

# YALE PEABODY MUSEUM

P.O. BOX 208118 | NEW HAVEN CT 06520-8118 USA | PEABODY.YALE. EDU

## JOURNAL OF MARINE RESEARCH

The *Journal of Marine Research*, one of the oldest journals in American marine science, published important peer-reviewed original research on a broad array of topics in physical, biological, and chemical oceanography vital to the academic oceanographic community in the long and rich tradition of the Sears Foundation for Marine Research at Yale University.

An archive of all issues from 1937 to 2021 (Volume 1–79) are available through EliScholar, a digital platform for scholarly publishing provided by Yale University Library at <https://elischolar.library.yale.edu/>.

Requests for permission to clear rights for use of this content should be directed to the authors, their estates, or other representatives. The *Journal of Marine Research* has no contact information beyond the affiliations listed in the published articles. We ask that you provide attribution to the *Journal of Marine Research*.

Yale University provides access to these materials for educational and research purposes only. Copyright or other proprietary rights to content contained in this document may be held by individuals or entities other than, or in addition to, Yale University. You are solely responsible for determining the ownership of the copyright, and for obtaining permission for your intended use. Yale University makes no warranty that your distribution, reproduction, or other use of these materials will not infringe the rights of third parties.



This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.  
<https://creativecommons.org/licenses/by-nc-sa/4.0/>



## **Assessing the energetics and dynamics of the Gulf Stream at 68W from moored current measurements**

by Melinda M. Hall<sup>1</sup>

### **ABSTRACT**

The energetics and dynamics of the Gulf Stream at 68W have been investigated using year-long time series of velocity and temperature throughout the water column. The major results that have emerged are as follows: (1) There is a net conversion of mean to eddy kinetic and potential energies, the barotropic mechanism being almost twice as strong as the baroclinic. These energy exchanges are dominated by what is happening in the upper 1000 m of the water column. (2) Comparison with other studies suggests that the mooring site may be characterized as a region of downstream spatial growth in eddy energy, with a growth rate of  $3\text{--}4 \times 10^{-3} \text{km}^{-1}$ . (3) Curvature changes due to the changing Stream path are sufficient to balance stretching of the water column below the thermocline, the dominant measurable effect in the vorticity equation. (4) A kinematic scheme including and relating barotropic cross-stream velocities, local temperature changes, stretching, and curvature changes is shown to be generally consistent with the observed data. (5) The vertical mass divergence  $\partial w/\partial z$  affects continuity at lowest order, and may be associated with along-stream changes in transport, a vertical redistribution of the alongstream momentum flux, or changing Stream width.

### **1. Introduction**

In a previous paper discussing this data set (Hall, 1986a, hereafter referred to as H1), the vertical and cross-stream structure of the average velocity fields for the Gulf Stream were constructed from data from a single current meter mooring at  $37^{\circ}37'N$ , 68W. The two key points to the success of the analysis were: (1) the well-defined relationship between temperature and cross-stream distance in the thermocline, enabling the use of the former as a horizontal coordinate; and (2) a daily-changing definition of Gulf Stream flow direction based on the shear between the thermocline and 2000 m depth. In addition, Bryden's (1980) method was used to obtain time-series of vertical velocities at the five standard depths (575, 875, 1175, 2000, and 4000 dbar), and it was suggested that their large magnitudes (maximum rms values in the thermocline of 0.08 cm/s) and associated vertical divergence could have implications for the dynamics of the flow. The magnitude of the vertical divergence  $\partial w/\partial z$  and of isotherm slopes was used to assess the applicability of quasi-geostrophic dynamics at the mooring site. A Rossby number  $\epsilon$  of about 0.3 was estimated for thermocline levels,

1. Scripps Institution of Oceanography, La Jolla, California, 92093, U.S.A.

so that while thermal wind might be used to relate density and velocity gradients, quasi-geostrophic dynamics might not be a good predictor for the time evolution of the Stream.

To shed more light on these aspects, both the energetics and dynamics of flow at the mooring site have been investigated. The merit of examining the energetic aspects of the flow is several fold. In the first place, the full energy equations may be obtained without assuming a particular dynamical framework for the flow (such as quasi-geostrophy), and the different terms examined for signatures of familiar processes such as barotropic or baroclinic instabilities. Second, for the first time concurrent records at thermocline and abyssal depths in the Stream are available for the analysis. Previous investigations of the energy exchanges between mean and eddy flows (Schmitz, 1977) have relied on deep measurements alone. Numerous analytic and numerical models (Warren, 1963; Robinson and Niiler, 1967; Orlanski, 1969; Johns, 1985) have sought to describe Gulf Stream variability via instability mechanisms; the predicted time and space scales of such models may be compared with observed characteristics of Gulf Stream meanders. However, there is no guarantee that the *particular* instability process of the model is occurring in the Gulf Stream. In fact, there is little (if any) direct evidence to answer the question of whether barotropic or baroclinic instabilities dominate the energetics of the Gulf Stream. The energetic analysis here seeks to determine the nature of energy exchanges between the mean and eddy fields; the results suggest that energy is being transferred from the mean to eddy field at the mooring site through both barotropic and baroclinic processes, the former about 2–3 times as large as the latter. Most of the contribution to these transfers comes from the upper part of the water column, pointing out the need for more data in the thermocline Gulf Stream.

A diagnostic dynamical analysis at the site is somewhat more difficult, as there is insufficient horizontal resolution to evaluate most terms in the vorticity equation. Most analytic models of the Stream rely on assumptions and simplifications which may or may not be justified. In a thin jet model of Gulf Stream meandering, for example, Luyten and Robinson (1974) assumed the meandering was vertically coherent, and that vortex stretching was unimportant in the vorticity balance. Results from H1 justify the former assumption, but suggest that the latter may not be true. Rather than trying to apply any particular model directly to the data, this work reports the outstanding qualitative features suggested by the data, and places them within a consistent kinematical framework. Thus, a major intent of this work is to present results which may help to guide the development of theoretical models addressing the time-evolution of the Gulf Stream.

## 2. Energetics at the mooring site

*a. Eddy energy equation.* In order to discuss energy exchanges between mean and eddy flow, it is necessary to be able to define the two. In H1 the Gulf Stream was

described as a feature with a well-defined horizontal and vertical velocity and temperature structure, and it was possible to deduce the time-averaged structure from data from a single current meter mooring. There is insufficient data to discuss deviations from that average, but it would be desirable to retain some degree of horizontal resolution, pertinent to a discussion of the role of barotropic energy exchanges. That is accomplished here by limiting horizontal resolution to two bins, corresponding to  $T_{575} < 13^\circ\text{C}$  (north of the Stream axis) or  $T_{575} > 13^\circ\text{C}$  (south of the axis), where  $T_{575}$  is the temperature at 575 dbars and is closely related to cross-stream position. Furthermore, in H1 it was recognized that the Gulf Stream may change its position and orientation continuously, and an along- and cross-stream coordinate system that changes daily was used to account for this feature. However, because the time-averaging involved in obtaining the kinetic and potential energy equations introduces an interpretative problem in such a system, the energetic analysis is carried out in a strictly Eulerian frame. This approach is more traditional and is more readily compared with past results as well. When they are used, variables in the rotated system will be denoted by  $\wedge$  notation.

