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The flow of equatorial Kelvin waves and the equatorial undercurrent around islands

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ABSTRACT

A linear theory is presented for the flow of equatorial Kelvin waves and of the equatorial undercurrent around islands situated on or near the equator. The island is modelled as an infinitesimally thin meridional barrier. It is shown that small islands affect the flow very little, while large islands effectively block the flow. To the east of such large islands the flow takes the form of a meandering current system, which is in agreement with various observations. To the west of the island nonlinearities are expected to be important.

1. The problem

In this paper the problem considered by Anderson and Rowlands (1976a), that of the response to an equatorial Kelvin wave incident on an eastern coast, is generalized. We ask the question: what happens when the coast is not of infinite extent, but only forms a partial barrier? If an equatorial Kelvin wave is incident on such an obstacle, we expect from the work of Anderson and Rowlands (1976a,b) that other equatorial waves will be reflected back along the equator, and that coastal Kelvin waves will be generated which will propagate away from the equator along the barrier (if the barrier comes within a distance of the order of the equatorial radius of deformation from the equator). When these Kelvin waves reach the tips of the barrier they will generate further coastal Kelvin waves, to the east of the barrier, which will propagate toward the equator and generate short equatorial

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Rossby waves and another equatorial Kelvin wave propagating eastward. The amplitude of this equatorial Kelvin wave will be less than that of the incident wave, as energy has been lost to other equatorial waves in the process of travelling round the barrier. The aim of this paper is to consider the disturbance caused by such a barrier, to calculate the change in amplitude of the equatorial Kelvin wave, and to indicate the nature of the flow due to the other equatorial waves which are generated.

It is apparent that a model for this flow could represent the propagation of equatorial Kelvin waves around islands situated on or near the equator. A better model for such islands would presumably be to give the barrier a thickness, making it into a rectangular obstacle. However, as discussed below, this would complicate the analysis; and anyway the main effects are expected to be similar to those for a thin barrier.

Another application is the study of the flow of the equatorial undercurrent around islands. McIntyre (personal communication) and Gill (1975) have suggested that the equatorial undercurrent has a meridional structure similar to that of a time-independent equatorial Kelvin wave in the second baroclinic mode. Thus we expect to be able to model the flow of this current around islands by seeking the longtime asymptotic response to an equatorial Kelvin wave of step function form (or in fact with any time dependence such that the amplitude tends to a constant nonzero value for large times).

There are several possible applications of the present theory to the real ocean. First, there is the flow of the undercurrent around the Galapagos Islands in the eastern Pacific. However, this flow is complicated by the proximity of the South American coast, and care must be exercised in interpreting the results for this case. A complete analysis of this problem would be possible by combining the present work with that given in Anderson and Rowlands (1976a) for a continuous coast. The main complication would be the continued reflection of equatorial waves back and forth between the island and the coast, in the absence of any dissipation, which would mean that one would have to solve the problem of Anderson and Rowlands (1976a) for incident short equatorial Rossby waves of all orders and including the full time dependence. We will not pursue this here, but simply will note that observations indicate that there is a region to the east of the Galapagos Islands where the effects of the coast are not obvious.

The second application arises from the GATE experiment of 1974. Some current meter moorings were placed just to the east of St. Peter and Paul Rocks in the Atlantic, on the assumption that any equatorial Kelvin waves would not be greatly affected by the presence of such small obstacles. The present analysis confirms this faith in intuition.

Finally, Hendry and Wunsch (1973) have considered the flow of the undercurrent around Jarvis Island in the Pacific. They attempt to explain the observed flow using a nonlinear model and neglecting Coriolis forces. They note that there are some regions, especially to the east of the island, where their theory is not satisfactory, although it does quite well to the west. It will be seen that the linear theory developed here, with account taken of the Earth's rotation, gives reasonable results to the east, but not to the west, of the island. It would appear that the inclusion of nonlinear terms in the present theory would give a much better agreement with observations, although it is not obvious how the analysis should proceed in this case.

In addition to the theory given by Hendry and Wunsch (1973), White (1971a) solved a problem similar to the present one. He considered a steady uniform (i.e., no meridional variation) zonal flow impinging on a circular island. His analysis was nonlinear and used the beta-plane approximation to represent the Earth's rotation. His results are qualitatively similar to ours with short Rossby waves apparent to the east of the island. To the west his solution is similar to that of Hendry and Wunsch (*loc. cit.*). Unfortunately, he has to make two ad hoc assumptions to enable a solution to be found. The first of these is that there should be no disturbance far upstream from the island. This is certainly not the case in the linear model discussed below. The second is an assumption of symmetry about the line of latitude through the center of the island. This is questionable as the beta effect certainly introduces an asymmetry into the problem. The assumption is needed because, away from the equator, there can be coastal Kelvin waves of arbitrary amplitude propagating around the island; in effect, White assumes a particular value for this amplitude, which may or may not agree with the required condition of no such waves when there is no incident current. In fact, unless the island is situated symmetrically about the equator, the solution is not expected to be symmetric (due to the definite direction of propagation of Kelvin waves) even for his assumed incident current which is independent of latitude. In the present analysis we are able to apply an initial condition of no flow around the island, which eliminates the need for any such assumption.

2. The model

We wish to study the effect of a partial meridional barrier on an incident equatorial Kelvin wave of step function form. As in Anderson and Rowlands (1976a,b), the barrier must be oriented exactly north-south, to enable the variables to be separated in the differential equations, and we consider a flat-bottomed ocean of infinite extent with no topography other than the barrier (in particular no continental shelf). Suppose the barrier is at $x = 0$ (where x is measured eastward, y northward, and $y = 0$ is the equator). Then for $x > 0$ or $x < 0$ separately we can expand the dependent variables in vertical and meridional modes. In the following analysis we concentrate on a single vertical mode, which could be any of the infinite set (for a general stratification), but we will be thinking of two of these in particular: the first baroclinic mode, as this is so relevant to upwelling studies, and the second baroclinic mode, as this is expected to be a reasonable model of the equatorial undercurrent. In the chosen vertical mode we then have to satisfy the differential equations for $x > 0$ or $x < 0$ separately and at $x = 0$ we must satisfy the boundary conditions

$$u = 0 \quad (\hat{b} < y < \hat{a}) \tag{1}$$

and

$$u,p \text{ continuous} \quad (y > \hat{a},\ y < \hat{b}) \tag{2}$$

where the latitudinal extent of the barrier is from $y = \hat{b}$ to $y = \hat{a}$, and the dependent variables are as in Anderson and Rowlands (1976a,b): u—zonal component of velocity, p—pressure anomaly. In the two modes of special interest to us, p may be interpreted as (i) vertical displacement of the thermocline (positive downward) in the first baroclinic mode, and (ii) change in thickness of the thermocline from its value in a state of rest (positive for increase of thickness) in the second baroclinic mode. Also we must satisfy the radiation condition for the reflected and transmitted equatorial waves.

