On combining satellite altimetry with hydrographic data

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ABSTRACT

It is shown, by random construction of many sea surfaces, each consistent with geostrophy and mass conservation in the underlying ocean, that absolute sea surface topography relative to a geopotential surface can be estimated to 10 cm accuracy from appropriate *in situ* measurements of density. If one were to use hydrography to remove the oceanographic signal from an *altimetric* geoid, then 10 cm accuracy is the limit of our present capability. If, on the other hand, altimetry is to be used with a *gravimetric* geoid in order to improve estimates of large-scale ocean circulation, then 10 cm is the upper bound on acceptable errors for the combined altimeter and geoid system, over the spatial scales of interest. A hypothetical smoothed altimetric sea surface is used to demonstrate improvement in sea surface estimation in the combined hydrography-altimetry-geoid problem. Linear inverse theory forms the computational framework.

1. Introduction

A proposed system of satellite altimetric and gravimetric measurements, combined with oceanic hydrographic surveys, could be used to estimate the general circulation of the oceans. This note addresses the question of how well the satellite measurements must be made in order to contribute information beyond that obtainable from hydrography alone.

The SEASAT project demonstrated that the height of a satellite can be measured with remarkable accuracy, relative to the sea surface, using a satellite altimeter. The precision is about 5 cm over 10 km horizontal scales (Born *et al.*, 1979). But for oceanographic applications, the height of the sea surface relative to a surface of constant geopotential is required; the gradient of this height being proportional to surface geostrophic velocity. Thus, in addition to the height of the spacecraft above the ocean, the position of the satellite in a fixed coordinate system and the height of the geopotential in the fixed system must be known. A summary of present day accuracy in orbit determination, geoid estimation and other error sources is given

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by Wunsch and Gaposchkin (1980). In the long term, the limiting factor in determining large-scale ocean circulation appears to be the geoid. At present, there is no global geoid of submeter accuracy. A proposed special purpose gravity measuring satellite, called GRAVSAT, would provide a global geoid with an accuracy of a few centimeters, but only as a weighted average over areas of several hundred kilometers in diameter. The use of such an average geoid with the quasi-linear altimetry track for the study of ocean circulation is not straightforward.

The topography of the sea surface relative to a geopotential can also be estimated from density measurements in the water column by the classical dynamic method (Sverdrup *et al.*, 1942). The sea surface slope is proportional to surface geostrophic velocity, and vertical geostrophic shear is proportional to the horizontal gradient of density. Thus density measurements yield surface slopes as well as geostrophic velocities at all depths, with an uncertainty due to an integration constant, or reference level velocity.

There exists a considerable body of oceanographic literature devoted to the problem of determining the reference level velocity as a function of position. Much of this work was done in the context of the general circulation of the North Atlantic. Historically, the problem evolved as attempts were made to define a surface on which velocity vanished, referred to as a "level of no motion." Examples of assumed levels of no motion in the North Atlantic are the 2000 m surface of Iselin (1936), the minimum shear layer used by Defant (1941), and the bottom or near bottom reference surface of Worthington (1976). Various studies have invoked higher order dynamics (e.g., Stommel, 1956; Sudo, 1965; Stommel and Schott, 1977), property conservation (e.g., Hidaka, 1940; Wunsch, 1978), or direct measurements (e.g., Warren and Volkmann, 1968) to obtain estimates of the reference level velocity. It is not our intention to summarize this extensive literature, but to indicate that the problem is one of considerable continuing interest. The proposed coupling of conventional *in situ* observations of the density field with satellite determinations of the absolute elevation of the sea surface relative to a known geoid is an extension of the much studied general circulation problem. Wunsch and Gaposchkin (1980) showed, in principle, how to accomplish this coupling, taking into account errors in all the observations.

Here we wish to explore two of the issues that remain in combining potential future altimetric and gravity measuring missions with the much more familiar dynamic method. Satellite missions will not operate in a conceptual vacuum; to the contrary, there is considerable knowledge extant of the nature of the ocean circulation and its variability. Thus while it is true that the total oceanographic "signal" relative to the geoid is approximately 1.5 m, the remaining uncertainty of the structure of that signal is far less than that over much of the ocean (see for example, Wunsch, 1981). One of the questions we pose here concerns the accuracy with which an altimetric *system* (the altimeter itself plus all the other components necessary) must measure the sea surface in order that it will add new, quantitative knowledge to the time average ocean circulation.

The spatial resolution of altimetric systems is between 10 and 25 km (Wunsch and Gaposchkin, 1980). But gravimetric satellite determinations of the global geoid will probably always be limited to determining the field only on spatial scales of several hundred kilometers and larger (Douglas *et al.*, 1980; Breakwell, 1979). But, conventional hydrographic surveys are set up to conform to the smallest expected spatial scales in the actual ocean flow field. Thus the second question we wish to address is the extent to which altimetric system data, whose short wavelength cutoff will be determined ultimately by that of the geoid, can be combined quantitatively with the hydrographic data.