The equation for eddy kinetic energy  $\overline{K'} = (1/2)\rho(\overline{u'^2} + \overline{v'^2})$  is obtained by adding  $u'x$ ( $u$ -momentum equation) +  $v'x$ ( $v$ -momentum equation) and then time-averaging:

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + \bar{v} \frac{\partial}{\partial y}\right) \overline{K'} = -\nabla_H \cdot \rho \left( \mathbf{u}_H \frac{\overline{u'^2} + \overline{v'^2}}{2} \right) - \overline{\rho u'v'}(\bar{v}_x + \bar{u}_y) - \overline{\rho u'^2} \bar{u}_x - \overline{\rho v'^2} \bar{v}_y - \overline{\mathbf{u}'_H \cdot \nabla_H p'} \quad (1)$$

where the subscript  $H$  is used to mean the horizontal components only. Similarly, the equation for eddy potential energy  $\overline{P'} = 1/2(g\alpha_o \overline{T'^2})/(\bar{\theta}_z)$  obtained by multiplying the heat equation by  $g\alpha_o T'/\bar{\theta}_z$  and time-averaging is:

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + \bar{v} \frac{\partial}{\partial y}\right) \overline{P'} = \frac{-g\alpha_o}{\bar{\theta}_z} (\overline{u'T'} \bar{T}_x + \overline{v'T'} \bar{T}_y) - g\alpha_o \overline{w'T'} - \nabla_H \cdot (\overline{u'T'^2} g\alpha_o / 2\bar{\theta}_z) \quad (2)$$

where  $\alpha_o = -d\rho/dT = -(\partial\rho/\partial T + \partial\rho/\partial S dS/dT)$  and is approximately  $10^{-4} \text{gm/cm}^3/^\circ\text{C}$ . Recalling that  $-\mathbf{u}'_H \cdot \nabla_H p' = -\nabla_H \cdot p'\mathbf{u}'_H - p'w'_z = -\nabla_3 \cdot p'\mathbf{u}'_3 + w'p'_z = -\nabla_3 \cdot p'\mathbf{u}'_3 - g\rho'w' = -\nabla_3 \cdot p'\mathbf{u}'_3 + g\alpha_o w'T'$ , (1) and (2) can be added to get the equation for the total eddy energy:

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + \bar{v} \frac{\partial}{\partial y}\right) (\overline{K'} + \overline{P'}) = -\nabla_H \cdot \overline{\mathbf{u}'(K' + P')} - \overline{\rho u'v'}(\bar{v}_x + \bar{u}_y) - \frac{g\alpha_o}{\bar{\theta}_z} (\overline{u'T'} \bar{T}_x + \overline{v'T'} \bar{T}_y) - \overline{\rho \bar{u}_x(u'^2 - v'^2)} - \nabla_3 \cdot \overline{p'\mathbf{u}'_3} \quad (3)$$

Local time changes and mean advection of total eddy energy are balanced by essentially two types of terms that appear on the RHS of Eq. (3): (1) exchanges

between the mean and eddy flows, represented by up- or down-gradient momentum and heat fluxes; and (2) “radiating” terms, which appear as divergences of quantities depending only on the eddy field: specifically, there is the eddy advection of eddy energy (first term on the RHS) and the pressure work term (last term on the RHS). From (1) and (2) it is clear that the exchange *between* eddy kinetic and potential energies is given by the term  $\pm g\alpha_o\overline{w'T'}$ , which appears with opposite sign in the two equations.

*b. Reynolds stresses and barotropic energy exchanges.* To assess the relative importance of terms on the RHS of (3) in a gross rather than localized sense, one ought to integrate over a volume. Integration in  $z$  is possible because of the mooring’s vertical resolution; to achieve integration in  $y$ , two separate temperature bins, corresponding to regions north and south of the Stream axis, have been used for the time-averaging process; integration in  $x$  poses some difficulty and will simply be ignored, since only a rough qualitative picture is sought. Furthermore, it is useful to recognize that in Eulerian as well as rotated coordinates, it is true that  $\overline{v_x} \ll \overline{u_y}$ , and  $\overline{v^2} \ll \overline{u^2}$  (see H1), so that the net mean-to-eddy momentum exchange is approximately given by  $-\rho\overline{u'v'}\overline{u_y} - \rho\overline{u^2}\overline{u_x}$ . To calculate the Reynolds stress  $\overline{u'v'}$ , all the data were divided into two bins according to whether  $T_{575} > 13^\circ\text{C}$  or  $T_{575} < 13^\circ\text{C}$  (where  $\overline{u_y}$  changes sign). Then, for each of the new data sets, mean and eddy velocities were computed, as was the product  $\overline{u'v'}$ . These values are given in Table 1. The accompanying values of  $\overline{u_y}$  were obtained by taking  $\overline{u_y}$  at 575 dbar equal to  $e^{-1}$  times the maximum value attained in the analytic expressions for  $\overline{u_y}$  deduced in H1. The magnitude is assumed to decay with depth on an  $e$ -folding scale of 1000 m. The width of the anticyclonic side is taken as 50 km, the distance from  $T_{575} = 13^\circ\text{C}$  to  $16.5^\circ\text{C}$ ; on the cyclonic side,  $\Delta y = 80$  km ( $T_{575} = 5^\circ\text{C}$  to  $13^\circ\text{C}$ ).

Previous estimates of exchanges between the mean and eddy kinetic energies in western boundary currents have relied either on long time-series from measurements in the deep water (4000 m) beneath the Gulf Stream (Schmitz, 1977) or on surface measurements alone (Webster, 1965; Schmitz and Niiler, 1969; Hager, 1977; Szabo and Weatherly, 1979; Nishida and White, 1982). Schmitz’s (1977) results show that  $\overline{u'v'}$  changes sign across the geographical average axis of the Stream over perhaps 2–3 degrees of latitude, such that there is a flux of eddy to mean kinetic energy ( $\overline{u'v'} > 0$  south of the Stream); directly under the axis,  $\overline{u'v'} < 0$  and has magnitude of 5–15  $\text{cm}^2 \text{s}^{-2}$ . The Reynolds stresses in the deep part of the water column at the GUSTO site are not terribly different for the two bins (Table 1); they are positive, and do not change sign across the Stream axis. In addition,  $(\overline{u'v'})_y > 0$  across the Stream, the opposite sense of Schmitz’s findings. However, the results in the upper 1000 m, which will make the greatest contribution to the net energy exchange because  $\overline{u_y}$  is strongest there, are remarkably different from all the deep water values. For both bins (i.e., on both “sides” of the Stream),  $\overline{u'v'}$  is large and negative, and  $(\overline{u'v'})_y > 0$ , so the more negative values occur on the warm side,  $T > 13^\circ\text{C}$ , where also

Table 1. Reynolds stresses and shear for two temperature bins.  $\overline{u'v'}$  is calculated from data, with 128 (242) data points contributing to averages for  $T_{575} < 13^\circ\text{C}$  ( $T_{575} > 13^\circ\text{C}$ ).  $\overline{u}_y$  obtained as described in text. Integration is trapezoidal, gives estimate of energy exchange due to down-gradient momentum flux. Correlation co-efficients are listed in columns headed 'C'.

Depth (db)	$\Delta z$ (db $\approx$ m)	$T_{575} < 13^\circ\text{C}$			$T_{575} > 13^\circ\text{C}$		
		$\overline{u'v'}$ ( $\text{cm}^2/\text{s}^2$ )	C	$\overline{u}_y$ ( $10^{-5} \text{ s}^{-1}$ )	$\overline{u'v'}$ ( $\text{cm}^2/\text{s}^2$ )	C	$\overline{u}_y$ ( $10^{-5} \text{ s}^{-1}$ )
575	725	-321.52	-.34	-.952	-424.70	-.49	2.234
875	300	-45.40	-.25	-.706	-115.28	-.38	1.655
1175	563	-34.56	-.27	-.523	-11.44	-.16	1.226
2000	1412	-12.64	-.14	-.229	-5.83	-.13	.537
4000	1688	10.65	0.16	-.031	8.71	0.22	.073

$\rho = 1.027 \text{ gm/cm}^3$

$\Delta y = 80 \text{ km}$

$\Delta y = 50 \text{ km}$

$$\begin{aligned}
 & - \sum_{T < 13} \Delta z \overline{u'v'} \overline{u}_y \rho = -251.9 \text{ gm/s}^3 \quad - \sum_{T > 13} \Delta z \overline{u'v'} \overline{u}_y \rho = 776.6 \text{ gm/s}^3 \\
 & - \int dy \int dz (\rho \overline{u'v'} \overline{u}_y) = 1.867 \times 10^4 \text{ kg m/s}^3
 \end{aligned}$$

$\overline{u}_y$  has the greater magnitude. The net effect is that of a down-gradient eddy momentum flux (see Table 1), which implies a growth of eddy energy at the expense of the mean kinetic energy via this mechanism. There is net southward eddy transport of eastward momentum across the Stream. The mean contribution  $\overline{u'v}$  indicates a similarly directed flux, an order of magnitude smaller;  $\overline{u'v} < 0$  may be related to the fact that the average flow was south of east. Fofonoff and Hall (1983) found that eastward momentum flux of the Gulf Stream is decreasing in this region;  $(\overline{u'v})_y > 0$  is one mechanism that can account for such a decrease but was not calculable in that work.