We can now see the extra difficulty involved in giving the island a finite zonal width. There would be two longitudes at which matching would have to be performed. This is quite possible to do, but it would add an unnecessary complication, as the results are expected to be similar to those derived here. More details of this comparison are given below.

3. The solution

The method used to solve this problem is an integral equation formulation similar to that used by Buchwald and Miles (1974) to describe the flow of coastal Kelvin waves around a partial barrier on an f-plane. A solution is sought for the long-time asymptotic response to an incident equatorial Kelvin wave of step function form (or indeed any form which tends to a steady value after some time). The response to such an incident wave does not itself become independent of time (at least in a linear nondissipative theory), the flow to the east of the island taking the form of a constantly narrowing boundary layer (c.f., the Somali Current, Lighthill (1969), or Anderson and Rowlands (1976b)). However, the response to the west of the island is expected to become independent of time (as in Anderson and Rowlands (1976a)) and thus so are u and p at $x = 0$.

As in Anderson and Rowlands (1976a,b), we introduce the variable $q \equiv p + cu$ where c is the speed of equatorial Kelvin waves in the vertical mode under consideration. We retain the variable u rather than introduce r as was done previously. The equations are nondimensionalized with length scale $\sqrt{c/2\beta}$ and time scale $\sqrt{1/2\beta c}$, where β is the rate of change of the Coriolis parameter (f) with latitude (y). Suppose that the nondimensional latitudes of the tips of the island are given by b and a, with $b < a$. Then if u were known as a function of latitude, y (now nondimensional), at the longitude of the island ($x=0$) we could write

$$q(x,y,t) = q_T(x,y,t) + \left(\int_{-\infty}^{b} + \int_{a}^{\infty}\right) Q_-(x,y,t,\eta)u(o,\eta)d\eta \qquad (x<0)$$

(3)

and

$$q(x,y,t) = \left(\int_{-\infty}^{b} + \int_{a}^{\infty}\right) Q_+(x,y,t,\eta)u(o,\eta)d\eta \qquad (x>0)\,, \tag{4}$$

where q_T is the long-time asymptotic value which q would take if there were a complete coast at $x=0$, and $Q_\pm$ are the Green functions for $x > 0$ or $x < 0$, respectively (i.e., they are the values which q (or q-q_T) would have for $x > 0$ or $x < 0$ for $u(o,y) = \delta(y\text{-}\eta)$). In equations (3) and (4) we have used the fact that $u(o,y)$ is expected to be independent of time and also the boundary conditions (1) and (2). The term q_T in equation (3) is known from Anderson and Rowlands (1976a). It is the asymptotic form of q for large times as derived in that paper. In particular, for an incident equatorial Kelvin wave of unit amplitude, i.e., if the incident wave is of the form

$$q \sim D_o(y) \equiv \exp(-y^2/4) \text{ as } t \to \infty \text{ for fixed } x\,, \tag{5}$$

where $D_o(y)$ is the lowest order parabolic cylinder function (Abramowitz and Stegun, 1965), then we have at the longitude of the island ($x=0$)

$$q_T(o,y,t) \sim \frac{1}{\sqrt{2}} \text{ as } t \to \infty\,. \tag{6}$$

It should be noted that the incident wave is included in q_T. The boundary condition (2) implies that q should be continuous at $x=0$ for $y<b$ and $y>a$. Applying this to equations (3) and (4) gives

$$\left(\int_{-\infty}^{b} + \int_{a}^{\infty}\right) \{Q_+(o,y,t,\eta) - Q_-(o,y,t,\eta)\}\, u(o,\eta)d\eta = q_T(o,y,t) \quad \text{for } y>b \text{ and } y<a\,. \tag{7}$$

In this equation $q_T(o,y,t)$ is known from equation (6). If we can calculate the Green functions, $Q_\pm$, equation (7) becomes a Fredholm integral equation of the first kind for $u(o,y)$ which is solvable, at least in principle, once the boundary conditions have been specified. The appropriate boundary conditions are apparently

$$u(o,y) \to 0 \text{ as } y \to \pm\infty\,, \tag{8}$$

for waves can exist only near the equator or when supported by a coast. When

$u(o,y)$ has been found this may then be substituted in equations (3) and (4) to give q everywhere. Then with u and q known at $x=0$, we may calculate $p \equiv q - u$ (in nondimensionalized units) at $x=0$. An expansion in terms of parabolic cylinder functions then enables p and u to be evaluated for all x,y, and t using the method of Anderson and Rowlands (1976a,b). The next step therefore is to calculate the Green functions, $Q_{\pm}$.

4. The Green functions

The Green functions can be thought of as the response to a steady flux of water through a pinhole at $y=\eta$ in an infinite barrier. We can expand q and r ($\equiv p-u$ in nondimensional form) as series of parabolic cylinder functions (Abramowitz and Stegun (1965), Ch. 19):

$$\left.\begin{aligned} q &= \sum_{m=0}^{\infty} q^m(x,t)D_m(y) \\ \text{and} \qquad & \\ r &= \sum_{m=0}^{\infty} r^m(x,t)D_m(y) \end{aligned}\right\} . \tag{9}$$

From equations (2.11) and (2.12) of Anderson and Rowlands (1976a) we have, after taking Laplace transforms with respect to time ($\tilde{q}^m = \int_o^\infty q^m e^{-st}\,dt$, etc.)

$$2s^2\hat{q}^{m+2} + 2k_m s\hat{q}^{m+2} + (m+2)\hat{q}^{m+2} = \hat{r}^m \quad (m=0,1,2,---) \tag{10}$$

where

$$\left.\begin{aligned} \tilde{q}^{m+2} &= \hat{q}^{m+2}(s)e^{k_m x} \\ \text{and} \qquad & \\ \tilde{r}^m &= \hat{r}^m(s)e^{k_m x}, \end{aligned}\right\} \tag{11}$$

with

$$\left.\begin{aligned} k_m &= \frac{-1}{4s} - \sqrt{\frac{1}{16s^2} + m + \frac{3}{2} + s^2} \qquad (x>0) \\ &= \frac{-1}{4s} + \sqrt{\frac{1}{16s^2} + m + \frac{3}{2} + s^2}\,. \qquad (x<0) \end{aligned}\right\} (m=0,1,2\ ---) \tag{12}$$

$$\left.\begin{aligned} \text{Also,} \qquad \tilde{q}^o &= \hat{q}^o(s)e^{-sx} \\ \text{and} \qquad \tilde{q}^1 &= \hat{q}^1(s)\exp\{-(s+1/2s)x\} \end{aligned}\right\} \text{for } x>0 \text{ only, owing to the radiation condition.} \tag{13}$$

In equations (12) the branch of the radical is chosen to satisfy the radiation condition, that q^m and r^m represent waves travelling away from the origin at large values of $|x|$.

