Journal of Marine Research, 42, 15–27, 1984

Dissipation and diffusion by internal wave breaking

by Ann E. Gargett[1] and Greg Holloway[1]

ABSTRACT

Recent direct observations of the rate of kinetic energy dissipation, ϵ, tend to vary systematically with buoyancy frequency, N. This note presents arguments leading to an expected relationship between these two parameters. We first suggest that the classical separation of velocity field into "turbulent" and "mean" (including internal waves) is inappropriate for a stratified system such as the ocean, in which nonlinear forces and buoyant restoring forces act over a wide range of space-time scales. Reconsidering the steady-state kinetic energy equation without this separation, we obtain $\epsilon \propto N^{1.0}$ or $\epsilon \propto N^{1.5}$, where the ambiguity in exponent is associated with uncertainty with regard to the appropriate form for the vertical velocity variance of the internal wave field. With similar assumptions in the steady-state equation for available potential energy (APE) it is shown that the rate of dissipation of APE, γ, also varies as $\gamma \propto N^{1.0}$ or $\gamma \propto N^{1.5}$, where ambiguity in exponent again derives from internal wave vertical velocity variance. If, in addition, the flux Richardson number is independent of N, the vertical eddy diffusivity for mass K_ρ associated with internal wave mixing varies as $K_\rho \propto N^{-1.0}$ or $K_\rho \propto N^{-0.5}$.

It has recently become possible to estimate the oceanic rate of kinetic energy dissipation ϵ from direct measurement of small-scale shear (Osborn and Crawford, 1980). Available data summarized in Figure 1 (Gargett and Osborn, 1981; Leuck *et al.*, 1983) suggest the approximate relation $\bar{\epsilon} = a_0 N^{+q}$, where $\bar{\epsilon}$ is an ensemble average,

$$N \equiv \left(\frac{-g}{\rho_0} \frac{\partial \bar{\rho}}{\partial z} \right)^{1/2}$$

is the buoyancy frequency, $q \sim 1$ and a_0 is "constant" within a factor of 2 or 3. The purpose of the present note is to suggest a possible explanation for this observation based upon the hypothesis that in the interior of mid-latitude oceans, turbulent dissipation originates through the processes of internal wave breakdown (double diffusion is not considered).

Consider the kinetic energy equation for a rotating stratified fluid under the

1. Institute of Ocean Sciences, Patricia Bay, P.O. Box 6000, Sidney, B.C., Canada, V8L 4B2.

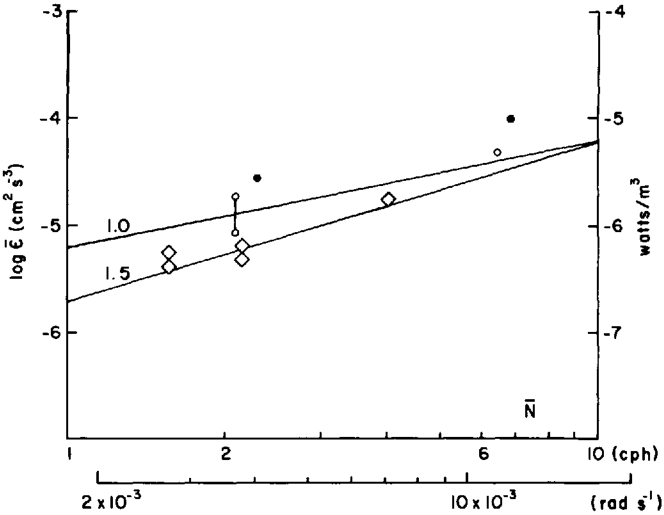

Figure 1. Ensemble-averaged values of kinetic energy dissipation rate ϵ as a function of N in logarithmic coordinates. The measurements of Gargett and Osborn (1981) were taken in the Sargasso Sea (open circles) and near Bermuda (filled circles); those of Leuck *et al.* (1983) are from continental shelf and slope waters off British Columbia, the far eastern North Pacific. The difference between joined symbols at the same N is due to finite instrumental noise levels. Lines of slope +1 and +1.5 are drawn for reference.

Boussinesq approximation

$$\frac{\partial}{\partial t}\left(\frac{1}{2}u_i^2\right) + u_i u_j \frac{\partial u_i}{\partial x_j} = -\rho_0^{-1} u_i \frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(u_i\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$$
$$-g\rho_0^{-1}\tilde{\rho}\, w - \frac{\nu}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 \quad (1)$$

where $u_i = (u_1, u_2, u_3 \equiv w)$ are components of the velocity vector (z is positive upward), p is pressure, ν is kinematic viscosity, the perturbation density $\tilde{\rho}$ is defined as the instantaneous departure from a slowly varying background field $\tilde{\rho} = \rho_0(1 - g^{-1}N^2 z)$, and repeated indices are summed over $i = 1, 2, 3$. Since Coriolis acceleration is perpendicular to the momentum vector, it is absent from the energy equation.

