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On the movements of deep mesoscale eddies in the North Atlantic

by Doron Nof[1,2]

ABSTRACT

A simplified three layer model is considered in order to examine the movements of deep mesoscale eddies such as the isolated Mediterranean eddies observed off the Bahamas. These anticyclonic eddies are found in the permanent thermocline more than 6000 km away from their parent water mass and are characterized by a lens-like cross section.

The model consists of two main layers, representing the upper and lower layer of the ocean, and an intermediate lens-like layer representing the eddy. It incorporates zonal eddy movements resulting from advection by uniform flows in both the upper and lower layer, and a β-induced westward translation. The movement of the eddy is assumed to be steady, frictionless and nondiffusive, but the motion is not constrained to be quasi-geostrophic. The combined β-induced movement and advection by the flows in the upper and lower layers is calculated analytically using the nonlinear equations of motion in an integrated form and a simple perturbation scheme.

It is found that the advection is a linear function of the different speeds within the two main layers, and is independent of the eddy's intensity, size and volume. The β-induced westward movement depends on the eddy's intensity, size and volume, and is rather slow for most deep anticyclonic eddies. For the Mediterranean eddies, the calculated β-induced movement is approximately 65 m per day. With such a slow self-propelled drift the eddies cannot cross the whole Atlantic Ocean because this would require a lifetime of over 250 years which is, obviously, impossible.

The presence of an advective flow of a few centimeters per second in either the lower or the upper layer can reduce the required lifetime to about 5-10 years. On this basis, it is suggested that the Mediterranean eddies are advected during most of their journey from the eastern to the western Atlantic.

1. Introduction

This paper has been motivated by the observations of McDowell and Rossby (1978) who identified an eddy containing Mediterranean water about 6000 km to
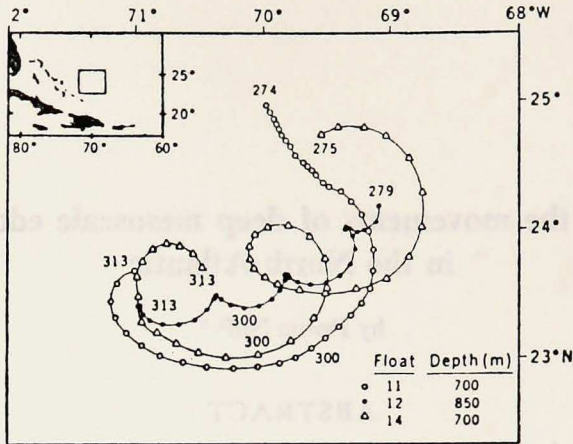
Figure 1. Trajectories of three SOFAR floats embedded in the Meddy [reproduced from McDowell and Rossby (1978)]. The floats were launched in October 1976 during RV *Oceanus* Cruise 15. Three-digit numbers adjacent to daily float positions denote the day of the year. The rectangle within the small scale physiographic inset illustrates the location of the survey relative to the Bahamas.

the west of its parent water mass. The Mediterranean eddy (hereafter, referred to as "Meddy") consisted of an anticyclonic lens with a radius of ~ 100 km and a maximum central depth of ~ 450 m (see Figs. 1 and 2). It had been observed in the permanent thermocline, and its orbital velocities reached a maximum of 30 cm/sec.

On the basis of water properties analysis, McDowell and Rossby (1978) concluded that the Meddy originated from the eastern Atlantic and was not related to the western part of the Mediterranean tongue which extends to large distances from the Straits of Gibraltar. The fact that the eddy was found more than 6000 km to the west of its original water mass raises the fundamental question regarding the mechanisms responsible for its movement within the ocean. McDowell and Rossby (1978) [hereafter, referred to as MR] suggested the interesting possibility that the β effect, which usually causes westward movement of mesoscale eddies, could be responsible for the existence of the eddy off the Bahamas. Because of the importance of such a process to the large-scale salt transport, it is of interest to examine the movements of isolated lens-like eddies in the deep ocean.

To do this, we shall consider a simplified model of an isolated anticyclonic eddy embedded in the permanent thermocline. We shall focus our attention on movements resulting from both advection and β-induced translation, and show that, as expected, the presence of β causes a westward drift of the entire eddy. We shall see, however, that the β-induced drift alone is so slow that it cannot be responsible for the existence of the eddy off the Bahama Islands. It will be suggested that advection,
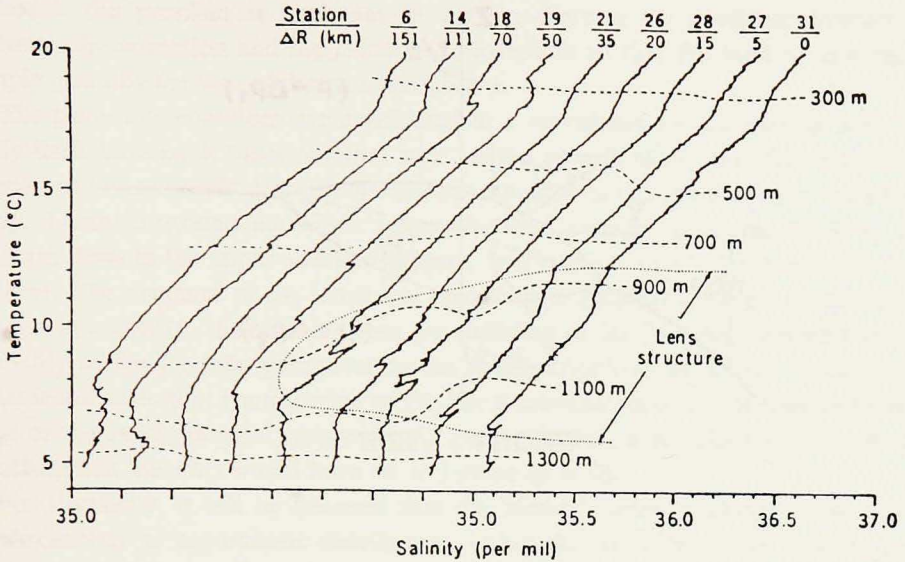
Figure 2. Temperature/salinity curves through the Meddy and the main thermocline for selected stations occupied during RV *Oceanus* Cruise 15 [reproduced from McDowell and Rossby (1978)]. The distance of each profile from the center of the Meddy is denoted by ΔR. Each successive profile has been displaced by 0.25 per mil.