If the Green functions are given by

$$Q_{\pm} = q \tag{14}$$

then at $x=0$ we require

$$u = \delta(y-\eta)H(t) \tag{15}$$

which may be written

$$q^m - r^m = 2\delta_m/s \text{ at } x=0 \tag{16}$$

where

$$\delta(y-\eta) = \sum_{m=0}^{\infty} \delta_m D_m(y)\,. \tag{17}$$

We seek the asymptotic solution for large time by letting $s \to 0$ in the Laplace transforms. Taking this limit for $x < 0$ and combining equations (10) and (16) we find that

$$(m+2)\hat{q}^{m+2} - \hat{q}^m = -2\delta_m/s \qquad (m=0,1,2,---)\,. \tag{18}$$

This is written as a differential equation as follows. Multiply equation (18) by $D_{m+1}(y)$ and sum over all m. Then using the recurrence relations for the parabolic cylinder functions:

$$\left.\begin{aligned} D_{m+1}(y) &= \frac{1}{2}\; yD_m(y) - \frac{dD_m(y)}{dy} \\ mD_{m-1}(y) &= \frac{1}{2}\; yD_m(y) + \frac{dD_m(y)}{dy} \end{aligned}\right\} \tag{19}$$

we find that

$$2\hat{q}_y = \hat{q}^o D_o(y) - \frac{y}{s}\; \delta(y-\eta) + \frac{2}{s}\,\delta_y(y-\eta) \tag{20}$$

where $\hat{q} \equiv \sum_{m=0}^{\infty} \hat{q}^m D_m(y)$ is the Laplace transform with respect to time of $q \equiv Q_-$ at $x=0$, and a subscript y indicates differentiation.

Now, the Green function is supposed to be forced only by the nonzero zonal component of velocity at $x=0$. That is, there are no incident equatorial waves. Equation (18) has eliminated all of these unwanted waves except for the equatorial Kelvin and Yanai waves (which correspond to nonzero $\hat{q}^0$ and $\hat{q}^1$, respectively). The Yanai wave can be neglected simply by setting $\hat{q}^1 \equiv 0$ in equation (20). This is easily shown by noting that, using the orthogonality of the parabolic cylinder functions, we need

$$\int_{-\infty}^{\infty} \hat{q} D_1(y) dy = 0\,.$$

Integrating by parts we have

$$0 = 2\int_{-\infty}^{\infty} \hat{q}_y \exp(-y^2/4)dy \quad (\text{as } D_1(y) \equiv y\exp(-y^2/4).)$$

$$= \int_{-\infty}^{\infty} \hat{q}^1 \Big[D_o(y)\Big]^2 dy - \frac{1}{s}\int_{-\infty}^{\infty} \{y\delta(y-\eta) - 2\delta_y(y-\eta)\}D_o(y)dy$$

from equation (20)

$$= \sqrt{2\pi}\,\hat{q}^1, \text{ integrating the last term by parts.}$$

The general solution of equation (20) is then

$$\hat{q} = \frac{A}{s} - \frac{\eta}{2s}H(y-\eta) + \frac{1}{s}\delta(y-\eta) \tag{21}$$

where $H(\xi) = 1$ for $\xi > 0$ and 0 otherwise, and A is a constant to be determined by eliminating the equatorial Kelvin wave. The orthogonality property of the parabolic cylinder functions gives the condition for this as

$$\int_{-\infty}^{\infty} \hat{q}D_o(y)dy = 0 .$$

Substituting from equation (21) into this yields

$$A = \frac{\eta}{2\sqrt{\pi}}\int_{\eta/2}^{\infty} e^{-\xi^2}\, d\xi - \frac{1}{2\sqrt{\pi}}e^{-\eta^2/4} . \tag{22}$$

Hence we have the Green function to the west of the island, evaluated at x=0, as $t \to \infty$:

$$Q_-(o,y,t,\eta) \sim \frac{\eta}{2\sqrt{\pi}}\int_{\eta/2}^{\infty} e^{-\xi^2}\, d\xi - \frac{1}{2\sqrt{\pi}}e^{-\eta^2/4}$$

$$-\frac{\eta}{2}H(y-\eta) + \delta(y-\eta) . \tag{23}$$

For x>0 the derivation is similar, but slightly more complicated. In this case equations (10) and (16) become

$$(m+1)\hat{q}^{m+2} - \hat{q}^m = -2\delta_m/s . \tag{24}$$

Multiplying this by $D_{m+1}(y)$ and summing over all m as before we find

$$2\hat{q}_y = \hat{q}^1 D_o(y) + \sum_{m=0}^{\infty} \hat{q}^{m+2}D_{m+1}(y) - \frac{y}{s}\,\delta(y-\eta) + \frac{2}{s}\,\delta_y(y-\eta) . \tag{25}$$

Multiplying equation (24) by $D_m(y)$ and summing over all m gives

$$\left(\frac{1}{2}\,y + \frac{d}{dy}\right)\sum_{m=0}^{\infty} \hat{q}^{m+2}D_{m+1}(y) - \hat{q} = \frac{-2}{s}\,\delta(y-\eta) . \tag{26}$$

Combining these last two equations leads to

$$2\hat{q}_{yy} + y\hat{q}_y - \hat{q} = \frac{-3}{s}\,\delta(y-\eta) - \frac{1}{2s}\,y^2\delta(y-\eta) + \frac{2}{s}\,\delta_{yy}(y-\eta)\,. \tag{27}$$

