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# A note on time-dependent spectra for chlorophyll variance<sup>1</sup>

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## ABSTRACT

Observations of the distribution of chlorophyll *a* along transects in central Long Island Sound have shown that the variance spectrum for chlorophyll can change significantly with time. The relative importance of growth and spectral transfer in producing observed changes in the chlorophyll spectrum is discussed in light of the dynamic equation for the spectrum of a passive contaminant (Hinze, 1975).

## 1. Introduction

During chlorophyll *a* mapping cruises in central Long Island Sound we noticed on several occasions that the small scale structure in the distribution of chlorophyll *a*, salinity, and to a lesser extent temperature could increase significantly from one day to the next. In this paper we present a set of observations collected after a wind event of moderate intensity. During the event increased mixing obliterated much of the small scale structure. In the relatively quiescent days immediately following the event the variance spectrum for both chlorophyll *a* and salinity showed a rapid increase in the spectral density at high wave numbers. The chlorophyll *a* spectrum showed, in addition, an increase in the spectral density at very low wave numbers, apparently representing the direct effect of increased phytoplankton growth.

## 2. Observations

Figure 1 shows the positions of the vertical stations and horizontal transects occupied during the week of 24 October 1977. The weather had been quite windy for several days before the cruise; on 20 and 22 October, for example, there were sustained wind speeds in excess of 20 knots for much of the day. Winds began to slacken on 23 October; the mean wind speed on 26 October was less than 5 knots.

The cruise of 24 October involved continuous underway sampling at a depth of 1 m for *in vivo* fluorescence, salinity and temperature along a transect from the current meter mooring (Station 3) to Station 5 (Fig. 1). On 25 October no underway

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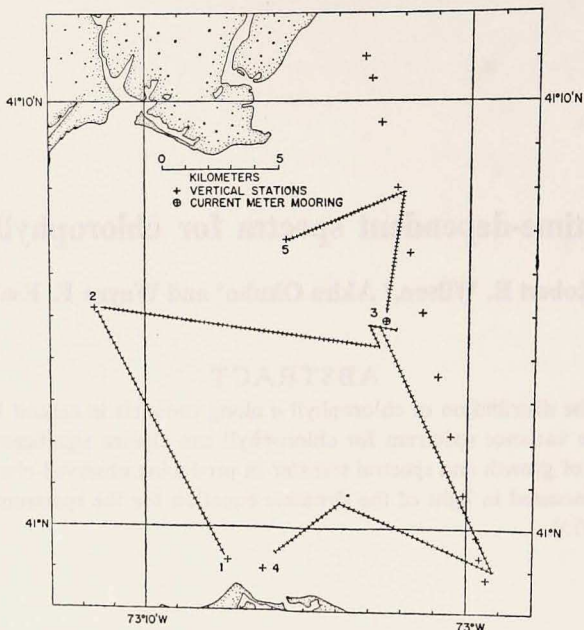


Figure 1. Positions of vertical stations and horizontal transects occupied in central Long Island Sound during the week of 24 October 1977 (see text).

sampling was performed; instead 11 vertical stations were occupied in a line across the Sound. On 27 October underway sampling was performed along the transects from Station 1 to Station 4. The instruments used were a Plessey 6600T Thermo-salinograph<sup>R</sup> and a Turner Design<sup>R</sup> model 10-005R flow through fluorometer.

Figure 2 provides a comparison of the records of *in vivo* fluorescence and salinity from 24 October with those from 27 October. The records shown for 27 October are from the transect from Station 2 to Station 3, nearest the transect of 24 October. The development of small scale structure in both *in vivo* fluorescence and salinity in this region of the Sound is apparent. *In vivo* fluorescence shows also a significant increase in mean value.