It has been customary (Osborn, 1980; Dillon, 1983) to separate the velocity field into "mean" and "turbulent" parts, assuming first that it is possible to separate internal waves from "turbulent" motions and second, that wave motions may be considered part of the "mean." (Velocity fields which are nonpropagating, cause fluid parcels on average to cross mean density surfaces, and dominate mean-square shear at dissipation scales are called turbulence; velocity fields which propagate, i.e., frequency and wavenumber, are connected through a dispersion relation, and are relatively nondissipative are termed waves.) It is becoming clear that a scale-dependent separation of the velocity field may not be possible in the stratified interior of the ocean. An example of a fully resolved spectrum is the vertical shear spectrum of Gargett *et al.* (1981) which is flat (k^0) at low vertical wavenumber, falls as k^{-1} between $k = k_0 = 0.1$ cpm and $k = k_b \equiv (N^3/\epsilon)^{1/2}$, then finally rises (no steeper than k^{+1}) to a dissipation-scale peak before falling steeply for $k \gtrsim k_s \equiv (\epsilon/\nu^3)^{1/4}$. When this shear spectrum is converted to a velocity spectrum (by dividing by the square of vertical wavenumber), the result is monotonic decreasing: there is no evidence of a spectral gap which might be used to separate mean and turbulent motions by defining a suitable averaging length. The velocity spectral slopes are sufficiently steep that the energy of the system resides at low vertical wavenumbers, the region of the spectrum which obeys WKB-scaling (Gargett *et al.*, 1981) and is interpreted as internal waves (Garrett and Munk, 1975). Attempting to distinguish between "mean" and "turbulent" velocities on the basis of scale-separation appears to be inappropriate for an environment in which both nonlinear interactions and buoyant restoring tendencies are active over a wide range of scales.

Our approach will be to defer making any "mean"/"turbulent" or "waves"/"turbulence" distinction throughout as much of our argument as possible. Assuming only a statistically steady-state, we time-average Eq. (1), obtaining

$$\overline{u_i u_j \frac{\partial u_i}{\partial x_j}} = -\rho_0^{-1} \frac{\partial}{\partial x_i} \overline{(u_i p)} + \nu \frac{\partial}{\partial x_j} \overline{\left(u_i \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)} - g \rho_0^{-1} \overline{\tilde{\rho} w} - \epsilon \qquad (2)$$

where

$$\epsilon = \frac{\nu}{2} \overline{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2}$$

is the kinetic energy dissipation rate, and the incompressibility condition $\partial u_i/\partial x_i = 0$ has been used. Time-averaging is denoted by an overbar; the averaging time must be sufficiently long to ensure the steady-state of the energy-containing (wave) motions. The second term on the right-hand side is negligible compared with ϵ (their ratio is $(\lambda/L)^2 \ll 1$, where λ is an internal length scale characteristic of the shear (dissipation) field and L is an external length scale characteristic of the velocity (energy-containing) field (Tennekes and Lumley, 1972, Sec. 3.2). The meager observational information on the pressure-velocity correlation (Elliott, 1972; Wyngaard and Cote, 1971) suggests that its divergence makes only a small contribution (of order 10%) to the kinetic energy budget of the highly anisotropic stable atmospheric boundary layer: in the more isotropic (horizontally at least) oceanic internal wave field, it should be an even smaller part. Thus the dominant balance is

$$\overline{u_i u_j \frac{\partial u_i}{\partial x_j}} = -g \rho_0^{-1} \overline{\tilde{\rho} w} - \epsilon. \qquad (3)$$

Although the oceanic shear tensor is dominated by terms involving the vertical derivative of horizontal velocities, the contribution of the purely horizontal stress/shear terms to the triple correlation in (3) is not known. Based upon this lack of information, we choose to discard the horizontal divergences (these are identically zero if the internal wave field exhibits horizontal homogeneity), and further simplify (3) to

$$\overline{u_i w \frac{\partial u_i}{\partial z}} = -g \rho_0^{-1} \overline{\tilde{\rho} w} - \epsilon \tag{4}$$

where now the repeated index summation is over $i = 1, 2$ only.