The boundary conditions to be applied to the solution of this differential equation arise from the fact that there must be no coastal Kelvin wave at $x=0$ which propagates in from high latitudes toward the equator. Thus we must have

$$\hat{q} \to 0 \text{ as } y \to \pm\infty\,. \tag{28}$$

The solution of these equations is fairly easy; it is

$$Q_+(o,y,t,\eta) \sim \frac{y}{4} + \frac{1}{2\sqrt{\pi}}\left(e^{-y^2/4} + y\int_0^{y/2} e^{-\xi^2}\,d\xi\right) + \delta(y-\eta) - \frac{y}{2}\,H(y-\eta) \tag{29}$$

$$\text{as } t \to \infty\,.$$

In equations (23) and (29) the delta functions are just the contributions to q from the imposed zonal component of velocity, u, at $x=0$. There is no such contribution from p, which is as we would hope, else this would give rise to a derivative of the delta function in the meridional momentum equation which could not be balanced by any other term in that equation. Away from the forcing (i.e., for $y \neq \eta$) Q_- is independent of y. It is interesting to note that there is always some response in the hemisphere where there is no forcing, and that this response increases as the forcing moves nearer the equator. This is, of course, a manifestation of the radius of deformation: a coastal Kelvin wave is forced which moves away from the equator, but there is always some disturbance equatorward of the forcing. This is expected to be concentrated within a distance of the order of the radius of deformation from the forcing and decays toward the equator. At the equator, however, there is always some disturbance, albeit usually exponentially small, which generates a coastal Kelvin wave in the other hemisphere. When the forcing is near the equator this other Kelvin wave is larger in amplitude, and this is shown in equation (23). The exponential decay expected equatorward of the forcing is smoothed out by the coastal Kelvin waves so that for large times the solution becomes independent of latitude away from the forcing region.

The behavior of Q_+ is much more surprising. If the forcing is sufficiently far from the equator $|\eta| >>$ equatorial radius of deformation $= 1$ in our nondimensional units) then there is a region in $0 < |y| < |\eta|$ in which Q_+ decreases linearly toward the equator. This unexpected difference between the asymptotic forms of the coastal Kelvin waves to the east and west of the barrier will later be seen to give rise to completely different behavior on opposite sides of the island in addition to the difference in wavelengths between the equatorial Rossby waves on opposite sides. There is, once again, in Q_+ a disturbance on the other side of the forcing to the direction of propagation of coastal Kelvin waves, although now this disturbance decays exponentially with latitude. There is also a disturbance in the other hemisphere, within a distance of the order of the equatorial radius of deformation of the equator, as would be expected.

The x dependence is easily introduced into $Q_\pm$. To the west of the barrier the solution becomes independent of x for large t, and so equation (23) is correct for all $x \leqslant 0$. Care must be exercised though as the convergence to this limit is non-uniform in x. To the east of the barrier all disturbances except the equatorial Kelvin wave have a Bessel function dependence on $\sqrt{xt}$, as was shown in Anderson and Rowlands (1976b). The equatorial Kelvin wave becomes independent of x. In equation (29) the equatorial Kelvin wave is given by the term $\frac{1}{2\sqrt{\pi}}\exp(-y^2/4)$, as can be verified by multiplying by $D_o(y)$ and integrating over all y. This term must be independent of x and t, while the other terms are multiplied by $J_o(\sqrt{2xt})$ giving

$$Q_+(x,y,t,\eta) \sim \frac{1}{2\sqrt{\pi}}\, e^{-y^2/4} + \left\{ \frac{y}{4} + \frac{y}{2\sqrt{\pi}} \int_0^{\frac{y}{2}} e^{-\xi^2}\, d\xi + \delta(y-\eta) - \frac{y}{2} H(y-\eta) \right\} J_o(\sqrt{2xt})\,. \tag{30}$$

where J_o is the Bessel function of the first kind and of order zero.

5. The solution of the integral equation

Now that the Green functions are known we can solve the integral equation (7) once we have specified q_T, that is, once we have specified the form of the incident wave in $x < 0$. When the incident disturbance is an equatorial Kelvin wave, q_T is just the asymptotic value of q as $t \to \infty$ at $x = 0$ which was calculated in Anderson and Rowlands (1976a). Thus for an equatorial Kelvin wave whose amplitude becomes independent of time as $t \to \infty$ we have

$$q_T = 1/\sqrt{2} \tag{31}$$

where because the problem is linear we have assumed, without loss of generality, that the amplitude of the incident equatorial Kelvin wave is unity, i.e.,

$$q \sim D_o(y) \text{ as } t \to \infty \text{ for fixed } x\,. \tag{32}$$

We now restrict attention to the case of an island situated symmetrically about the equator, i.e., $b = -a$. This gives all the features for the general case but simplifies the analysis somewhat. First we consider a large island, $a >> 1$. For $\eta \geqslant a$ we then have

$$\left.\begin{aligned} Q_+ &= \frac{y}{2} + \delta(y-\eta) - \frac{y}{2}H(y-\eta) + 0(\eta^{-2}e^{-\eta^2/4}) \\ \text{and} \qquad & \\ Q_- &= \frac{-\eta}{2}H(y-\eta) + \delta(y-\eta) + 0(\eta^{-2}e^{-\eta^2/4}), \end{aligned}\right\} \tag{33}$$

and the integral equation (7) is

$$\frac{1}{2}\int_a^y \eta u(o,\eta)d\eta + \frac{1}{2}\int_y^\infty yu(o,\eta)d\eta = \frac{1}{\sqrt{2}} \text{ for } y \geqslant a. \tag{34}$$

This has the obvious solution

$$u(o,y) = \frac{\sqrt{2}}{a}\delta(y-a) \qquad \text{for } y \geqslant 0 \tag{35}$$

where we take the delta function to be applied at a value of y which is very slightly larger than a, in order to make the first term in equation (34) meaningful. By symmetry we have

$$u(o,y) = \frac{\sqrt{2}}{a}\{\delta(y-a) + \delta(y+a)\} \qquad \text{for all } y. \tag{36}$$

Then just to the west of the island we have, from equation (3),

$$q = \frac{\sqrt{2}}{a}\{\delta(y-a) + \delta(y+a)\} + \frac{1}{\sqrt{2}}H(y+a) - \frac{1}{\sqrt{2}}H(y-a), \tag{37}$$

and just to the east of the island equation (4) gives

$$q = \frac{\sqrt{2}}{a}\{\delta(y-a) + \delta(y+a)\} + \frac{1}{a}\sqrt{\frac{2}{\pi}}\left(e^{-y^2/4} + y\int_o^{y/2} e^{-\xi^2}d\xi\right) \left(H(y+a) - H(y-a)\right). \tag{38}$$

It should be noted that there is no disturbance for latitudes greater than a. This result is surprising as we would normally expect a coastal Kelvin wave at the tip of the island decaying in a distance of the order of the radius of deformation away from the tip. There are two possible explanations for the calculated behavior. Firstly, we have $a >> 1$ and so the radius of deformation at $y = a$ is very small ($2/a$, in fact) and we would, perhaps, not expect to be able to resolve distances of this magnitude. In fact $Q_\pm$ as given by equation (33) are accurate to within an exponentially small term and so if the disturbance were in fact spread over a radius of deformation this should be apparent in the solution (36). The second possibility, and almost certainly the correct interpretation, is that the effect is due to the vanishing thickness of the barrier[2]: there is no zonal coast to support a proper coastal Kelvin wave.