Calibrations for *in vivo* fluorescence versus extractable chlorophyll *a* show that this increase in fluorescence represented an increase in phytoplankton biomass from approximately  $1.4 \text{ mg m}^{-3}$  on 24 October to  $2.8 \text{ mg m}^{-3}$  on 27 October. The regression for *in vivo* fluorescence as a function of extractable chlorophyll *a* ( $\text{mg m}^{-3}$ ) based on 37 near-surface samples collected on 25 October was  $y = 1.533x + .777$  with  $r^2 = .84$ ; the regression based on 12 near-surface samples collected on 27 October was  $y = 1.782x + .657$  with  $r^2 = .97$ .

To better examine the relative contribution from structure of different length scales to the total variance in the fluorescence and salinity records, and the change of this structure with time, we computed the spectrum for each of the records in



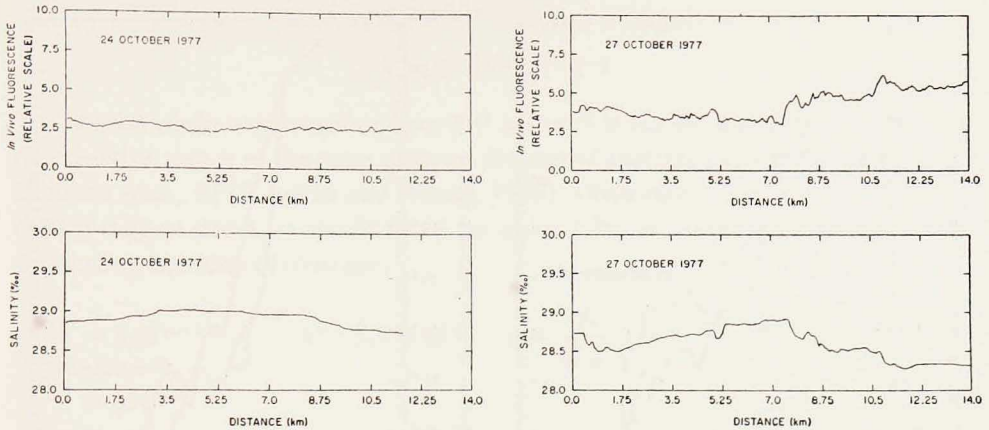


Figure 2. Records of *in vivo* fluorescence and salinity from transects on 24 October and 27 October 1977 at 1 m depth.

Figure 2 (Platt *et al.*, 1970; Platt, 1972; Denman and Platt, 1975; Denman and Platt, 1976; Denman *et al.*, 1977). We digitized the analog signals at a time interval of 15 s; for a mean ship speed of approximately 7 knots this corresponds to a spatial interval of approximately 54 m. We then windowed the data and added zeroes to extend each record to 256 points. The raw spectral estimates were subsequently smoothed by averaging over three adjacent estimates.

The variance spectra in Figure 3 show clearly the increase in small scale structure in the distributions of both *in vivo* fluorescence and salinity from 24 October to 27 October. The behavior of the fluorescence spectra at very low wave numbers confirms the overall increase in phytoplankton biomass from 24 October to 27 October.

### 3. Discussion

We present the following discussion to provide some insight into the relative importance of growth and spectral transfer processes in producing the observed changes in the chlorophyll *a* (*in vivo* fluorescence) spectrum.

Hinze (1975) has presented a derivation of the dynamic equation for the three-dimensional spectrum for a dynamically passive quantity for the case of isotropic turbulence. It is a simple matter to extend this equation to represent a quantity which exhibits a first order growth with a constant growth rate:

$$\frac{\partial}{\partial t} \int_0^k Sdk' = -2 \left\{ D + \beta \int_k^\infty (E k'^{-3})^{\frac{1}{2}} dk' \right\} \int_0^k k'^2 Sdk' + 2 \alpha \int_0^k Sdk' \quad (1)$$

We may use equation (1) to simulate the behavior of the chlorophyll spectrum with time. In equation (1)  $t$  is time,  $k$  is wave number,  $E = E(k, t)$  is the turbulent velocity

