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Construction of net isopleth plots in cross-sections of tidal estuaries

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ABSTRACT

Construction of isopleth plots of net velocity, material concentration, or material flux in cross-sections of tidal estuaries is not a trivial matter. To construct a flux-preserving isopleth plot requires that each instantaneous measure is weighted by the subarea for which the measure is representative. This area-weighted averaging procedure is outlined. Without area-weighting, net isopleth plots typically yield misleading results in tidal estuaries. In our example, net fluxes of total nitrogen are over-estimated without area-weighting.

1. Introduction

During estuarine flux studies, it is attractive to illustrate spatial variability by graphing isopleths of time-averaged or net quantities in an estuarine cross-section. Quantities such as velocity, material concentration, and material flux are often plotted on a cross-sectional area equal to the mean area of the transect in question. This requires special consideration in the case of tidal estuaries, where the tidal range is an appreciable fraction of the water depth at mean tide. In particular, we will outline how the averaging procedure can be carried out such that net cross-sectional distribution plots properly conserve net discharge, net cross-sectional concentration, and net material flux, respectively. Our main objective is to ensure that the isopleth plot is flux-preserving. However, it should be mentioned that the procedure could either under- or over-estimate the time-averaged quantity itself depending on the phase angles between current, area, and concentration (Kjerfve, 1975). Thus, our technique should be used with discrimination.

2. Analysis and interpretation

Consider a quantity, $f(y, z, t)$, tidally induced to oscillate with time (t), which due to physical or biological processes varies laterally (y) or vertically (z) in the cross-section. The quantity $f(y, z, t)$ could be velocity, material concentration, or

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material flux. The cross-sectional area,

$$A(t) = \iint dydz \tag{1}$$

oscillates more or less sinusoidally around the net cross-sectional area, A_0, because of tidal action. The degree to which $A(t)$ oscillates can be quantified as a percentage of variation, $\epsilon = 100 \times$ rms of $A(t)/A_0$.

In constructing graphs of net distribution of $f(y, z, t)$ in the estuarine cross-section, we first calculate time-averages of $f(y, z, t)$ everywhere in the cross-section, i.e.,

$$f_0(y, z) = \int_0^T f(y, z, t)\, dt/T \tag{2}$$

where T is one or more complete tidal cycles. However, this proves not to be straightforward, as we require that the integration of $f_0(y, z)$ over the mean cross-sectional area

$$F_* = \iint_{A_0} f_0(y, z)\, dA \tag{3}$$

equals the time-average of the instantaneous discharges, concentrations, or material fluxes, respectively, such that $F_* = F_0$ where

$$F_0 = \int_0^T \iint_{A(t)} f(y, z, t)\, dA\, dt/T. \tag{4}$$

The problem lies in the numerical evaluation of f_0 in tidal estuaries where ϵ exceeds a few percent. For each cross-sectional measurement position, Kjerfve (1975, 1979) has proposed time-averaging of f at nondimensionalized depths between surface and bottom. As the tide varies, the depth is telescoped up or down. A f_0-value is the average of n sequential f-values. Each of these f-values is estimated for the same position relative to the total depth. However, they are at the same time estimates at varying absolute depths below the surface, depending on tidal stage. F_* is then estimated from Eq. (3) numerically by multiplying each f_0-value by a percentage of A_0 and summing the products over the cross-section. The resulting F_*-value will in general not equal F_0 from Eq. (4) except in the absence of tides. From experience, we know that the disagreement between F_* and F_0 increases with increasing ϵ. It is our intention here to modify Kjerfve's (1975, 1979) averaging scheme for estuaries with substantial ϵ values in such a way that $F_* = F_0$.

Ideally, we would like to rewrite Eq. (3) analytically, exchange the order of integration, and put Eq. (3) in a form similar to Eq. (2). In general, transposing the order of integration cannot be done analytically as the limits of the area-integration in Eq. (4) are functions of time. However, with a sufficiently dense set of f-measurements in the cross-section, repeated an adequate number of times, F_0 can easily be evaluated numerically by first computing instantaneous area-integrations of f and then average these over time (Kjerfve, 1979). Similarly, F_* can be calculated numerically from Eq.

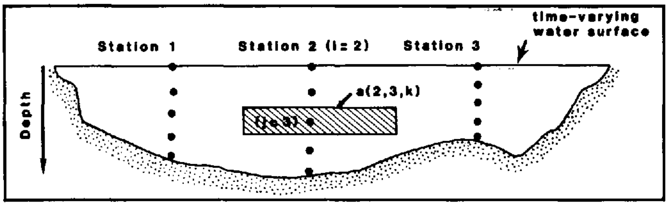

Figure 1. Illustration of a subarea, $a(2, 3, k)$, of the cross-section.

(3) by area-integration of f_0. For consistency in analysis and interpretation of field data, it seems reasonable to require and force F_* to be equal to F_0. The difficulty lies in the construction of an appropriate isopleth plot of $f_0(y, z)$ or the computation of the time-average of $f(y, z, t)$ with $\epsilon \neq 0$.