2. In the following paper Cane and du Penhoat show this results from the assumption of steady state inviscid flow, and not the thinness of the island as hypothesized here.

Here then we see the main drawback of our model. If the island were taken to be a rectangle, the solution would include the expected coastal Kelvin wave, and the zonal component of velocity to the north and south of the island given by equation (36) would be spread, meridionally, over a distance of the order of the radius of deformation.

One parameter of interest in this problem is the amplitude of the transmitted equatorial Kelvin wave. This gives a measure of the magnitude of the disturbance introduced by the island. It is given by

$$\frac{\int_{-\infty}^{\infty} q\, e^{-y^2/4}\, dy}{\int_{-\infty}^{\infty} e^{-y^2/2}\, dy} \quad \text{at } x = 0^+ . \tag{39}$$

From equation (37) this is

$$\frac{2}{a}\sqrt{\frac{2}{\pi}} . \tag{40}$$

Thus for large a the amplitude of the transmitted equatorial Kelvin wave is $0(a^{-1})$, and a large island is seen effectively to block the equatorial Kelvin wave. This result was not obvious *a priori* as it was expected that the coastal Kelvin waves might travel right around the island with almost constant amplitudes and generate an equatorial Kelvin wave at the east whose amplitude was comparable with that of the incident wave.

With such a small transmitted equatorial Kelvin wave we expect the flow to the east of the island to be dominated by short Rossby waves. In this case p would take the form of a series of cells of alternating sign due to the Bessel function behavior of the waves. We have $p = q - u$, and so just to the east of the island,

$$p = \frac{1}{a}\sqrt{\frac{2}{\pi}}\left(e^{-y^2/4} + y \int_{o}^{y/2} e^{-\xi^2}\, d\xi \right)\left(H(y+a) - H(y-a) \right) . \tag{41}$$

The reason for the smallness of the transmitted Kelvin wave is now apparent. The decrease of Q_+ toward the equator at $x=0$ causes p to decrease similarly, and for a large island p is very small by the time the equator is reached. For a disturbance which has p independent of time at $x=0$, the short Rossby waves have x-t dependence of the form $J_o(\sqrt{2xt})$ for $x \geqslant 0$ (see Anderson and Rowlands, 1976b), while the equatorial Kelvin wave is independent of x and t. If we separate the short Rossby waves from the equatorial Kelvin wave using equation (40), remembering that for the equatorial Kelvin wave $p = u = q/2$, we find that

$$p = \frac{1}{a}\sqrt{\frac{2}{\pi}}\, y \int_{o}^{y/2} e^{-\xi^2}\, d\xi \left[H(y+a) - H(y-a) \right] J_o(\sqrt{zxt}) + \frac{1}{a}\sqrt{\frac{2}{\pi}}\, e^{-y^2/4} \quad \text{for } x \geqslant 0 . \tag{42}$$

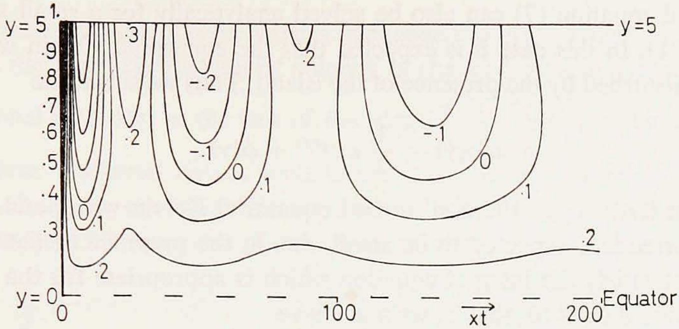

Figure 1. The pressure anomaly (p) in $x > 0$ and $y > 0$ for a large island ($a = 5$) showing the dominance of the short Rossby waves, linear variation of p at the coast, and no disturbance for $y > a$.

This solution is plotted for $a = 5$ in $y > 0$ in Figure 1, where the dominance of the short Rossby waves is obvious. As in Anderson and Rowlands (1976b), the flow round these cells is approximately geostrophic away from the island. It can be seen that p decreases linearly toward the equator at the coast, from 1 at $y = a$, and the dependence on xt indicates the westward propagation of the series of cells and the constant narrowing of the boundary layer. For an incident equatorial Kelvin wave in the first baroclinic mode with eastward flow in the upper layer, there is a downward displacement of the thermocline along the eastern coast of the island, and this is a maximum near the tips of the island. For the equatorial undercurrent, which has eastward flow in the middle layer of the second baroclinic mode, positive p corresponds to a thickening of the thermocline.

To the west of the island we have from equation (3), and noting that there is no x-dependence in the limit of long time,

$$q(x,y,t) \sim \frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{a}\left\{\frac{-a}{2\sqrt{\pi}}\int_{\frac{-a}{2}}^{a/2} e^{-\xi^2}\,d\xi - \frac{1}{\sqrt{\pi}}\,e^{-a^2/4} + \frac{a}{2}\left[H(y+a)-H(y-a)\right]\right.$$

$$\left. + \delta(y+a) + \delta(y-a)\right\}, \qquad x \leqslant 0 \tag{43}$$

$$= \frac{1}{\sqrt{2}}\left[H(y+a) - H(y-a)\right] + \frac{\sqrt{2}}{a}\left[\delta(y+a) + \delta(y-a)\right] + 0(e^{-a^2/4})\,. \tag{44}$$

Thus

$$p(x,y,t) \sim \frac{1}{\sqrt{2}}\left[H(y+a) - H(y-a)\right] + 0(e^{-a^2/4}) \text{ in } x \leqslant 0 \text{ as } t \to \infty\,. \tag{45}$$

Once again there is no disturbance outside the latitudes of the island, and within these latitudes the pressure anomaly becomes independent of position.

