rest of the section should be minor. (Also, the velocity fluctuations were somewhat less there than elsewhere.) It should be emphasized that there is virtually no tidal component in the velocity profiles. This is well-established from spectral analysis of current meter records obtained in the area (Halkin and Rossby, 1985). Although the scale of the tidal velocity field is large, the velocity profiles are not taken simultaneously, but about every 4–6 hours, and thus could seriously contaminate the pressure estimates. While a correction for tidal motion could have been made, it was not necessary.

6. Results

a. The mean field. We begin with a presentation of the mean fields of temperature, downstream velocity and lateral shear. These, as well as subsequent calculations, will be shown in both geographical and stream coordinates as described above. We emphasize that the information at each site (geographic) or interval (stream) is completely independent of neighboring points. The top panels in Figure 4 show the mean temperature field, in geographical (left) and stream (right) coordinates. There is little difference, except that the slope of the isotherms appears to be somewhat greater in the right panel. Similarly, the downstream velocity field (middle panels) is somewhat more sharply focussed and stronger when the meandering of the stream is removed. The third pair, showing the lateral shear normalized by the planetary vorticity is much more intense in stream coordinates. This is not surprising since the shear, being a lateral derivative, emphasizes the smaller scales, which are easily blurred by the meandering. It is noteworthy that $f^{-1}d\langle v\rangle/dx$ is as large as 0.5 after averaging over 15 sections with a lateral resolution of 20 km. The $\langle\ \rangle$ denote ensemble averaging.

b. The eddy field. Figure 5 shows the variance of temperature in C^2. The tongue of large variance coincides with the location of strongest lateral gradient and reflects the meandering of the path of the stream. This variance is substantially reduced in the stream coordinate system, and would probably be even less had we allowed for a continuously changing temperature field rather than one which is piecewise constant for each 20 km interval. (I.e., in computing the statistics in stream coordinates no allowance was made for a continuously changing mean field across each 20 km interval.) Note that the maximum variance in the main thermocline to the south of the stream reflects vertical displacements and is thus insensitive to choice of coordinate system. The bottom panels of Figure 5 show the pressure variance in dbars^2. The variance at the surface is about 0.01 (outside the current) and corresponds well with other studies (cf. Rossby and Rago, 1984). The reduction in variance in the center of

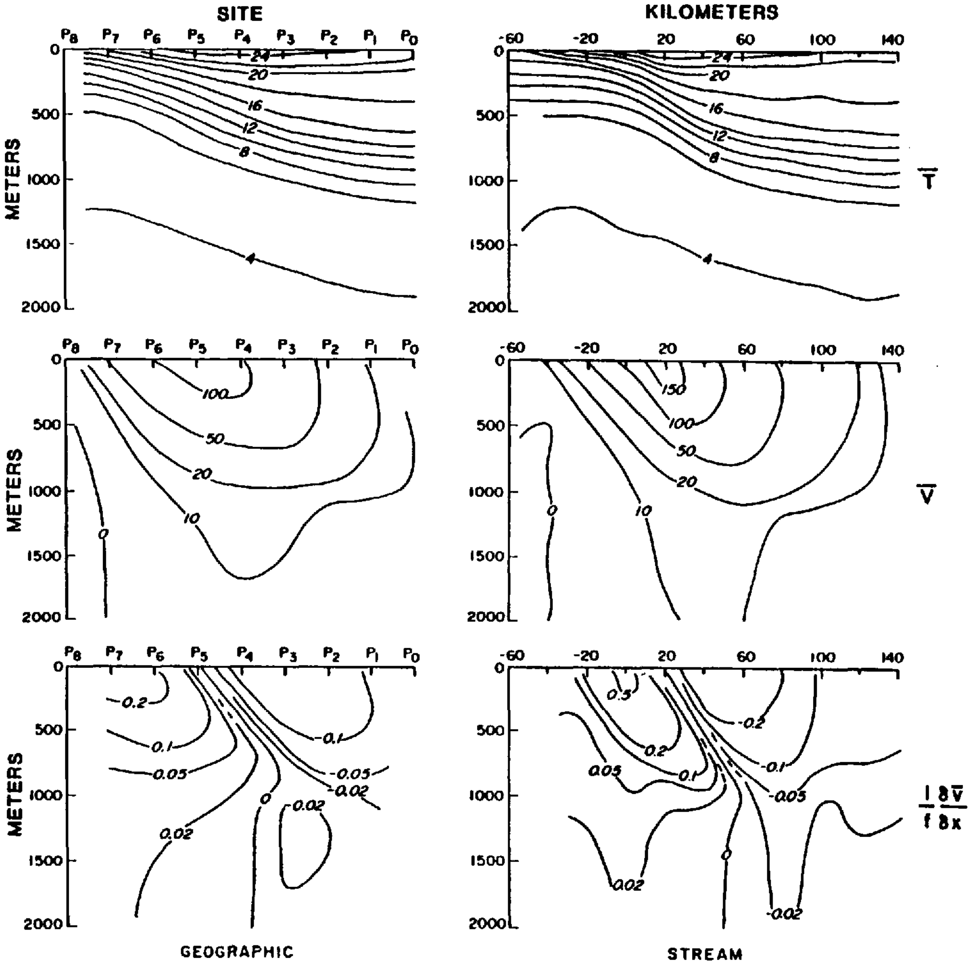

Figure 4. The mean fields of temperature (C, top), downstream velocity (cm s^{-1}, middle), and normalized lateral shear (s^{-1}, bottom). The left and right panels show the fields in geographical and stream coordinates respectively. See text for discussion.

the current from 0.05 to 0.01 $dbars^2$ is, like that for temperature, due to the removal of meandering.

