Cooling processes in deep, temperate lakes: A review with examples from two lakes in British Columbia

by Eddy C. Carmack[1] and David M. Farmer[2]

ABSTRACT

The cooling period in deep, temperate lakes includes the breakdown of the seasonal thermocline, isothermal overturn as the lake cools through 4°C, and winter restratification.

The first part of this paper is taken up with a review of the relatively few studies on the seasonal cooling of lakes. From this it is clear that our understanding is weakest with regard to the mixing processes whereby heat is transferred from the interior of a lake to the surface where loss to the atmosphere takes place. There follows a comparison of the thermal history and mixing characteristics of two deep lakes in central British Columbia.

Some features of thermal structure evolution can be explained within the framework of mixed-layer theory. At temperatures near 4°C, however, it is necessary to take into account the unique PVT properties of freshwater. Two- and three-dimensional processes can also influence circulation at various times and/or locations: e.g., enhanced mixing associated with large-amplitude internal seiches; retardation of mixed-layer advance by river-induced upwelling; thermal bar circulation driven by the influx of cold river water; thermal bar circulation driven by the differential cooling of water in shallow, tributary bays; and horizontal density flows resulting from differential wind mixing in sheltered arms of the lake prior to freezing.

1. Introduction

A lake in which the water passes through the temperature of maximum density twice yearly, in autumn and in spring, is called a temperate lake. The majority of studies on such lakes have been carried out during summer, so that the physical processes associated with cooling the water to and below 4°C are still poorly understood. In this paper we attempt to provide an overview of cooling process in deep, temperate lakes, firstly by reviewing the literature on this subject, and secondly by describing wintertime data from two deep lakes in British Columbia.

1. National Water Research Institute, 4160 Marine Drive, West Vancouver, British Columbia, Canada, V7V 1N6.

2. Institute of Ocean Sciences, 9860 Saanich Road, Sidney, British Columbia, Canada, V8L 4B2.

NOTATION

A — area
b — width
c — phase speed
f — Coriolis parameter
g,g' — gravity, reduced gravity
Gr — Grashoff number
h — upper layer depth
H — total depth
N_h — horizontal eddy diffusion coefficient
m — proportionality coefficient
Q — heat flux
R — Rossby radius $(g'H)^{1/2}/f$
T — temperature
T_i — inflow temperature
T_s — surface (mixed-layer) temperature
T_{max} — temperature of maximum density
T_p — internal wave period
u_* — friction velocity
V — volume
V_i — river discharge rate
V_b — production rate of 4°C water
V_h — hydraulic transport
W_e — Wedderburn number
ρ — density

2. Review

Studies on the cooling of lakes have been carried out using a wide variety of methods. One common approach involves heat budget considerations in which either the change in heat content (Gorham, 1964; Ragotzkie, 1978) or surface energy exchange (Anderson, 1952; Elder *et al.*, 1974) is estimated from field observations. Timms (1975) noted that the effects of interannual variability appear to be greater in deep lakes than in shallow lakes. Bennett (1978) points out that the colder a lake becomes in winter, the greater the fraction of the heat income in spring that must be used to warm the lake to 4°C; hence lakes with large heat losses in winter experience a longer period of convective overturn in spring and correspondingly shorter growing season in summer. Wiegand and Carmack (1981) discuss the influence of river inflow on winter heat budgets. When calculating the winter portion of annual heat budgets, it is also necessary to take into account the frozen cover (Adams and Lasenby, 1978). While these lines of study are useful in describing the integrated response of a lake to external forcing and determining the relative importance of heat flux components, they are of limited use in understanding mixing processes.

Descriptions which focus specifically on cooling processes in deep temperate lakes are rare. According to classical descriptions (c.f. Hutchinson, 1957, for review), autumn cooling leads to an initial deepening of the surface mixed-layer, and a reduction in overall temperature stratification. As cooling proceeds, the mixed-layer eventually includes the entire lake. Whipple (1895) was evidently the first to apply the term 'turning over' to this stage of full vertical circulation. Further cooling of surface water to, and below, 4 °C leads finally to inverse stratification and, in some cases, ice formation. Thus three stages of cooling can be recognized: the breakdown of the seasonal thermocline, isothermal overturn, and the onset of inverse stratification.

One-dimensional models of thermocline structure based on the original formulation of Kraus and Turner (1967) have been applied to lake studies by a number of authors (Edwards and Darbyshire, 1973; Stefan and Ford, 1975; Tucker and Green, 1977; Spigel and Imberger, 1980). This approach relates mixed-layer behavior to the combined action of the wind and surface buoyancy flux. Farmer (1975) used one-dimensional arguments to describe the effects of springtime convection in an ice-covered lake in the absence of horizontal flow. Farmer and Carmack (1981) developed a one-dimensional model of the restratification that occurs as a lake cools below 4 °C.

An especially clear description of the initial breakdown of stratification has been given for Lake Windermere by Mortimer (1955), who notes that the exposure of deeper layers at the windward end of the lake during high winds strongly enhanced vertical mixing. His observations also show that the magnitude of the static stability parameter following a strong wind may vary by an order of magnitude from one end of the lake to the other. Similar observations of large vertical displacements (50-70 m) have been presented by Hollan and Simons (1978) for Lake Constance. These observations and others reviewed by Mortimer (1974) suggest that large displacements in thermocline depth play an important role in autumn cooling, both in changing the proximity of the thermocline to wind stirring and in periodically altering the depth to which convective cooling takes place.

The timing and manner by which a lake reaches full vertical circulation and cools through 4 °C is dependent upon its shape, altitude, through-flow characteristics and local climate (see Hutchinson, 1957, for comparative examples). At this time, differential cooling of the water column often results in large horizontal gradients in temperature (Nobel, 1965) so that lateral, as well as vertical, flow patterns must be taken into account (cf. Patterson and Imberger, 1980). Since the change in density which results from a given temperature increment near 4 °C is much smaller than at higher temperatures, it may be important to consider how density is altered by salinity (Chen and Millero, 1977; Wiegand and Carmack, 1981), silicate concentrations (Marmorino *et al.*, 1980), and suspended material (Pharo and Carmack, 1979). There is also reason to believe that contraction on mixing, the so-called cabbeling instability, (Foster, 1972, for discussion) may influence circulation during this period. For example, spring overturn in many large lakes is influenced by thermal bar formation (Tikhomirov, 1963, Rodgers, 1965; Bennett, 1971; Brooks and Lick, 1972; Scavia and Bennett, 1980). This convective phenomenon is driven by the mixing together of warm ($T > 4°C$) inshore water with cold ($T < 4°C$) offshore water to form a common, dense water mass. Although a similar circulation in autumn was predicted by Forel (1895) for Lac Léman and observed by Tikhomirov (1963) in Lake Ladoga, the few available data for the Great Lakes suggest that the autumn thermal bar is a much weaker hydrographic feature (Rodgers, 1966). In this connection Bowman and Okubo (1978) have argued that the higher level of wind stirring in autumn inhibits the formation of thermal bar fronts. On the other hand, a circulation analogous to the thermal bar, in which river inflow drives the cabbeling instability, has been observed in both spring and autumn (Carmack *et al.*, 1979; and see below).

Farmer and Carmack (1981) describe two additional ways in which the unique properties of freshwater influence vertical convection at temperatures near that of maximum density. First, because of the quadratic dependence of density on temperature, the buoyancy flux associated with a constant cooling rate depends on surface temperature. Thus, even with a constant ratio of heat loss to the input of mechanical energy, continued cooling at temperatures below 4°C results in a decrease in mixed-layer depth with time. Second, because the temperature of maximum density decreases with depth, an inversely stratified lake can be rendered conditionally unstable if the base of the mixed-layer is lowered to a transition depth where its temperature matches that of maximum density. Above this transition depth the wind must work against buoyancy forces during cooling, while below it the water is gravitationally unstable. This may explain the frequent occurrence of bottom water colder than 4°C at depths greater than would be expected by wind mixing alone. As the lake cools further, stability becomes less sensitive to pressure and a given heat flux produces a progressively greater buoyancy flux leading to the process of restratification. In general, lakes subjected to relatively more intense wind mixing during the onset of inverse stratification have lower interior temperatures.