The present study entails two calculations. First a hydrographic section along 50W in the North Atlantic (see Fig. 1) is used to estimate sea surface topography by an inverse method. This method yields smooth reference level velocities which conserve mass and which give demonstrably smaller errors in surface topography than a traditional assumption of zero velocity at the reference level. The magnitude of these errors from the inversion, about 7 cm (r.m.s.), indicates how accurately the satellite measurements must be made in order to add new information to that available from hydrography for the large-scale circulation problem.

The second calculation is a direct demonstration of altimetric constraints in the inverse problem. A plausible hypothetical sea surface is used to construct altimetry 'data'. This surface is smoothed over a 250 km scale and assumed to have an expected error of 3 cm on the broad scale. These data, in addition to the density data, are used to constrain the inverse problem in order to illustrate the resulting improvement of solutions.

2. Method and results

The IGY hydrographic section at 50W (Fuglister, 1960, see Fig. 1) was obtained in 1956 and closes off an area of the North Atlantic to the west. This section crosses the Gulf Stream and the recirculation region to the south, running from the Grand Banks to the coast of South America. By coincidence, it is the same section studied by Luyten and Stommel (1982) elsewhere in this issue and the reader is referred to their work for a discussion of the section and the geostrophic flow through it. The section also forms part of the much larger data set used by Wunsch and Grant (1982) in their discussion of the circulation of the entire North Atlantic. In the present paper, we are not concerned with the flow details as such; the section is being used merely as a representative example of real hydrography.

We briefly sketch the inverse method as applied to this section; the procedure is

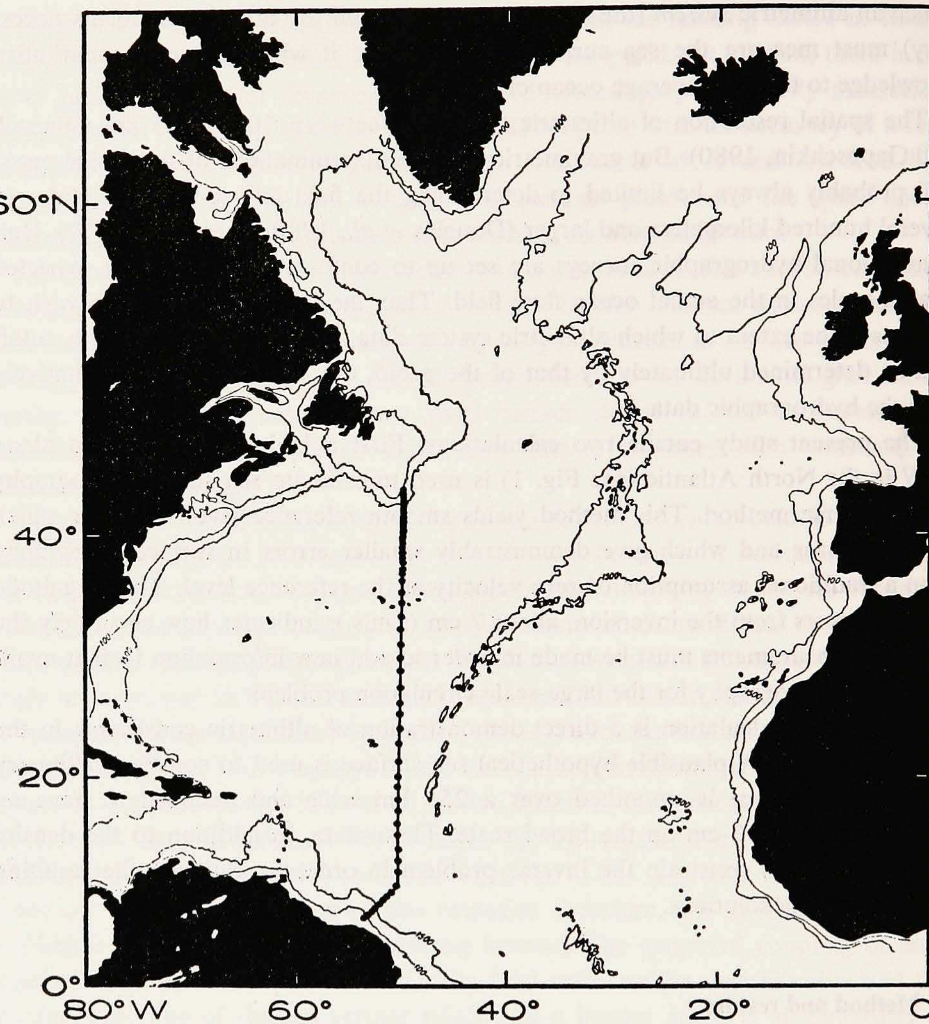

Figure 1. Station locations of the 50W hydrographic survey, Grand Banks to South America.

described in much greater detail in Wunsch (1978) and Roemmich (1979). The inverse method should be regarded as nothing more than a mathematical statement of the purely conventional dynamic method (e.g., Sverdrup *et al.*, 1942). As discussed in detail in Wunsch (1982), the objections which have been raised against the inverse method are really complaints about either the dynamic method or the inadequacies of the existing data base, or both.