The distribution of eddy potential and kinetic energy across the stream is shown in Figure 6. Since EPE essentially reflects the variance of density divided by the stratification, it is not surprising that it resembles the temperature variance in Figure 5. The ten-fold reduction in EPE in stream coordinates reflects the reduction in lateral (meandering) variance and *not* vertical motion. The lower left panel of eddy kinetic energy shows two maxima that do not appear on the right panel. These maxima

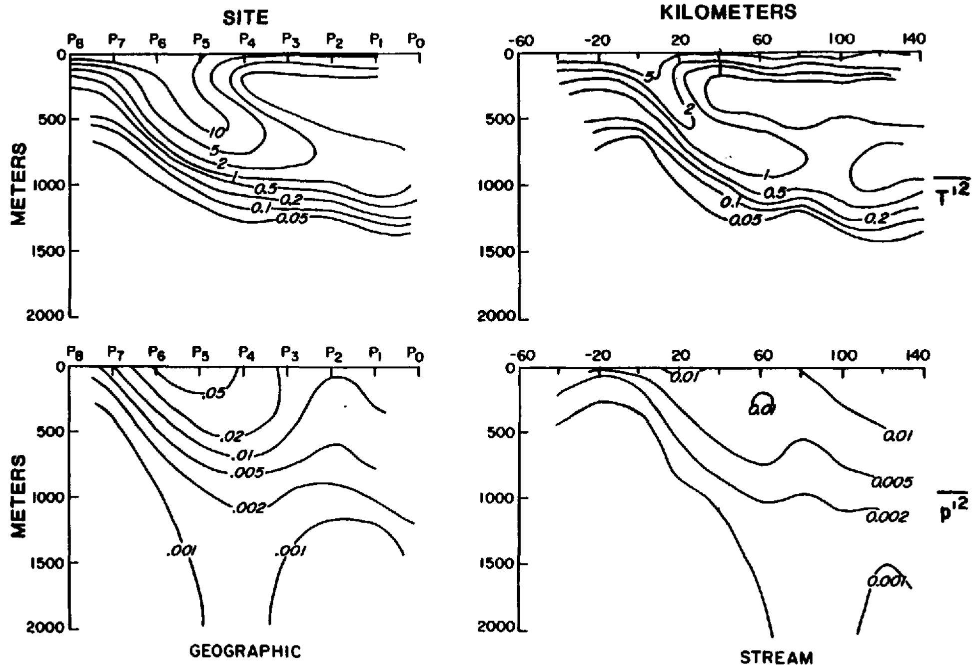

Figure 5. The upper and lower panels show the variance of temperature (C^2), and pressure ($dbars^2$) respectively in geographical (left) and stream coordinates (right).

coincide with the regions of maximum cyclonic and anticyclonic shear and hence velocity variability due to meandering. The reduction of variance in stream coordinates is not as striking as for EPE and is undoubtedly due to variations in the shape of the current from transect to transect. (Velocity is a derivative of the density field.) Note that there is an upper limit to the local eddy kinetic energy, which is simply $0.5*(\overline{v})^2$, where $\overline{v}$ is the cross-stream maximum of downstream speed at the depth of interest. In summary, by showing the eddy variability both in geographic and stream coordinates, the latter obtained from the former merely by rotation and translation of the section, we find substantial reduction in eddy variability from one to the other, clearly indicating that the large eddy variability is primarily due to the movement or meandering of the entire current and not to variability of the structure itself. The cross-stream averages of eddy energy are given in Table 1.

c. The conversion terms. We focus our discussion on the three largest terms, namely the cross-stream components of the baroclinic, the pressure work, and the barotropic terms. The first term in line 1c was found to be an order of magnitude smaller than the others so it is not discussed further. Similarly, the first term in line 1f, the conversion of eddy energy by eddy advection, was quite negligible with a weak and erratic

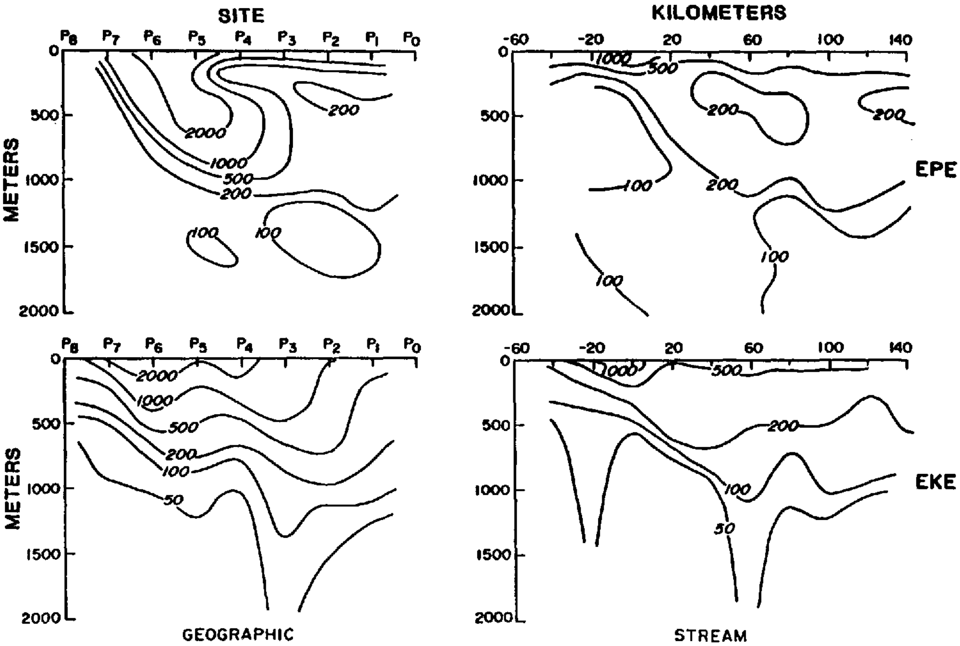

Figure 6. The eddy potential (upper) and eddy kinetic (lower) energy in geographical (left) and stream coordinates (right) in ergs gm^{-1}.