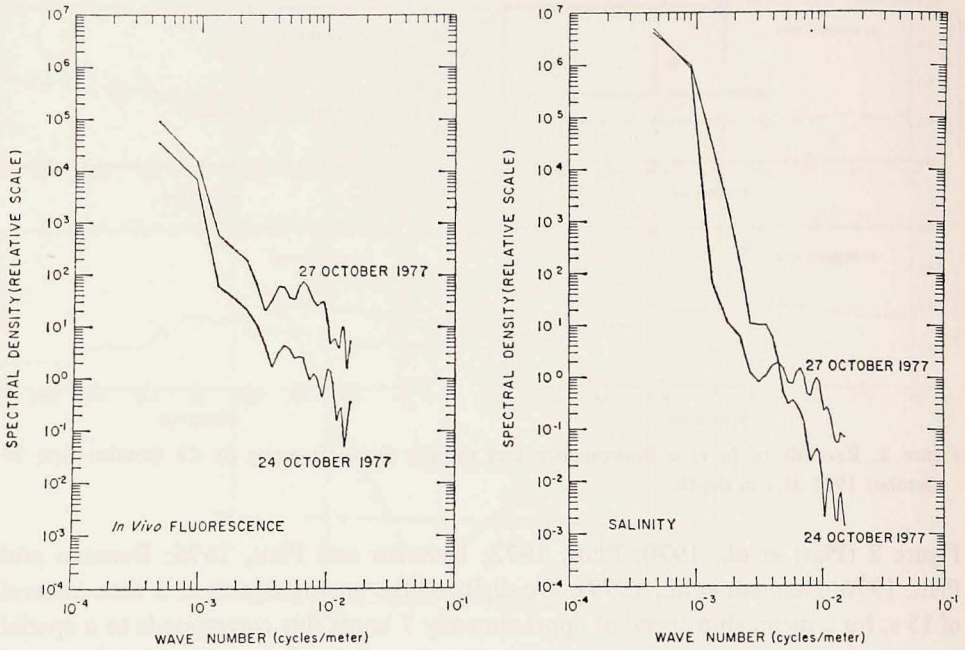


Figure 3. Power spectra for *in vivo* fluorescence and salinity for 24 October and 27 October 1977 computed from records in Figure 2.

spectrum, and  $S=S(k,t)$  represents the chlorophyll spectrum.  $D$  is a diffusivity (not necessarily molecular) associated with the dissipation of chlorophyll variance at high wave number,  $\alpha$  is the chlorophyll growth rate, and  $\beta$  is a numerical constant. Implicit in equation (1) is Heisenberg's hypothesis whereby spectral transfer from larger to smaller eddies behaves as if a certain turbulent diffusivity were present (Hinze, 1975). This diffusivity is represented in equation (1) by the expression

$$\beta \int_k^\infty (E k'^{-3})^{\frac{1}{2}} dk' .$$

If we differentiate equation (1) with respect to  $k$  we obtain

$$\frac{\partial S}{\partial t} = 2\beta (E k^{-3})^{\frac{1}{2}} \int_k^0 k'^2 S dk' - 2\{D + \beta \int_k^\infty (E k'^{-3})^{\frac{1}{2}} dk'\} k^2 S + 2\alpha S \tag{2}$$

which may be integrated to determine  $S(k,t)$  for given initial conditions when  $E(k,t)$  is specified. Ichiye (1967) has integrated an equation similar to (2) minus the growth term to simulate the temporal behavior of the concentration spectrum for a dye tracer. For expediency we take  $E(k,t)$  to be the Kolmogoroff spectrum

$$E(k) = \epsilon^{2/3} k^{-5/3}$$

where  $\epsilon$  is the turbulent energy dissipation rate. We nondimensionalize as follows:



$$t = t^* / \alpha$$

$$k = k^* 3^{3/2} \alpha^{3/2} \epsilon^{-1/2}$$

The characteristic wave number  $k_c = 3^{3/2} \alpha^{3/2} \epsilon^{-1/2}$  is the wave number at which the rate reactive gain is of the same order or the rate of spectral transfer (Corrsin, 1961; Denman et al., 1977; Lekan and Wilson, 1978). Observations by Lekan and Wilson (1978) suggest that  $k_c^{-1}$  should be of the order 1 km in Long Island Sound. Under this scaling equation (2) becomes

$$\frac{\partial S}{\partial t^*} = 6\beta k^{*-7/3} \int_0^{k^*} k^{*'} S(k^*) dk^{*'} - \left\{ 54 \frac{D}{\epsilon/\alpha^2} + \frac{9}{2} \beta k^{*-4/3} \right\} k^{*2} S(k^*) + 2S(k^*) . \quad (2')$$