In particular, notice that in this paper we make no claim that the 50W section represents the time average ocean; rather we make the opposite assumption that it adequately sampled the ocean at one moment in time.

Conservation of mass is imposed by requiring that the net flow across the section be small in each of 10 layers separated by surfaces of constant potential density. In the ith layer,

$$\sum_{j=1}^{N} a_{ij} b_j \approx -\gamma_i$$

where a_{ij} is the area of layer i at station pair j, b_j is the reference level velocity at pair j, and γ_i is the transport layer i relative to the reference level (2000 m). The γ_i are calculated from the data using the thermal wind equation. If there the M equations and N station pairs, the linear system to be solved is

$$\mathbf{A}_{M\times N}\, b_{N\times 1} \approx -\Gamma_{M\times 1} \;. \tag{1}$$

The ($\approx$) sign indicates that the equations are not to be solved exactly. The exact problem is, in general, ill-conditioned because of near linear dependency of two or more rows of the matrix **A**, owing primarily to observational noise.

A singular value decomposition of **A** is written

$$\mathbf{A}_{M\times N} = \mathbf{U}_{M\times M}\, \mathbf{L}_{M\times N}\, \mathbf{V}^T{}_{N\times N}$$

where

$$\mathbf{A}\,\mathbf{A}^T\mathbf{U} = \mathbf{L}\,\mathbf{L}^T\mathbf{U}, \mathbf{A}^T\mathbf{A}\mathbf{V} = \mathbf{L}^T\mathbf{L}\mathbf{V} \,.$$

L is a diagonal matrix of positive singular values of the matrix **A** arranged in decreasing order. A solution to (1) is

$$\hat{b} = -\mathbf{V}_{N\times K}\, \mathbf{L}_{K\times K}{}^{-1}\, \mathbf{U}^T{}_{K\times M}\, \Gamma_{M\times 1} \,. \tag{2}$$

Small singular values l_i, $K < i \leqslant M$, are discarded together with the corresponding eigenvectors to control residuals and minimize the effect of noise. For further details, see Wunsch (1978) and Roemmich (1979). In order to remove the bias resulting from variations in station spacing and water depth, the matrix **A** is subjected to a column weighting. The weighting is such that if the only constraint was conservation of total mass, then the solution velocity would be the same at all station pairs.

In this problem there are 10 equations and 55 stations pairs. The solution (2) is computed with $K = 6$, leading to residuals in (1) of order 1×10^6 m^3 sec^{-1}. The system of 10 equations (of which about 6 were determined to be independent) in 55 unknowns is underdetermined and has an infinite number of solutions. The solution (2) is the particular solution which has the minimum sum of squared reference level velocities.

We wish to systematically examine and compare many solutions to (1). An arbitrary solution to (1), which includes all possible solutions, may be written as