Table 1. Table of eddy energy and energy conversion rates (area averages, 0 – 2000 m, across the current as defined).

Eddy energy level	Geographic	Stream
kin. (ergs gm^{-1}	344	119
pot. (ergs gm^{-1})	504	205
	848	324
Conversion terms:		
baroclinic (ergs $gm^{-1}s^{-1}$	.0029 (±.0027)	.0002 (±.0018)
press. work (ergs $gm^{-1}s^{-1}$)	.0010 (±.0037)	.0007 (±.0045)
barotropic (ergs $gm^{-1}s^{-1}$)	.0008 (±.0014)	.0002 (±.0009)
	.0047 (±.0048)	.0011 (±.0049)
doubling time (days)	2.1	(3.4)
(or for a mean flow of 25 cm s^{-1} is equivalent to a distance of	~50 km.)	

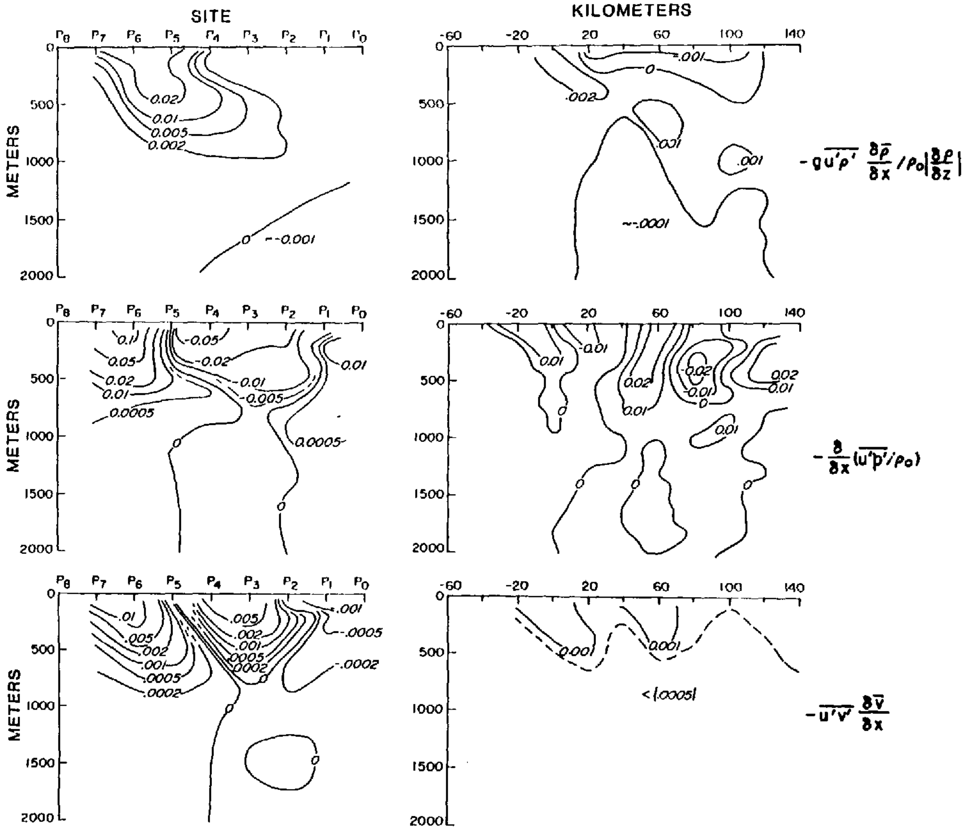

Figure 7. Energy conversion rates in ergs gm^{-1} s^{-1}. The top, middle and bottom panels show the baroclinic, pressure-work, and barotropic transformation rates in geographical (left) and stream (right) coordinates respectively. See text.

cross-stream field. The cross-stream averages of these two terms were <0.0001 and 0.0002 ergs gm^{-1} s^{-1} respectively. This is not surprising since $\langle u'^2 \rangle$ is less than $\langle v'^2 \rangle$. The sign convention adopted in Figure 7 is such that positive contour values correspond to an increase in eddy energy and vice versa. The left-hand panels show the fields in geographic coordinates, and the right-hand panels show the corresponding quantities in stream coordinates. Here we digress to discuss the meaning of Reynolds stress calculations in stream coordinates.

In the development of the Reynolds equations the dependent variables are decomposed into mean and fluctuating terms where the mean field is assumed on physical grounds to be constant over the ensemble of independent observations. When working in stream coordinates the mean field is not stationary, but is meandering about in physical space. However, this meandering is slow compared to the velocities observed in the stream. Whereas typical lateral meander speeds are of the order of 5 cm s^{-1}, the rms velocities in stream coordinates are of order 7–10 cm s^{-1} in the deep waters and 20 cm s^{-1} at 500 m. Since the movement of the coordinate system is slow compared to the local velocity field we shall assume that the ensemble of sections is unaffected by the movement of the coordinate system. This unorthodox step is motivated by the idea that by removing the meandering we might be able to focus more clearly on processes in the current itself. This is not to say that the remaining variance is uncorrelated with the meandering of the current. Indeed, it is known from other work (Rossby *et al.*, 1985) that there is considerable lateral and vertical motion associated with changes in curvature of the current.