We finally assume that the processes in this system are characterized by a small flux Richardson number,

$$0 < R_f \equiv \frac{\overline{-g \rho_0^{-1} \tilde{\rho} w}}{\overline{u_i w \frac{\partial u_i}{\partial z}}} \ll 1 \tag{5}$$

as suggested by the laboratory results of Britter (1974) and McEwan (1980) and the oceanic measurements of Oakey (1982). Note that although R_f as defined above differs from its more normal definition (i.e., where the denominator is written as $\overline{u_i w}\, \partial \overline{U}_i/\partial z$), the denominator still represents the source of energy, hence the argument of Stewart (1959) that R_f must be considerably less than 1 still holds. Using (5), Eq. (4) becomes

$$0 < \overline{-u_i w \frac{\partial u_i}{\partial z}} = \frac{\epsilon}{1 - R_f} = \epsilon \text{ since } R_f \ll 1. \tag{6}$$

Introducing a nondimensional triple correlation coefficient C_1, Eq. (6) can be written as

$$\overline{-u_i w \frac{\partial u_i}{\partial z}} = C_1 \left[\overline{u_i^2}\, \overline{w^2}\, \overline{\left(\frac{\partial u_i}{\partial z}\right)^2} \right]^{1/2} \simeq \epsilon. \tag{7}$$

We now examine the N-dependence of the variances in Eq. (7), first noting that the correlation coefficient C_1 cannot depend on N since there is no other intrinsic time scale with which to form a nondimensional variable.

It is at this point that we introduce the hypothesis that wavelike motions determine the variances appearing in (7), supplying the energy which is dissipated at the smallest scales. If, as suggested by Munk (1981), total shear variance in the internal wave field is Richardson number limited, a characteristic wave Richardson number

$$Ri_w \equiv \frac{N^2}{\left[\left(\frac{\partial u_i}{\partial z} \right)^2 \right]_w} \sim 0(1), \tag{8}$$

where the subscript implies that the shear variance

$$\left[\overline{\left(\frac{\partial u_i}{\partial z}\right)^2}\right]_w = \int_0^{k_u} \phi_s \, dk$$

is due to wavelike motions. This suggestion (hence (8)) has been supported by the observations of Gargett *et al.* (1981), which indicate that $Ri_w \sim 1$ if $k_u = 0.1$ cpm (the wavenumber of the observed change in slope of ϕ_s from k^0 to k^{-1}) and $Ri_w \sim 1/4$ if $k_u = k_b \equiv (N^3/\epsilon)^{1/2}$ (at and beyond k_b, the "turbulence" parameter ϵ is necessary for scaling shear spectral level). Thus the appropriate shear variance to use in (7) is

$$\left[\overline{\left(\frac{\partial u_i}{\partial z}\right)^2}\right]_w \sim N^2$$

regardless of the exact nature of wave breaking.

The N-dependence of the product $\overline{u_i^2}\,\overline{w^2}$ depends upon the character of the wave field. We identify two extremes:

Type (1) is the narrow-band case of internal waves of a (nearly) single frequency, and includes internal seiches in closed basins and various classes of topographic lee waves. Wave velocities obey WKB scaling (Phillips, 1977) with

$$\overline{u_i^2} \sim N \qquad \text{and} \qquad \overline{w^2} \sim N^{-1}$$

Since from (8)

$$\overline{\left(\frac{\partial u_i}{\partial z}\right)^2} = \left[\overline{\left(\frac{\partial u_i}{\partial z}\right)^2}\right]_w \sim N^2$$

Eq. (7) yields

$$\epsilon \sim (N\,N^{-1}N^2)^{1/2} \sim N^{+1} \tag{9}$$

Type (2) is the broad-band case of a multi-wave environment in which frequency band-width as well as velocity magnitude varies with N: the classical example is the deep-ocean internal wave field as described by the GM model spectrum (Garrett and Munk, 1975, 1979). Munk (1981, see Eq. (9.24)) shows that for the semi-empirical spectrum GM79,

$$\overline{u_i^2} = \overline{u_1^2} + \overline{u_2^2} \sim N$$

A straightforward calculation similar to those carried out by Munk for other mean-square quantities shows that for GM79,

$$\overline{w^2} \sim \overline{N^0}$$

So that Eq. (7) gives

$$\epsilon \sim (N\,N^0\,N^2)^{1/2} \sim N^{+1.5} \tag{10}$$

Although it seems likely that the actual oceanic interval wave field is associated with $\epsilon = a_0 N^{+q}$, where $1 \leq q \leq 1.5$, the appropriate choice for q is far from clear, due to uncertainty as to the appropriate scaling for $\overline{w^2}$. In fact, observed oceanic vertical velocity spectra are often not well described by GM79, which assumes that vertical velocity is white with frequency between f and N. Many spectra, such as that derived from a rotating drifting float (Voorhis, 1968), or those derived from isotherm displacements at depths greater than 200 m (Desaubies, 1975; Pinkel, 1981) are all reasonably flat for an interval above f, but rise to a broad peak at N (c.f. Fig. 2a). When plotted in variance-preserving form (Fig. 2b), the dominance of the vertical velocity variance by motion of (more nearly) a single frequency is obvious. The dominance is further enhanced in the upper 200 m, where observed vertical velocity spectra are depleted at low frequencies relative to the deeper spectra (Pinkel, 1981). If this observed Väisälä "ringing" of the ocean is due to wave groups which individually obey WKB scaling, and if it dominates the time-averaged vertical velocity field, then the oceanic internal wave field would be closer to Type (1) (hence $\epsilon \propto N^{+1}$) than Type (2) above. If not, we must await experimental determination of the appropriate scaling for $\overline{w^2}$. Whatever the value of q, the "constancy" of the proportionality factor a_0 should be of the same degree as the "universality" (factor of 2-3) of the internal wave field if, as we are assuming, those processes producing dissipation are just those responsible for maintaining the statistically steady-state of the wave field.