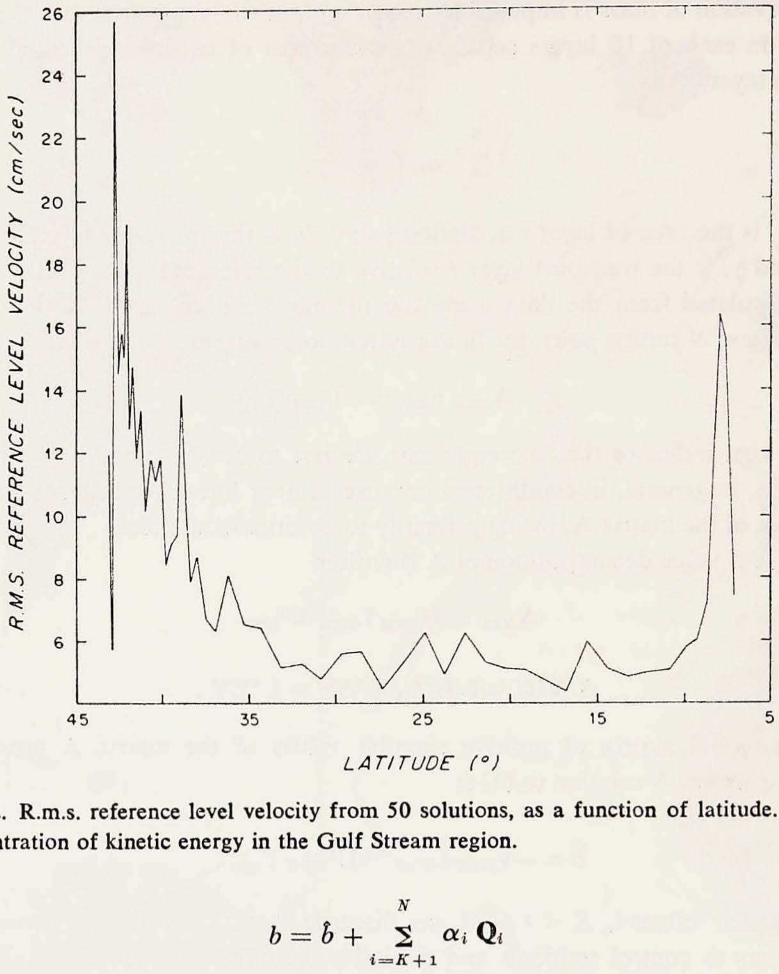

Figure 2. R.m.s. reference level velocity from 50 solutions, as a function of latitude. Note the concentration of kinetic energy in the Gulf Stream region.

$$b = \hat{b} + \sum_{i=K+1}^{N} \alpha_i \mathbf{Q}_i \tag{3a}$$

where the (N-K) vectors $\mathbf{Q}_i$ lie in the nullspace of the matrix **A**, i.e.,

$$\mathbf{A}\,\mathbf{Q}_i \approx 0 \tag{3b}$$

and the α_i are arbitrary scalars. The null space vectors $\mathbf{Q}_i$ are simply the eigenvectors $\mathbf{V}_i(i=K+1,N)$ associated with negligibly small or zero eigenvalues.

For purposes of comparison, 50 solutions of the form (3a) were constructed. In generating the α_i, we take advantage of the fact that the distribution of mean and low frequency kinetic energy in the deep ocean is, within bounds, known. For example, Schmitz (1978) discussed this distribution using moored array data along 55W from 41N to 28N. We assume that deep energy along 50W is not greatly different from that at 55W, with large expected velocities (tens of centimeters per second) near the Gulf Stream decaying to a few centimeters per second in the interior. The solutions (3a) were obtained by generating the α_i as independent random numbers, uniformly distributed over the interval −1 to 1, and then collectively normalizing them to satisfy the energy constraint

$$\left(\frac{1}{N} \sum_{j=1}^{N} b_j^2\right)^{1/2} = 10\ \mathrm{cms}^{-1}\,. \tag{4}$$

Figure 2 shows the r.m.s. reference velocity from the 50 solutions, plotted as a function of latitude. The energy constraint (4) applies to the total energy level and has no effect on its latitudinal distribution, which is controlled by the hydrography. Nevertheless, Figure 2 shows a marked increase in the level of kinetic energy near the Gulf Stream, which is at about 38N, in agreement with the moored array results. This is a statement about the structure of the null space of **A**. Null space components are large where isopycnal surfaces are nearly parallel over a large depth range—the case in the Gulf Stream.

In addition to (4), other constraints could be applied which would further limit the possible solutions. For example, the net heat flux into the area could be specified or it might be required that the meridional wavenumber spectrum at the reference level agree with observations or models. The procedure of generating many solutions and selecting those which satisfy additional constraints is related to the Monte Carlo method of Press (1968). (Other methods are more direct and efficient; but we have thought it useful here to actually display some members of the ensemble of acceptable solutions.) We have not applied constraints other than (1) and (4) in the present work.

Figure 3a shows all 50 estimates of the sea surface topography, with the northern end point arbitrarily set to zero. The change in sea level ($\Delta\zeta$) between a pair of stations separated by a distance Δx is

$$\Delta\zeta = (f\,\Delta x/g)v_s \tag{5}$$

where f is the Coriolis parameter and v_s the total surface geostrophic velocity. All solutions show the sharp sea level rise across the Gulf Stream with more or less diffuse recirculations to the south.

Assuming that the sampling along 50W represents the ocean in 1956, then any one of the solutions displayed in Figure 3a could have been closest to the actual sea surface at that time. Without further information, the dynamic method is unable to distinguish any one of these solutions from any other and the envelope of surface elevations (and the corresponding kinetic energies) at least roughly represents our true degree of ignorance about the ocean circulation determined from hydrography.

At each latitude, a mean value of the sea surface height was calculated. For the 50 solutions, the standard deviation about the mean is plotted as a function of latitude in Figure 3b. The section as a whole has a standard deviation of 6.9 cm. Of course, any point along the section could be chosen as the zero point for sea level, with sea level differences referenced to that point. In each case, the standard