The top panels in Figure 7 show the field corresponding to the first term in Eq. 1d. The tongue of strong positive conversion corresponds to the region of large temperature variance caused by the meandering. In stream coordinates the conversion rate is greatly reduced due to the loss of temperature variance as shown in Figure 5. The reason for the reduction is very interesting and will be discussed later, but it should be borne in mind that the baroclinic conversion consists of two horizontal terms, the left panel shows only the component perpendicular to the mean path. The downstream component may be quite large as well. In the transformation to stream coordinates the cross-stream component is much decreased due to the 'stiffness' of the current itself; the downstream component, had we been able to estimate it, would be much reduced due to the uniformity of the current in that direction. The middle panels show the pressure-work term (the first term in Eq. 1b). Locally, it can be very large, substantially exceeding the baroclinic component, but it is of opposite sign to each side: on the cyclonic side it is apparently destabilizing, and conversely on the anticyclonic side. When averaged across the entire current, the pressure-work term is about one-third as large as that of the baroclinic field, Table 1, but it is not significantly different from zero. This is because it depends only on the endpoints, which are at the edge or outside the current where nonlinear effects are small. (Moreover, the number of independent observations is somewhat less there than in the center of the current.) The pressure fluctuations include both the geostrophic and ageostrophic components; we cannot separate them apart. Most of the variance is geostrophic, which means that the observed pressure-work field must, locally and everywhere, be nearly balanced by the corresponding downstream component. The bold, coherent pattern to the pressure-work field reflects the dominance of the first baroclinic mode, m_1. The x-integral of $\langle u'p' \rangle_x$ is very similar in shape to that of $\langle u'T' \rangle$. In fact, the correlation coefficient between p' and T' is typically .75–.9. Similar to the top panels, the pressure-work term all but disappears in stream coordinates, middle right panel of Figure 7 (the positive and negative regions on scales not much larger than the resolution of the section, 20 km, are probably measurement noise). The bottom panels show the barotropic term (the third term in Eq. 1c). There are two maxima: one each in the cyclonic and anticyclonic region separated by a narrow zone down along the velocity maximum

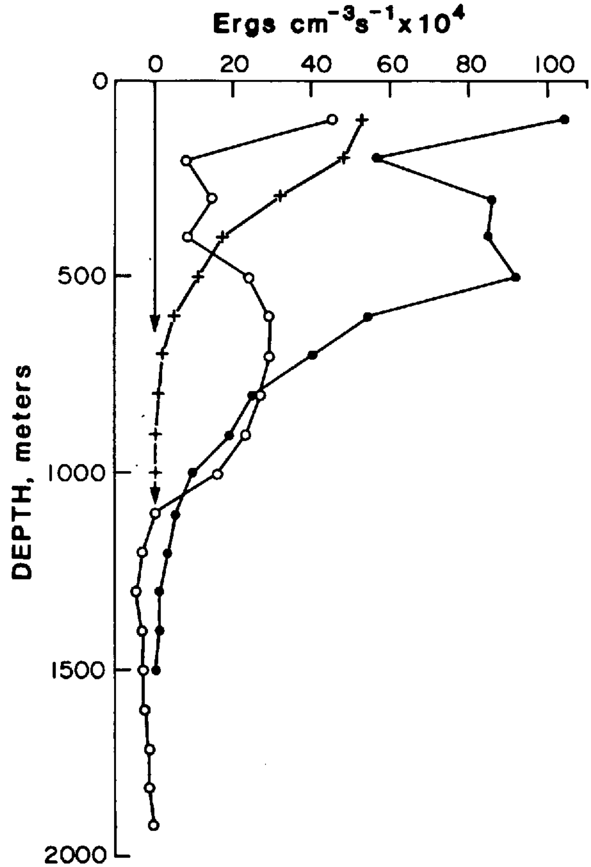

Figure 8. Cross-stream averages of the baroclinic (•), pressure-work (○), and barotropic conversion terms (+) respectively in ergs gm^{-1} s^{-1} as a function of depth.

where the horizontal shear is zero. Since the $\langle u'v' \rangle$ correlation also changes sign the conversion is positive everywhere. Similar to the top panels the barotropic contribution largely disappears in stream coordinates. The bottom panels show graphically what a misnomer 'barotropic' is: the vertical attenuation of this term is faster than either of the above. This can be seen in Figure 8 where the cross-stream average of the three fields is shown as a function of depth. Figure 8 also shows that none of the fields evince much activity below 1000 m. The patterns of the conversion fields in Figure 7 are highly significant: the isopleths contour smoothly from one site to another, each one with completely independent observations. The correlation coefficients for $\langle u'T' \rangle$, $\langle u'p' \rangle$, and $\langle u'v' \rangle$ typically range between .4 and .7 with the high values near the surface.

It is now clear from Figures 5–7 that there is considerable structure to the eddy fields and the conversion terms, and locally the magnitudes of the conversion terms can be quite large. One might think that these would lead to rapid erosion and breakdown of the current. The local rate of conversion, >0.02 ergs gm^{-1} s^{-1}, in the cyclonic shear would lead to a doubling of eddy energy in that part of the current in a day. This is much greater than is observed, either locally or in the surrounding regions. It is also much greater than the observed dissipation rates in the Gulf Stream or elsewhere in the Sargasso Sea (Gargett and Osborn, 1981), notwithstanding the evidence for some small scale (presumably mostly inertial) eddy energy as shown in Figure 3. In short, there is no evidence for a local production of eddy energy of this magnitude. In view of the observed fact that the structure of the front is very rigid in the downstream direction (in the sense that it does not change shape, or scale, or dissipate) it seems more useful to examine what is the net effect of the transformation processes on the front as a whole.