rather than self-propulsion due to β, is the probable mechanism responsible for large scale movements of isolated eddies in the deep ocean.

The dynamics of mesoscale eddies received a considerable amount of attention during the last two decades. In particular, there have been a large number of both analytical and numerical investigations of the β-induced movements and the decay of mesoscale eddies. Among these studies are those of Warren (1967), Stern (1975), Firing and Beardsley (1976), Larichev and Reznik (1976), and Flierl (1977). These investigations are informative, but they do not deal directly with the movement of deep isolated eddies which is addressed in this study. The same can be said of Flierl (1979), Mied and Lindemann (1979), McWilliams and Flierl (1979), Flierl *et al.* (1980), and Nof (1981). None of these investigations have dealt specifically with a deep lens-like eddy which is subject to both the influence of β and the influence of advective flows in the environment.

To model the movements of the eddy, we shall consider a mean field consisting of two homogeneous layers with slightly different densities. Each layer contains a uniform flow which is directed eastward or westward. A third isolated layer which represents the eddy itself is embedded in the flow, underneath the upper layer and above the lower as shown in Figure 3. We shall assume that all motions are friction-
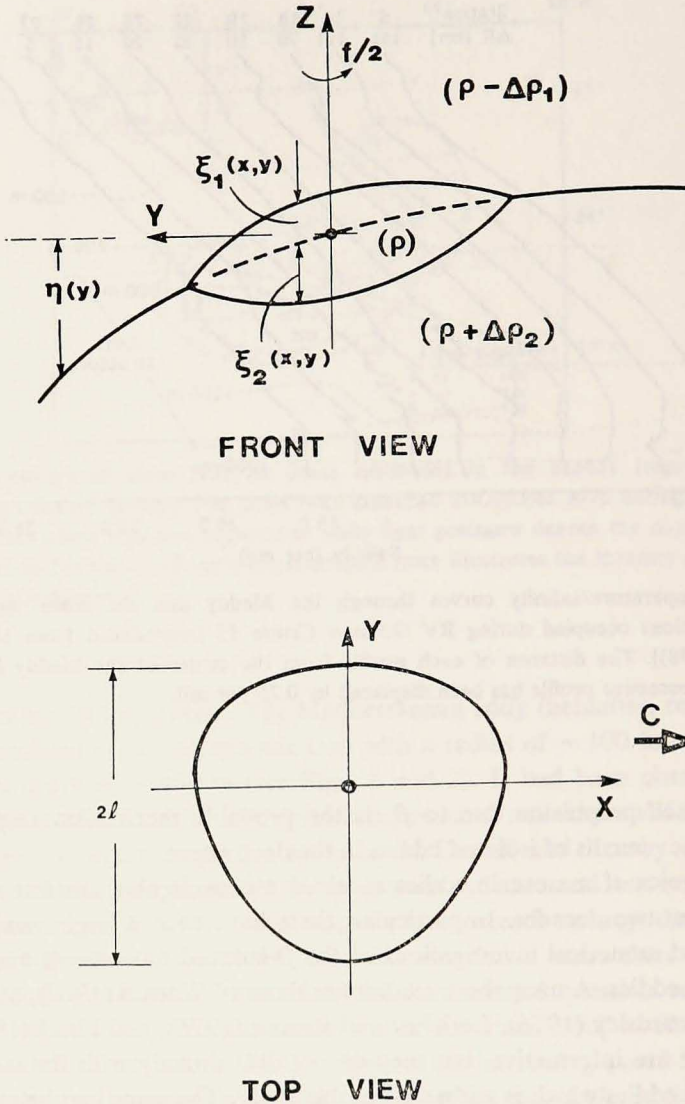
FRONT   VIEW

TOP   VIEW

Figure 3. Schematic diagram of the model under study. Each of the two main layers [whose densities are $(\rho - \Delta\rho_1)$ and $(\rho + \Delta\rho_2)$] contain uniform zonal flow and is infinitely deep. The vertical displacement of the main interface $\eta(y)$ is measured downward from the origin $[\eta(O) = 0]$. The vertical displacements of the eddy's upper and lower interfaces [$\xi_1(x,y)$ and $\xi_2(x,y)$] are measured upward and downward from the main interface.

less and nondiffusive and that the eddy translates at a constant speed without changing its shape and structure with time. The translation consists of two components, β-induced drift and advection by the ambient fluids. We shall see that

although the problem is nonlinear in nature, there is no nonlinear interaction between the advection and the β-induced movement so that the total translation is simply given by the sum of the two movements.

The governing equations are considered in a coordinate system moving with the eddy itself because it turns out that with such a coordinate system the translation speed can be easily calculated by integrating the governing equations over the whole eddy. These integrated equations relate the total translation of the eddy to the uniform flows in the upper and lower layers, and to the eddy's intensity, its size and volume. The structure of the integrated equations is simplified using a perturbation scheme in $\epsilon = \beta l / f_o$, the ratio between the variation of the Coriolis parameter across the eddy to the Coriolis parameter in the center. Application of this perturbation scheme indicates that computation of the zonal movements does not require knowledge of the exact eddy's structure on a β plane. Rather, it is sufficient to know the structure that the eddy would have on an f plane ($\beta = 0$).