The energy transfer from Table 1 is opposite of that found in the Florida Current by Webster (1965), who reported a net flux of eddy to mean kinetic energy at four sections between Florida and Cape Hatteras. He found that the energy transfer, estimated as  $\overline{u'v'} \overline{v}_x$  where  $v$  is alongstream velocity, occurred primarily as a result of strong up-gradient fluxes in the cyclonic portion of the Stream. Schmitz and Niiler (1969) confirmed this general pattern, but concluded that the net exchange integrated across the current was zero, so that only a redistribution of energy was occurring. In the Florida Current, of course, the flow is accelerating, while at the GUSTO site it may already be decelerating, so the structure of the Reynolds stresses must change—either gradually or abruptly—along the portion in between.

In fact, Hager (1977) used ship drift data to estimate all the components of kinetic energy exchange, between Florida and about 70W. Although the general pattern he deduced for the cross-stream component  $\overline{u'v'} \overline{v}_x$  confirmed Webster's results, Hager found that values of the downstream component  $\overline{v'^2} \overline{v}_y$  were several times larger and patchy in distribution through the Gulf Stream, leading to a similar picture for the net

exchange. Thus, it is possible that this term (equivalent to  $\overline{u'^2} \overline{u}_x$  for the GUSTO site) could alter significantly the results of Table 1. The effect of this term is hard to determine from the mooring. Some historical data (Worthington, 1976; Knauss, 1969) suggest that Gulf Stream transport may still be increasing at 68W, and if  $\overline{u}_x > 0$  as well, then  $-\rho \overline{u'^2} \overline{u}_x$  could indeed offset the effect of the Reynolds stresses in this region. Fofonoff and Hall (1983) found  $\overline{u}_x < 0$  at this longitude, however, in which case  $-\rho \overline{u'^2} \overline{u}_x$  could enhance the exchange due to Reynolds stresses.

The above studies demonstrate the complexity of the kinetic energy exchange terms in the surface flow alone; yet Table 1 suggests that there is considerable vertical structure to the exchange as well. For example, the vertical structure of Reynolds stresses within the southern portion of the Stream at 68W is substantially different from that found by Schmitz (1980) 200–300 km south of the Gulf Stream axis at 55W. Schmitz identified a weakly depth-dependent eastward flow regime at 37.5N, where values of  $\overline{u'v'}$  ranged from about  $30 \text{ cm}^2 \text{ s}^{-2}$  at 600 m depth to  $35 \text{ cm}^2 \text{ s}^{-2}$  at 4000 m depth. (Notice that these values are opposite in sign to the upper water values reported in Table 1.) The shear  $\partial \overline{u} / \partial y$  is small, so that little energy exchange is associated with the momentum flux. Similar studies in the Kuroshio (Szabo and Weatherly, 1979; Nishida and White, 1982) suggest that the question of kinetic energy exchange in strong western boundary currents is one of great complexity.

*c. Heat fluxes and baroclinic energy exchanges.* The mean to eddy potential energy flux at the mooring,  $\overline{u'T'} \overline{T}_x + \overline{v'T'} \overline{T}_y$ , can be estimated without the necessity of separating the data into bins. This flux is down-gradient as well, with an integrated magnitude about half as large as the barotropic conversion, dominated by values from the upper 1500 m or so of the water column (see Table 2). As expected,  $\overline{v'T'} > 0$ , but all the correlation coefficients for the heat flux calculations are small. The buoyancy flux term  $g\alpha_o \overline{w'T'}$  is positive (although again the correlation coefficients are very small), and may be estimated from values at 575 dbar alone, since its magnitude falls off rapidly to negligible values. At 575 db,  $\overline{w'T'} = 24.979 \times 10^{-3} \text{ }^\circ\text{C cm/s}$ ; taking  $\Delta z = 725 \text{ m}$ ,  $\Delta y = 130 \text{ km}$ :

$$\int dy \int dz (g\alpha_o \overline{w'T'}) = (130 \text{ km})(725 \text{ m}) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) \left( 10^{-4} \frac{\text{gm}}{\text{cm}^3 \text{ }^\circ\text{C}} \right) \\ \times (24.979 \times 10^3 \text{ }^\circ\text{C cm/s}) = 2.310 \times 10^4 \text{ kg m/s}^3.$$

Since this term appears with a minus sign in (2), eddy potential energy evidently is being converted into eddy kinetic energy; moreover, it is much larger than the release of mean to eddy potential energy, so that the net tendency of eddy potential energy is to decrease. The energy pathway—mean potential to eddy potential to eddy kinetic energy—tantalizingly suggests the presence of baroclinic instability at the mooring site. Caution is warranted, however, for all of the calculations involved are rather noisy:

Table 2. Estimate of mean to eddy potential energy conversion. Temperature gradients are estimated geostrophically from shear. Shear at 575 dbar is calculated from top 2 instruments; at 875 and 1175 dbar, cubic spline fits to average velocity were used to estimate shear. Variables are not rotated.  $\bar{\theta}_z$  is from analytic fits of Raymer *et al.* (1984). Integration is down to 1500 dbar only because contribution below is negligible. Correlation coefficients for heat fluxes are all less than 0.15, except for  $\overline{u'T'}$  at 575 dbar, where  $C = .20$ .

Depth (db)	$\Delta z$ (db $\approx$ m)	$\bar{u}_z$ (cm/s/m)	$\bar{T}_y$ ( $10^{-3}$ °C/m)	$\overline{v'T'}$ (°C cm/s)	$\bar{v}_z$ ( $10^{-3}$ cm/s/m)	$\bar{T}_x$ ( $10^{-5}$ °C/m)	$\overline{u'T'}$ (°C cm/s)	$\bar{\theta}_z$ (°C/m)
575	725	.0710	-6.61	1.547	.303	.0282	31.039	.0178
875	300	.0531	-4.90	3.901	-.241	-.0224	2.963	.0127
1175	475	.0171	-1.59	.055	-.532	-.0495	-.539	.00348

$$\begin{aligned}
 & - \sum \frac{g\alpha}{\bar{\theta}_z} (\overline{u'T'} \bar{T}_x + \overline{v'T'} \bar{T}_y) \Delta z = 82.51 \text{ gm/s}^3 \\
 & - \int dy \int dz \left[ \frac{g\alpha}{\bar{\theta}_z} (\overline{u'T'} \bar{T}_x + \overline{v'T'} \bar{T}_y) \right] = 1.073 \times 10^4 \text{ kg m/s}^3 \\
 & \alpha = 10^{-4} \text{ gm/cm}^3/\text{°C} \quad \Delta y = 130 \text{ km}
 \end{aligned}$$



the correlation coefficients are generally small and not significantly different from zero for the year-long data record.

The net results may be compared with those found by Hall (1985b) in a recent energetic analysis of a numerical model. That work investigated the kinetic energy equation integrated over open-bounded volumes, and identified a variety of energetically different regimes, among them an accelerating and decelerating jet (analog of the Gulf Stream), as well as a western and eastern recirculation. The energy pathways in the decelerating jet (and to an extent in the accelerating portion as well) suggested that barotropic instability is important for transferring energy from the mean to eddy field, while throughout the recirculation baroclinic instability dominates. In the GUSTO results, the signatures of both processes are found within the Stream, the barotropic mechanism dominating, as in the numerical model. On the other hand, Flierl and Robinson's (1984) thin-jet model of meandering suggests that for Gulf Stream-type parameters, the meandering instability gains available potential energy at a rate about 2.4 times greater than it gains kinetic energy. However, that model is formulated not in Cartesian coordinates, but in coordinates that move with the Stream axis, so that it is not clear the energy exchanges should appear the same as those calculated here.