We should point out that the relatively small variation in N between the abyssal ocean ($N \sim 1$ cph) and the seasonal thermocline ($N \sim 6$ cph), coupled with noise and/or sampling problems associated with microscale measurements, makes it difficult to test the proposed weak dependence of ϵ (or K_ρ) on N: we need more measurements with lower noise levels at all depths. The stronger dependence of χ (as N^{+3} or $N^{+3.5}$), may prove easier to test. Because even a weak dependence of ϵ, hence K_ρ, upon N has dynamical implications for ocean circulation (Gargett, 1983), it seems imperative to carry out such measurements.

As an aside, we note that a result reminiscent of the Type (1) expression for ϵ as a function of N has been derived for application to the stably stratified atmosphere (Businger and Arya, 1974; Zeman and Tennekes, 1977) by using energetic arguments to obtain

$$\epsilon = C_0 \sigma_w{}^2 N$$

where σ_w^2 is the variance of the "turbulent" vertical velocity. Weinstock (1981) determined a numerical value of 0.4 for the constant C_0, under the assumptions of isotropic and locally inertial spectral forms for $k > k_b = (N^3/\epsilon)^{1/2}$, and claimed satisfactory agreement between measured ϵ and $C_0 \sigma_w{}^2 N$. Understanding the connection between the atmospheric and oceanic cases is complicated by two factors. First, the value of N is constant ($N = 2.1 \times 10^{-2}\,\mathrm{s}^{-1}$, the isothermal stratospheric value) for the atmospheric measurements, so they do not constitute a critical test of the

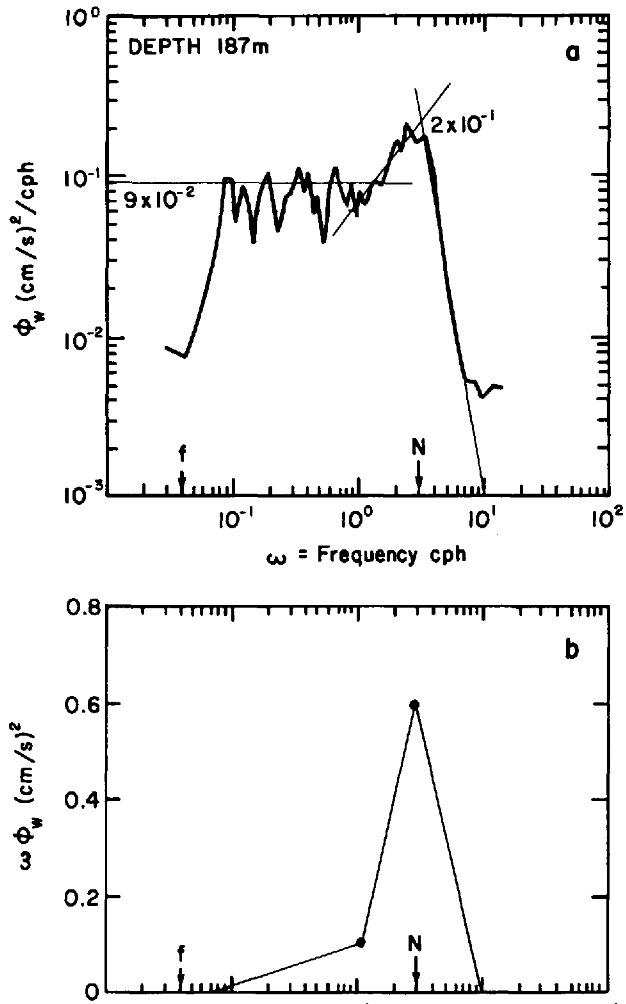

Figure 2. (a) Spectrum of vertical velocity as a function of frequency, determined from isotherm displacement measurements (Pinkel, 1981). The variance is confined to internal wave frequencies between f and local N, and the peak at the buoyancy frequency is typical. (b) A variance-preserving plot of this spectrum (values taken approximately from (a)) makes it clear that vertical velocity fluctuations are dominated by motions of near buoyancy frequency.