For simplicity, it will be assumed that the Meddy's orbital velocity corresponds approximately to a parabolic distribution. Using this velocity profile and the perturbed integrated equations, the total zonal movement (consisting of both advection and β-induced movement) is expressed in terms of the eddy's orbital speed, the densities in the field, the eddy's depth and the speeds in the upper and lower layers.

The results of the mathematical model are then combined with the data presented by MR to give the β-induced drift, and the advection by the two environmental layers. The analysis is concluded with an examination of the calculated translation speeds and the corresponding eddy's lifetime. This analysis reveals that the β-induced drift is so slow that an eddy with a lifetime of several years cannot translate across the whole Atlantic Ocean unless it is advected by local currents.

This paper is organized as follows: the formulation of the problem is presented in Section 2 and the perturbation analysis in Section 3. Section 4 contains the applicability of the model to the Meddy and Section 5 summarizes this study.

2. Formulation

As an idealized formulation of the problem, consider the three-layer model shown in Figure 3. The intermediate lens-like layer, whose density is ρ, represents the eddy itself and translates zonally at speed C (positive for eastward movement and negative for westward movement). The upper layer [whose density $(\rho - \triangle \rho_1)$ is slightly lower than that of the eddy] moves zonally at a constant uniform speed U_1, and the lower layer $(\rho - \triangle \rho_1)$ contains a uniform flow U_2. These two layers are of infinite depth and represent the movements above and below the permanent thermocline.

We shall consider a coordinate system moving with the eddy itself at speed C. The origin of this coordinate system is located at the center of the eddy. The x and y axes are directed eastward and northward (respectively), and the system rotates

with angular velocity $\frac{1}{2}f$ about the vertical axis. It is assumed that the translation is steady and that the eddy shape does not change much in time so that in our moving coordinate system the motion is steady. This assumption of negligible changes in time is plausible, but it is not a priori obvious under what conditions it is an adequate approximation. Using scaling arguments it will be shown later that the assumption is adequate as long as $(\beta l/f_o)^2 \ll R_o$, where R_o is the Rossby number, f_o the Coriolis parameter at $y = 0$, β the linear variation of the Coriolis parameter with latitude and l is the eddy length scale defined as half the distance between the northernmost and southernmost edges.

We shall focus our attention on frictionless and nondiffusive movements and assume that all motions are hydrostatic. For hydrostatic motions the pressure in each layer is a linear function of z so that the horizontal pressure gradients depend on x and y alone. Therefore, the horizontal velocity components in each layer are also independent of z.

The governing equations for our zonally moving coordinate system are obtained by applying the transformations $x_* \rightarrow x + Ct$ and $y_* \rightarrow y$ to the time-dependent equations in a stationary coordinate system (x_*, y_*). For the conditions mentioned above, the equations governing the movements of the eddy are found to be:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - (f_o + \beta y)v = - \frac{1}{\rho} \frac{\partial P}{\partial x} \tag{2.1}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + (f_o + \beta y)(u + C) = - \frac{1}{\rho} \frac{\partial P}{\partial y} \tag{2.2}$$

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 , \tag{2.3}$$

where u and v are the depth independent $[u = u(x,y), v = v(x,y)]$ horizontal velocity components in the x and y direction, $h(x,y)$ is the total eddy's depth and P is the deviation of the hydrostatic pressure from the hydrostatic pressure associated with a state of rest. Note that the term $(f_o + \beta y)C$ on the left-hand side of 2.2 results from the fact that in a moving coordinate system there is an additional force acting on all fluid parcels.

It is further assumed that the eddy is thin and shallow (i.e., $h \ll l$) so that the movement of the eddy through the environment has a negligible effect on the flow in both the upper and lower layer. This negligible effect results from the fact that the ratio between the eddy depth to the depth of the environmental layers vanishes. For these conditions, potential vorticity conservation for the upper and lower layer implies that the influence of the eddy movement is vanishingly small. In this respect, the behavior of the upper and lower layer is equivalent to the hydrostatic flow of a homogeneous infinitely deep fluid over a bump whose height is small; under such conditions the effect of the bump is negligible [see e.g., Ingersoll (1969), eq. (27)

with $h_o \to 0$ since $H \to \infty$]. Thus, the shallow eddy moves through the environment as a "blade" leaving the flow in the upper and lower layer unaltered. It is important to note, however, that the situation would have been quite different had the upper and lower layers not been infinitely deep. With a finite upper and lower layer there will probably be important interactions between the eddy and its surroundings so that the subsequent analysis may not be applicable.

Within the eddy, at $z = 0$, the deviation of the hydrostatic pressure from the hydrostatic pressure associated with a state of rest is:

$$P = P_u + g\Delta\rho_1 (\xi_1 - \eta),\tag{2.4}$$

where P_u is the pressure deviation in the upper layer, $\eta(y)$ is the vertical displacement of the main interface [measured downward from the origin (i.e., $\eta(0) = 0$)] and $\xi_1(x,y)$ is the vertical displacement of the eddy upper interface [measured upward from the main interface (see Fig. 3)]. The flow in both the upper and the lower layer is in geostrophic balance so that for a Boussinesq fluid:

$$(f_o + \beta y)U_1 = - \frac{1}{\rho} \frac{\partial P_u}{\partial y}\tag{2.5}$$

and

$$(f_o + \beta y) (U_2 + U_1) = \frac{(\Delta\rho_1 + \Delta\rho_2)}{\rho} g \frac{\partial \eta}{\partial y} ,\tag{2.6}$$

where $U_1 > 0$ and $U_2 > 0$ for an eastward flow in both layers, and $U_1 < 0$ and $U_2 < 0$ for a westward flow. Note that (2.6) was obtained by expressing the pressure in the lower layer in terms of $\eta(y)$ and the velocity in the upper layer, and that U_1 and U_2 are the absolute velocities (relative to a fixed coordinate system). Eqs. (2.5-2.6) do not involve the translation speed C because in a moving coordinate system its effect on a zonal flow is cancelled out.