The uncertainties in these estimates are large. In computing the standard error only the variance associated with the covariance quantities is considered; the errors in the mean field gradients are by comparison unimportant. Figure 9 shows the cross-stream structure of the conversion terms at 300 m with the standard errors added. There is little question that the basic structure of the fields is significant, but the uncertainties are such that the accuracy of the cross-stream area estimates is limited. Table 1 gives the area averages of Figures 6 and 7 with the error associated with the conversion terms computed as the rms standard error for the section. The errors shown are conservative in the sense that they are averaged over the entire section to 2000 m. Had the means and errors been computed only over the top 1000 m, the conversion terms would have doubled roughly, but the corresponding errors would increase only by a factor 1.4 (since there is no covariance in the lower 1000 m).

The sum of the three conversion terms, when divided into the sum of the eddy energies translates into an eddy energy doubling time of 2.1 days, which given a mean flow of 25 cm s^{-1} is equivalent to a downstream distance of 50 km. There is no question that the eddy energy increases in the downstream direction, but certainly not at such a fast rate. Figure 1 shows the envelope of the north wall meandering as reported by Cornillon (1986). A doubling of eddy energy (in a cross-stream averaged sense) would require that the rms velocity field increase by 40%. While we do not have any direct information on this we do know that the velocity field in stream coordinates does not change appreciably downstream (Shaw and Rossby, 1984). Assuming instead that, to a first approximation, the EKE following the mean flow is proportional to the width of the meander envelope, the latter would have to double over a distance of 50 km. The observed rate of growth of the meander envelope, $W^{-1}dW/dy$, where W is the width and y is the downstream direction at 73.5 W can be estimated from Cornillon (1986) to give a doubling distance of about 260 km, which is much greater (or slower) than our result. The source of inconsistency is almost certainly due to the fact that the budget in Table 1 is incomplete: only cross-stream terms have been included. There is plausible

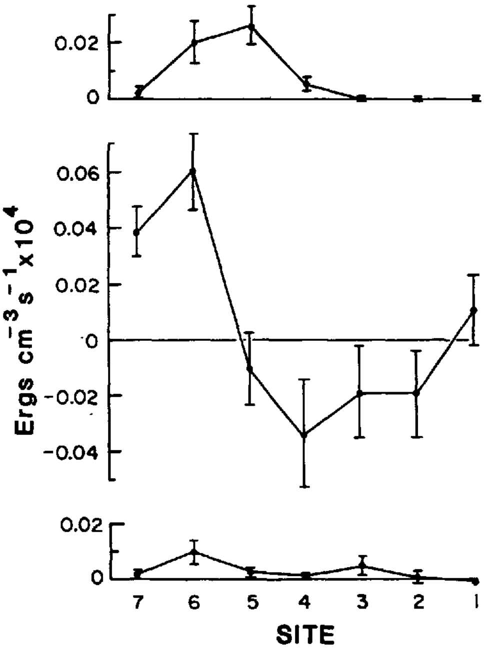

Figure 9. The baroclinic, pressure work and barotropic terms (ergs gm^{-1} s^{-1}) as a function of cross-stream position at 300 m in geographic coordinates (top, middle, and bottom panels). The standard errors are indicated.

evidence that the downstream term in line 1d is of the opposite sign to the cross-stream term. On the assumption that the meander envelope is broadening here, the downstream gradient of density $\partial\langle \rho \rangle/\partial y$ will be negative to the north of the mean path and positive to the south. At the same time $\langle v'\rho' \rangle$, which can be measured, is negative to the north and positive to the south. Hence the downstream conversion should be negative and, as a result, the net contribution by line 1d would be less than what is indicated in Table 1. In principle the downstream term in line 1d can be estimated by replacing $\langle \rho_y \rangle$ with $(\rho f g^{-1})$ $\langle u_z \rangle$ (thermal wind balance). Unfortunately both $\langle u \rangle$ and $\langle u_z \rangle$ are small and cannot be estimated with any accuracy. Furthermore, the assumption that the downstream momentum balance is geostrophic to the same degree as that of the cross-stream is not clear. In the Brooks and Niiler (1977) study these terms were of the opposite sign, suggesting approximate horizontal nondivergence.

It is not difficult to visualize why the baroclinic conversion terms must be nearly horizontally nondivergent. The controlling factor in line 1d is the quantity $\langle u'\rho' \rangle$, the eddy heat flux. As the meandering increases in the downstream direction, there is a transfer of heat from the Sargasso Sea to the Slope Waters, i.e. there is warm water to the north (west) of the mean path and vice versa. In the cross-stream plane this is evident as a broadening of the mean field properties going downstream. The front