The energy transfer of mean to eddy fields implies local growth of eddy energy, mean or eddy advection of eddy energy away from the mooring site, or radiation of eddy energy away from the mooring site. The term  $-(\partial/\partial y)(\overline{v'(K' + P')})$  may be estimated with the "two-bin method" used on the Reynolds stresses, and although it has the desired sign to balance the momentum and heat fluxes, it is at least an order of magnitude smaller than those terms. The remaining radiation type terms cannot be estimated from the data. It is plausible that this region is one of either temporal or spatial eddy growth, and corresponding time and space scales for the implied growth may be estimated from the numbers so far derived, along with an estimate of net eddy energy from values in Table I of H1. The average along-stream velocity  $\bar{u}$  is an integrated value over the depth. Then:

$$\begin{aligned} \int dy \int dz \frac{1}{2} \left( \rho(\overline{u^2} + \overline{v^2}) + \frac{g\alpha_o \overline{T'^2}}{\theta_z} \right) &\approx 8.06 \times 10^{10} \text{ kg m/s}^2; \\ - \int dy \int dz (\rho \overline{u'v'} \bar{u}_y) &\approx 1.87 \times 10^4 \text{ kg m/s}^3; \\ - \int dy \int dz \left( \frac{g\alpha_o}{\theta_z} \right) (\overline{v'T'} \bar{T}_y + \overline{u'T'} \bar{T}_x) &\approx 1.07 \times 10^4 \text{ kg m/s}^3; \\ \bar{u} &\approx 9 \text{ cm/s.} \end{aligned}$$

The above values imply either a growth rate  $r$  (calculated from energy/conversion)

$$r \sim \frac{1.87 \times 10^4 + 1.07 \times 10^4}{8.06 \times 10^{10}} \text{ s}^{-1} = 3.6 \times 10^{-7} \text{ s}^{-1} \rightarrow \frac{1}{r} = 32 \text{ days}$$

or a downstream scale  $L_x$  for eddy energy growth (calculated from  $L_x \sim (\bar{u} \times$

energy)/conversion) of

$$L_x \sim \frac{(9 \text{ cm/s})(8.06 \times 10^{10} \text{ kg m/s}^2)}{(1.87 \times 10^4 + 1.07 \times 10^4) \text{ kg m/s}^3} = 247 \text{ km.}$$

Watts and Johns (1982) have presented observational dispersion curves for Gulf Stream meanders from data spaced 100 to 200 km downstream (northeast) of Cape Hatteras, and thus about 450 km upstream of the GUSTO site. They suggested that shorter period meanders ( $T \leq 9$  days) were best described by a spatial growth rate, those with  $T \geq 14$  days by temporal growth. The temporal growth rate calculated here of  $3.2 \times 10^{-2} \text{ d}^{-1}$  falls below the minimum values found by Watts and Johns of  $4 \times 10^{-2} \text{ d}^{-1}$ . However, a spatial growth rate of  $4 \times 10^{-3} \text{ km}^{-1}$ , which corresponds to the scale  $L_x$  estimated above, is found for meanders of wavelength 350 km, with phase speeds of 30 km/d and a period  $T \approx 9$ –14 days. Halliwell and Mooers (1983), in an analysis of satellite data reaching from Cape Hatteras to 1000 km downstream, found that the most energetic meanders in the downstream portion had a spatial growth rate of  $(3.2 \pm 1.3) \times 10^{-3} \text{ km}^{-1}$ , a wavelength of 330 km, periods of 1.5 months, and phase speeds of about 7 km/d. Such phase speeds are, in fact, more typical of 68W (Hansen, 1970). In spite of the crudeness of the estimates from the GUSTO data, the comparison with other estimates suggests that the mooring site is in fact a region of downstream spatial growth, characterized by a growth rate of  $3$ – $4 \times 10^{-3} \text{ km}^{-1}$ .

### 3. A kinematic framework for interpreting the flow

Because the equations for mean and eddy energies do not depend on small amplitude expansions, different terms in them may be examined for signatures of familiar processes such as barotropic or baroclinic instabilities. Analysis of dynamical balances is less tractable, however, because the relevant terms in the vorticity equation, for example, involve so many derivatives. Moreover, with thermocline Rossby numbers of about 0.3 (see H1), it may be necessary to search for a new dynamical framework that explains the flow. Although such a dynamical framework has not been fully developed, a kinematic framework has been explored that is consistent with the data. It is just one interpretation of what is occurring at the mooring site, and is not necessarily unique.

The cornerstone of this kinematic framework is the importance of vertical stretching to the dynamics. Northward velocity and estimates of  $\partial w / \partial z$  are significantly correlated throughout the water column: correlation coefficients range from  $C = 0.3$ – $0.4$ , significant at the 95% level for 30 degrees of freedom; however, the term  $\beta v$  is generally an order of magnitude smaller than  $f \partial w / \partial z$ . Suppose, for example, there is a change in  $w$  of  $50 \times 10^{-3} \text{ cm/s}$  from 875 dbar to 4000 dbar. The  $v$  required to balance  $f \partial w / \partial z$  is then

$$v = \frac{f \partial w / \partial z}{\beta} \approx \frac{(0.89 \times 10^{-4} \text{ s}^{-1})(50 \times 10^{-3} \text{ cm/s})}{3125 \text{ m} \times 1.9 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}} = 75 \text{ cm/s,}$$

which is greater than the maximum value of  $v$  in that part of the water column by a factor of about three.

Further investigation reveals even stronger correlations between  $v$  and  $w$  (as well as cross-stream velocity  $\hat{v}$  and  $w$ ) at all depths, with correlation coefficients ranging from 0.5 to 0.9. These results can be interpreted in two ways. It was found in H1 that the first empirical mode for the vertical structure of  $w$  closely resembles a first baroclinic mode. If indeed the system behaves as such, then  $w$  and  $\partial w/\partial z$  should display the same behavior in time, but because the latter is a much noisier term,  $v$  and  $w$  are better correlated than  $v$  and  $\partial w/\partial z$ . The second interpretation depends on water parcels conserving their temperature: to the extent that  $w = \mathbf{u} \cdot \nabla z_T \approx \hat{v} \partial z_T / \partial \hat{y}$  (where  $z_T$  is the depth of an isotherm,  $\partial z_T / \partial \hat{x}$  is assumed to be zero, and  $\partial z_T / \partial \hat{y} \approx \text{constant}$ ),  $w$  and  $\hat{v}$  ought to be correlated. In fact, the close relation between the two interpretations becomes evident in a scenario based on the supposition that primarily the lower layer of a two-layer system is observed, as suggested by the results of H1. As vertical velocities are induced at the bottom by flow up or down the bottom slope, higher in the water column water parcels moving vertically must also move horizontally—primarily cross-stream—to remain on isotherms. Given that cross-stream velocities are nearly barotropic, as was shown empirically in H1, then the vertical velocities will be greatest where the isotherm slopes are greatest, that is, in the thermocline or at its equivalent, the interface in the two-layer model. Thus,  $\partial w/\partial z$ ,  $w$ , and  $\hat{v}$  should all behave much the same. The orientation of isotherm slopes may be quite different from the direction of the bottom slope, whence the bottom vertical velocity appears as an independent forcing mechanism.

The local temperature change is important too, however, and may be represented as  $T_t = -VT\hat{y}$ , where  $V$  is then related to the cross-stream translational velocity, so that  $w = (\hat{v} - V)\partial z_T / \partial \hat{y}$ . Thus, when local temperature change is compensated primarily by horizontal advection, vertical velocities remain small. However, as discussed in H1, there are events for which the apparent translational velocity is opposite in sign to cross-stream velocity; then  $|\hat{v} - V| > |\hat{v}|$  and  $w$  must be relatively large to balance the temperature equation. In fact, the first empirical mode for the vertical structure of cross-stream velocity is highly correlated with  $w_B$  (vertical velocity at 4000 m), with  $C = 0.8$ , indicating that the mechanism suggested in the previous paragraph is operative. However, it is not at all correlated with  $\partial T/\partial t$  at 575 dbar ( $C = 0.05$ ): thus, there is no significant tendency for horizontal advection to balance local temperature change. Now if the barotropic cross-stream velocity arises in response to  $w_B$ , the size of the vertical velocities throughout the remainder of the water column then depends in part on the Gulf Stream's translation: in the case of opposing translational and measured velocities,  $\partial w/\partial z$  below the thermocline should have the same sign as  $w_B$ :

$$\frac{\partial w}{\partial z} \approx (\hat{v} - V) \left( \frac{\partial z_T}{\partial \hat{y}} (575) - \frac{\partial z_T}{\partial \hat{y}} (4000) \right); (\hat{v} - V)w_B > 0; \left. \frac{\partial z_T}{\partial \hat{y}} \right|_{4000}^{575} > 0.$$