Since the presence of the eddy does not alter the flow in the upper and lower layers, it does not introduce any pressure deviations in the lower layer so that:

$$\xi_2 = \xi_1 \frac{\Delta\rho_1}{\Delta\rho_2} ,\tag{2.7}$$

where $\xi_2(x,y)$ is the vertical displacement of the eddy's lower interface [measured downward from the main interface (see Fig. 3)]. Substitution of (2.7), (2.6) and (2.5) into (2.4), and inserting the resulting equation into (2.1) and (2.2) give:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - (f_o + \beta y)v = -g^* \frac{\partial h}{\partial x}\tag{2.8}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + (f_o + \beta y) \left(u + C - \frac{U_1\Delta\rho_2 + U_2\Delta\rho_1}{\Delta\rho_1 + \Delta\rho_2} \right) = -g^* \frac{\partial h}{\partial y}\tag{2.9}$$

where $h = \xi_1 + \xi_2$ and g^* corresponds to a "reduced gravity" defined by:

$$g^* = \frac{\Delta \rho_1 g}{\rho(1 + \Delta \rho_1 / \Delta \rho_2)} \ . \tag{2.10}$$

The set (2.8-2.9) is subject to the following boundary conditions:

$$h = 0 ; \qquad \phi(x,y) = 0 \tag{2.11a}$$

$$(\mathbf{i}u + \mathbf{j}v) \cdot \nabla \phi = 0 ; \qquad \phi(x,y) = 0 \tag{2.11b}$$

where ∇ is the horizontal (two-dimensional) del-operator and ϕ denotes the eddy's outer edge. Condition (2.11a) states that $h = 0$ along a curve which is not known in advance (ϕ) and (2.11b) requires that the edge will be a streamline. These conditions correspond to the fact that the location and shape of the eddy's outer edge are not known a priori, but rather must be determined as part of the problem.

Before concluding the present discussion, it is appropriate to comment on the class of solutions which are admittable by (2.8-2.11). By writing (2.8), (2.9) and (2.11) in polar coordinates, it is easy to show that if $\beta = 0$ and $U_1 = U_2 = 0$ the system always possesses solutions which are radially symmetric corresponding to purely tangential motion. This means that an eddy on an f plane which is not subject to any external forcing is always circular. It will be shown later (Section 3) that the presence of advection $[U_1 \neq 0, U_2 \neq 0]$ causes only translation and does not affect the eddy's structure in any way. Consequently, an eddy which is advected on an f plane will have the same circular structure that it would have in the absence of advection.

On the other hand, once β is introduced to the problem, the system (2.8-2.11) no longer possesses radially symmetric solutions. Therefore, an eddy translating on a β plane cannot have an exact circular structure. However, since the perturbations imposed by β are small ($\beta l << f_o$) it is expected that on a β-plane the eddy's structure will not deviate much from that of an f plane so that its geometry will correspond to slightly distorted circles. The structure of (2.8-2.11) indicates that the small horizontal distortions (from an exact circle) are of $0(\epsilon l)$ at the eddy's outer edge. These theoretical considerations regarding the influence of β are supported by the observations of MR who state that the "nearly circular geometry" was evident as expected theoretically.

3. Solution

In this section, we shall derive an expression for the nonlinear translation speed and show that the translation can be calculated without finding the detailed eddy structure on a β plane. To do so, eq. (2.9) is multiplied by h and integrated over the whole eddy:

$$\iint_s \left(hu \frac{\partial v}{\partial x} + hv \frac{\partial v}{\partial y} \right) dxdy + \iint_s (f_o + \beta y) uh dxdy$$

$$+ \left(C - \frac{U_1 \Delta \rho_2 + U_2 \Delta \rho_1}{\Delta \rho_1 + \Delta \rho_2} \right) \iint_s (f_o + \beta y) h dxdy = -\frac{g^*}{2} \iint_s \frac{\partial}{\partial y} (h^2) dxdy \tag{3.1}$$

where s denotes the entire area of the eddy. Using (2.3) and Stokes' theorem, (3.1) can be expressed as:

$$-\int_\phi hv^2 dx + \int_\phi huv dy + \iint_s (f_o + \beta y) uh dxdy$$

$$+ \left(C - \frac{U_1 \Delta \rho_2 + U_2 \Delta \rho_1}{\Delta \rho_1 + \Delta \rho_2} \right) \iint_s (f_o + \beta y) h dxdy = -\frac{g^*}{2} \int_\phi h^2 dx, \tag{3.2}$$

where ϕ denotes the eddy's outer edge (2.11a). Since $h = 0$ along ϕ, the line integrals over the nonlinear terms and the pressure term vanish identically. With the aid of the transport function:

$$\frac{\partial \psi}{\partial y} = -uh \; ; \frac{\partial \psi}{\partial x} = vh \;, \tag{3.3}$$

the resulting equation can be written in the form:

$$-\iint_s \left\{ \frac{\partial}{\partial y} \left[(f_o + \beta y) \psi \right] - \beta \psi \right\} dxdy$$

$$+ \left(C - \frac{U_1 \Delta \rho_2 + U_2 \Delta \rho_1}{\Delta \rho_1 + \Delta \rho_2} \right) \iint_s (f_o + \beta y) h dxdy = 0 \tag{3.4}$$

which by defining ψ to be zero on the boundary [i.e., $\psi = 0$ along ϕ], and using Stokes' theorem gives:

$$C = \frac{U_1 \Delta \rho_2 + U_2 \Delta \rho_1}{\Delta \rho_1 + \Delta \rho_2} - \frac{\beta \iint_s \psi dxdy}{\iint_s (f_o + \beta y) h dxdy}. \tag{3.5}$$