The integral equation (7) can also be solved analytically for a small symmetrical island ($a << 1$). In this case it is expected that the equatorial Kelvin wave will be only slightly disturbed by the presence of the island. Thus we substitute

$$u(o,y) = \frac{1}{2} e^{-y^2/4} + \phi(y) , \tag{46}$$

where the first term is just the undisturbed equatorial Kelvin wave, and the second term is the correction, expected to be small, due to the presence of the island. The first term must satisfy the integral equation which is appropriate for the case where no island is present ($a = 0$); that is, we must have

$$\frac{1}{2}\int_{-\infty}^{\infty} (Q_+ - Q_-) e^{-\eta^2/4}\, d\eta = \frac{1}{\sqrt{2}} \quad \text{for all } y . \tag{47}$$

It is easy to verify explicitly that this is indeed so. Then for $|y| > a$ the integral equation (7) becomes

$$\left(\int_{-\infty}^{-a} + \int_{a}^{\infty}\right)(Q_+ - Q_-)\phi(\eta)\, d\eta = \frac{1}{2}\int_{-a}^{a} (Q_+ - Q_-) e^{-\eta^2/4}\, d\eta \tag{48}$$

which for $a << 1$ can be written as

$$\left(\int_{-\infty}^{a} + \int_{a}^{\infty}\right)(Q_+ - Q_-)\phi(\eta)\, d\eta = \left[\frac{a}{2} + 0(a^3)\right]\left[\frac{1}{\sqrt{\pi}} e^{-y^2/4} + \frac{y}{\sqrt{\pi}}\int_{o}^{y/2} e^{-\xi^2}\, d\xi - \frac{y}{2} + \frac{1}{\sqrt{\pi}}\right] \quad \text{for } |y| > a . \tag{49}$$

The solution of this equation is

$$\phi(y) = \frac{a}{2}\left[\delta(y+a) + \delta(y-a)\right] + 0(a^3) \tag{50}$$

and so we have

$$u(o,y) = \frac{1}{2} e^{-y^2/4} + \frac{a}{2}\left[\delta(y+a) + \delta(y-a)\right] + 0(a^3) \text{ for } |y| > a << 1 . \tag{51}$$

Substituting this result in equation (4) it is found that just to the east of the island

$$q = e^{-y^2/4} + \left(\frac{y^2}{4} - \frac{1}{2}\right)\left[H(y+a) - H(y-a)\right] + \frac{a}{2}\left[\delta(y+a) + \delta(y-a)\right] + 0(a^3) . \tag{52}$$

Combining this with equation (51) we have

$$p = \frac{1}{2} e^{-y^2/4} + 0(a^2) \text{ for all } y , \tag{53}$$

just to the east of the island. Thus it is seen that as far as the pressure anomaly is concerned the equatorial Kelvin wave is unchanged to $0(a^2)$. However, equation (51) shows that the zonal component of velocity is changed by a quantity $0(a)$, but that the total zonal transport to the east of the island $\left(\int_{-\infty}^{\infty} u dy\right)$ is the same as it was in the incident equatorial Kelvin wave to order a^3. From equation (52) we find that the amplitude of the equatorial Kelvin wave in $x > 0$ is $1 + 0(a^3)$. Separating this contribution from u in equation (51) we have

$$u(x,y,t) = \frac{1}{2} e^{-y^2/4} + 0(a^3) + \left\{\frac{a}{2}\Big[\delta(y+a) + \delta(y-a)\Big] - \frac{1}{2}\Big[H(y+a) - H(y-a)\Big] + 0(a^2)\right\} J_o(\sqrt{2xt}) \,. \tag{54}$$

In particular, at the equator ($y = 0$) we have

$$u(x,o,t) = \frac{1}{2} + 0(a^3) - \left\{\frac{1}{2} + 0(a^2)\right\} J_o(\sqrt{2xt}) \,. \tag{55}$$

Thus the zonal component of velocity is seen to oscillate about $\frac{1}{2}$ (its value in the incident wave) with a maximum of 0.7 at $\sqrt{2xt} = 3.8$ and a subsequent minimum of 0.35 at $\sqrt{2xt} = 7.0$.

Similarly, for $x < 0$ we find

$$p = \frac{1}{2} e^{-y^2/4} + 0(a^2) \tag{56}$$

which demonstrates that to the west of the island, also, the pressure anomaly is changed only by a term $0(a^2)$ from that in the incident wave.

For other values of a, the integral equation (7) must be solved numerically. For this purpose the equation can be written in a rather more convenient form:

$$\frac{1}{\sqrt{\pi}} \int_a^{\infty} \left\{ e^{-y^2/4} + y \int_o^{y/2} e^{-\xi^2} d\xi + \eta \int_o^{\eta/2} e^{-\xi^2} d\xi + e^{-\eta^2/4} \right\} u(o,\eta) d\eta - \frac{y}{2} \int_a^{y} u(o,\eta) d\eta - \frac{1}{2} \int_y^{\infty} \eta u(o,\eta) d\eta = \frac{1}{\sqrt{2}} \quad \text{for } y \geqslant a \,. \tag{57}$$

The solution for $y \leqslant -a$ is obtained by symmetry. In this form the integral equation has a symmetric kernel. The equation is well conditioned in the sense of Baker *et al.* (1964). That is, the eigenvalues of the kernel do not decrease rapidly to zero as the order of the eigenfunction increases. This means that the naive method of solution given by Acton (1970) is suitable, as the high order eigenfunctions are not overemphasized by numerical errors. This method consists of writing equation (57) as a set of simultaneous equations for u at given values of y by expressing the integrals as sums. In practice these integrals may be evaluated satisfactorily using the trapezoidal rule with an interval of 0.1.

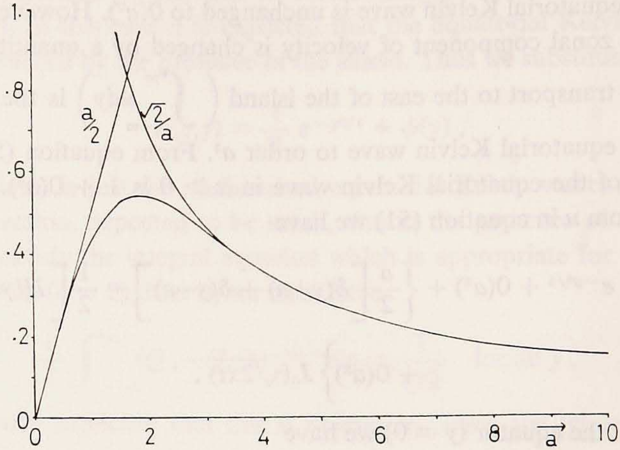

Figure 2. The amplitude of the delta function in the zonal component of velocity at $x = 0$ as a function of a. Also shown are the asymptotic curves for $a << 1$ and $a >> 1$.