Thermodynamic considerations relating to the depression of the temperature of maximum density with depth and water column stability have been proposed to account for temperature structure at depths well removed from the influence of surface stirring (e.g. Strøm, 1945; Eckel, 1949; Eklund, 1965; Bennett, 1975). These models are based on the common assumption that the temperature profile of the system will be found in whatever state exhibits minimum potential energy, i.e., maximum resistance to mixing. However, they do not explain explicitly the dynamical processes leading to a given temperature profile, nor do they allow for variations in surface buoyancy flux.

Cooling during the inverse stratification stage depends strongly upon whether or not (and when) ice formation occurs (see Pivovarov, 1972; Michel, 1971, for reviews). Most studies on ice-covered systems have been carried out in shallow lakes (e.g. Woodcock, 1965; Likens and Ragotzkie, 1965; Palmer and Izatt, 1972). Ice formation requires a period of both cold temperatures and calm atmospheric conditions, and many lakes continue to circulate until cooled well below 4°C (Yoshimura, 1936; Stewart, 1973). A relationship between wind fetch and the temperature at which lakes freeze over was presented by Scott (1964). Once ice is formed, the rate of cooling is drastically reduced. Because ice- and snow-cover insulate the underlying waters, the shallow parts of some lakes actually warm during winter due to solar radiation (Hutchinson, 1941). That circulation under ice may also be driven by the release of heat and dissolved solids from bottom sediments into the overlying water (which subsequently sink to greater depths as thin density currents) was shown by Mortimer and Mackereth (1958); they also observed that the bottom contact water also lost oxygen, so that depending on local sediment characteristics, the final winter-end temperature, salinity and oxygen characteristics in topographically separated depressions showed distinct differences. Inflow of river water during winter may further influence water circulation and ice distribution patterns (Tesaker, 1973). In Lake Sperillen, Norway (an ice-covered lake with through-flow) Stigebrandt (1978) found that almost all vertical mixing and heat exchange occurred in the ice free region adjacent to the inflow.

The approach, of thermal-structure modelling has proven to be relatively successful in predicting temperature distributions during the cooling cycle (Huber *et al.*, 1972; Rahman, 1978; Killworth and Carmack, 1979). However, since numerical models require specification of mixing parameters they stand to benefit from new information on mixing processes.

These, then, are the considerations generally used to account for the cooling of deep, temperate-latitude lakes, and it seems clear that our understanding is weakest with regard to the mixing processes whereby heat is transferred from the interior of the lake to the surface where exchange with the atmosphere takes place. Clearly such processes directly influence the effectiveness of reoxygenation and the redistribution of nutrients during the cooling period, and must be understood before realistic models of the annual temperature cycle in lakes can be produced.

3. Observations

The lakes described here are located in the interior of British Columbia (Fig. 1). The first, Babine Lake, is long (150 km), narrow (4 km), deep (maximum depth,

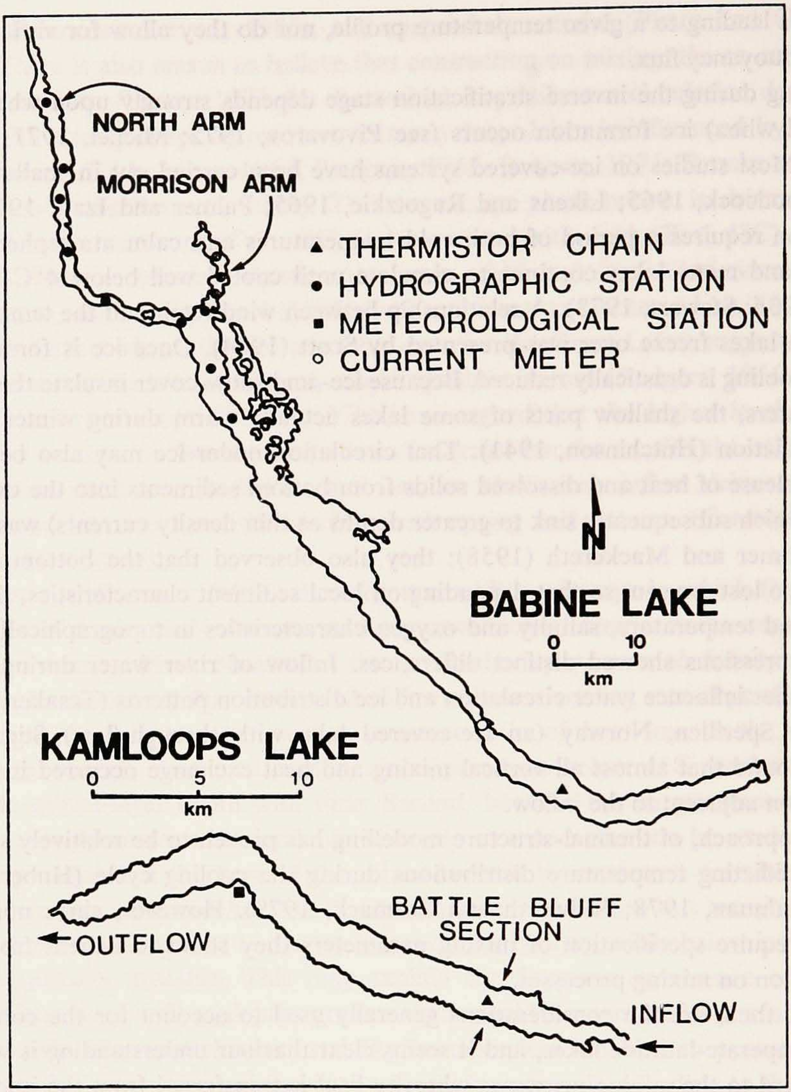


Figure 1. The study area: locations of Babine and Kamloops Lakes, and positions of those moorings and hydrographic stations relevant to discussions in this paper.

230 m), and characteristically ice-covered (80 cm) for four months in winter. The annual mean residence time for this system is approximately twenty years and almost no inflow takes place in winter. The second, Kamloops Lake, is morphologically similar to Babine Lake (length, 25 km; width, 3 km; maximum depth, 140 m), but remains ice-free throughout winter. Its annual residence time is about two months, and while inflow declines in winter, it is still sufficient to maintain winter flushing times of about one year. Babine Lake then is a deep temperate lake with ice-cover

but no winter through-flow, and Kamloops Lake is a deep temperate lake with winter through-flow but no ice-cover. The complementary case, that of a deep temperate lake with both ice-cover and winter through-flow, has been nicely treated by Stigebrandt (1978).

The emphasis throughout this paper will be on seasonal or cumulative effects of cooling, rather than on processes that are fine-scale or episodic in character (for comparison see Thorpe, 1977; Dillon and Powell, 1979).

Full descriptions of the observational methods used in studying these two lakes are given by Farmer (1975, 1978) and Farmer and Carmack (1981) for Babine Lake and by Carmack *et al.* (1979) for Kamloops Lake. Briefly, the data were obtained using temperature/depth profiling systems for lake-wide hydrographic cruises, moored thermistor chains recording temperature at 20 to 30 fixed depths at time intervals of 20 to 80 minutes, thermographs recording inflow and outflow temperatures (Kamloops Lake only), a current meter mooring (Babine Lake only), and meterological stations (Fig. 1).

4. Vertical structure

a. Thermal history of Babine Lake. Time series plots of daily-averaged temperatures from selected depths for the main basin of Babine Lake (Fig. 2) show the separate stages of the cooling period: stratification breakdown, isothermal overturn, and the onset of inverse stratification.

Initially, the surface water cools at about 0.2°C day^{-1}, and the mixed-layer depth progressively increases at about 0.4 m day^{-1}. As the base of the mixed-layer descends to include each successive thermistor, the observed temperature initially rises, showing that some of the mechanical energy made available by the wind and buoyancy flux is used to entrain water at the foot of the mixed-layer. This effect is most important in late autumn and is easily seen at depth; e.g., water at 52 m begins to warm from 4.4°C on 20 October, increases to 5.0°C on 30 October, and then cools steadily thereafter. It is somewhat surprising that this warming interlude lasts so long, suggesting that material redistribution by penetrative convection occurs throughout a fairly thick depth interval.

Mixed-layer deepening is not, however, the only important physical process. Mid-depth thermistors show large temperature fluctuations associated with internal seiche motions. These waves grow progressively larger as the density difference across the thermocline diminishes and reach heights of 30 m or more in late October. Eventually a point is reached at which a strong wind can raise the thermocline to the surface and mix it still further.