The first term on the right-hand side represents the advection by the upper and lower layers (C_a) and the second is the β-induced westward drift (C_β). The former (C_a) results from the pressure which is directly transmitted from the environment to the eddy, whereas the latter (C_β) represents a balance between the net southward force due to β and the northward force due to the westward translation and the

Coriolis force. Note that when the procedure outlined above is applied to eq. (2.8), one finds that all the integrals vanish identically indicating that the integrated forces in the x direction are in balance for any translation speed.

To examine the role of the environmental flows, it is convenient to express the advection (C_a) in terms of the displacements ξ_1 and ξ_2. Let ξ_{10} and ξ_{20} denote the displacements of the eddy upper and lower boundaries at the center [i.e., $\xi_{10} = \xi_1(0,0)$, $\xi_{20} = \xi_2(0,0)$, and $\xi_{10} + \xi_{20} = h_o$]. By (2.7) one finds that in terms of these variables the advection by the upper and lower layer is:

$$C_a = \frac{U_1\xi_{10} + U_2\xi_{20}}{\xi_{10} + \xi_{20}} \tag{3.6}$$

indicating that the advection is simply a weighted average of the mean speeds. In other words, the effect of each layer on the eddy translation is proportional to its relative displacement (ξ_o/h_o) within the layer. Thus, an eddy may be advected in one direction even if the flow in one of the layers is directed in the opposite direction.

It is easy to show that the flow in the environmental layers causes only translation and does not introduce any changes in the structure of the eddy. To show this, (3.5) is substituted into (2.8) and (2.9) and β is set equal to zero ($U_1 \neq 0$, $U_2 \neq 0$). This reduces the equations to the same form that they would have in the absence of advection ($U_1 = U_2 = 0$) indicating that the advection does not influence the internal structure of the eddy nor does it affect the eddy's shape.

Eq. (3.5) shows that knowledge of $\psi(x,y)$ and $h(x,y)$ is required for calculating the β-induced drift; to determine the specific necessary information regarding these functions, a perturbation scheme will be applied to (3.5). For this purpose, the following nondimensional variables are defined:

$$x^* = x/l; \qquad y^* = y/l; \qquad h^* = h/h_o; \qquad \psi^* = \psi/Vh_ol$$

$$\epsilon = \beta l/f_o; \qquad R_d = (g^*h_o)^{\frac{1}{2}}/f_o; \qquad C_\beta^* = C_\beta/V \tag{3.7}$$

$$R_o = V/f_ol$$

where V is the velocity scale, l is the eddy length scale (defined earlier in Section 2), R_d is the internal deformation radius and R_o is the Rossby number. The parameter ϵ is small for most eddies of practical interest; for the Meddy, $l \sim 100$ km, $\beta \sim 2 \times 10^{-11}$m^{-1}sec^{-1}, and $f_o \sim 10^{-4}$sec^{-1} so that $\epsilon \sim 0.02$.

In terms of these nondimensional variables, the β-induced translation is:

$$C_\beta^* = - \frac{\epsilon \int\int_{g^*} \psi^* dx^* dy^*}{\int\int_{g^*} (1 + \epsilon y^*)h^* dx^* dy^*} \tag{3.8}$$

where S^* is the nondimensional area of the eddy ($S/\pi l^2$). It is further assumed that the dependent variables possess power series expansions in ϵ:

$$\psi^*(x^*,y^*,\epsilon) = \psi^{(0)} + \epsilon\psi^{(1)} + 0(\epsilon^2) + \ldots$$

$$h^*(x^*,y^*,\epsilon) = h^{(0)} + \epsilon h^{(1)} + 0(\epsilon^2) + \ldots \qquad (3.9)$$

$$C_\beta^* = \epsilon C^{(1)} + \epsilon^2 C^{(2)} + \ldots$$

where the zeroth-order state corresponds to the structure that the eddy would have in the absence of β. As pointed out earlier in Section 2, the zeroth-order state ($\psi^{(0)}$, $h^{(0)}$) is always radially symmetric while the perturbations ($\epsilon\psi^{(1)}$, $\epsilon h^{(1)}$) correspond to small distortions of the circular geometry.

By substitution of (3.9) into (3.8) and collecting terms of $0(\epsilon)$ one finds:

$$C_\beta^* = \frac{-\epsilon \displaystyle\iint_{S^{(0)}} \psi^{(0)} dx^* dy^*}{\displaystyle\iint_{S^{(0)}} h^{(0)} dx^* dy^*} + 0(\epsilon^2) \qquad (3.10)$$

where $S^{(0)}$ is the area of the eddy corresponding to the zeroth-order state ($\beta = 0$). We see that in order to compute the β-induced translation it is sufficient to know the structure that the eddy would have on an f plane; it is not necessary to seek the detailed structure on a β plane. This results from the fact that the influence of the distortions on the translation speed is of $0(\epsilon^2)$.