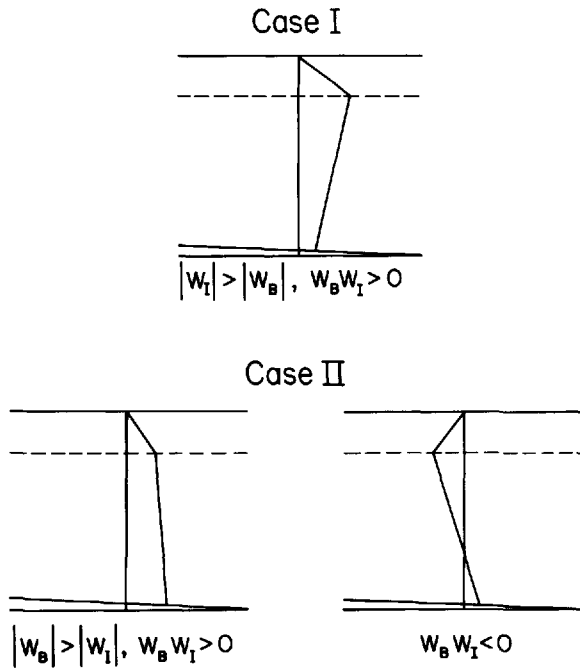


Figure 1. Vertical velocity amplitude structures for a two-layer system for cases discussed in text. Slight bottom slope allows  $w \neq 0$  at the bottom. Interface (dashed line) represents thermocline of real ocean;  $w_B$  = vertical velocity at bottom;  $w_I$  = vertical velocity at interface. In Case II both a “barotropic-like” (left) and baroclinic (right) response are possible. The cases corresponding to Ia or IIa in the text occur when  $w_B < 0$ , so that the pictured amplitude structure is negative.

The first inequality states that cross-stream velocities, which are enhanced by  $V$ , are correlated with  $w_B$ . The second inequality states that isotherm slopes are positive, and stronger at 575 dbar than at depth. Thus, if  $w_B > 0$  (whence  $\hat{v} > 0$ ) then  $T_i < 0$  ( $V < 0$ ) should imply  $\partial w / \partial z > 0$ ; while  $w_B < 0$  ( $\hat{v} < 0$ ) and  $T_i > 0$  implies  $\partial w / \partial z < 0$ . The fact that  $\partial T / \partial t$  at 575 dbar and  $\Delta w = w_{575} - w_{4000}$  are negatively correlated with  $C = -.75$  confirms these implications. That result is based on the entire time series, yet  $\hat{v}$  and  $T_i$  are oppositely signed only about half the time: evidently there are two different flow regimes that can be summarized as follows:

Case I	(Case Ia)	Case II	(Case IIa)
$w_B > 0$	$(w_B < 0)$	$w_B > 0$	$(w_B < 0)$
$\hat{v} > 0$	$(\hat{v} < 0)$	$\hat{v} > 0$	$(\hat{v} < 0)$
$T_i < 0$	$(T_i > 0)$	$T_i > 0$	$(T_i < 0)$
$w_z > 0$	$(w_z < 0)$	$w_z < 0$	$(w_z > 0)$

Figure 1 shows what the vertical velocity structure would be in the two-layer system for

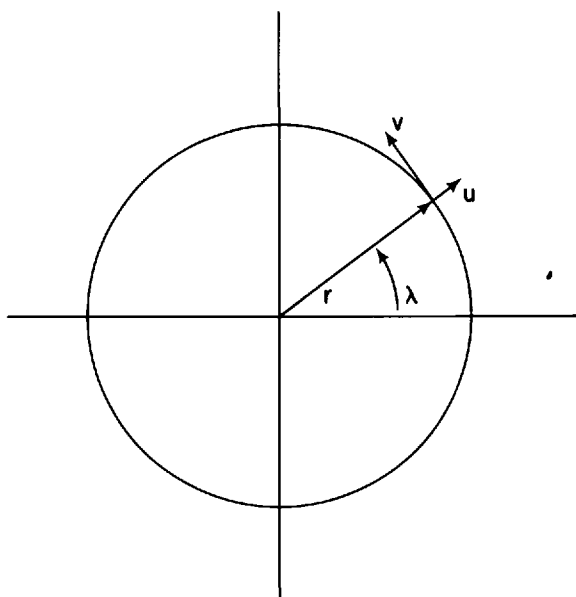


Figure 2. Definition sketch for variables in cylindrical coordinates. Radius  $r > 0$  always;  $v > 0$  when motion is cyclonic.

these two cases; in Case II, there are two possibilities, since  $w$  at the interface need not have the same sign as  $w$  at the bottom; but the data suggest the baroclinic response (shown on the right in Fig. 1 under Case II) is more typical. Johns and Watts (1985) present a linear analysis of the temperature equation, from data just downstream of Cape Hatteras, which yields results analogous to Case I described here; but in that study, Case I evidently described most of the data, and Case II was not considered at all. The vertical structures shown for Case I and for the baroclinic possibility in Case II resemble that of the first empirical mode for vertical velocity, which accounts for about 80% of the variance (see H1).

Physically, the cases may be described as follows. In Case I, as the Gulf Stream moves southward (as inferred from  $\partial T/\partial t$ ), water parcels below the thermocline move essentially northward and upward, remaining on isotherms; the increasing slope of isotherms from the bottom to the thermocline implies a net stretching in the lower part of the water column. In Case II, if the Gulf Stream moves northward ( $\partial T/\partial t > 0$ ), water parcels in the Stream tend to do the same; the northward upslope movement induces a positive vertical velocity at the bottom, but depending on the relative strengths of  $\hat{v}$  and  $V$ , vertical velocities above the bottom may even be negative. The general tendency in this case is the squashing of the water column.

Now consider the implications of  $\partial w/\partial z$  to the vorticity balance. It has been shown that the term  $\beta v$  is insufficient to compensate the stretching term  $f \partial w/\partial z$ . However, examination of Gulf Stream paths produced by the National Weather Service from

satellite data for the year suggest the Stream's curvature was quite strong at times. To assess its possible importance to the vorticity balance, consider the vorticity equation in cylindrical coordinates. Figure 2 gives a definition sketch for the variables. Then

$$\zeta = \frac{1}{r} \frac{\partial}{\partial r} (vr) - \frac{1}{r} \frac{\partial u}{\partial \lambda} = \frac{v}{r} + \frac{\partial v}{\partial r} - \frac{1}{r} \frac{\partial u}{\partial \lambda}.$$

$$\mathbf{u} \cdot \nabla \zeta = u \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv) - \frac{1}{r} \frac{\partial u}{\partial \lambda} \right) + \frac{v}{r} \frac{\partial}{\partial \lambda} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv) - \frac{1}{r} \frac{\partial u}{\partial \lambda} \right)$$

(Note that now  $v$  is long-stream velocity, and may be negative or positive according to the curvature of the Stream, since  $r$  must be positive.) With

$$v \gg u, \frac{\partial}{\partial r} \gg \frac{1}{r} \frac{\partial}{\partial \lambda},$$

then

$$\zeta \sim \frac{v}{r} + \frac{\partial v}{\partial r}$$

and the vorticity equation becomes

$$\frac{\partial}{\partial t} \left( \frac{v}{r} + \frac{\partial v}{\partial r} \right) + \frac{u}{r} \frac{\partial v}{\partial r} - \frac{uv}{r^2} + u \frac{\partial^2 v}{\partial r^2} + \frac{v}{r^2} \frac{\partial v}{\partial \lambda} + \frac{v}{r} \frac{\partial^2 v}{\partial \lambda \partial r} + \beta(u \sin \lambda + v \cos \lambda) = f \frac{\partial w}{\partial z}. \quad (4)$$

The local change of curvature explicitly appears, and its size can be estimated by referring to the NWS maps. Figure 3 shows schematically how the curvature changes from May 27 when  $r$  is roughly 70 km, to June 1, when the flow has straightened out so  $r$  is essentially infinite. During this time, the along-stream velocity at 575 dbar  $v \sim 30$  cm/s whence

$$\frac{\partial}{\partial t} \left( \frac{v}{r} \right) \approx \frac{0 - (30 \text{ cm/s}) / (70 \text{ km})}{5 \text{ days}} = -.99 \times 10^{-11} \text{ s}^{-2}.$$

Meanwhile,  $\partial w / \partial z$  between the bottom and thermocline is negative, and has an estimated magnitude:

$$f \frac{\partial w}{\partial z} \sim (.89 \times 10^{-4} \text{ s}^{-1}) \times \frac{(-30 \times 10^{-3} \text{ cm/s})}{3425 \text{ m}} = -.78 \times 10^{-11} \text{ s}^{-2}.$$

Thus, the effect of changing curvature is more than enough to balance the squashing in the lower part of the water column. Notice that the same balance cannot obtain above the thermocline, where  $\partial w / \partial z$  must be  $< 0$ ; this point is addressed below.