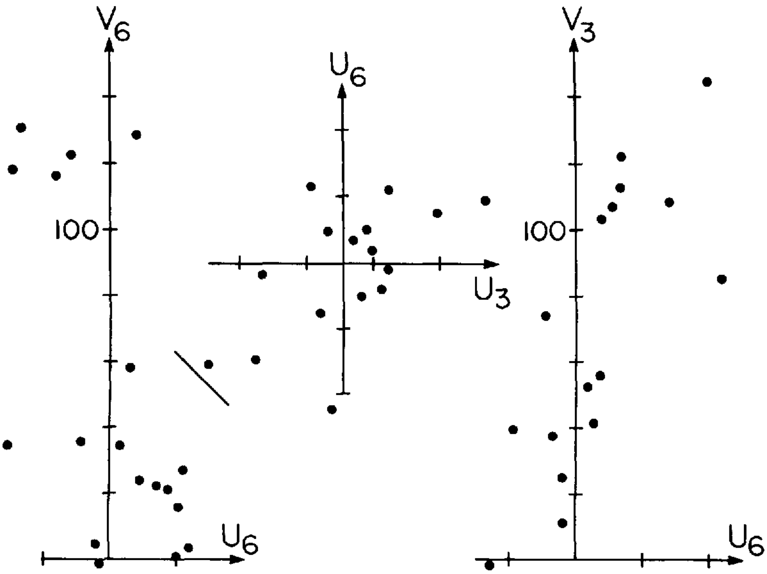

Figure 10. The side panels show the relationship between cross-stream and downstream velocities (cm s^{-1}) on the cyclonic side (site 6) and on the anticyclonic side (site 3). They are clearly negatively (positively) correlated on the (anti-) cyclonic side. The center panel shows that the cross-stream velocities at the two sites are positively correlated.

itself, however, remains intact, so there is no net flux of heat across it. The rate and direction of 'eddy' heat transport reflects whether the reservoirs of warm water to the north and cold water to the south are expanding or contracting, and not stirring or mixing. In other words the flux of heat is not a thermodynamic process, but an expression of the rate of downstream change in meander amplitude or envelope. In the region south of Cape Hatteras where the meandering is decreasing downstream, large volumes of water are being moved back up the mean gradient to the Sargasso Sea. This provides a simple geometric explanation for the large upgradient heatfluxes south of Cape Hatteras reported by Oort (1964) and Hood and Bane (1983).

Barotropic conversion of energy is probably also close to horizontally nondivergent, but this may be harder to demonstrate than for baroclinic conversion since we are dealing with a vector quantity instead of a scalar like temperature. In Figure 7 (bottom left panel) it was noted that the cross-stream component of barotropic energy conversion was positive everywhere even though the mean shear changes sign. This is because the covariance $\langle u'v' \rangle$ also changes sign. However, u' is positively correlated across the stream so v' must be positively and negatively correlated with u' in the cyclonic and anticyclonic regions, respectively. Figure 10 gives evidence of such a correlation at sites P_3 and P_6, respectively. The figure also shows that u' is positively correlated between P_3 and P_6 (middle panel). Thus by changing the sign of u' in relation to the v' field, the direction of transfer of energy to or from the eddy field across the entire current can be altered. This can be arranged by pointing the current primarily away from its mean path or toward it; i.e., growing or decaying meanders. Thus, the sign of the cross-stream component can be given a simple geometric interpretation, as many authors have shown (e.g. Starr, 1968; Brooks and Bane, 1983).

The previous discussion sought to show that the conversion of energy between the mean and eddy fields in the Gulf Stream can be interpreted in terms of path geometry. Moreover, we have shown that the net conversion rates—had we been able to assess the downstream components—would be much lower due to their approximate horizontal nondivergence. It should be noted that the Reynolds calculations are ensemble averages: there is no assumption about how fast the fluctuations are. Thus the stream can (and often does) vary in position very slowly, but as long as the observations are widely separated in time, they are independent. Thus the covariances of u, v, p, and T may be large, positive or negative, depending on whether the meandering is growing or decaying downstream without there being much time dependence. This makes it appear as if there is much energy being converted from one state to the other, without this necessarily being the case. There is, of course, a transfer of energy between scales. In a nonmeandering system all of the energy will be at low wavenumbers, in the presence of meanders some of that energy will be accounted for at high wavenumbers.

To summarize this section, we find the baroclinic conversion term to be the largest in an integrated sense, but it is also clear that consideration of the cross-stream terms alone leads to an overestimate of the net conversion rates. The baroclinic conversion term may be the most important contributor to the production of eddy energy, but this conclusion hinges critically on the assumption that the net (horizontal) contributions are proportional to the terms themselves. Finally, it is also clear that any future measurement program to test this assumption must put great stress on estimating all terms with considerable accuracy since the net rates are smaller than what we have observed. In the next section we attempt to estimate how much energy is required to transform the stream from a straight to a meandering state.

7. A kinematic model of potential and kinetic energy conversion due to meandering

The broadening of the mean temperature field due to the widening meander envelope suggests the loss of potential energy, but this is misleading. The reason is that an estimate of (available) potential energy involves an integral in *both* horizontal dimensions of displacement squared of the density field from what it would be if the system were at rest. If the Gulf Stream were an infinitely thin front, no potential energy would be released as a result of increased meandering as long as the areas of warm and cold waters to either side remain unchanged. However, the front has finite width, so as the meandering increases the potential energy of the region should decrease and the kinetic energy should increase.

Consider a two-layer system. This is amply justified from the modal analysis in section four. The fluid is at rest everywhere except in the upper layer in a narrow band which meanders from west to east. The interface separating the upper and lower layer in this band shoals according to a prescribed cubic polynomial:

$$\begin{aligned} h &= h_o + ax + cx^3 & -x_o < x < x_o \\ h &= h_o + d & x > x_o \\ h &= h_o - d & x < -x_o \end{aligned} \tag{2}$$

where x is cross-stream position in stream coordinates. Thus the structure of the current remains the same regardless of meander position. This form approximates the Gulf Stream reasonably well and was chosen instead of the two-layer constant potential vorticity model (Stommel, 1958) in order to insure that the mean depth remains constant. The coefficients are chosen so that $h - h_o = \pm d$ and $h'(x) = 0$ at $x = \pm x_o$. The velocity field in the upper layer is geostrophic:

$$v(x) = (g'/f)\,(3cx^2 + a). \tag{3}$$

A control volume is established around (outside) the meandering domain such that changes in potential and kinetic energy are solely due to changes in path length since the fluid outside the current is everywhere at rest. Regardless of the path of the current the areas to each side should equal each other so that at all times the mean depth of the upper layer in the control volume is constant, i.e.