dependence of ϵ on N. Secondly, in the atmosphere as well as the ocean it is not clear how to separate the "turbulent" part from a total vertical velocity variance which is dominated by wave motions, hence has an implicit N-dependence. Weinstock (1981) suggested that the turbulent part should be defined as the variance contribution from wave numbers greater than k_b, a suggestion which is consistent with the interpretation of the oceanic spectra advanced by Gargett *et al.* (1981) and accepted here. However actual atmospheric variances often appear to be calculated using a fixed lower wavenumber for convenience, regardless of the local value of ϵ, hence k_b (for example, Lilly *et al.* (1974) use $k = 2\pi/(640\text{ m})$ as the low wavenumber limit for calculation of σ_w^2). These features, coupled with possible effects due to anisotropy of "turbulent" velocity fields in a stratified fluid, leave the value and constancy of the coefficient C_0 in some doubt.

The present analysis can be extended to include dissipation rates of available potential energy (APE). However, this raises some new and potentially disturbing questions.

The evolution equation for perturbation density $\tilde{\rho}$ is

$$\frac{\partial \tilde{\rho}}{\partial t} + u_i \frac{\partial \tilde{\rho}}{\partial x_i} - w \frac{N^2}{g \rho_0^{-1}} = \kappa \frac{\partial^2 \tilde{\rho}}{\partial x_i \partial x_i} \tag{11}$$

where κ is the effective diffusivity for $\tilde{\rho}$ (if density is controlled by temperature, κ is thermal conductivity times volumetric coefficient of thermal expansion) and repeated indices are summed over $i = 1, 2, 3$. From (11), perturbation density variance is governed by:

$$\frac{\partial}{\partial t}\overline{\left(\frac{\tilde{\rho}^2}{2}\right)} + \overline{\tilde{\rho} u_i \frac{\partial \tilde{\rho}}{\partial x_i}} - \overline{\tilde{\rho} w} \frac{N^2}{g \rho_0^{-1}} = \kappa \overline{\frac{\partial}{\partial x_i}\left(\tilde{\rho}\frac{\partial \tilde{\rho}}{\partial x_i}\right)} - \kappa \overline{\left(\frac{\partial \tilde{\rho}}{\partial x_i}\right)\left(\frac{\partial \tilde{\rho}}{\partial x_i}\right)} \tag{12}$$

Perturbation density variance is related to APE by considering the hypothetical vertical displacement ζ which a fluid element would experience if moved adiabatically to that depth at which its perturbation density vanishes. Thus

$$\zeta = -\tilde{\rho} g N^2 \tag{13}$$

Gravitational energy released by such hypothetical displacement is

$$-\tfrac{1}{2}\tilde{\rho} g \zeta = \tfrac{1}{2}\tilde{\rho}^2 g^2 N^{-2} \tag{14}$$

where a coefficient $\tfrac{1}{2}$ enters because $\tilde{\rho}$ relaxes linearly to zero over the total displacement ζ. Identifying APE as $\tfrac{1}{2}\overline{\tilde{\rho}^2} g^2 N^{-2}$, we see from (12) that $g\overline{\tilde{\rho} w}$ acts as a source term for APE just as $g\overline{\tilde{\rho} w}$ acts as a sink in the kinetic energy equation (2).

The second term on the right side of (12) is a dissipation term for density variance. The density variance dissipation rate $\chi \equiv \kappa\overline{(\partial\tilde{\rho}/\partial x_i)(\partial\tilde{\rho}/\partial x_i)}$ is related to the dissipation rate γ for APE through (14):

$$\gamma = g^2 N^{-2} \chi \tag{15}$$

Our goal is to seek a relationship between γ (or χ) and N.

The first term on the right side of (12) is small compared with the second by a ratio $(\lambda_\theta/L) < (\lambda/L) \ll 1$, as discussed in connection with Eq. (1). Thus a stationary balance in (12) requires

$$\overline{\tilde{\rho} u_i \frac{\partial \tilde{\rho}}{\partial x_i}} - \overline{\tilde{\rho} w} \frac{N^2}{g \rho_0^{-1}} = -\chi \tag{16}$$

It has been customary to cast the triple correlation term as a flux divergence

$$\frac{\partial}{\partial x_i} \left(\overline{\frac{1}{2} u_i \tilde{\rho}^2} \right)$$

and to suppose that such terms are negligible, permitting the estimation of vertical mass flux from direct observations of $\partial\bar{\rho}/\partial z$ and χ. As first introduced by Osborn and Cox (1972), the technique actually estimates vertical heat flux from measurements of $\partial\overline{T}/\partial z$ and microscale temperature gradient variance, then assumes that the turbulent eddy diffusivity for density $K_\rho \equiv -\overline{\tilde{\rho} w}/(\partial\bar{\rho}/\partial z) = g\rho_0^{-1}\overline{\tilde{\rho} w}/N^2$ is equal to that for temperature $K_T \equiv -\overline{\tilde{T} w}/(\partial\overline{T}/\partial z)$. This technique has been widely used (Gregg *et al.*, 1973; Gregg, 1977; Gargett, 1976; Oakey and Elliott, 1977; Caldwell *et al.*, 1980 are but a few examples) and the inferred values of K_ρ have found wide use in climate studies (ocean heat storage) and in studies of geochemical distribution balances. However just as we have considered the role of a triple correlation term in the kinetic energy budget, we are obliged to consider this term in the APE budget or alternatively, the budget of fluctuation density variance (16).