Coincident with the rapid growth in internal seiche amplitude, the rate of mixed-layer deepening is tremendously accelerated. Between 1 and 6 November the deepening rate averages nearly 20 m day^{-1} (see insert, Fig. 2).

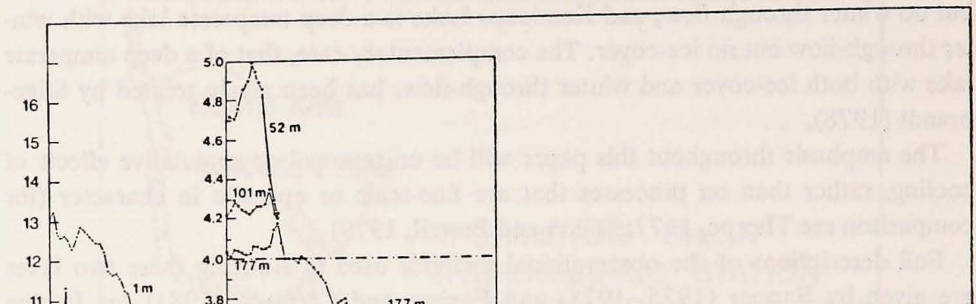


Figure 2. Time-series of daily-averaged temperature at selected depths in the main basin of Babine Lake between 10 September 1972 and 30 April 1973. The insert shows an expanded scale plot for the period of near-isothermal conditions.

By 7 November the lake is isothermal with surface and bottom temperatures differing by less than 0.1°C, and on 10 November the water column cools through 4°C. From this point onward, the sign of the surface buoyancy flux due to cooling changes. Recall, however, the temperature of maximum density decreases with depth, at a rate of about −0.02°C bar^{-1}. If wind stirring is sufficient to overcome the buoyancy flux and mix colder surface water down to a depth where its temperature matches that of maximum density, then the underlying water will convect to the bottom. Thus, it is to be expected that bottom temperatures will usually be colder than 4°C (cf. Farmer and Carmack, 1981).

The end of convective mixing and the start of winter (inverse) stratification is signalled by temperatures in the deeper portion of the lake beginning to level off at constant values, i.e., toward the end of November. This occurs when the bottom temperature is about 3.6°C. By 30 December the mixed-layer has retreated virtually to the surface of the lake, as shown by temperatures at 7 m; this yields an average rate of mixed-layer retreat of 3 m day^{-1}.

As the temperature profile changes from positive gradient through isothermal conditions to inverse stratification, we expect changes in the structure of the baroclinic response, and these can be seen in the current vectors plotted in Figure 3. The temperature profile of November 5 showed isothermal conditions down to the 35 m instrument with small positive stratification from 45-55 m. At this time the currents were generally NW down to 35 m and the reverse below. This would be consistent with a first mode internal response having a node just below 35 m. On the other hand, by December 25 negative stratification had appeared between 5 and 25 m with isothermal conditions below. The current again was NW near the surface and SE below, but the current reversal now occurs between 15 and 25 m. Although on both occasions the surface flow was northerly, on November 5 the net horizontal heat flux was north while on December 25 it was to the south since the temperature stratification had changed sign.

Subsequent to the development of full ice-cover in early January, the lake becomes isolated from the principal mechanical energy source. Internal wave energy decays, and currents at all depths decrease rapidly. The heat content of the lake and probably the distribution of other properties remains virtually unaltered throughout the remainder of winter. This behavior is in sharp contrast to ice-covered lakes with river through-flow (Stigebrandt, 1978).

The temperature profile changes little until spring when a new stage in the cycle begins. Solar radiation then warms the water beneath the ice causing a convective mixing layer that progressively deepens, a process described in detail by Farmer (1975).

It would appear, therefore, that with the exception of the large internal waves in late autumn, most features of the temperature structure in the main basin at Babine Lake can be explained by one-dimensional considerations of mixing. We shall later show that certain boundary-related processes occur in this lake, but that the magnitude of their effect on the lake's thermal structure is small.

b. Thermal history of Kamloops Lake. In describing the time-series record for Kamloops Lake (Fig. 4) we will emphasize the differences from Babine Lake that are due to advective processes.

Although Kamloops Lake is smaller and shallower than Babine Lake, its summer thermocline extends much deeper into the water column. This is because river-dominated mixing in early summer entrains cold water from depths greater than would otherwise occur by wind-driven mixing alone. The overall cooling rate of surface water is slightly less than that of Babine Lake, roughly 0.15°C day, while the rate of mixed-layer advance remains about 0.4 m day^{-1}. However, because Kamloops Lake has an initially thicker and deeper thermocline, the period of stratification breakdown extends much later into the year than that of Babine Lake. In early and middle autumn the mean temperature at a given depth remains more-

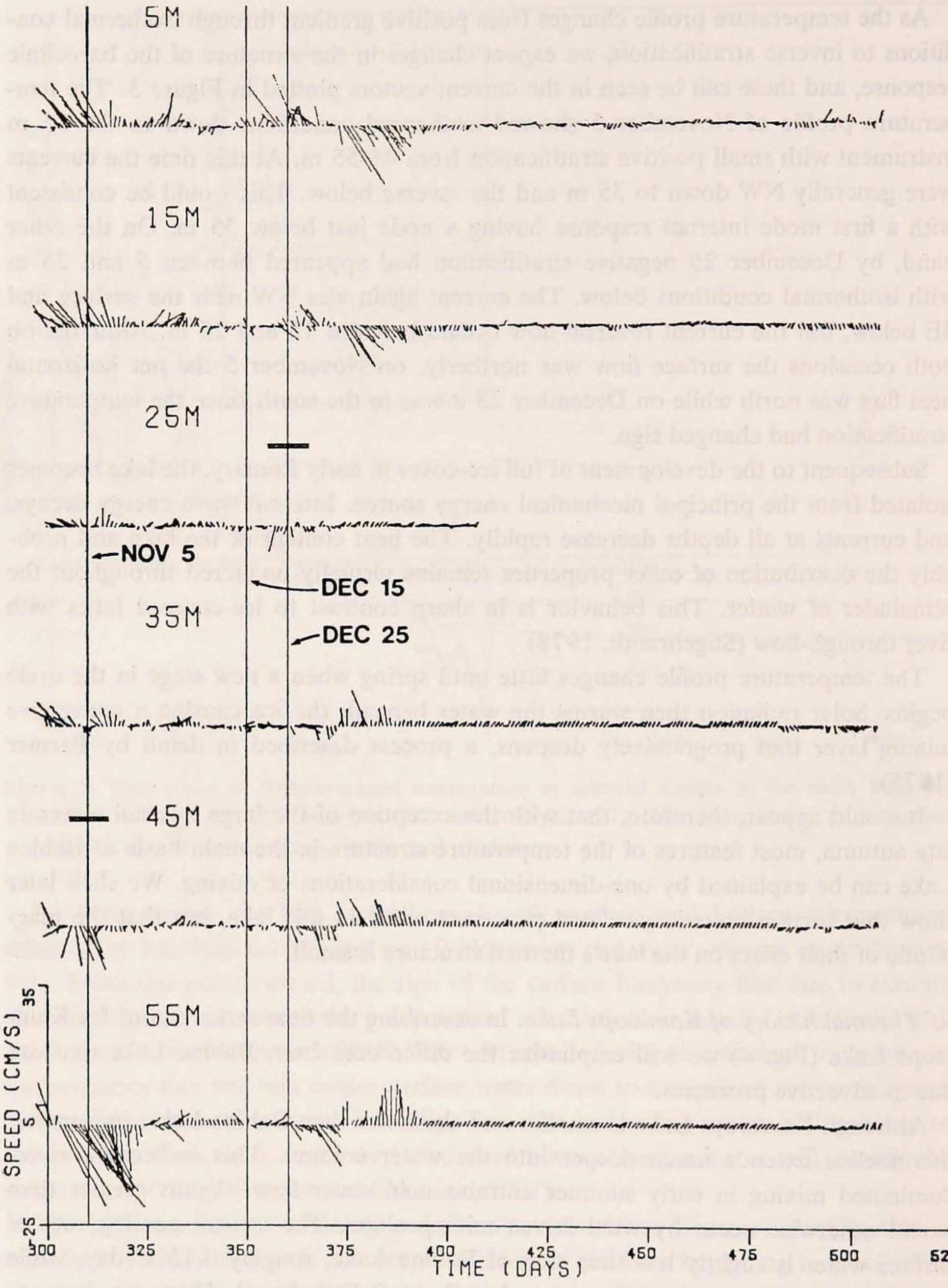
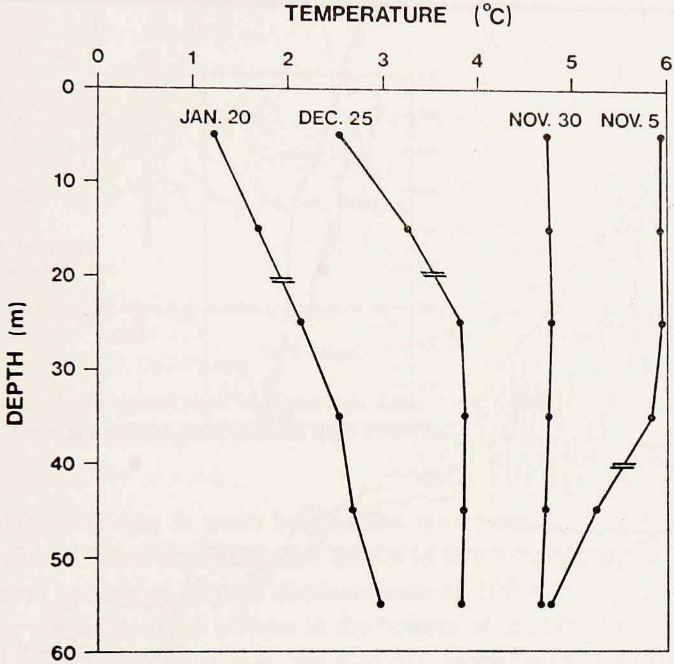