At this point, it is appropriate to mention that $f(y, z, t)$ can represent either velocity $[LT^{-1}]$, material concentration $[ML^{-3}]$, or material flux $[\mathrm{ML}^{-2}T^{-1}]$. In an estuarine flux study, velocity measurements should be made simultaneously with concentration determinations (e.g., from water samples) at each cross-sectional sampling point. The material flux is simply the product of velocity and concentration (Kjerfve, 1979). We (cf. Chrzanowski *et al.*, 1979; Kjerfve and Proehl, 1979) usually measure velocity and concentration at several depths for each location in an estuarine cross-section. Vertical profiles are interpolated with cubic splines from surface to bottom at p equidistant points. Depending on water depth and number of measurements with depth, we have commonly selected p to be 5, 11, or 21, including interpolated values at the surface and bottom. The number of lateral sampling locations, m, has varied from 1 to 11, depending on cross-sectional width, tidal prism, and logistic capability (Kjerfve *et al.*, 1981, 1982). The constant sampling rate, Δt, has varied from 20 min to 1.5 lunar hours with the constraint that the minimum sampling duration, $n\Delta t$, equals the period of a complete tidal cycle. However, we have usually taken the sampling duration to be an even number of consecutive tidal cycles (usually two or four) to minimize effects of diurnal tidal inequality.

Let $f(i, j, k)$ represent the f-value at lateral position $i = 1, 2, \ldots m$; at interpolated depth $j = 1, 2, \ldots p$; and at sampling time $k = 1, 2, \ldots n$. This f-value is representative for a subarea, $a(i, j, k)$, of the total instantaneous cross-sectional area (Fig. 1). The time-average of $a(i, j, k)$ is given by

$$a_0(i, j) = \sum_{k=1}^{n} a(i, j, k)/n \qquad (5)$$

which sum to the net cross-sectional area, i.e.,

$$A_0 = \sum_{i=1}^{m} \sum_{j=1}^{p} a_0(i, j). \qquad (6)$$

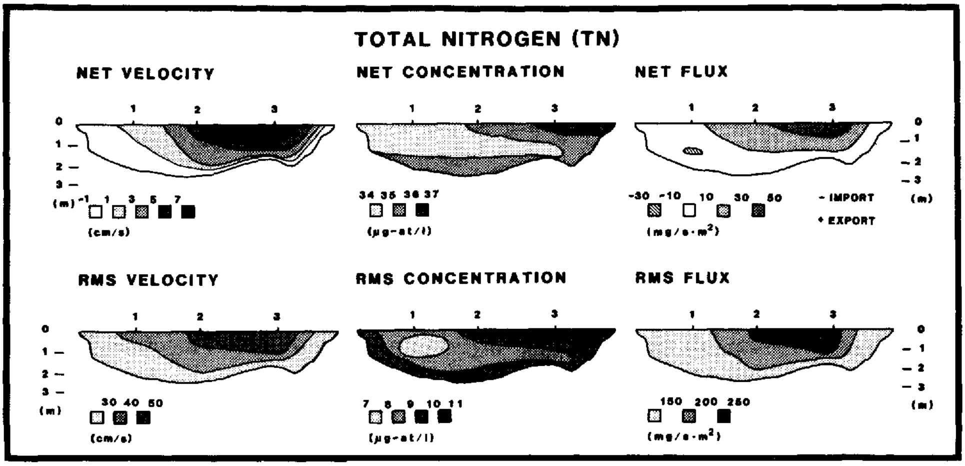

Figure 2. Isopleth plots of net and rms distributions of velocity, total nitrogen concentration, and total nitrogen flux without area-weighting.

Eqs. (3) and (4) may be rewritten as

$$F_* = \sum_{i=1}^{m} \sum_{j=1}^{p} f_0(i,j)\, a_0(i,j) \tag{7}$$

and

$$F_0 = \sum_{k=1}^{n} \left[\sum_{i=1}^{m} \sum_{j=1}^{p} f(i,j,k)\, a(i,j,k) \right] \Big/ n \tag{8}$$

respectively. It is then easy to show that if the time-average of $f(i, j, k)$ is written

$$f_0(i,j) = \left[\sum_{k=1}^{n} f(i,j,k)\, a(i,j,k) \right] \Big/ [n \cdot a_0(i,j)] \tag{9}$$

the condition that F_* equals F_0 is satisfied. Similarly, the root-mean-square (rms) deviation of $f(i, j, k)$ may be expressed

$$f_{\rm rms}(i,j) = \left(\sum_{k=1}^{n} [f(i,j,k)\, a(i,j,k) - a_0(i,j) f_0(i,j)]^2 / [(n-1)\, a_0^2(i,j)] \right)^{1/2}. \tag{10}$$

The summation of $f_{\rm rms}(i, j)$ over the entire cross-section is similarly identical to the temporal rms deviation of the instantaneous cross-sectional flux from the F_0-value.