As previously, we will neglect horizontal contributions to the balance (16), although this is not easily justified: we depend upon horizontal isotropy. The triple correlation can be expressed as

$$\overline{\tilde{\rho} w \frac{\partial \tilde{\rho}}{\partial z}} = C_2 \left[\overline{\tilde{\rho}^2}\, \overline{w^2}\, \overline{\left(\frac{\partial \tilde{\rho}}{\partial z}\right)^2} \right]^{1/2} \tag{17}$$

where C_2 is another nondimensional triple correlation coefficient. By WKB scaling, consistent with GM79,

$$\overline{\tilde{\rho}^2} \sim \overline{\zeta^2}\, N^4 \sim N^3$$

We have indicated that $\overline{w^2}$ may take one of two forms, namely

$$\text{Type (1): } \overline{w^2} \sim N^{-1}$$
$$\text{Type (2): } \overline{w^2} \sim N^0$$

Finally,

$$\overline{\left(\frac{\partial \tilde{\rho}}{\partial z}\right)^2_w} \sim \overline{\left(\frac{\partial \zeta}{\partial z}\right)^2_w} N^4$$

where the condition

$$\overline{\left(\frac{\partial \zeta}{\partial z}\right)_w^2} = 0(1) \tag{18}$$

is a gravitational overturning condition, as (8) is a shear instability condition. For a broad spectrum of waves, the two conditions may not be distinct and can be expected to occur together (Munk, 1981). Thus the right side of (17) is expected to vary as

$$\begin{aligned} &\text{Type (1): } N^3 \\ &\text{Type (2): } N^{3.5} \end{aligned} \tag{19}$$

The term $\overline{\tilde{\rho} w}\ N^2/g\rho_0^{-1}$ is easily seen to have the same N variation as the triple correlation term.

The relative importance of the two terms cannot be decided *a priori,* since it depends upon the relative magnitudes of the triple correlation coefficient C_2 and the correlation coefficient for $\overline{\tilde{\rho} w}$, both unknown. Holloway (1983) conjectures that the shapes of both kinetic and potential energy wavenumber spectra can be explained assuming relative dominance of the triple correlation term; in this case, estimates of K_ρ based on temperature microstructure observations will be overestimates.

Since both terms on the left side of (16) vary as (19), the dissipation rate γ for APE varies as

$$\begin{aligned} &\text{Type (1): } N^1 \\ &\text{Type (2): } N^{1.5} \end{aligned}$$

just as does ϵ. The ratio γ/ϵ should be nearly constant, as has been observed by Oakey (1982) from simultaneous observations of shear and temperature gradient fine structure.

We are now in a position to examine the question of density diffusion produced by internal wave breaking processes. We introduce the standard concept of gradient diffusion by the usual definition of a "turbulent" diffusivity for density

$$K_\rho \equiv \frac{\overline{-\tilde{\rho} w}}{\dfrac{\partial \bar{\rho}}{\partial z}} = \frac{g\,\rho_0^{-1}\,\overline{\tilde{\rho} w}}{N^2} \tag{20}$$

Using this definition alone with expression (5) for R_f, the kinetic energy Eq. (4) may be rewritten as

$$K_\rho = \frac{R_f}{1 - R_f}\frac{\epsilon}{N^2}. \tag{21}$$

This relationship was most recently derived by Osborn (1980), who argued from the usual viewpoint of separated mean and turbulent velocity scales. It still holds without this separation and, moreover, we now know the N-dependence of ϵ. If the flux Richardson number R_f is now taken to be nearly constant, the N–dependence of K_ρ is

$$\text{Type (1): } K_\rho \sim N^{-1}$$
$$\text{Type (2): } K_\rho \sim N^{-0.5}$$