It is noted that for both large and small islands the solution contains a delta function at the tip of the island, and so this is expected to be the case also for islands of intermediate size. That this is indeed so is indicated by the fact that when no special provision is made to include behavior in a numerical scheme, u has a large maximum at $y = a$. There is in fact no need to make special provision for this as the magnitude of the delta function can be inferred from the simple scheme by the following device. The solution near $y = a$ is extrapolated to give the amplitude of the contribution to u at $y = a$ from the term not including this delta function. The magnitude of the delta function is then obtained by subtracting this extrapolated value from that actually calculated at $y = a$ and noting that the trapezoidal rule "smears out" the delta function over half an interval in the integration scheme. Thus if the extrapolated value of u at $y = a$ is u_e, the calculated value is u_c, and the step length in the integration scheme is h, the amplitude of the delta function is given by

$$\frac{2}{h}(u_c - u_e)\,. \tag{58}$$

The amplitude of the delta function calculated in this way is shown in Figure 2, together with the known asymptotic values for very small and very large a. The numerical calculations merge smoothly into these asymptotic values in the appropriate limits, thus strongly suggesting the accuracy of the numerical calculations. It can be seen from this figure that the special cases of large and small islands treated above are probably of more general application than was expected to be the case. If we judge by the way in which the amplitude of the delta functions approaches the asymptotic value, it would appear that $a << 1$ means $a < 0.5$, and that $a >> 1$ means $a > 3.5$. In particular, most real islands on or near the equator would qualify

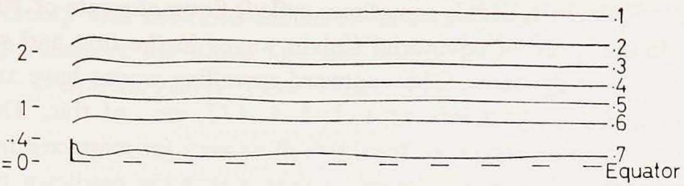

Figure 3. The pressure anomaly (p) in $x > 0$ and $y > 0$ for a small island ($a = 0.4$) showing the almost unchanged equatorial Kelvin wave.

as small islands, for in dimensional terms a small island means one less than 100 km in diameter for the first baroclinic mode or about 75 km for the second baroclinic mode.

The pressure anomaly, p, is calculated in $x > 0$ as before, and typical contours are shown in Figure 3 (for $a = 0.4$) and Figure 4 (for $a = 2$). The transition from the case of a small island, with virtually undisturbed equatorial Kelvin wave, to that of a large island, with a series of cells stretching eastward from the tips of the island, is apparent from these diagrams.

6. Interpretation and comparison with observations and existing theory

In interpreting the preceding results and applying them to the real ocean, the limitations of the theory must always be borne in mind. These approximations consist of the use of a linear model, the neglect of any form of dissipation, the neglect of bottom topography, and the simplified representation of the island.

In the absence of any form of dissipation the linearization must become invalid at some time, at least to the east of the island where the short Rossby waves cause the length scale to decrease without limit. Nonlinearities are expected to prevent this decrease from continuing below some value. To the west of the island nonlinearities would have a different effect. It was shown in Anderson and Rowlands (1976b) that a current flowing against the direction of propagation of a coastal Kelvin wave can prevent that wave from propagating if the current is sufficiently strong. A similar process is expected here with the eastward flowing undercurrent impeding the westward propagation of equatorial waves. The Pacific undercurrent

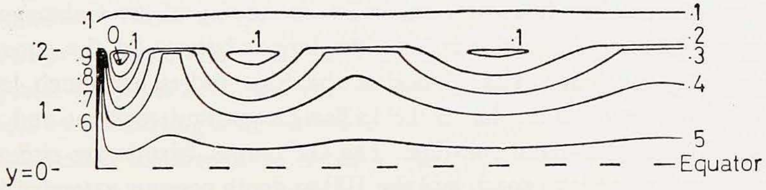

Figure 4. The pressure anomaly (p) in $x > 0$ and $y > 0$ for a medium-sized island ($a = 2$).

(or the Cromwell Current, as it is sometimes called) flows at a rate of 100 cm/sec^{-1} typically, while the speed of equatorial Kelvin waves in the first and second baroclinic modes is about the same. The westward travelling waves have velocities, for large times (i.e., low frequencies), of 1/3, 1/5, 1/7, etc., of this. Thus all these waves are expected to be prevented from travelling very far westward by the undercurrent. Comparison with the coastal case suggests that the predicted thickening of the thermocline to the west of the island will occur only within a certain distance of the island, and not infinitely far upstream as the linear theory predicts.

Dissipation is expected to affect mainly the narrowing boundary layer to the east of the island. The result will, of course, be similar to that for nonlinearity, that the narrowing will cease after some time and the flow will become steady. The sharp peaks in the velocity predicted near the tips of the island would also be affected, as the analysis indicates that they would be concentrated in a very short distance. However, this prediction has already been argued to be rather unrealistic due to the modelling of the island as a thin barrier. If such large velocities were to occur the effect of viscosity would be to broaden the peaks and perhaps to cause a reduction of the flow to the east of the island, in conjunction with nonlinear effects. With a more realistic model of the island which has some thickness, it is expected that the peaks would broaden into proper coastal Kelvin waves which could still be affected by dissipation and might even be caused to separate from the island, in conjunction with nonlinearities, to give two cores to the undercurrent to the east of the island. This replacing of the delta function peaks by coastal Kelvin waves is not expected to affect the other features of the flow very much. The general pattern of meandering currents to the east, with larger eddies for larger islands, will persist.

The inclusion of bottom topography in the vicinity of the island would lead to vertical mode mixing. An incident equatorial Kelvin wave in one vertical mode would generate disturbances in the other modes as it crosses the region of depth change. For instance, a change in the strength of the undercurrent (in the second baroclinic mode) would lead to disturbances being generated in the first baroclinic mode which would cause the thermocline, and thus the undercurrent as a whole, to change in depth. The inclusion of this topographic effect in the model is extremely difficult.