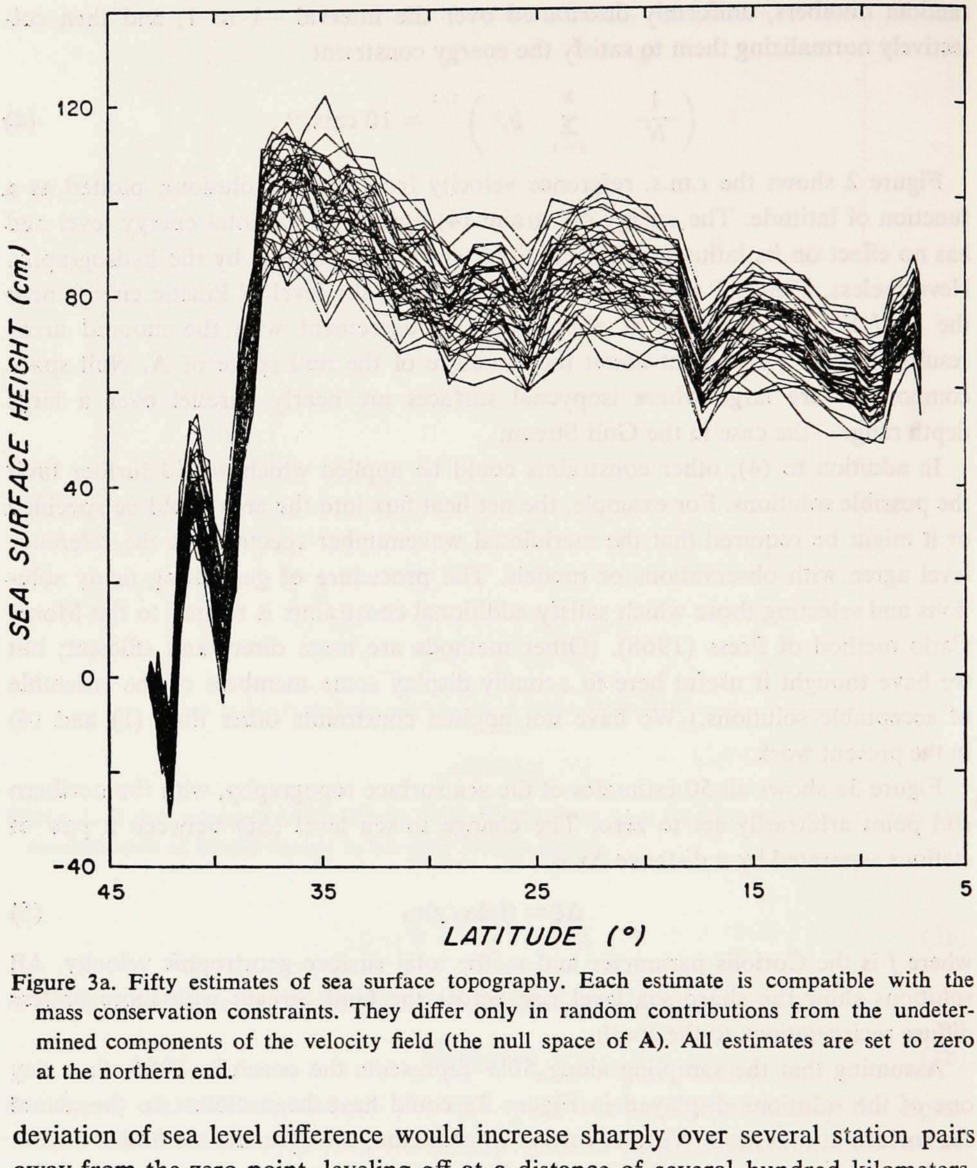

Figure 3a. Fifty estimates of sea surface topography. Each estimate is compatible with the mass conservation constraints. They differ only in random contributions from the undetermined components of the velocity field (the null space of **A**). All estimates are set to zero at the northern end.

deviation of sea level difference would increase sharply over several station pairs away from the zero point, leveling off at a distance of several hundred kilometers, and then gradually diminishing over scales of a thousand kilometers and longer. The important feature of Figure 3b is that the standard deviation of sea level difference does not simply increase monotonically with distance, as it would if the reference velocities were totally random and not subject to the mass conservation constraints. The information derived from our imposed constraints is predominantly of basin scale, so the uncertainty diminishes as the scale increases. We conclude from this study that the difference between the particular solution chosen by the