Since $\psi^{(0)}$ and $h^{(0)}$ are always radially symmetric, it is convenient to express (3.10) in polar coordinates; for clarity we shall use these coordinates in dimensional variables. In terms of these dimensional variables the total translation speed is:

$$C = \frac{U_1\Delta\rho_2 + U_2\Delta\rho_1}{\Delta\rho_1 + \Delta\rho_2} - \frac{\beta \displaystyle\int_0^{r_o}\int_{r_o}^r \bar{V}_\theta(r)\bar{h}(r)dr dr}{f_o \displaystyle\int_0^{r_o} \bar{h}(r)r dr} + 0\left[\left(\frac{\beta r_o}{f_o}\right)^2 \bar{V}_\theta\right] \qquad (3.11)$$

where $\bar{V}_\theta(r)$ and $\bar{h}(r)$ are the swirl velocity and depth corresponding to the basic state, and r_o is the eddy's radius corresponding to the same state [$\bar{h}(r_o) = 0$]. Eq. (3.11) includes the full nonlinear terms and hence allows calculation of the translation speed for any Rossby number.

As pointed out earlier, our analysis is valid as long as the eddy translates steadily without changing its shape and structure with time. Since we have not found the complete first-order solution within the eddy nor have we shown that such a

solution exists, we cannot find the exact necessary conditions for such a dynamical behavior. However, using scaling arguments it will be demonstrated below that even if the structure of the translating eddy changes in time, the changes are small and can be neglected for the Meddy and for other mesoscale eddies.

To illustrate this point, it is recalled that the β-induced distortion of the eddy's outer edge (from an exact circle) is of $O(\epsilon l)$. If the eddy changes its shape due to β, then the associated time scale is $O(1/\beta l)$ which together with the above length scale (ϵl) gives a velocity scale of $O(\epsilon \beta l^2)$. This velocity scale represents the velocity component which is associated with time-dependent movements if such movements exist.

The governing equation (2.2) indicates that time dependent motions can be neglected as long as: $\dfrac{\partial u}{\partial t} \ll f_o C$. When the scales mentioned above and $C \sim O\left(\dfrac{\beta l}{f_o} V\right)$ are substituted into this relationship, one finds that the neglect is justified as long as: $R_o \gg (\beta l/f_o)^2$. This condition is satisfied by the Meddy because the parameter $(\beta l/f_o)^2$ is about 4×10^{-4} whereas the Rossby number is about 0.1. We see, therefore, that the assumption of steady translation and an approximately steady shape is probably adequate for the Meddy.

4. Analysis

As a simplification of the Meddy structure, we shall consider an eddy whose f plane structure corresponds to a parabolic velocity distribution. The adopted velocity profile is shown in Figure 4 and is given by:

$$\bar{V}_\theta = 2R_o f_o r \left(\frac{r}{r_o} - 1\right), \qquad (4.1)$$

where R_o is the Rossby number defined on the basis of the maximum velocity and its distance from the center of the eddy ($r_o/2$). It is important to note that there is an upper bound to the degree of nonlinearity which can be associated with (4.1). The Rossby number (R_o) cannot assume values larger than $1/4$ because otherwise the negative relative vorticity $\left[\dfrac{1}{r}\dfrac{d}{dr}(r\bar{V}_\theta)\right]$ would be larger than f_o (as $r \to 0$) which is impossible. In addition, note that the ratio between the centrifugal acceleration and the Coriolis force ($\bar{V}_\theta/f_o r$) reaches a maximum at the eddy center ($r = 0$) where it equals $2R_o$. The depth of the eddy $h(r)$ is found by integrating the nonlinear momentum equation:

$$\frac{\bar{V}_\theta^2}{r} + f_o \bar{V}_\theta = g^* \frac{dh}{dr} \qquad (4.2)$$
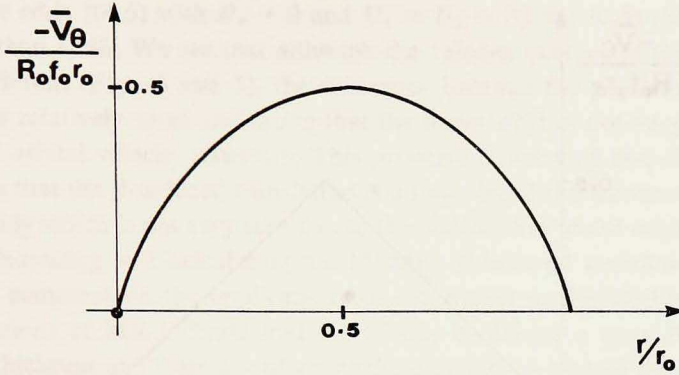
which gives:

Figure 4. The assumed tangential velocity structure for the Meddy.

$$h(r) = h_o + R_o f_o^2 r^2 (2R_o - 1)/g^* + 2R_o f_o^2 r^3 (1 - 4R_o)/3g^* r_o$$
$$+ R_o^2 f_o^2 r^4 / g^* r_o^2 . \tag{4.3}$$

The corresponding eddy radius (r_o) is obtained by setting $h(r_o) = 0$ in (4.3) and solving for r_o:

$$r_o = \left[\frac{3g^* h_o}{R_o(1 - R_o)} \right]^{\frac{1}{2}} /f_o . \tag{4.4}$$