Proceeding in a similar but qualitative manner for the four individual events suggests that the observed flow patterns can be accounted for by quasi-fixed spatial

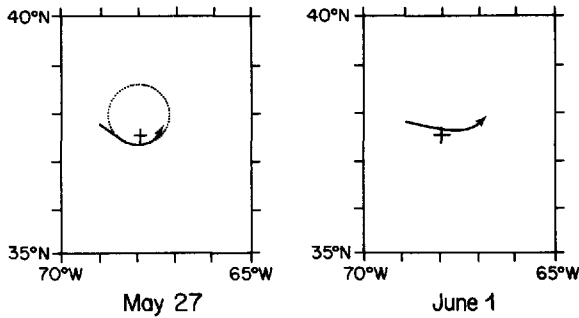


Figure 3. Schematic showing change of curvature in Gulf Stream from May 27, 1983 to June 1, 1983. Cross indicates mooring site. Path is adapted from northern edge of front as shown on satellite composites. Dotted circle has radius of about 70 km and approximately matches curvature of Stream at mooring site on May 27.

patterns like meanders moving past the mooring site. There is qualitative agreement between the calculated along-stream direction of flow and the apparent direction from the satellite pictures, indicating that surface patterns broadly reflect structure in the deeper flow (Fofonoff, personal communication). Figure 4 shows how this idea of moving patterns is consistent with all the calculations from the data for those events. In addition, the events can all be characterized by either Case I or Case II above. In the figure, single line arrows are selected daily along-stream directions, which when placed end to end suggest the spatial pattern which could account for flow direction at the mooring site if the feature passes over the mooring site in the general direction shown by the double dashed line arrows. The X's qualitatively show successive positions of the mooring relative to the propagating features. With each feature is a summary of the behavior of relevant quantities during the event, and its classification according to the above cases. March and early September are good examples of Case I: temperatures are decreasing, but  $\hat{v} > 0$ ; examination of the vertical velocity time series shows that  $w_B > 0$  and  $\partial w / \partial z > 0$  (where  $\partial w / \partial z$  is taken between the thermocline and the bottom). The sign of  $\partial / \partial t (v/r)$  is consistent with the overall stretching between thermocline and bottom during those events. As the meanders propagate past the mooring site, there is a shift in each case from anti-cyclonic to cyclonic flow. The June event is a combination of two cases. In late May/early June, bottom vertical velocities were negative, and accordingly cross-stream velocities were negative as well. However, temperature was locally increasing so that evidently Case Ia is occurring. Consistent with this conclusion,  $\partial w / \partial z < 0$  during that time frame, and (as calculated above).

$$\frac{\partial}{\partial t} \left( \frac{v}{r} \right) < 0$$

as well. Between June 5 and 7,  $w_B$  and  $\hat{v}$  change sign and the flow straightens out to a steady direction of about  $90^\circ$  true, while  $\partial T / \partial t$  remains positive; this case is like II if

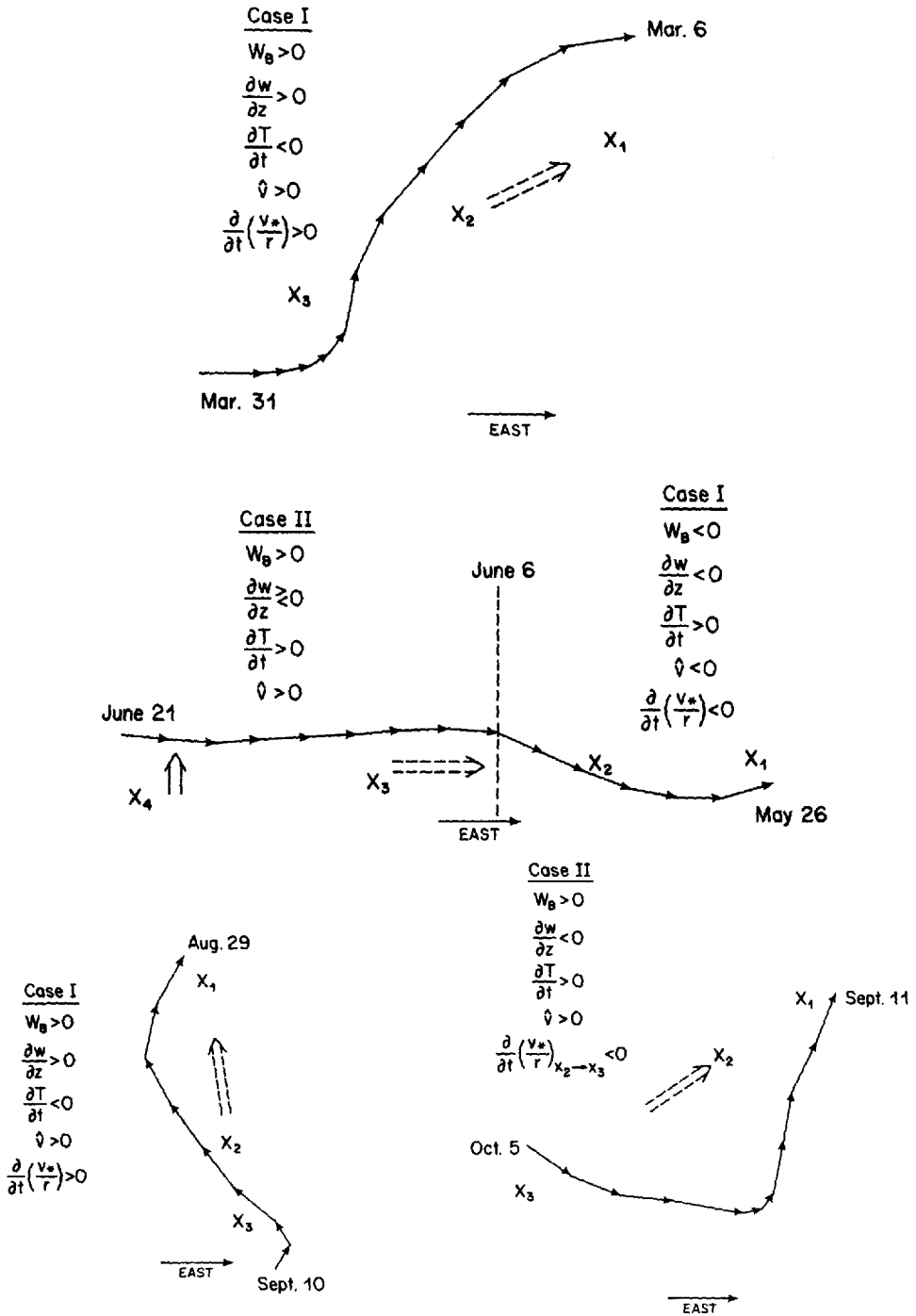


Figure 4. Single line arrows point in direction of flow for selected successive dates during each Gulf Stream passage event; length is proportional to time between successive arrows. Doubled dashed lines are velocities of meanders with shapes outlined by single arrows, required to account for flow at mooring site. Successive qualitative positions of site are indicated by X's. Along-stream velocity in cylindrical coordinates is indicated by  $v_*$ . Relevant information on each event according to classification scheme discussed in text is listed with each feature.



$\partial w/\partial z$  is estimated from 575 to 4000 dbar, which yields negative or *small* positive values. (However,  $\partial w/\partial z$  between 875 dbar and the bottom is decisively positive for the remainder of the event.) In later September, Case II is observed, although a clear indication of the curvature tendency for this event is lacking. Going in detail through the data, one can find other isolated examples of shorter duration that are also consistent with the schematic interpretation presented and fall into one of the four cases enumerated above.