Distribution of time-averaged and rms quantities in tidal estuarine cross-sections is best and most consistently displayed by plotting $f_0(i, j)$ and $f_{\rm rms}(i, j)$, respectively, for the m stations and p interpolated depths on the net cross-sectional area, A_0. The advantage of this new way [Eq. (9)] of computing time-averages is that it properly weights each flow, concentration, or flux measurement by the appropriate, instantaneous cross-sectional subarea in such a way that consistency is maintained whether time-averaging is performed before area-averaging or vice versa. Similarly, the rms

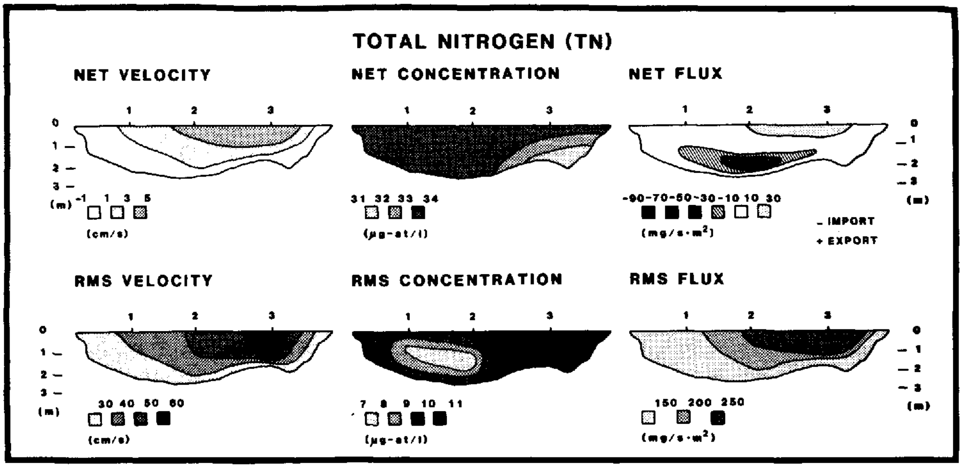

Figure 3. Isopleth plots of net and rms distribution of velocity, total nitrogen concentration, and total nitrogen flux with area-weighting according to Eqs. (9) and (10).

computations [Eq. (10)] yield consistency. The f_{rms}-values are most readily interpreted as the intensity of the water or material tidal transport, or in the case of concentration, the time-variability of the tidal concentration variation.

3. Example

As an example of the averaging procedures, we will carry out comparative net and rms computations on the same data sets of velocity and total nitrogen concentration. The data along with analytical procedures and interpretations will be covered in a future paper. The measurements were made in Bly Creek of the North Inlet estuary in South Carolina. The ϵ value measures 33%. Three stations in the 45 m wide cross-section were covered simultaneously every 40 min for two consecutive tidal cycles 15–16 October 1982. Velocity measurements were made with current crosses (Kjerfve, 1982) every 0.5 m from surface to bottom and water samples for total nitrogen analysis were drawn at three depths at each station, and then spline-fit from surface to bottom according to Chrzanowski *et al.* (1979). For each cross-sectional sub-section (Fig. 1), the material flux is calculated as the product of instantaneous velocity, material concentration, and subarea. It is recognized that each of these parameters is a time-varying function.

Figure 2 shows the net and rms distributions of velocity, total nitrogen concentration, and total nitrogen flux without weighting each value according to the corresponding subarea (cf. Kjerfve, 1975). In comparison, Figure 3 shows the same plots using the area-weighted averaging scheme according to Eqs. (9) and (10). It is apparent that the two sets of figures differ significantly.

In general, differences between the two sets of rms plots are fairly minor (Table 1), the area-weighting causing only qualitative changes in location and intensity of cores

Table 1. Comparison of numerical values for this example with and without area-weighting. Net and rms values were computed similarly to Eq. (7).

	Net comparisons		rms comparisons	
Parameter	area-weighting	no area-weighting	area-weighting	no area-weighting
Discharge (m^3/s)	+ 1.2	+ 3.5	25.6	21.3
Total N conc. (μg-at/l)	33.2	34.8	9.3	8.7
Total N flux (g/s)	0.0	+10.0	12.0	9.3

of high variability. However, differences between the two sets of net plots is both more apparent and of greater significance (Table 1). This is particularly pronounced in the net total nitrogen flux, where the cross-section integration of f_0-values (Eq. 7) yields a vastly different picture (Fig. 3) than averaging without area-weighting (Fig. 2). Similarly, the discrepancy between the net velocity and net concentration graphs, respectively, is quite large.

In general, we have found that the net values with area-weighting according to Eq. (7) yield smaller values than without area-weighting. However, the rms-values with area-weighting exceed those without area-weighting. The discrepancy generally increases with increasing ϵ value or tidal range. We recommend that Eqs. (9) and (10) be used in construction of net and rms isopleth plots in tidal estuaries, especially where ϵ exceeds 10%.

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