It can be shown from Richardson's 4/3-power law of diffusion (Richardson, 1926; Obukhov, 1941) that  $\epsilon/\alpha^2$  is a characteristic turbulent diffusivity  $D_c$  associated with length scale  $k_c^{-1}$ ; substitution of  $k_c \sim \alpha^{3/2} \epsilon^{-1/2}$  into  $D_c \sim \epsilon^{1/3} k_c^{-4/3}$  leads to  $D_c \sim \epsilon/\alpha^2$ . The parameter  $\frac{D}{\epsilon/\alpha^2}$  thus represents the ratio of a molecular diffusivity to the turbulent diffusivity  $D_c$ .

The increase in spectral density with time described by equation (2') at any wave number below the dissipative range is due both to chlorophyll growth with time scale  $1/\alpha$  and to spectral transfer from low wave numbers to high wave numbers with time scale  $t_k$ . The rate of spectral transfer is proportional to  $\beta$ . From dimensional arguments the time scale  $t_k$  for spectral transfer in the vicinity of wave number  $k$  is  $t_k \sim \epsilon^{-1/3} k^{-2/3}$ . For a typical magnitude for  $\epsilon$  of  $10^{-4} \text{ cm}^2 \text{ s}^{-3}$  and for  $k$  of the order  $k_c = 10^{-5} \text{ cm}^{-1}$ ,  $t_k \sim \alpha^{-1}$  is of the order  $10^5 \text{ s}$ . For wave numbers greater than  $k_c$ ,  $t_k$  decreases rapidly with increasing wave number and chlorophyll growth becomes unimportant. At very high wave numbers ( $k^* \gg 1$ ) the dissipation term in equation (2') involving the parameter  $\frac{D}{\epsilon/\alpha^2}$  becomes relatively more important. For wave numbers less than  $k_c$ ,  $t_k$  is increased and chlorophyll growth becomes more important. At very low wave numbers (2') becomes

$$\frac{\partial S}{\partial t^*} = 2S(k^*) ,$$

and so the increase in spectral density is independent of both  $\beta$  and  $\frac{D}{\epsilon/\alpha^2}$  and due entirely to growth. Presumably the observed changes in the fluorescence spectrum at very low wave numbers (Fig. 3) are due primarily to growth.

We have integrated (2') numerically with  $\beta$  and  $\frac{D}{\epsilon/\alpha^2}$  as parameters using as an initial condition the fluorescence spectrum for 24 October (Fig. 3). In light of experimental results by a number of investigators (Hinze, 1975) we chose 0.1-0.5 as