Meanders at this location typically have phase speeds of 8–10 km/d (Hansen, 1970), or 9–12 cm s<sup>-1</sup>. Water parcels above the thermocline certainly have speeds greater than 10 cm s<sup>-1</sup>, however. Thus, rather than the meander passing by a water parcel, the water parcel moves *through* the meander, so the effective change in curvature that it “sees” is opposite to what is “seen” by water parcels below the thermocline. The tendency of  $\partial w/\partial z$  has opposite sign above and below the thermocline as well, so that curvature and stretching terms can consistently balance in both “layers.”

In a study of the dynamics of strongly meandering currents, Chew (1974) shows that changes in path curvature  $v/r$  are balanced by four terms: (1) advection of planetary vorticity; (2) horizontal divergence, or equivalently the stretching term; (3) a curvature-acceleration term, the product of path curvature and downstream changes in speed; and (4) a “banking” term due to the baroclinic mass distribution. The last two terms appear with opposite sign in the tendency equation for lateral shear vorticity  $\partial v/\partial r$  and thus represent the mechanism by which vorticity is exchanged between these two components. In several case studies based on XBT data and free drifting parachute drogues at ~40 m depth (in the Loop Current system in the Gulf of Mexico, and in the Florida Current at its beginning), Chew (1974) consistently found that the  $\beta$ -effect, or advection of planetary vorticity, could be ignored in the dynamical balances. On the other hand, all the other terms were important in contributing to curvature change in one or more of the case studies. Thus, in cases where the banking or curvature acceleration terms are important, changes in lateral shear vorticity are also implied, whence the cross-stream velocity structure is *not* fixed through a meander. Chew also found that as the drogues passed through the cyclonic into the anti-cyclonic part of a meander, they evidently ascended, and on departing the anti-cyclonic turn, descended. Since his results come from data above the thermocline, equivalent to the upper layer in the GUSTO data, ascending motion corresponds to squashing and descending to stretching, so

$$\frac{d}{dt} \left( \frac{v}{r} \right) \quad \text{and} \quad f \frac{\partial w}{\partial z}$$

tended always to be compensating.

Now consider the importance of stretching to the mass balance. In ordinary quasi-geostrophic dynamics, to lowest order  $u_x + v_y = 0$ . At the GUSTO site, however, it is possible that  $\partial w/\partial z$  affects the mass balance at lowest order. To test this idea

quantitatively, continuity is integrated over a cross-section normal to the Stream, from 575 to 4000 dbar:

$$\int_{y_s}^{y_N} \int_{4000}^{575} \frac{\partial \hat{u}}{\partial \hat{x}} dy dz + \int_{4000}^{575} (\hat{v}_N - \hat{v}_s) dz + \int_{y_s}^{y_N} (w_{575} - w_{4000}) dy = 0. \quad (5)$$

If it is assumed that the Stream maintains a steady width (an implicit assumption of the analysis thus far), then  $\hat{v}_N = \hat{v}_s$ , and:

$$\frac{\partial M}{\partial \hat{x}} = - \int_{y_s}^{y_N} (w_{575} - w_{4000}) dy, \quad M = \int_{y_s}^{y_N} \int_{4000}^{575} \hat{u} dy dz. \quad (6)$$

Thus, the transport calculated for the “lower layer” can change in the downstream direction if there is squashing or stretching in that part of the water column. The RHS of (6) has been estimated for the March and June events. For March, when  $\partial w / \partial z$  was basically positive, the RHS has a value of  $-59.7 \text{ m}^2/\text{s}$ . For June,  $\partial w / \partial z < 0$  and  $\text{RHS} = 49.8 \text{ m}^2/\text{s}$ . The widths of the two events from Table III of H1 are nearly the same. Estimated transport for the two events differs by  $32 \times 10^6 \text{ m}^3/\text{s}$ , about half of which occurs below 575 db. With  $\Delta M = 16 \times 10^6 \text{ m}^3/\text{s}$ , a downstream distance  $\Delta x$  can be estimated, over which squashing or stretching must act to produce the observed transport difference:

$$\Delta x = \frac{16 \times 10^6 \text{ m}^3/\text{s}}{(49.8 \text{ to } 59.7) \text{ m}^2/\text{s}} = 268 \text{ to } 321 \text{ km}.$$

Over a length scale of about 300 km, a change in transport *below* 575 db can occur that is comparable to the observed differences between the March and June events. Since velocities have merely been extrapolated to the surface to obtain the total transports, they reflect the changes observed below 575 dbar. However, in the situation described above, in the “upper layer”  $\partial w / \partial z$  generally should have the opposite sign as in the lower layer, and a compensating change in transport ought to be observed in that layer if instruments were there to measure it.

Alternatively, the assumption of a fixed-width Stream may be violated. Then, balancing the last two terms on the LHS of (5) would give (assuming  $\hat{v}_N, \hat{v}_s$  are approximately barotropic):

$$(\hat{v}_N - \hat{v}_s) (3425 \text{ m}) \approx 0(50 \text{ m}^2/\text{s}) \Rightarrow \hat{v}_N - \hat{v}_s \sim 0(1.5 \text{ cm/s}),$$

so that in the presence of stretching (squashing), the Stream would be narrowing (widening) at a rate of 1.5 cm/s or about 1.5 km/day. The situation is again obscured by the possibility that the opposite tendency occurs in the “upper layer,” i.e., above the thermocline. If the cross-stream velocities are indeed barotropic, then stretching and narrowing in the lower layer imply squashing and narrowing in the upper, so that to maintain continuity, along-stream velocities above the thermocline would have to

increase in the downstream direction. Thus one would observe a surface-intensified flow if measurements existed to the surface. On the other hand, if  $v$  is not barotropic (contrary to the indications of H1), a narrowing Stream in the lower layer would imply a widening Stream in the upper layer, and vice versa.

Although these possibilities are difficult to test with the GUSTO data, comparison with Chew's (1974) results suggest that all three terms are important. Keeping in mind that his results are for the upper layer, the following description may be deduced from his results in conjunction with the GUSTO results. Chew found that the drogues on approaching an anti-cyclonic turn tended to slow down and ascend, corresponding to squashing in the upper layer, while the opposite occurred as the drogues left the anti-cyclonic turn. In addition, a spreading of isotherms was observed when the current slowed down, implying the current was widening, while there was evidently contraction where the current accelerated. Thus, in the upper layer,  $\partial\hat{u}/\partial\hat{x}$  and  $\partial w/\partial z$  have the *same* tendency, and the combination is countered by a narrowing or widening current in the term  $\partial\hat{v}/\partial\hat{y}$ . In the lower layer,  $\partial w/\partial z$  has the opposite sign as in the upper, but if a barotropic cross-stream velocity structure is assumed,  $\partial\hat{v}/\partial\hat{y}$  has the same tendency in both layers. Thus, below the thermocline  $\partial w/\partial z$  and  $\partial\hat{v}/\partial\hat{y}$  are the same sign, and must be opposed by downstream changes in speed,  $\partial\hat{u}/\partial\hat{x}$ . This conclusion agrees with what is observed in the GUSTO data, that is, that  $\partial w/\partial z$  and  $\partial\hat{u}/\partial\hat{x}$  are (partially) compensating in the lower layer. This scenario suggests that vertical redistribution of the momentum flux occurs in the meandering process, and that the current does not maintain a fixed width at all times.

#### 4. Summary and conclusions

The GUSTO mooring data, consisting of year-long time-series of velocity and temperature from thermocline to bottom depths, on a single mooring at the mean position of the Gulf Stream at 68W, have added considerably to our understanding of that current. In H1, the descriptive aspects of the velocity field are presented, and some of their dynamical implications discussed. This work extends the analysis to the energetics and dynamics (or kinematics) of the flow, as deduced from the data set, and raises a number of unanswered questions that remain to be addressed.

Perhaps one of the most important results to come out of the energetic analysis is that regarding the sign and relative importance of the barotropic and baroclinic energy conversions between mean and eddy fields. Schmitz (1977), relying on measurements at 4000 m, suggested that the mean field in the vicinity of the Gulf Stream gains kinetic energy from the eddies. Numerous studies using surface data in the Gulf Stream and Kuroshio have shown that the pattern of barotropic energy exchanges is spatially quite complex, and that any of the terms contributing to the exchange may be important. Tables 1 and 2 demonstrate the importance of the upper water column in determining the energy conversions: because the horizontal and vertical shears are so strong in the thermocline, kinetic and potential energy conversions are dominated by

what goes on there. The result, integrated throughout the water column, is a net conversion of mean to eddy energy via both barotropic and baroclinic mechanisms, the former more than twice as great as the latter. This is the first time such complete observations have been available to answer directly the question of what instability mechanism is operative in the eastward flowing Gulf Stream. The result, that there is transfer of mean to eddy kinetic energy, is contrary to what is found far upstream in the Florida Current, which is still accelerating.