Figure 3. Data from current meter moorings over the southern sill in Babine Lake showing (a) daily averaged velocity vectors from current meter mooring, and (b) vertical temperature profiles from current meter mooring plotted at selected time intervals. Record begins on 17 October.



or-less constant, except for internal wave fluctuations, until engulfed by the deepening mixed-layer. Hence, the initial stage of mixing is characteristic of convective adjustment. However, during late autumn, an abrupt jump in deep-water temperatures occurs prior to cooling, a signature of penetrative convection.

The periods of internal seiches in Kamloops Lake are less than those of Babine Lake; the observed amplitudes, however, are even larger, reaching 100 m or more in late November. As in Babine Lake, the occurrence of these waves appears to be associated with intense vertical mixing.

In contrast to Babine Lake, Kamloops Lake is never isothermal until the entire lake cools below 4°C. That is, the deep waters actually cool more rapidly than surface waters, a condition inconsistent with one-dimensional convection. We later show that this feature is the result of a deep-water renewal process by river inflow.

Incoming river waters at temperatures near 0°C tend to remain in the upper layers of the lake, and thus act as an additional source of buoyancy. Minimum heat content and maximum inverse stratification occur in late February. Since Kamloops Lake remains ice-free, the water column is continually stirred, so that bottom temperatures continue to decline throughout the full winter period. In fact, strong winds during the last week in March effectively mix the lake from top to bottom to produce isothermal water at 1.8°C. The subsequent warming period has been described by Carmack (1979).

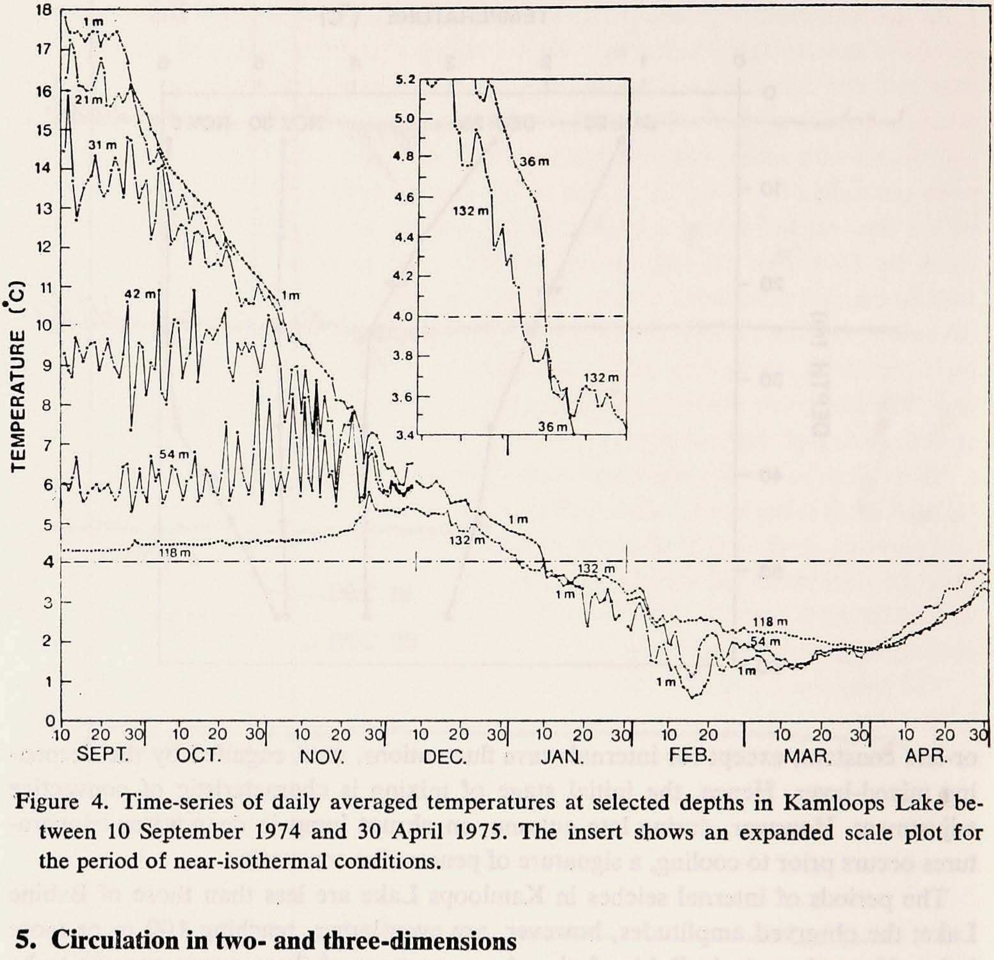


Figure 4. Time-series of daily averaged temperatures at selected depths in Kamloops Lake between 10 September 1974 and 30 April 1975. The insert shows an expanded scale plot for the period of near-isothermal conditions.

5. Circulation in two- and three-dimensions

The one-dimensional view afforded by observations from fixed moorings provides a good starting point for descriptions of the cooling season. In many cases, however, additional processes may become important, especially in lakes with significant river discharge, with extensive shallow regions, or with variable ice-cover. To illustrate this we present a variety of examples from both lakes in which the temperature structure is influenced by two- and three-dimensional processes.

a. Internal seiche motions. The isotherm-depth history for Kamloops Lake (Fig. 5) shows that the destruction of the seasonal thermocline in late autumn is coincident with the rapid growth of long-period internal waves. As the density difference across the thermocline decreases, internal wave behavior is modified in three ways. First, the height of an internal wave generated by a given wind stress is increased. Second, the phase speed is decreased so that waves propagate more slowly within the basin. Third, the baroclinic Rossby radius of deformation is diminished resulting in a

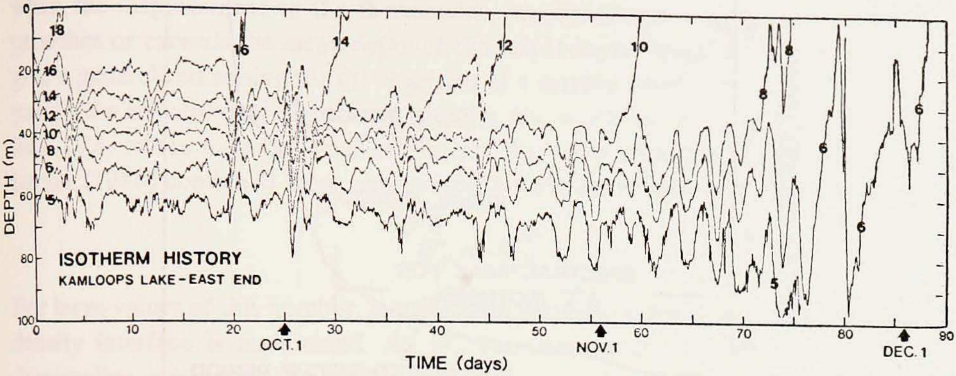


Figure 5. Isotherm-depth versus time in Kamloops Lake for the period 7 September to 5 December 1974. Data are plotted at 20-minute time intervals.

greater cross-lake variation in wave height. The most important of these effects is the rapid increase in wave amplitude that occurs in late autumn. By mid-November the 6°C isotherm undergoes vertical displacements of 100 m or more (Fig. 5), and thus alternately moves from the surface to the bottom of the lake. At the same time, isotherms within the thermocline (e.g., the 8 to 5°C isopleths) begin to diverge, suggesting that turbulent diffusion is intensified throughout the water column. This latter effect is so pronounced that the 5°C isotherm, rather than following preceding isotherms to the surface, is eventually forced downward to the lake bed.