It should be emphasized that this derivation of the functional form of K_ρ does *not* involve the APE equation from which the Osborn and Cox (1972) method develops, hence is unaffected by the problem of not knowing relative magnitudes of the double and triple correlation terms in this equation. It *is* dependent upon the assumption of near constant R_f. This has been a point of considerable discussion recently, because of apparently contradictory laboratory results. Britter (1974) found that, regardless of the value of gross Richardson number Ri, the transport properties of a temperature-stratified channel flow were best described as the result of turbulence existing at a constant (critical) "Richardson flux number" $Rf \equiv \dot{p}/\epsilon$, where $\dot{p} \equiv g\rho_0^{-1}\overline{\tilde{\rho}w}$. The critical value of $Rf \simeq 0.20$ determined by Britter corresponds to a value of the flux Richardson number (as defined by (5)) of $R_f = \dot{p}/(\dot{p} + \epsilon) \simeq 0.17$. Britter's results appeared to contradict the laboratory data collated by Linden (1979, Fig. 4). which show a maximum $R_f \sim 0.20$–0.25 near $Ri \simeq 0.10$, followed by an abrupt decrease to near-zero values of R_f at only slightly higher Ri. McEwan (1980) has recently suggested that resolution of this contradiction lies in recognizing that viscous losses due to wall friction and internal (but nonturbulent) friction are unavoidable in laboratory experiments of modest scale, and contribute a sizeable fraction of the total energy dissipation in the regime of sparse instabilities which prevails as $Ri \rightarrow 1$. He differentiates between flux Richardson number R_f and mixing efficiency η, defined respectively as

$$R_f = \frac{\dot{p}}{\dot{p} + \epsilon + \epsilon_\nu} \quad \text{and} \quad \eta = \frac{\dot{p}}{\dot{p} + \epsilon}$$

Here ϵ is dissipation in the (small) volume of fluid containing turbulence, while ϵ_ν is viscous dissipation, unaccompanied by buoyancy flux, in the remaining (large) fluid volume (including a large component from sidewall boundaries). His experiments were carried out in a regime of sparse instabilities produced by forcing the fundamental internal wave mode in a rectangular box to sporadic and localized "breaking." The results show that although $R_f \rightarrow 0$ as Ri increases beyond $Ri \sim 0.10$ (due to increased importance of ϵ_ν relative to ϵ), the mixing efficiency η remains constant at a value of ~ 0.26. This value of η, the appropriate flux Richardson number for the turbulence (as opposed to the entire system), is in reasonable agreement with values of R_f obtained from experiments (such as that of Britter) where turbulence occupies a much larger fraction of the experimental volume. In the ocean it is clear that dissipation and diffusion due to turbulence dominate that due to molecular processes (Munk, 1966): thus our assumption of constant $R_f \simeq 0.20$–0.25 seems justified.

In closing we would like to emphasize the most important qualitative feature of the arguments presented here, namely the idea that in a stably stratified fluid, it may not be possible to separate "turbulence" from "mean" (including internal waves) in the *velocity* field, but that some progress may be made by introducing a constraint on the *shear* field in the form of a statistically constant Richardson number. The subsequent examination of the connection between internal waves and the enhanced dissipation and diffusion which result when they "break" should be regarded as a preliminary one. A major contribution of such an attempt is to define specific questions which should be investigated by appropriate observations. We suggest a few such questions. What is the importance of the horizontal terms which we have ignored in Eqs. (3) and (16)? What is the appropriate time-averaged form for $\overline{w^2}$, and how should it be interpreted? What are the wave space-time scales at which the velocity triple-correlation arises? How can we resolve the question of the relative importance of triple and double correlation terms in the density (temperature) variance equation (16), hence the associated question of the accuracy of eddy diffusivities determined from direct microstructure measurements?

Acknowledgments. Steve Pond was a great support during our first attempts to give up the crutch of the Reynolds decomposition. This paper has benefited from conversations with Rob Pinkel and correspondence with Jerry Weinstock, and the hospitality of the Institute of Geophysics and Planetary Physics and the La Jolla Institute during preparation of a final revision.

REFERENCES

Britter, R. E. 1974. An experiment on turbulence in a density-stratified fluid. Ph.D. thesis, Monash University, Victoria, Australia.

Businger, J. A. and S. P. S. Arya. 1974. Height of the mixed layer in a stably stratified planetary boundary layer. Adv. Geophys., *18A*, 73–92.

Caldwell, D. R., T. M. Dillon, J. M., Brubaker, P. A. Newberger and C. A. Paulson. 1980. The scaling of vertical temperature gradient spectra. J. Geophys. Res., *85*, (C4), 1917–1924.

Desaubies, Y. J. F. 1975. A linear theory of internal wave spectra and coherences near the Väisälä frequency. J. Geophys. Res., *80*, 895–899.

Dillon, T. 1983. The energetics of overturning structures: implications for the theory of fossil turbulence. J. Phys. Oceanogr., (submitted).

Elliott, J. A. 1972. Microscale pressure fluctuations measured within the lower atmospheric boundary layer. J. Fluid Mech., *53*, 351–383.