A comparison of the implied temporal or spatial growth of eddy energy with observed dispersion characteristics and growth rates (Watts and Johns, 1982; Halliwell and Mooers, 1983) suggests that the GUSTO site is a region of spatial growth, characterized by a growth rate of  $3\text{--}4 \times 10^{-3} \text{ km}^{-1}$ . Both Watts and Johns (1982) and Halliwell and Mooers (1983) found these growth rates to be associated primarily with wavelengths of 330 km, though the other dispersion characteristics (frequency and phase and group speeds) differ greatly. Clearly the GUSTO data are inadequate to determine a wavelength in the downstream direction.

In fact, a complete dynamical analysis of the GUSTO data is rendered difficult by the lack of horizontal resolution. The following observations have been made, however. Estimates of stretching in the water column below the thermocline and of the local change in curvature of the Stream (as deduced from satellite data) suggest that the two may be compensating. Above the thermocline, where  $\partial w/\partial z$  must have the opposite sign, water parcels presumably have velocities greater than typical meander phase speeds, so that they move through the meanders and "see" an opposite change in curvature to that experienced in the lower layer: thus, a balance between stretching and curvature changes is consistent in both layers. There appear to be two flow regimes, characterized by whether local temperature changes are compensated or reinforced by horizontal advection, and relying on the primarily barotropic nature of the cross-stream flow. Comparison with a study by Chew (1974) shows that meandering currents display a redistribution of mass and momentum flux, as well as changes in total width. The kinematic scheme proposed in section 3 invokes the bottom vertical velocity  $w_b$  as forcing; it remains to determine the dynamics governing  $w_b$ .

There are pressing reasons to extend the GUSTO data set vertically as well as horizontally. Many of these are discussed in H1, and still more have arisen as a result of the energetic and dynamical analyses. In particular, the vigorous and strongly sheared flow above the thermocline may be key to energetic conversions, and further confirmation for the two-layer interpretation of the flow is sought. In particular, it may turn out that redistribution of momentum in the vertical, as suggested above, accounts for apparent calculated transport differences (see H1).

Understanding of the flow at the mooring site is clearly incomplete; but the emphasis of this work has been to present the results that should guide future development of analytical models of the Stream, both by providing justification for certain assumptions and by demonstrating the salient features that should emerge as a consequence. In

the former category, the results of H1 have justified modelling the Gulf Stream as a vertically coherent flow. It has also been demonstrated that the along-stream flow should not necessarily be assumed quasi-geostrophic, particularly in the upper part of the water column. Thus, it is to be expected that models such as those of Flierl and Robinson (1984) are more appropriate for this portion of the Gulf Stream than, for example, quasi-gastrophic channel models of barotropic or baroclinic instability. Determining the success of these models is more difficult, since they predict the evolution of the Stream path (axis) in time, rather than anything about the vertical structure, which has been emphasized here. Although much work remains to be done on the problem, the rapid concurrent advancement of observational technology, as well as analytical and numerical modelling tools, could answer many of the interesting questions raised by the GUSTO data.

*Acknowledgments.* This work was supported by grant OCE82-08746 and OCE85-00598, a graduate fellowship from the National Science Foundation, and contract N00014-82-C-0019, NR 083-004 from the Office of Naval Research. Final preparation of the manuscript took place at Scripps Institution of Oceanography, where the author was supported by grant OCE82-17915 from NSF and contract N00014-85-C-0104 from ONR. H. Bryden offered helpful suggestions throughout the development of this work. Discussions with W. Johns clarified several ideas concerning the kinematic descriptions. N. Fofonoff suggested the use of satellite pictures to estimate the impact of changing Stream curvature.

#### REFERENCES

- Bryden, H. L. 1980. Geostrophic vorticity balance in midocean. *J. Geophys. Res.*, *85*, 2825–2828.
- 1982. Sources of eddy energy in the Gulf Stream recirculation region. *J. Mar. Res.*, *40*, 1047–1068.
- Chew, F. 1974. The turning process in meandering currents: A case study. *J. Phys. Oceanogr.*, *4*, 27–57.
- Flierl, G. R. and A. R. Robinson. 1984. On the time-dependent meandering of a thin jet. *J. Phys. Oceanogr.*, *14*, 412–423.
- Fofonoff, N. P. and M. M. Hall. 1983. Estimates of mass, momentum, and kinetic energy fluxes of the Gulf Stream. *J. Phys. Oceanogr.*, *13*, 1868–1877.
- Hager, J. G. 1977. Kinetic energy exchange in the Gulf Stream. *J. Geophys. Res.*, *82*, 1718–1724.
- Hall, M. M. 1986a. Horizontal and vertical structure of the Gulf Stream velocity field at 68W. *J. Phys. Oceanogr.*, (in press).
- 1986b. A diagnostic investigation of kinetic energy budgets in a numerical model. *J. Geophys. Res.*, *91*, 2555–2568.
- Halliwell, G. R., Jr. and C. N. K. Mooers. 1983. Meanders of the Gulf Stream downstream from Cape Hatteras 1975–1978. *J. Phys. Oceanogr.*, *13*, 1275–1292.
- Hansen, D. V. 1970. Gulf Stream meanders between Cape Hatteras and the Grand Banks, *Deep-Sea Res.*, *17*, 495–511.
- Johns, W. E. 1985. Dynamics and structure of Gulf Stream meanders northeast of Cape Hatteras, N.C. Ph.D. thesis, University of Rhode Island, 232 pp.
- Johns, W. E. and D. R. Watts. 1985. Gulf Stream meanders: Observations on the deep currents. *J. Geophys. Res.*, *90*, 4819–4832.

- Knauss, J. A. 1969. A note on the transport of the Gulf Stream. Frederick C. Fuglister Sixtieth Anniversary Volume, *Deep-Sea Res.*, 16 (Suppl.), 117–123.
- Luyten, J. R. and A. R. Robinson. 1974. Transient Gulf Stream meandering. Part II: Analysis via a quasi-geostrophic-time dependent model. *J. Phys. Oceanogr.*, 4, 256–269.
- Nishida, H. and W. B. White. 1982. Horizontal eddy fluxes of momentum and kinetic energy in the near-surface of the Kuroshio extension. *J. Phys. Oceanogr.*, 12, 160–170.
- Orlanski, I. 1969. The influence of bottom topography of the stability of jets in a baroclinic fluid. *J. Atmos. Sci.*, 26, 1216–1232.
- Raymer, M. E., A. Spencer and H. L. Bryden. 1986. Temperature correction for mooring motion on GUSTO current meter time series. W.H.O.I. Technical Report (in prep.).
- Robinson, A. R. and P. P. Niiler. 1967. The theory of free inertial currents. I. Path and structure. *Tellus*, 19, 269–291.
- Schmitz, W. J., Jr. 1977. On the deep general circulation in the western North Atlantic. *J. Mar. Res.*, 35, 21–28.
- 1980. Weakly depth-dependent segments of the North Atlantic circulation. *J. Mar. Res.*, 38, 111–133.
- Schmitz, W. J., Jr. and P. P. Niiler. 1969. A note on the kinetic energy exchange between fluctuations and mean flow in the surface layer of the Florida current. *Tellus*, XXI, 814–819.
- Szabo, D. and G. L. Weatherly. 1979. Energetics of the Kuroshio south of Japan. *J. Mar. Res.*, 37, 531–556.
- Warren, B. A. 1963. Topographic influences on the path of the Gulf Stream. *Tellus*, 15, 167–183.
- Watts, D. R. and W. E. Johns. 1982. Gulf Stream meanders: observations on propagation and growth. *J. Geophys. Res.*, 87, 9467–9476.
- Webster, F. 1965. Measurements of eddy fluxes of momentum in the surface layer of the Gulf Stream. *Tellus*, XVII, 239–245.
- Worthington, L. V. 1976. On the North Atlantic Circulation. *The Johns Hopkins Oceanographic Studies* 6, 110 pp.

Received: 10 January, 1986.