The theory developed in the preceding sections will now be compared with observations of the flow of the equatorial undercurrent around islands in the Pacific Ocean. Most observations have been made in the vicinity of the Galapagos Islands in the eastern Pacific. These islands form the largest barrier on the equator apart from continental coastlines. The islands of the East Indies are much larger, but they form an impenetrable barrier to the Indian Ocean undercurrent and may thus be considered as a continental coastline. The Galapagos Islands are situated about 10W of the South American coast, and the 100 m depth contour extends from about 0 to 1S. As these islands are rather near the coast, care must be taken in comparing the present theory with the observations to make sure that the effects observed are due to the presence of the island and not to any interaction with the coast. This is ensured by considering only those observations which are well away from the coast.

Knauss (1960, 1966) measured the undercurrent to the west of the Galapagos Islands. He found that the maximum speed decreased as the Galapagos were approached, from a value above 100 cm/sec^{-1} to rather more than 70 cm/sec^{-1} just to the west of the islands. The core of the undercurrent is at a depth of about 100 m in this region. To the east of the Galapagos he found maximum speeds of about 20 cm/sec^{-1} at depths below 150 m. He thus concluded that the effect of the islands is to reduce the speed of the undercurrent and to deepen its core. He also claims that the main part of the undercurrent flows around the north of the islands.

Stevenson and Taft (1971) compare measurements made at various times and conclude that the undercurrent speed to the east of the Galapagos is 1/2 to 1/3 of that to the west. However, they disagree with Knauss (*loc. cit.*) over his proposal that the core of the undercurrent is deeper to the east of the islands than it is to the west.

The properties described in the previous paragraphs compare favorably with our theoretical predictions. In the second baroclinic mode the horizontal length scale is about 160 km, which means that the Galapagos Islands, which have a latitudinal extent of about 100 km at the depth of the undercurrent (~100 m), cannot properly be considered a "small" island (latitudinal extent ≤80 km). Nevertheless, the flow is expected to be qualitatively similar (as indicated by our figures), the main differences being an increase in the meandering to the east of the island as compared with our predictions for a small island, and a greater reduction in the strength of the undercurrent. The analytical small island results are more convenient to use than are the numerical results for larger islands. First, from equation (55) we expect the zonal component of the current at the equator to vary with distance from the island, having a maximum of 1.4 times its value in the undercurrent far to the west of the island, and a minimum of 0.7 times the same amplitude. As the island is rather larger than the small island solution requires, we expect an oscillation about a lower mean value to be observed. Thus the observations of Stevenson and Taft (1971) are consistent with the present theory, at least at some longitudes. As was noted earlier, the theory cannot predict any change in the depth of the undercurrent core. The observations certainly suggest that this is a time-dependent phenomenon, and as such it is presumably due to either equatorial waves in the first baroclinic mode travelling past the islands, or to waves in other vertical modes which generate such disturbances by interacting with bottom topography.

A more detailed picture of the observed flow is given by White (1971b, 1973). His observations show that in the flow to the west of the islands there is a small region where the core of the undercurrent is thickened in the vertical. It was argued earlier that the inclusion of nonlinear effects in the analysis of the reflected long Rossby waves would lead to this result. The observed flow to the east of the islands is encouragingly similar to that shown in our figures, with an undercurrent speed of 15 cm/sec^{-1}, similar to that measured by Knauss (*loc. cit.*). White notes that the observations were taken over a sufficiently short period (~70 days) for any time dependence in the meandering flow not to be apparent. Once again, it is expected that nonlinearities and dissipation will inhibit the shortening of the zonal length scale predicted here. White (1971a) analyzed such a flow around islands using a nonlinear beta-plane model with barotropic flow and no meridional variation of the incident current. While the application of his model to the equatorial undercurrent is questionable on the last two aspects, he does show that nonlinearities prevent the time variation of the flow to the east of the island. This result is expected to apply equally to the present model. Observations over a larger period would be required to establish whether the actual flow is steady or not. The alternative hypothesis given by White (1973) for the observed cellular nature of the flow, that it is a von Kárnán vortex street, has been dismissed by Hendry and Wunsch (1973), who note that such a flow is essentially a two-dimensional (i.e., barotropic) phenomenon, while that under consideration at present is certainly three-dimensional (i.e., baroclinic). A point to note in White's observations is that the undercurrent appears to have two separate cores to the east of the Galapagos Islands. These appear to extend, approximately, from the northern and southern tips of the islands. This could well be a manifestation of the strong currents predicted at the tips of the islands in the present analysis. If nonlinearities are indeed important, as seems to be the case, these currents would be expected to separate from the island to give twin cores very much as observed. It is probably dangerous to try to deduce much more from the present analysis, as the presence of the South American coast must have a considerable effect on the flow to the east of the Galapagos Islands.

Further detailed observations of the flow of the undercurrent around an island are given by Hendry and Wunsch (1973). They consider the flow round Jarvis Island, which is located at 00°23′S, 160°00′W, in the central Pacific Ocean. This island has a latitudinal extent of only about 2 km, and thus certainly qualifies as a "small" island. They show that upstream of this island there is a small region where the core of the undercurrent is thickened in the vertical, and that immediately downstream it is thinnest. If geostrophic flow is assumed, their diagrams indicate that further downstream there is a pattern of meandering currents corresponding to the observed variation of the thickness of the core similar to that shown in our Figure 3. They develop a nonlinear theory, ignoring the Earth's rotation, which agrees reasonably well with the upstream flow but which gives completely erroneous results downstream. The success of their theory to the west of the island again suggests that nonlinear effects are important there. However, the flow to the east of the island, which they are completely unable to predict, is very similar to that given by the present theory, at least qualitatively, with a thinning of the undercurrent core just downstream and a subsequent oscillation in thickness further to the east. We thus conclude that even for very small islands the rotation of the Earth is important, giving rise to short Rossby waves which cause the observed wake of meandering currents.

7. Concluding remarks

This paper has described a possible mechanism for the generation of the observed flow of the equatorial undercurrent around islands. The work can also be used to describe the upwelling pattern due to equatorial waves in the first baroclinic mode which are incident upon such islands. The calculated solutions for this case are the same but the interpretation is slightly different. The length scale is now larger (~200 km) and the pressure anomaly is to be interpreted as vertical thermocline displacement, positive downward for eastward flow in the upper layer.

Owing to the anisotropy of Rossby waves the meandering wake behind an island can appear only to the east of the island. Barkley (1972) describes a similar wake to the northwest of an island at mid-latitudes. This wake, presumably, cannot be due to the mechanism described here; even though short Rossby waves would give a similar meandering pattern at mid-latitudes, this should occur only to the east of an island. Thus there is certainly some other mechanism at work in mid-latitudes, and whatever it is, it may well also occur at the equator.

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