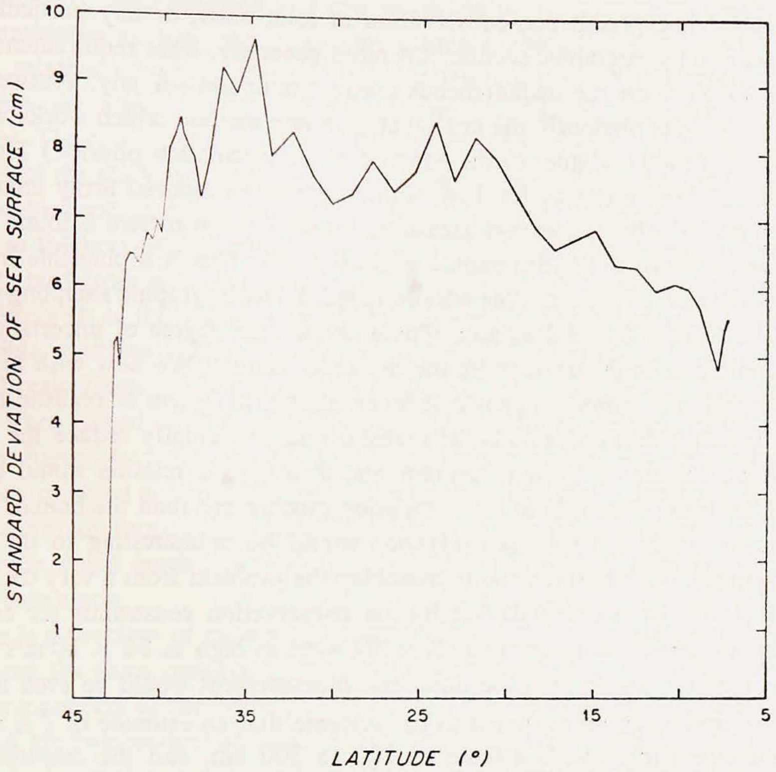

Figure 3b. Standard deviation about the mean of the estimates in Figure 3a, as a function of latitude. The largest scales of motion are the best determined, accounting for the decrease in uncertainty as one approaches the southern end of the transect.

inverse calculation (e.g., Eq. 2) or by any other selection rule will, with high probability, differ from the "true" solution by equivalent sea level changes somewhere within the envelope of Figure 3a. Although one can make elaborate statistical statements about the distribution of elevations as a function of position and separation, we wish to simply note that an altimetric determination of height differences of 10 cm or better over spatial scales of several degrees and longer would more tightly constrain the range of possible acceptable solutions.

The reader should be aware that two choices made in the calculation can significantly affect the standard deviation of sea level differences. The first is the size of residuals in solutions to (1). Solutions are sufficiently stable that changes in the rank, K, of matrix $\mathbf{A}$ of, say 50% will not greatly affect the answers. However, Luyten and Stommel (1982) argue elsewhere in this issue that large residuals, up to $30 \times 10^6 m^3 s^{-1}$, may be realistic. Such large residuals, implying rank 0 or 1, would raise the level of Figure 3b by about a factor of 3. The problem of applying conservation laws to quasi-synoptic hydrographic sections requires further attention. (To what extent should one permit non-conservation of total mass, or any particular water mass type, in a hydrographic section? Or more generally, what requirements are we prepared to place on the instantaneous ocean circulation—if any? Answering this type of question is obviously the crucial step in any method which works with real, quasi-synoptic data and goes to the heart of ocean circulation physics.) The second choice is the allowed energy level (4). In comparing to moored array data, the assumption is made that the energy measured by the current meters is found predominantly on scales of the hydrographic grid or larger. This is a plausible hypothesis related to the skill of the observer who designed the hydrographic sampling program.

We take Figures 2 and 3 as an estimate of the true degree of uncertainty in the ocean circulation as determined by the dynamic method. We now wish to examine the worth of the information provided by an altimetric system of realistic capability toward reducing that uncertainty. If realistic missions greatly reduce the limits of solutions consistent with our measurements, then such a mission would be highly worthwhile. If we cannot limit the circulation much more than the bounds given by the dynamic method, then such missions would be uninteresting to the physical oceanographer. Note that we are approaching the problem from a very conservative point of view: if the residuals left by the conservation constraints (or equivalent residuals in other types of constraint) really were as high as $30 \times 10^6 m^3 s^{-1}$ rather than the $1 \times 10^6 m^3 s^{-1}$, then the altimetric measurement would be even more valuable than we are going to find it to be. Assume that an estimate of ζ is available, smoothed along track with a scale of 200 to 300 km, and the constraints were applied as successive differences of the smoothed height; here we follow Wunsch and Gaposchkin (1980). We assume (and this is realistic) that the altimetric surface is constructed from data obtained over a time period coincident and of the same duration as that of the hydrography. For a 1956 section of course, we can only simulate the corresponding altimetry. Let v'_{sj} be the surface velocity in station pair j as determined from the thermal wind equation and the initial level of no motion. Then the total surface velocity is $v_{sj} = v'_{sj} + b_j$ where b_j is the j^{th} component of **b**. With the altimetric measurement of ζ_i (ζ evaluated at position i) we can then write

$$\sum_j \alpha_{ij} b_j = g/f\,(\zeta_i - \zeta_{i-1}) - \sum_j \alpha_{ij} v'_{sj}$$

a set of equations which may now simply be appended to equations (1) to augment the constraints, where the α_{ij} are constants representing the proper averaging interval for b_j to be consistent with the averaging interval for ζ_i. (Note that one need *not* use the sea surface as the reference level to use the altimetric data.) The smoothed sea surface used here was taken from one of the 50 solutions of the previous calculation, chosen arbitrarily, and subjected to a running mean filter of length 300 km to simulate a realistic altimeter mission.