$$h_o = \iint h\,dx\,dy.$$

Suppose first that the band is infinitely thin so that the interface depth changes abruptly from $h_o + d$ to $h_o - d$. The potential energy of the control volume is then

$$\mathrm{PE} = \iint \tfrac{1}{2} g'\rho (h - h_o)^2 dx\,dy = \tfrac{1}{2} g'\rho d^2(\text{area}).$$

where g' is the reduced gravity and ρ is the density of the fluid. Now, since the band has a finite width, the loss of potential energy (LPE) can be written

$$\mathrm{LPE} = \tfrac{1}{2} g'\rho \int dy \int (d^2 - (h - h_o)^2)\,dx,$$

which, using Eq. 2, becomes

$$\mathrm{LPE} = \tfrac{1}{2} g'\rho \int 36/35\, d^2 x_o\,dy. \tag{4}$$

Thus, as the path length, $\int dy$, increases, so does the loss in potential energy.

Similarly, the kinetic energy across the current can be estimated using Eqs. 2 and 3,

$$\mathrm{KE} = \tfrac{1}{2}\rho \int dy \int v^2 h\,dx$$

which becomes

$$\mathrm{KE} = \tfrac{1}{2}\rho h_o(g'/f)^2 d^2/x_o{}^{12}\!/\!_5 \int dy. \tag{5}$$

Since the integrals 4 and 5 are evaluated along the path, the potential energy of the system will decrease and the kinetic energy will increase as the path length is increased. We now define the ratio

$$\mathrm{KE/LPE} = \tfrac{7}{3}(g'h_o/f^2)/x_o^2 = (x_c/x_o)^2.$$

For $x_o < x_c$, a narrow current, an increase in meandering will lead to an increase in kinetic energy that cannot be supplied locally. Conversely, if the current is wider, $x_o > x_c$, increased meandering will lead to an excess of energy. This suggests that a minimum width is required for meandering to occur spontaneously through local energy conversion. The critical width, x_c, is the radius of deformation $* \sqrt{7/3}$, which for $g' = 2$ cm s^{-2}, $f = 10^{-4}$ s^{-1}, and $h_o = 1000$ m equals 48 km.

How much energy is released from a meandering current? Let the current be 50% wider, i.e. 150 km and that to begin with it is straight, and that after an elapsed time, t, the path length due to meandering has increased from L to $L + dL$. The net energy released per unit time is Eq. 4 − Eq. 5 times the change in path length divided by the elapsed time, t, i.e.,

$$\tfrac{1}{2}\sigma g' d^2 x_o{}^{12}\!/\!_5(\tfrac{3}{7} - (g'h_o/f^2)/x_o^2)dL/t.$$

If this rate of growth is divided by the volume, L(long) × H(wide) × D(deep), we obtain the rate of release per unit volume and time. Assuming $L = 300$ km, $H = 200$ km, and $D = 1000$ m, $t = 10$ days and $dL = 30$ km, corresponding to a meander wavelength and amplitude of 300 km and 30 km, we obtain a 'growth rate' of $\sim 2 * 10^{-4}$ ergs gm^{-1} s^{-1}. Had we chosen $D = 2$ km as used in the data analysis in this study, the growth rate would have been a factor 2 smaller. These are very small numbers compared to the terms shown in Table 1. The model is obviously very restrictive: no time dependence, constant width, and no motion outside the current. Nonetheless, the fact is that at least 60–70% of the eddy variance (Table 1) is due to the meandering of a permanent structure of nearly constant width (~10% variations). Furthermore, apart from the presence of rings, the eddy energy outside the current is very low. Kim and Rossby (1979) showed from XBT sections between New York and Bermuda there is little increase in eddy potential energy as one approaches the Gulf Stream if one excludes the contributions from cold core rings. Thus, this calculation suggests that the amount of energy associated with changes in meander path length is quite small.

8. Discussion and summary

A major point that we have tried to make is that the Gulf Stream is a well-defined structure with substantial stability to the cross-stream structure regardless of position and time at 73W. Earlier Shaw and Rossby (1984) had shown that the peak velocity in the current (specifically at 700 m, the depth of the Sofar floats) was remarkably uniform between Cape Hatteras and ~57W. This means that the cross-stream scale is essentially invariant over 2000 km. Since the mean dynamic height field to each side of the current changes only slowly to the east, it is fair to say that the current is two-dimensional, in the vertical and cross-stream, with only little downstream change. This is not a new observation, but it is reinforced by the present study.

A direct consequence of this is that much of the variability that we call eddy kinetic and eddy potential energy in the immediate vicinity of the current is due to the meandering of the current. Nothing is said about the temporal content of this eddy energy. It is immaterial how fast or slow the meandering is as long as the ensemble statistics span a sufficient number of different path states. A corollary of this is that it should be possible to estimate eddy energy levels from a knowledge of meander statistics, which are readily derived from satellite observations (Cornillon, 1986). (To this one could in principle add the eddy contributions from migrating warm and cold core rings.) The above argument is of course limited to the baroclinic (upper) part of the stream.