The dominant nature of internal seiches in the temperature field is confirmed by comparison of a long-period internal wave model to spectral estimates of the thermistor chain records. The seiche period for a two-layered system is given by $T_p = 2L (H/g'h(H-h))^{1/2}$, where L is length of the lake, h is the depth of the upper layer, H is total depth, $g' = g\Delta\rho/\rho$ is reduced gravity, and ρ is density. It is significant that the seiche period computed in this manner (Fig. 6a) increases slowly until mid-November, and then increases abruptly as the lake enters the strong mixing phase. Prior to this break-point, the period of the spectral peak (Fig. 6b) and that calculated from the seiche equation are in good agreement; almost all energy is associated with periods of two to three days. (The observed spectral peak is somewhat longer. However, Hamblin (1978) has shown that the effect of rotation is to increase the period of first vertical mode by about 10%. Further, calculations which allow a thermocline of finite thickness (cf. Roberts, 1978) increase the fundamental period by an additional 5%.) Subsequent to the break-point, the background stratification changes far too fast to allow meaningful spectral analysis. There is, however, fair agreement between visual estimates of seiche period in Figure 5 and those predicted by the seiche equation.

These observations suggest, collectively, that a critical period of mixing occurs

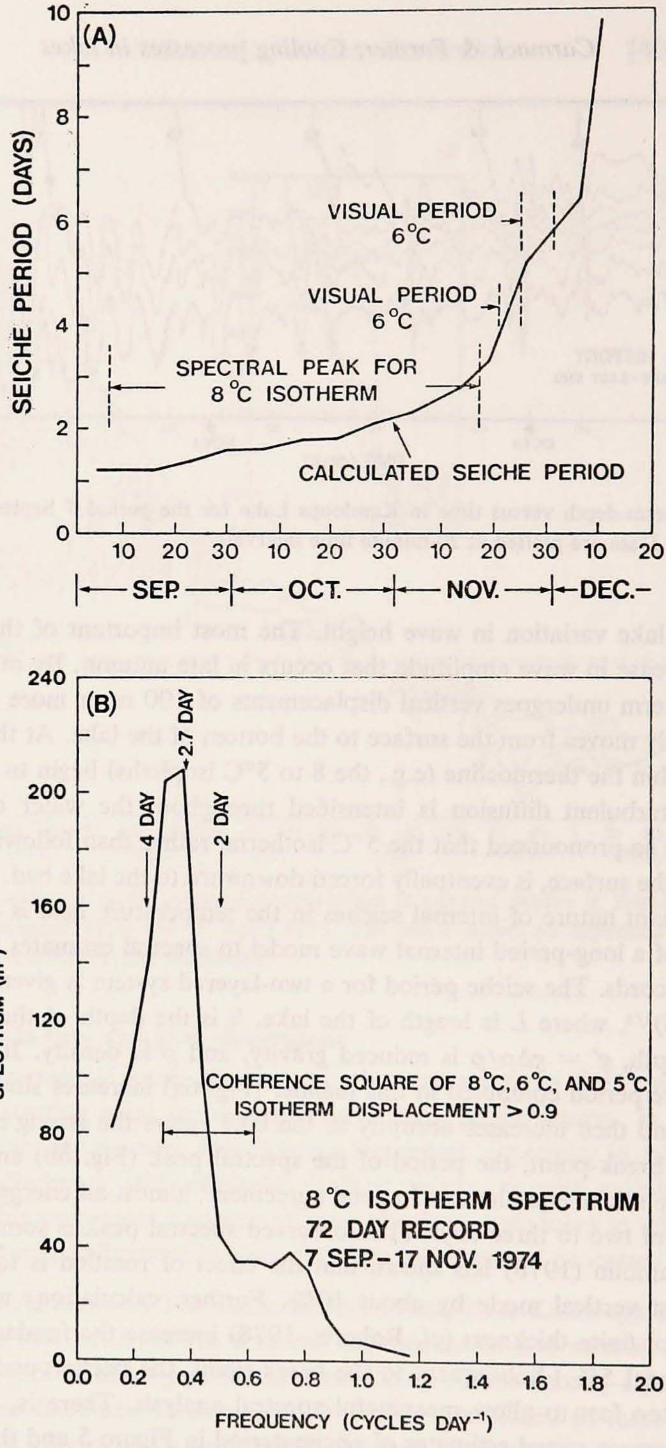


Figure 6. Oscillation characteristics of long-period internal waves in Kamloops Lake for the period 7 September to 5 December 1974: (a) Oscillation period calculated by means of an internal seiche model applied to hydrographic cruise data; (b) Spectral analysis based on the time series of depth of the 8 °C isotherm.

when the displacement of the thermocline, resulting from a given wind stress, approaches or exceeds the mean depth of the mixed-layer. Spigel and Imberger (1980) give a generalized setting for the response of a density interface to the wind by comparing the mixed-layer Richardson number $Ri_* = g'h/u_*^2$ to the aspect ratio of the lake L/h, where u_* is the friction velocity due to wind stress. Thompson and Imberger (1980) combined these parameters to form the so-called Wedderburn number

$$W_e = \frac{g'h^2}{u_*^2 L}$$

For large values of this number, stratification dominates over wind stress and a sharp density interface is maintained. As W_e becomes smaller the upwind end of the thermocline approaches the surface where it is weakened by convection and wind mixing. In Kamloops Lake the violent mixing phase, marked by the break-point in Figure 6a, begins at surface temperatures of about 8°C ($g' \cong 10^{-1}$ m s^{-2}), a mixed-layer depth of 50 m, and at mean wind speeds of about 7 m s^{-1} ($u_*^2 \cong 8 \times 10^{-1}$ m s^{-1}) yielding a Wedderburn number of order 1. Thompson and Imberger (1980) suggest that for $W_e \cong 2$ the entire interface will be exposed at the upwind end of the lake and rapid mixing will ensue; this in good agreement with our observations.

It is also possible for internal waves to grow, as free oscillations, even in the absence of additional inputs of wind energy. That is, if wave energy is preserved over several cycles, and if the stratification changes rapidly during this time, then conservation of energy requires an increase in wave amplitude to offset the decrease in stratification. Heap and Ramsbottom (1965) estimated a damping time for Lake Windermere roughly equal to five wave cycles. If a similar time scale holds for Kamloops Lake, then the combined prerequisites for the growth of free oscillation, e.g., rapidly changing stratification and preservation of wave energy over several cycles, would appear to hold in Kamloops Lake.

b. Effects of river interflow on mixed-layer deepening. In short residence-time systems such as Kamloops Lake, the conventional picture of a turbulent mixed-layer advancing into quiescent water is modified by the riverine circulation. Incoming river water has a temperature which is between that of lake surface and bottom water. Upon entering the lake it sinks to its equilibrium depth, and then spreads downlake as an interflow. At any given level conservation of mass requires that the total downward mass flux must vanish, i.e., that the downward transport due to the sinking of river water near the inflow region is compensated by upwelling in the interior of the lake. Mixed-layer water, itself, is continually being removed to supply the downstream river.