Gargett, A. E. 1983. Vertical eddy diffusivity in the ocean interior. J. Mar. Res., (submitted).

—— 1976. An investigation of the occurrence of oceanic turbulence with respect to fine structure. J. Phys. Oceanogr., *6*, 139–156.

Gargett, A. E., P. J. Hendricks, T. B. Sanford, T. R. Osborn, and A. J. Williams III. 1981. A composite spectrum of vertical shear in the upper ocean. J. Phys. Oceanogr., *11*, 1258–1271.

Gargett, A. E. and T. R. Osborn. 1981. Small-scale shear measurements during the Fine and Microstructure Experiment (FAME). J. Geophys. Res., *86*, 1929–1944.

Garrett, C. J. R. and W. Munk. 1975. Space-time scales of internal waves: a progress report. J. Geophys. Res., *80*, 291–297.

—— 1979. Internal waves in the ocean. Ann. Rev. Fluid Mech., *11*, 339–369.

Gregg, M. C. 1977. Variations in the intensity of small-scale mixing in the main thermocline. J. Phys. Oceanogr., *7*, 436–454.

Gregg, M. C., C. S. Cox and P. W. Hacker. 1973. Vertical microstructure measurements in the central North Pacific. J. Phys. Oceanogr., *3*, 458–469.

Holloway, G. 1983. A conjecture relating oceanic internal waves and small-scale processes. Atmosphere-Ocean, *21*, 107–122.

Lilly, D. K., D. E. Waco and S. I. Adelfang. 1974. Stratospheric mixing estimated from high altitude turbulence measurements. J. Appl. Meteor., *13*, 488–493.

Linden, P. F. 1979. Mixing in stratified fluids. Geophys. Astrophys. Fluid Dyn., *13*, 3–23.

Lueck, R., W. R. Crawford and T. R. Osborn. 1983. Turbulent dissipation over the continental slope off Vancouver Island. J. Phys. Oceanogr., *13*, 1809–1818.

McEwan, A. D. 1980. Mass and momentum diffusion in internal breaking events, *in* Stratified Flows 1980 Proceedings, Vol. II, T. Carstens and T. McClimans, eds., Tapir, N-7034 Trondheim, Norway, 1095 pp.

Munk, W. 1966. Abyssal recipes. Deep-Sea Res., *13*, 707–730.

—— 1981. Internal waves and small-scale processes, *in* The Evolution of Phys. Oceanography: Scientific Papers in Honour of Henry Stommel, B. A. Warren and C. Wunsch, eds., MIT Press, 264–291.

Oakey, N. S. 1982. Determination of the rate of dissipation of turbulent energy from simultaneous temperature and velocity shear microstructure measurements. J. Phys. Oceanogr., *12*, 256–271.

Oakey, N. S. and J. A. Elliott. 1977. Vertical temperature gradient structure across the Gulf Stream. J. Geophys. Res., *82*, 1369–1380.

Osborn, T. R. 1980. Estimates of the local rate of vertical diffusion from dissipation measurements. J. Phys. Oceanogr., *10*, 83–89.

Osborn, T. R. and C. S. Cox. 1972. Oceanic fine structure. Geophys. Fluid Dyn., *3*, 321–345.

Osborn, T. R. and W. R. Crawford. 1980. An airfoil probe for measuring turbulent velocity fluctuations in water, Chapter 19 *in* Air-Sea Interaction: Instruments and Methods, F. Dobson, L. Hasse and R. Davis, eds., Plenum, New York 801 pp.

Phillips, O. M. 1977. The Dynamics of the Upper Ocean (second edition). Cambridge Univ. Press, 309 pp.

Pinkel, R. 1981. Observations of the near-surface internal wavefield. J. Phys. Oceanogr., *11*, 1248–1257.

Stewart, R. W. 1959. The problem of diffusion in a stratified fluid. Adv. Geophys. *6*, Academic Press, 303–311.

Tennekes, H. and J. L. Lumley. 1972. A First Course in Turbulence. The MIT Press, Cambridge, Mass., 300 pp.

Voorhis, A. D. 1968. Measurements of vertical motion and the partition of energy in the New England slope water. Deep-Sea Res., *15*, 599–608.

Weinstock, J. 1981. Energy dissipation rates of turbulence in the stable free atmosphere. J. Atmosph. Sci., *38*, 880–883.

Wyngaard, J. C. and O. R. Cote. 1971. The budgets of turbulent kinetic energy and temperature variance in the atmospheric surface layer. J. Atmos. Sci., *28*, 190–201.

Zeman, O. and H. Tennekes. 1977. Parameterization of the turbulent energy budget at the top of the daytime atmospheric boundary layer. J. Atmos. Sci., *34*, 111–123.

Received: *28 April, 1983;* revised: *23 August, 1983.*