The central part of this study was concerned with estimating the *rate* of transfer of energy between the mean and fluctuating fields. Of the observable processes, the baroclinic conversion term was clearly the largest, suggesting that in this region where the meandering of the current is indeed increasing in the downstream direction, the current is baroclinically unstable. This conclusion would be in agreement with Watts and Johns' (1982) study. The difficulty here is that the estimate of energy transfer is incomplete: it includes only the cross-stream production term and neglects the downstream term, which although it could not be measured, can arguably be shown to be of the opposite sign. Specifically, it is suggested that the sum of the two baroclinic productions is probably very nearly horizontally nondivergent. This was the case in the study by Brooks and Niiler (1977). Since there is a simple geometric argument for why this should be so and since a similar geometric argument might apply to the barotropic conversion term as well (the pressure-work terms we know must be approximately nondivergent), one is obliged to assume that the horizontal divergence from each of these processes is proportional to the cross-stream term itself if the interpretation of the relative importance of the production terms is to make sense; a possible but not very satisfying result. Clearly, quantitative studies of the conversion processes must include their measurement in both horizontal dimensions with considerable care. This conclusion is further borne out by a simple model in which it is shown that very little energy is apparently required to accommodate a change in path length. Given the total amount of energy present, one can't but marvel at the coherence of the current and its resistance to disruption and collapse.

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REFERENCES

Armi, L. and N. A. Bray. 1983. A standard analytic curve of potential temperature versus salinity for the Western North Atlantic. J. Phys. Oceanogr., *12,* 384–387.

Bane, J. M., Jr. and D. A. Brooks. 1979. Gulf Stream meanders along the continental margin from the Florida Straits to Cape Hatteras. Geophys. Res. Lett., *6,* 280–282.

Briscoe, M. G. and R. A. Weller. 1984. Preliminary Results from the Long-Term Upper Ocean Study (LOTUS). Dyn. Atm. and Oceans., *8,* 243–265.

Brooks, D. A. and J. M. Bane, Jr. 1983. Gulf Stream Meanders off North Carolina during winter and summer 1979. J. Geophys. Res., *88,* 4633–4650.

Brooks, I. H. and P. P. Niiler. 1977. Energetics of the Florida Current. J. Mar. Res., *35,* 163–191.

Cornillon, P. 1986. The effect of the New England Seamounts on Gulf Stream meandering as observed from satellite IR imagery. J. Phys. Oceanogr., *16,* 386–389.

Gargett, A. E. and T. R. Osborn. 1981. Small-scale shear measurements during the fine and microstructure experiment. J. Geophys. Res., *86,* 1929–1944.

Halkin, D. and T. Rossby. 1985. The structure and transport of the Gulf Stream at 73W. J. Phys. Oceanogr., *15,* 1439–1452.

Hood, C. A. and J. M. Bane. 1983. Subsurface energetics of the cyclonic frontal zone off Onslow Bay, North Carolina. J. Geophys. Res., *88*(C8), 4651–4662.

Johns, W. E. 1984. Dynamics and Structure of the Gulf Stream Meanders off Cape Hatteras, North Carolina. Ph.D. thesis, University of Rhode Island.

Kim, K. and T. Rossby. 1979. On the eddy statistics in a ring-rich area: A hypothesis of bimodal structure. J. Mar. Res., *37,* 201–213.

Leaman, K. D., T. Rossby, R. Molinari and D. Halkin. 1984. The Gulf Stream in the Florida Straits and off Cape Hatteras: A comparison of two Pegasus Programs. EOS Transactions., *65,* 224. Amer. Geophys. Union. (abstract only.)

Oort, A. H. 1964. Computations of the eddy heat and density transports across the Gulf Stream. Tellus, *16,* 55–63.

Orlanski, I. 1969. The influence of bottom topography on the stability of jets in a baroclinic fluid. J. Atmos. Sci., *26,* 1216–1232.

Richardson, W. S. and W. J. Schmitz. 1965. A technique for the direct measurement of transport with application to the Straits of Florida. J. Mar. Res., *23,* 172–185.

Rossby, T. 1974. Studies of the vertical structure of horizontal currents near Bermuda. J. Geophys. Res., *79,* 1781–1791.

Rossby, T., A. B. Bower and P. T. Shaw. 1985. Particle pathways in the Gulf Stream. Bull. Am. Meteor. Soc., *66,* 1106–1110.

Rossby, T. and T. Rago. 1984. Hydrographic evidence for seasonal and secular change in the Gulf Stream. Time series of ocean measurements, Vol. 2, IOC Tech. Series 30. Unesco.

Schmitz, W. J. and P. P. Niiler. 1969. A note on the kinetic energy exchange between fluctuations and mean flow in the surface layer of the Florida Current. Tellus, *21,* 814–819.

Shaw, P. T. and T. Rossby. 1984. Towards a Lagrangian description of the Gulf Stream. J. Phys. Oceanogr., *14,* 528–540.

Spain, P. F., D. L. Dorson and H. T. Rossby. 1981. Pegasus, a simple, acoustically tracked velocity profiler. Deep-Sea Res., *28,* 1553–1567.

Starr, V. 1968. Physics of Negative Viscosity Phenomena: McGraw-Hill Book Company, NY; 256 pp.

Stommel, H. 1958. The Gulf Stream. Univ. of California Press, Berkeley, CA, 202 pp.

Warren, B. A. 1963. Topographic influence on the path of the Gulf Stream. Tellus, *15,* 167–183.

Watts, D. R. and W. E. Johns. 1982. Gulf Stream meanders, observations on propagation and growth. J. Geophys. Res., *87,* 9467–9476.

Webster, F. 1961. The effect of meanders on the kinetic energy balance of the Gulf Stream. Tellus, *13,* 392–401.

—— 1965. Measurements of eddy fluxes of momentum in the surface layer of the Gulf Stream. Tellus, *17,* 239–245.

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