Vertical profiles of temperature (Fig. 7) show that river interflow takes place in the upper part of the thermocline during autumn. (The position of the riverine layer was inferred from corresponding turbidity profiles, cf. Hamblin and Carmack, 1978.) Clearly, until mid-December, the riverine layer actually forms the base of the mixed-

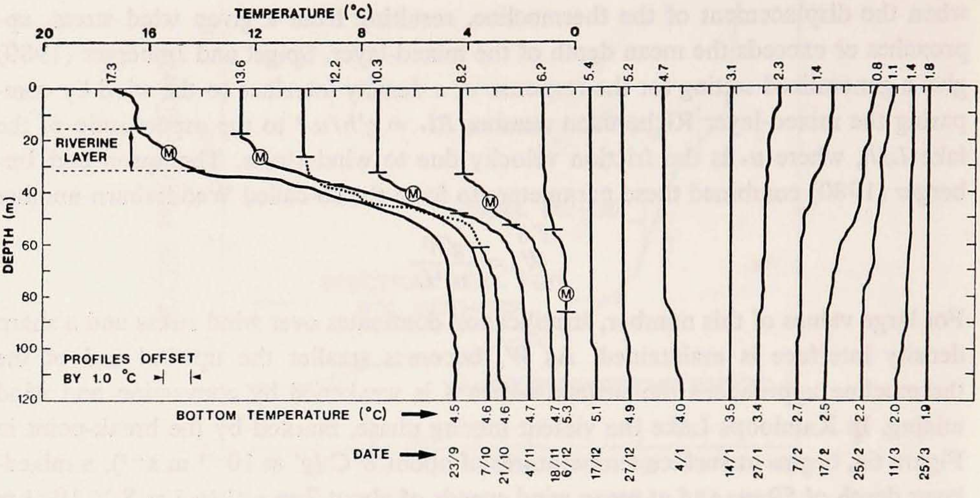


Figure 7. Successive temperature-depth profiles obtained with a continuous temperature/turbidity/depth profiler at approximately 2-week intervals in Kamloops Lake. The profiles are staggered by 1.0°C to avoid overlay, and plotted with a reversed temperature scale to suggest a sense of density increasing with depth during the period of stratification breakdown. The riverine layer, as identified by a layer of high turbidity water, is demarked by short horizontal marks on the profiles; the 'M' denotes the depth of maximum turbidity.

layer. As cooling continues, the temperatures of surface and inflow water fall together, so that the river sinks to progressively greater depths within the lake. Still, the requirement for upward velocity from the riverine layer holds.

A rough estimate of the effects of riverine circulation on mixed-layer deepening can be obtained by comparing the magnitude of the velocity of river-induced upwelling to that of mixed-layer deepening. As a first approximation it is sufficient to consider the initial deepening process as one of convective adjustment. In other words, the depth of mixing h is precisely where the temperature of the mixed-layer and the interior temperature are equal; at greater depths the temperature structure remains unchanged. Heat conservation over the mixed-layer depth implies that the change in heat content is due to direct loss to the atmosphere. Thus, for a mixed-layer of temperature T_m cooling at a rate dT_m/dt, and advancing into thermocline of constant gradient dT/dz, the rate of deepening $dh/dt \cong (d\,T_m/dt)/(dT/dz)$. With typical values $dT_m/dt = 0.15°\text{C day}^{-1}$ and $dT/dz = 0.25°\text{C m}^{-1}$, we have $dh/dt \cong 0.6$ m day. For comparison, a bulk estimate of the opposing velocity due to upwelling is given by the river discharge ($\cong 200\ \text{m}^3\ \text{s}^{-1}$) divided by the horizontal area of the lake at the depth of interflow (40 km^2), yielding a value of 5×10^{-6} m s^{-1} or about 0.5 m day^{-1}. The two mechanisms thus yield similar velocities, suggesting that riverine upwelling will likely influence mixed-layer advance.

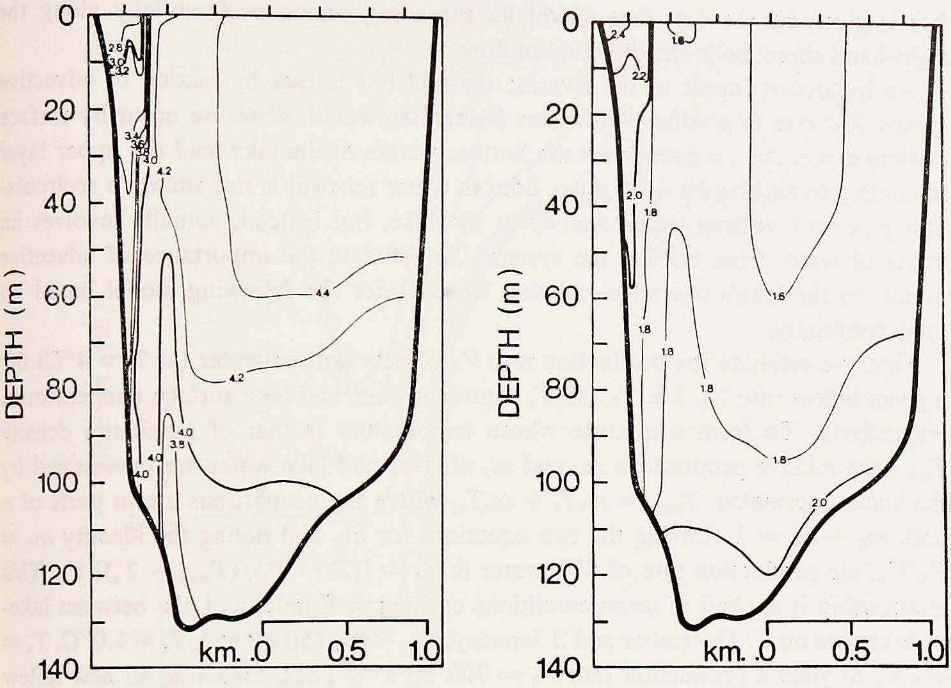


Figure 8. Transverse sections of temperature and turbidity (in J.T.U. units) in Kamloops Lake on 3 January 1980.

c. Riverine thermal bar. Carmack (1979) described a spring circulation phenomenon in which incoming river water, already warmer than 4°C, mixed with open lake water, still colder than 4°C, to effect thermal bar formation. In the following we show that a similar circulation occurs in autumn.

Figures 8a, b show transverse sections of temperature and turbidity, respectively, obtained in Kamloops Lake near the river mouth approximately one week before the lake cooled below 4°C. The temperature of the inflow is near 0°C, while the mean temperature of the lake is about 4.6°C. High values of turbidity demark mixtures rich in river water. Upon entering the lake, river water is initially less dense than lake water so it spreads across the surface as a buoyant plume. However, mixtures $3.4 < T < 4.6$°C, i.e., symmetrical to the temperature of maximum density, are denser than either lake or river water. Convergence and sinking of these dense mixtures is indicated by sharp gradients and the vertical arrangement of isotherms near 4°C. Inclusion of lake water into the sinking plume requires a compensating flow directed toward the river mouth. This, in turn, reinforces the surface temperature gradients and halts the downlake spreading of water in the buoyant plume. A suggestion that the Coriolis acceleration affects the dynamics of the riverine thermal bar is given by the fact that downlake spreading occurs preferentially along the right-hand shoreline in the direction of flow.

An important aspect of the riverine thermal bar is that by mixing of advective inputs, it serves to produce 4°C water faster than would otherwise occur by surface cooling alone. As a consequence the bottom waters of the lake lead the upper layer in cooling to and below 4°C. Also, bottom water renewal is not simply a recirculation process involving water already in the lake, but instead, actually involves an influx of water from outside the system. To estimate the importance of advective inputs on the lake's overall circulation we consider the following model based on mass continuity.