The β-induced translation and the associated total translation can now be calculated by using (4.4), (4.3), (4.1) and (3.11). For the Meddy, the Rossby number is of $0(0.1)$ [see MR] so that one may keep terms of $0(R_o)$ and neglect terms of $0(R_o^2)$ in calculating the β-induced drift. Under these conditions, (4.4), (4.3), (4.1) and (3.11) give:

$$C = \frac{U_1 \triangle \rho_2 + U_2 \triangle \rho_1}{\triangle \rho_1 + \triangle \rho_2} - \frac{\beta(0.285 + 0.306\, R_o) \triangle \rho_1 g h_o}{\rho(1 + \triangle \rho_1 / \triangle \rho_2) f_o^2}$$
$$+ 0 \left(\frac{\beta^2 r_o \triangle \rho_1 g h_o}{\rho f_o^3} \right) + 0 \left(\frac{R_o^2 \beta \triangle \rho_1 g h_o}{\rho f_o^2} \right) , \tag{4.5}$$

which enables one to compute the total translation on a β plane. Because of the dependence of (3.11) on $\bar{V}_\theta(r)$, it is appropriate to examine the sensitivity of this result to the assumed orbital velocity structure (4.1). To do so, we shall calculate the β-induced translation for a different velocity distribution and compare the predicted translation speeds. For this purpose, consider the wedge-like velocity structure shown in Figure 5. The corresponding velocity distribution is:

$$\bar{V}_{\theta 1} = -R_o f_o r \quad ; \quad 0 \leqslant r \leqslant r_o/2$$
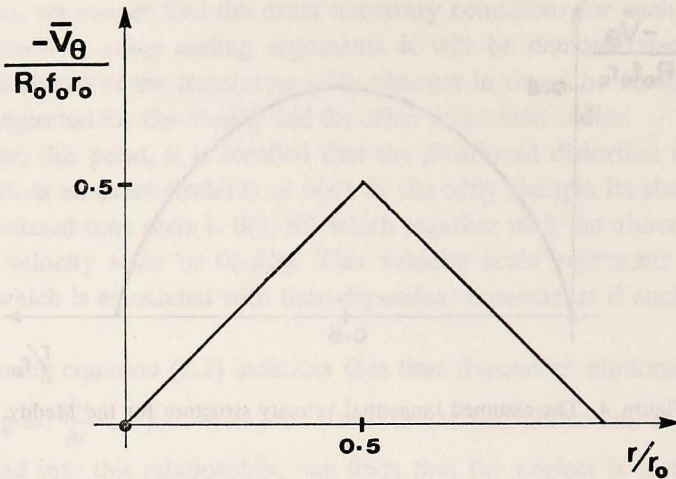$$\bar{V}_{\theta 2} = R_o f_o (r - r_o) ; \quad r_o/2 \leqslant r \leqslant r_o , \tag{4.6}$$

Figure 5. A wedge-like velocity structure used for the sensitivity study.

where the subscripts 1 and 2 indicate that the variable in question is associated with the inner portion of the eddy $(r \leqslant r_o/2)$ and the outer portion $(r \geqslant r_o/2)$, respectively. Since the Meddy's Rossby number is of $0(0.1)$ and we are interested now merely in examining the sensitivity of C_β to the assumed velocity profile, it is sufficient to consider the linear β-induced drift $(R_o << 1)$. For linear movements the depth distribution [corresponding to (4.6)] is:

$$h_1 = h_o - \frac{R_o}{2} f_o^2 r^2/g^*$$

$$h_2 = h_o + \frac{R_o f_o^2}{g^*}\left(\frac{r^2}{2} - r_o r + \frac{r_o^2}{4}\right) \tag{4.7}$$

and the eddy's radius $[\bar{h}_2(r_o) = 0]$ is:

$$r_o = \left(\frac{4g^* h_o}{R_o}\right)^{\frac{1}{2}} /f_o. \tag{4.8}$$

In a similar fashion to previously, the β-induced translation (C_β) is calculated with the aid of (4.8), (4.7), (4.6) and (3.11). By substituting (4.7), (4.6) and (4.8) into (3.11) and integrating in two steps (from $r = 0$ to $r = r_o/2$ and from $r_o/2$ to r_o) one obtains:

$$C_\beta = -\frac{0.329\,\beta\Delta\rho_1 g h_o}{\rho(1 + \Delta\rho_1/\Delta\rho_2)f_o^2} + 0\left(\frac{R_o\beta\Delta\rho_1 g h_o}{\rho f_o^2}\right)$$

$$+ 0\left(\frac{\beta^2 r_o \Delta\rho_1 g h_o}{\rho f_o^3}\right) \tag{4.9}$$

When this velocity is compared to the linear β-induced translation computed for

the parabolic eddy [(4.5) with $R_o \to 0$ and $U_1 = U_2 = 0$] one finds that the difference is less than 15%. We see that although the velocity profiles of the two eddies are quite different (Figs. 4 and 5), the difference between the predicted β-induced translation is relatively small indicating that the linear drift is not very sensitive to the assumed orbital velocity structure. This property is not very surprising because (3.10) shows that the β-induced translation is a function of the transport circulating within the eddy which is not very sensitive to the distribution of the orbital velocity.

Before proceeding and calculating the Meddy's β-induced translation, it is appropriate to comment on the applicability of our model to the Meddy's structure. The observations of MR indicate that the Meddy contained a quasi-homogeneous core with a thickness and diameter of approximately 200 m and 20 km respectively. Trajectories of neutrally buoyant (SOFAR) floats show, however, that the associated dynamical structure was considerably larger than this central core and extended to about 80-100 km (see MR and Fig. 1). This suggests that active mixing occurred at the outer edges of the eddy. For the purpose of computing the advection and the β-induced drift we shall neglect this mixing along the edges and assume that the whole eddy consists of seawater whose properties are identical to those found in the central core. It will become clear shortly that this choice may cause an overestimate, but not an underestimate, of the translation speed.