The residuals of the altimetric constraints, relative to those of the mass conservation constraints, were adjusted by a row weighting of the augmented matrix **A**. One simply multiplies an equation by a number greater than 1 to decrease its expected residual relative to others or by a number less than 1 for an increase. In this calculation, there were 31 altimetric equations, in addition to the 10 conservation equations. The row weighting of the **A** matrix was chosen to give a realistic standard deviation of the sea level estimates of 3 cm, representing a highly accurate altimeter-geoid combination. A choice of rank $K = 16$ reduced the mass residuals to the same level as in the previous calculation. Thus the available information has been effectively doubled from what it was using the hydrography alone. Once again the null space vectors $\mathbf{Q}_i$ of the homogeneous problem were calculated and used to construct 50 possible solutions subject to the energy normalization (4). Figures 4a and 4b show 50 realizations of the sea surface and the standard deviation as a function of latitude when forced to be consistent with the altimetry as well as the hydrography. Compare Figure 4a to Figure 3a. The scope for ambiguity is vastly reduced. The large eddy centered at about 20N is clearly brought out and indeed all of the larger scale features are more clearly defined. At least in this example then, the altimetric information would much more fully constrain the sea surface than does the hydrographic data alone.

There is a problem of mismatch between the sampling capabilities of a satellite system and the ships required to measure the hydrography. But if the long wavelength components of the hydrography do not change rapidly with time, then the $\mathbf{V}_i$, $i=1,K$ are independent of time over the duration of section observations.

Long hydrographic sections will not be repeated very often, whereas a satellite would happily continue to measure the sea surface elevation every few days for many years. To the extent that long wavelength components of the ocean circulation are independent of time over the duration of an entire altimetric mission, the single hydrographic section can be used in conjunction with the altimeter to deduce the time average components of flow. It is possible (but unproven) that the time-dependent ocean circulation is dominated simply by temporal changes in the coefficients of our null space vectors. It would be useful to know if this is true, but only experience will serve. In practice, the determination of the *mean* flow from hydrography and altimetry will probably be carried out through the use of self-consistent numerical models constructed to conform with the altimetry and any other available data. Although this procedure is under active study, we will not describe it here.

3. The spatial average problem

Up to this point we have neglected the fact that the geoid will only be known as a spatial average over some region of diameter L of order 200 km rather than as

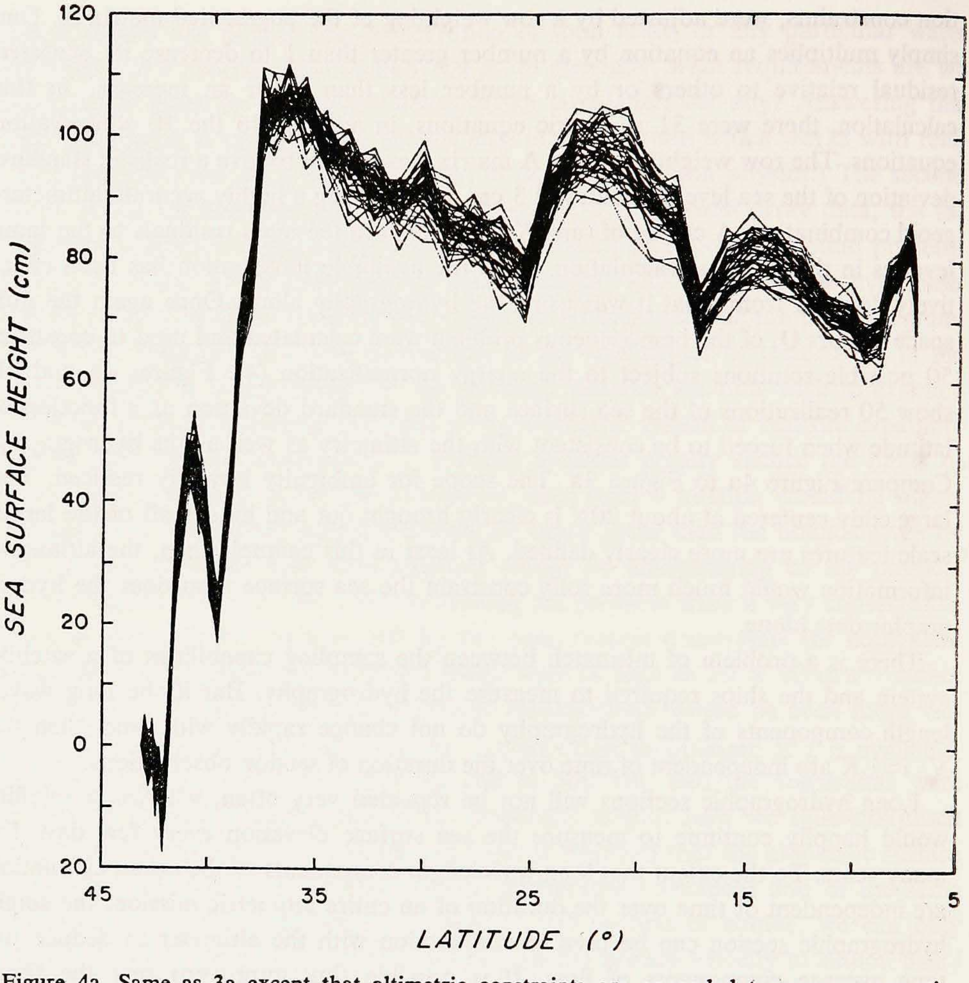

Figure 4a. Same as 3a except that altimetric constraints are appended to mass conservation constraints. Improved estimation is seen as a decrease in the size of the envelope containing the estimates.