First, we estimate the production rate V_b of new bottom water (at $T = 4°C$) for a given inflow rate V_i. Let T_i and T_s represent river and lake surface temperatures, respectively. To form a mixture whose temperature is that of maximum density T_{max}, the relative proportions m_i and m_s of river and lake water are determined by the linear expression, $T_{max} = m_iT_i + m_sT_s$, where the proportions are in parts of a unit, $m_i + m_s = 1$. Solving the two equations for m_i, and noting the identity $m_i = V_i/V_b$, the production rate of 4°C water is $V_b = [(T_i - T_s)/(T_{max} - T_s)]\, V_i$. This relationship is applied to mean conditions existing in Kamloops Lake between lake-wide cruises on 27 December and 3 January; i.e., $V_i = 150$ m^3 s^{-1}, $T_i = 1.0°C$, $T_s = 4.8°C$, to yield a production rate $V_b = 700$ m^3 s^{-1}. Thus, assuming all new inflow water is used to form bottom water, the total production of new bottom water for the above 7 day period is 0.4 km^3. This predicted value is in good agreement with lake-wide cruise data which show a net increase in the volume of water in the class interval $T < 4°C$ of 0.3 km^3. Hence, the production of new bottom water by convection in the riverine thermal bar replaces approximately 10% of the lake's total volume in a one-week period.

The estimate of V_b may now be used to calculate a mean sinking velocity. The frontal region marking the location of the riverine thermal bar was about 2 km in length and 60 m in width giving a horizontal area of 12×10^4 m^3. Through this area the flow $V_b = 700$ m^3 s^{-1} must pass, therefore yielding a sinking velocity of 0.6 cm s^{-1}.

d. Autumnal thermal bar in tributary bays. When a lake cools, the temperature of nearshore water changes more rapidly than that of offshore water since the heat loss is distributed through a shallow depth. It follows that a given isotherm appears first near the shore, and then propagates seaward. When nearshore waters reach 4°C a region of sinking, the thermal bar, forms between the stratified water columns on either side. Flows from the nearshore and open lake regions converge and mix to form additional volumes of 4°C water, and thus maintain the sinking motion. On a smaller scale, thermal bar circulations can also be driven by differential cooling of shallow water in tributary bays. Such conditions are found in Morrison Arm of

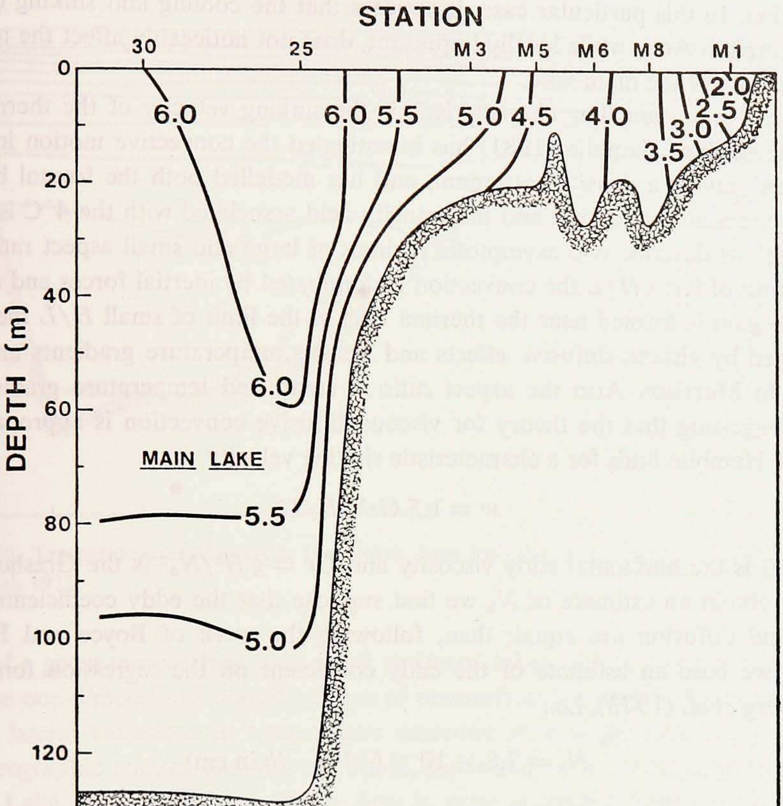


Figure 9. Temperature section from Morrison Arm into the main basin of Babine Lake on 18 November 1972.

Babine Lake. A typical section taken on 18 November (Fig. 9) reveals isotherms receding from head to mouth as the shallow waters of the bay cool more rapidly than those of the open lake. The 4°C isotherm, which defines the position of the thermal bar, is located at mid-length along the bay. Open lake water is still weakly stratified, while the inshore water is just starting to develop inverse stratification. Admittedly, fine-scale features of the circulation cannot be seen with such widely spaced stations. However, it is clear that the lateral thermal gradients are quite weak, between 0.2 to 0.4 × 10^{-4} °C m^{-1}. Examination of data obtained prior to and after 18 November (not shown) reveals that the thermal bar, demarked by an outcropping of the 4°C isotherm, migrates lakeward at a rate of approximately 0.5 km day^{-1}. Some fraction of the water sinking within the bar probably drains into the main lake. However, neither the lake-wide cruise data nor the time-series record of temperature in the main lake (Fig. 2) showed evidence of lateral circulation during this period. In this particular case, it appears that the cooling and sinking of water from Morrison Arm, while locally important, does not noticeably affect the temperature structure of the main lake.

Another important flow characteristic is the sinking velocity of the thermal bar. Along these lines Hamblin (1981) has investigated the convective motion in a confined fluid having a density extremum, and has modelled both the frontal behavior of the temperature structure and the velocity field associated with the 4°C isotherm. His solutions describe two asymptotic regimes of large and small aspect ratio H/L. In the limit of large H/L the convection is dominated by inertial forces and a strong frontal region is formed near the thermal bar; in the limit of small H/L the flow is dominated by viscous-diffusive effects and weaker temperature gradients are maintained. In Morrison Arm the aspect ratio is small and temperature gradients are weak, suggesting that the theory for viscous-diffusive convection is appropriate. In this case Hamblin finds for a characteristic sinking velocity

$$w = 1.5\, GrN_h H/L^2$$

where N_h is the horizontal eddy viscosity and $Gr = g'H^3/N_h^2$ is the Grashoff number. To obtain an estimate of N_h we first suppose that the eddy coefficients of viscosity and diffusion are equal; then, following the work of Boyce and Hamblin (1975), we base an estimate of the eddy coefficient on the regression formula of Kullenberg *et al.* (1973), i.e.,

$$N_h = 7.8 \times 10^{-3}\, b^{1.2} \qquad (b \text{ in cm})$$

where b is the width of the basin. For $b = 1.5$ km we have $N_h \cong 1.2$ m^2 s^{-1}. Then, with appropriate values for Morrison Arm of $\Delta\rho/\rho = 10^{-4}$, $L = 2$ km, and $H = 20$ m, we find for the sinking velocity $w \cong 5$ m day^{-1}. This yields a transit time for a parcel of water to sink from the surface to the bottom of about four days.

Unfortunately, direct measurements of sinking velocities within thermal bars do not appear to have been made. However, Elliott and Elliott (1970) found sinking speeds of up to 1 cm s^{-1} in their laboratory simulation of the thermal bar, and Elliott (1971) suggests that similar speeds may occur in lakes. Garrett and Horne (1978) estimated vertical velocities of about 1 m day^{-1} for frontal circulation driven by cabbeling in the ocean.

e. Interbasin exchange beneath the ice cover. Farmer and Carmack (1981) show that the shape of the inverse temperature profile left behind during restratification at temperatures below 4°C is related to a Monin-Obukhov length scale, defined by the ratio of wind to buoyancy inputs. When strong winds are present, cold surface water is mixed to greater and greater depths. The stronger the winds relative to heat loss (buoyancy gain) the lower will be the resulting heat content, and the weaker the resulting temperature profile. In general, large exposed lakes have deeper iso-