The data presented by MR suggest that the average density of the Meddy was $\rho \approx 1.02745$ and that the densities above and below the Meddy were, $(\rho - \Delta\rho_1) \approx 1.02680$ and $(\rho + \Delta\rho_2) \approx 1.02790$ (see Fig. 2 in MR). These values correspond to a $g^* \left[= \dfrac{g\Delta\rho_1}{\rho(1 + \Delta\rho_1/\Delta\rho_2)} \right]$ of about 0.27 cm/sec^2 which with the observed maximum depth of $h_o \approx 450$ m gives an internal deformation radius $[(g^*h_o)^{\frac{1}{2}}/f_o]$ of approximately 11 km. Based on MR's direct measurement of the maximum velocity (0.3 m/sec at ~ 50 km from the center), one finds that the corresponding Rossby number [as defined by (4.1)] is about 0.06. For the above deformation radius and Rossby number, the predicted eddy's radius (4.4) is approximately 80 km which is fairly close to the observed radius (~ 100 km). This agreement supports our assumption that the velocity profile can be approximated by a parabolic distribution.

For the above numerical values, the computed β-induced translation [given by the second term on the right-hand side of (4.5)] is:

$$C_\beta \approx -7.4 \times 10^{-2} \text{ cm sec}^{-1} = -64 \text{ m/day}$$

and the corresponding total translation $(C_\alpha + C_\beta)$ is:

$$C \approx [0.41U_1 + 0.59U_2 - 7.4 \times 10^{-2}] \text{ cm sec}^{-1} . \qquad (4.10)$$

We see that the predicted β-induced translation (C_β) is rather slow; with such a slow drift it would take the eddy over 250 years to cross the whole Atlantic Ocean (~ 6000 km). Since such a long lifetime is obviously impossible, it is reasonable to

suppose that the Meddy was advected from the eastern Atlantic to the west. This suggestion is consistent with the analysis presented by Sverdrup *et al.* (1942), and Worthington (1976) who require westward flow of a few centimeters per second in the subtropical Atlantic. If the flow in the upper and lower layer is taken to be, say, 3 cm sec^{-1} and 2 cm sec^{-1} respectively, then the total translation speed (C) is about 2.4 cm sec^{-1} corresponding to a transit time of approximately 8 yrs. This time scale is somewhat long, but is more realistic for the Meddy lifetime than the previously predicted 250 years corresponding to a purely self-propelled movement.

However, although these lifetime considerations are plausible, it is not entirely clear that in the western part of the subtropical gyre the flow is directed westward everywhere as required by Sverdrup *et al.* (1942) and Worthington (1976). In contrast to their requirement of a westward flow in the whole subtropical gyre, recent (SOFAR) float tracks (MR) and the calculations of Leetmaa *et al.* (1977) suggest that in the western part of the subtropical gyre, portions of the flow are directed eastward. There is no doubt that in most of the area between the Straits of Gibraltar and the Bahamas the mean flow is directed westward so that the Meddy could be advected during most of its journey but, in view of these recent studies, it is not entirely clear how the eddy crossed the western part of the subtropical gyre.

With our present contradictory knowledge of the mean flow within the western part of the subtropical gyre it is impossible to determine the actual behavior of the Meddy in this region. We may speculate, however, that the Meddy could have crossed the western region by being advected around or underneath the area of eastward flow. This conjecture is supported by the behavior of the Meddy which according to MR moved during the survey to the southwest at a rate of $\sim$ 6 cm/sec. As pointed out earlier, such a relatively fast drift can only be a result of advection.

5. Summary and conclusions

Apart from demonstrating that the nonlinear translation of deep lens-like eddies can be calculated analytically, our results give important information on the movements of Mediterranean eddies. Before listing our conclusions, it is appropriate to mention again the limitations involved in the analysis. The most important assumptions which have been made throughout the analysis are that the eddy is frictionless and nondiffusive, that it has a lens-like cross-section, that it translates steadily due to a combined effect of advection and β-induced movement and that its depth is small compared to the depth of both the upper and lower layers so that there is no induced movement underneath or above the eddy. Scaling arguments support the validity of these assumptions for the Meddy, but under different circumstances the assumptions may not be valid. The results of the study can be summarized as follows:

i) The translation of a lens-like eddy, embedded in the interface between two layers, consists of two independent components, advection by the flows in the two main layers and β-induced westward movement.

ii) The advection component is a weighted average of the speed within the two layers. It is proportional to the relative displacement of the eddy in each layer so that an eddy may move in one direction even if one of the layers flows in an opposite direction.

iii) The β-induced component is a function of the eddy intensity, size and volume. When the Rossby number (R_o) is of $0(0.1)$ or smaller and the swirl velocity profile is approximately parabolic, the westward β-induced translation is:

$$C_\beta \approx - \frac{(0.285 + 0.306\, R_o)\beta g h_o \triangle\rho_1}{\rho(1 + \triangle\rho_1/\triangle\rho_2)f_o{}^2} \ ,$$

where $\triangle\rho_1$ and $\triangle\rho_2$ are the differences between the density of the eddy and the densities of the upper and lower layer (respectively), and h_o is the maximum eddy depth.

Application of (ii) and (iii) to the Mediterranean eddies observed off the Bahamas indicate that the β-induced westward drift is rather slow and amounts to no more than 7.4×10^{-2} cm/sec. The complementary translation corresponding to advection by the two layers can be much larger and may amount to a few centimeters per second. The slow speed of the β-induced movement suggests that the Meddy could not have crossed the Atlantic Ocean due to its self-propulsion alone because this would require a lifetime of over 250 years. The presence of advection can reduce this required lifetime to about 8 years suggesting that the Meddy was advected across the ocean.

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