the point values we have been using here. Let us consider the problem in one dimension first. The differential form of equation (5) is

$$v_S = (g/f)\, \partial\zeta/\partial x \;. \tag{6}$$

Suppose we have measured with our altimetric system (which includes a geoid determination) the sea surface elevation ζ averaged over a distance L which we can represent as

$$\zeta(x) = 1/L \int_{x}^{x+L} \zeta(x')dx' \,. \tag{7}$$

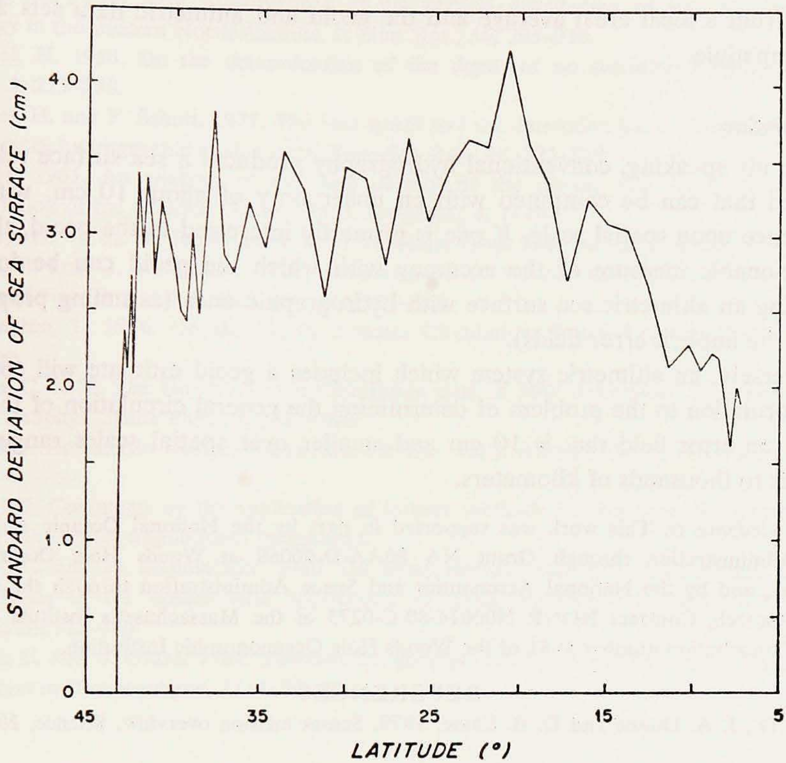

Figure 4b. Same as 3b though for the problem with altimetric constraints appended to mass conservation constraints.

If we average (6) over a distance L, we obtain

$$\bar{v}_s(x) = g/Lf\,[\zeta(x+L) - \zeta(x)]. \quad (8)$$

which involves ζ, not $\bar{\zeta}$. If we average (8) again over a distance L as

$$1/L \int_x^{x+L} \bar{v}_s(x')dx' = \bar{v}_s(x) = g/Lf\,[\bar{\zeta}(x+L) - \bar{\zeta}(x)] \quad (9)$$

since $\bar{v}_s$ is its own average. Thus the mean surface velocity may be obtained from the mean slopes in a way which reduces back to the case treated in Section 2.

But the geoid will be an areal not a linear average. On the other hand, in practice, the altimetric measurement of the sea surface, although it is intrinsically made along lines, will almost certainly be fitted to local surfaces as well. The reason for the surface fitting is to reduce the errors through a crossing-arc analysis as in Marsh *et al.* (1980). Once a surface is found, the full altimetric field can be treated as derived from a local areal average and the geoid and altimetric data sets are thus fully compatible.

4. Discussion

Generally speaking, conventional hydrography produces a sea surface relative to the geoid that can be computed with an uncertainty of about 10 cm, with some dependence upon spatial scale. If one is primarily interested in the geoid, this then is a reasonable measure of the accuracy with which the geoid can be found by combining an altimetric sea surface with hydrographic ones (assuming proper handling of the implicit error fields).

Conversely, an altimetric system which includes a geoid estimate will contribute new information to the problem of determining the general circulation of the ocean if it has an error field that is 10 cm and smaller over spatial scales ranging from hundreds to thousands of kilometers.

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