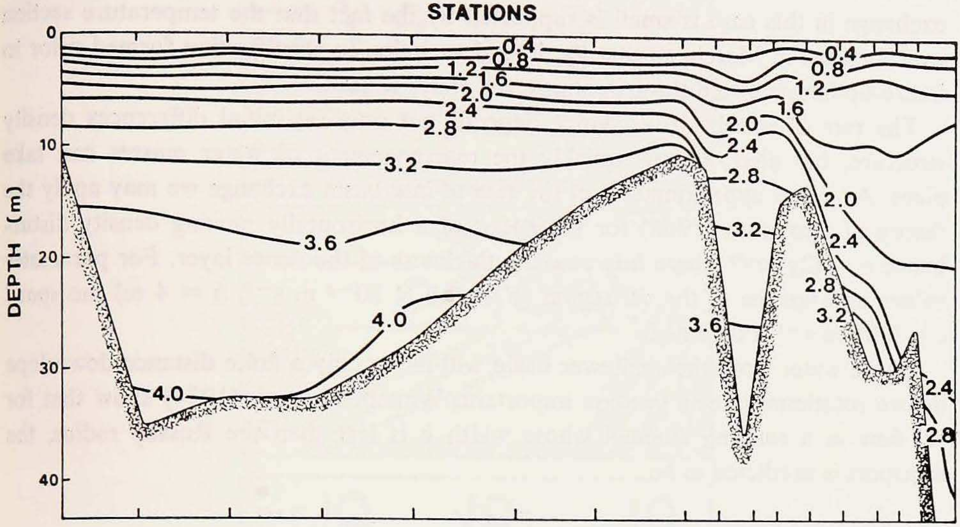


Figure 10. Temperature section from the North Arm into the main basin of Babine Lake on 5 March 1973.

pleths of a given temperature than small sheltered lakes. While these arguments are based on one-dimensional considerations of restratification, they also can be used to explain lateral variations in temperature structure in a single lake having two or more orographic and/or bathymetric zones, separated by a constriction in flow. In Babine Lake, for example, the North Arm is more protected from storms than the main lake. These two basins, in turn, are separated by a sill approximately 10 m deep. Surface cooling in the North Arm should lead the main lake, and the corresponding temperature profile should have sharper gradients. In addition, local knowledge is that the North Arm always leads the main lake in ice formation. As soon as ice forms, the covered portion will be cut off from the wind and heat loss altogether, thus accentuating the effects of differential cooling.

The anticipated results of wind-sheltering on thermal structure are clearly shown in a section extending from the North Arm into the main lake (Fig. 10). Stratification is relatively pronounced in the North Arm, and the thermocline is quite shallow. By contrast, much weaker temperature gradients are observed in the main lake; for example, the 2.0°C isotherm, which lies at about 5 m depth in the North Arm, extends to over 15 m in the main lake. Because of this, a density difference exists across the interbasin sill. The resulting pressure gradient appears to drive a deep circulation in the main lake, as evidenced by the strongly inclined isotherms, e.g., 2.2 to 2.4°C, leading down the flank of the sill. Comparison with data from the main basin shows that the sinking plume represented by these isotherms has reached its equilibrium approximate depth, 30 to 40 m (cf. Fig. 10). That the rate of interbasin exchange in this case is small is supported by the fact that the temperature section was obtained in March, so that the lateral variation in stratification formed prior to freeze-up has persisted through almost the entire winter.

The rate of interbasin exchange depends not only on initial differences density structure, but also on how quickly the rearrangement of water masses can take place. As a first approximation of the rate of interbasin exchange we may apply the theory of Benjamin (1968) for the speed of a horizontally moving density disturbance $c = (2g'h)^{1/2}$ where h is now the thickness of the dense layer. For parameter values appropriate to the sill region ($g' = 2.6 \times 10^{-4}$ m s^{-2}, $h = 4$ m) the speed $c = 0.05$ m s^{-1} is obtained.

Dense water from the shallower basin will move only a finite distance downslope before rotational effects become important. Whitehead *et al.* (1974) show that for sill flow in a rotating channel whose width b is less than the Rossby radius, the transport is predicted to be

$$V_h = b(g')^{1/2}\left[\frac{2}{3}\left(h - \frac{f^2b^2}{8g'}\right)\right]^{3/2}$$

Taking $b = 100$ m as the minimum width of the channel we have $V_h \cong 12$ m^3 s^{-1}. If this transport is supposed to take place in the lower half of the channel, through a cross-sectional area of 4×10^2 m^2, then a mean speed of 0.03 m s^{-1} over the sill is predicted. This value is slightly less than that obtained for a horizontally spreading density disturbance.

A rough check on the reasonableness of the above transport estimate may be obtained from the observation that the temperature (density) structure shown in Figure 9 changed little subsequent to freeze-up. This means that the total volume outflow during the intervening 100 days must have been significantly less than the volume of water in the North Arm. Now, the length of the North Arm is 40 km and its width is 2 km; hence the volume of water above sill depth 10 m, is approximately 1.2 km^3. Dividing this value by the outflow rate 12 m^3 sec, we find an effective residence time of 10^8 s or three years. This residence time is far greater than the duration of winter so that persistence of the above flow situation can be expected throughout the ice-covered period.

6. Summary

We have described the evolution of thermal structure in two temperate lakes, one a long residence-time lake with seasonal ice-cover, and the other a rapidly flushed lake with no ice-cover. The mixing regime of both lakes is strongly influenced by the rapid increase in internal seiche amplitude that occurs in late autumn. The main basin of Babine Lake is well-described by mixed-layer theory, providing the special consequences of the nonlinear and pressure-dependent terms in the equation of state

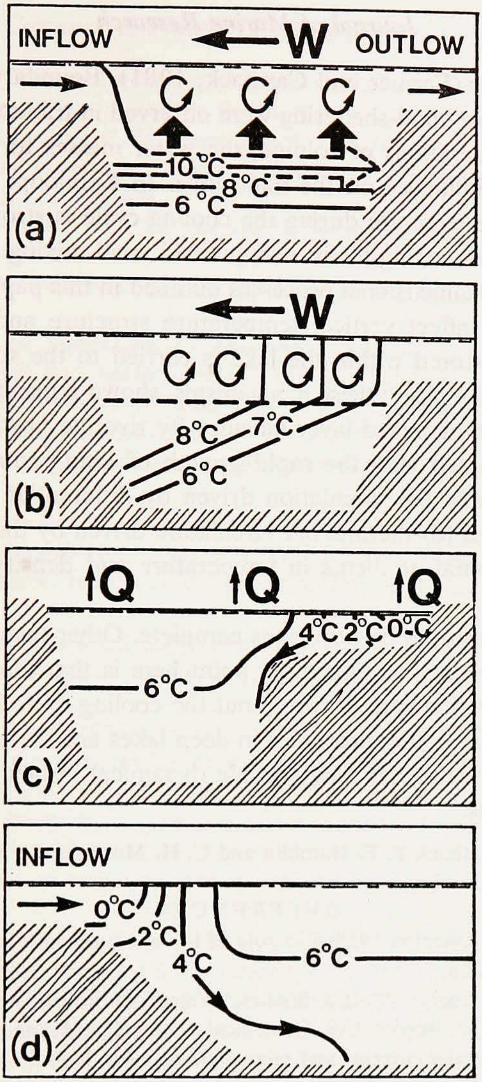


Figure 11. Schematic diagram of specific processes which may influence circulation during the cooling period: (a) effect of riverine upwelling on mixed-layer deepening; (b) enhanced vertical mixing of the thermocline due to large amplitude internal seiches; (c) thermal bar circulation in tributary bays; (d) thermal bar circulation due to river inflow; and (e) gravity flows resulting from differential wind-sheltering.

are taken into account (Farmer and Carmack, 1981). Boundary effects due to differential cooling and/or wind-sheltering were observed in tributary arms of the lake. However, from considerations of volume, the water masses formed in this manner appear to have little influence on the main lake. In Kamloops Lake, on the other hand, the temperature structure during the cooling cycle is strongly affected by additional considerations relating to river inputs and wind-mixing throughout winter.

The two- and three-dimensional processes outlined in this paper lead to horizontal variability which can affect vertical temperature structure and, subsequently, the means whereby heat stored within the lake is carried to the surface where loss to the atmosphere can take place; these mechanisms, shown schematically in Figure 11, are (a) the retardation of mixed-layer advance by riverine upwelling; (b) enhanced vertical mixing associated with the rapid growth of long-period internal waves in late autumn; (c) thermal bar circulation driven by differential cooling of water in shallow, tributary bays; (d) thermal bar circulation driven by the influx of cold river water; and (e) horizontal gradients in temperature and density results from wind sheltering.

This list of mechanisms is by no means complete. Other lakes may reveal wholly different convective processes. Our main point here is that much thought must be given before generalizations are drawn about the cooling cycle in any given lake. It is also clear that while cooling processes in deep lakes are important from a strictly limnological point of view, they also provide dynamical insight to related problems occurring in the oceans.

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