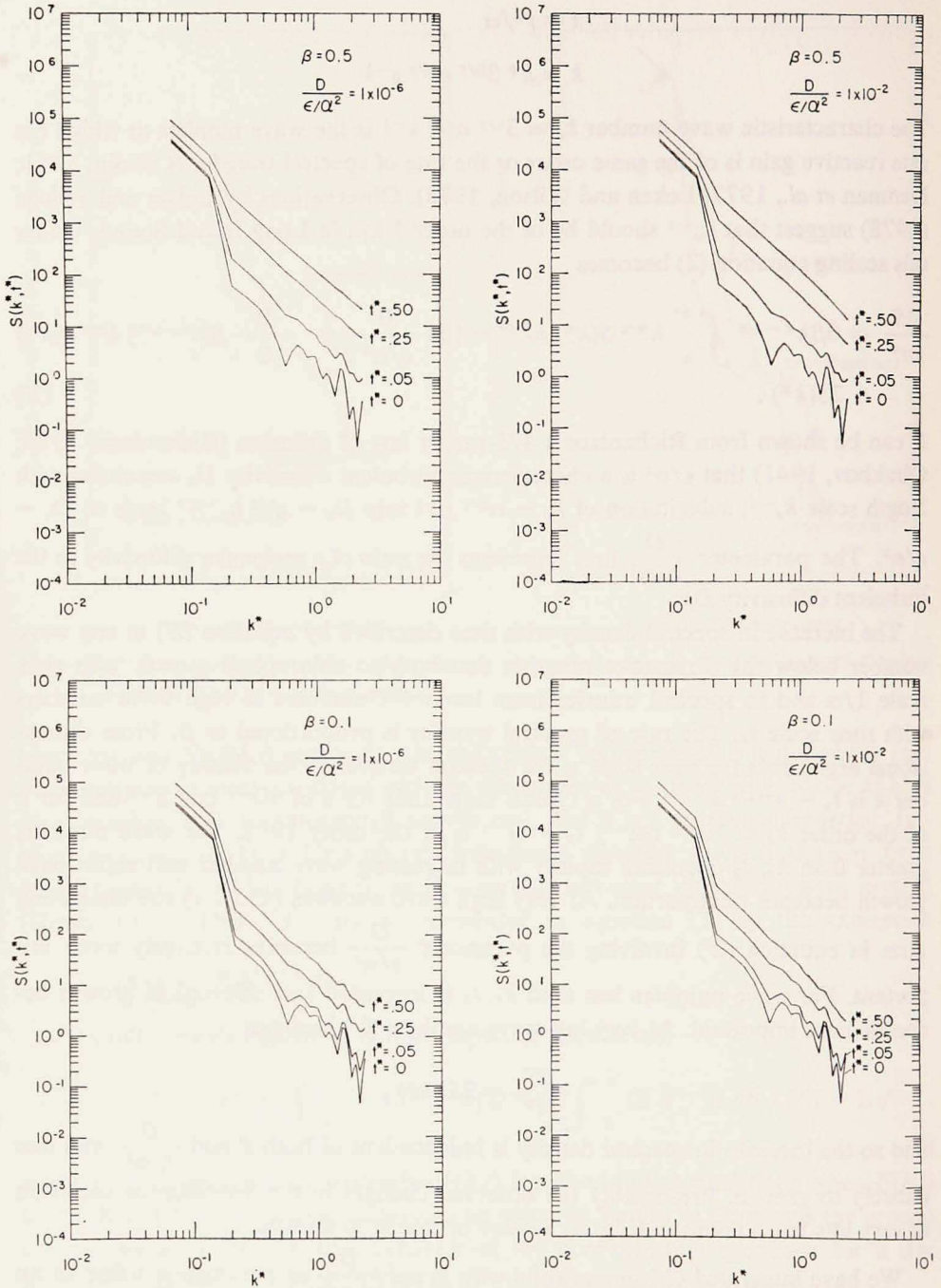


Figure 4. Predicted development of chlorophyll spectrum for values of  $\beta$  and  $\frac{D}{\epsilon/\alpha^2}$  indicated (see text).

the range for  $\beta$ . We chose  $1 \times 10^{-2} - 1 \times 10^{-6}$  as the range for  $\frac{D}{\epsilon/\alpha^2}$ . Figure 4 shows the predicted development of the chlorophyll spectrum for different values of  $\beta$  and  $\frac{D}{\epsilon/\alpha^2}$ . It suggests a strong dependence of the rate of spectral increase at high wave numbers on  $\beta$ . As  $\beta$  is diminished the rate of spectral transfer from lower wave numbers to higher wave numbers decreases and the rate of increase of spectral density at higher wave numbers is reduced. The magnitude of the parameter  $\frac{D}{\epsilon/\alpha^2}$  affects the structure of the spectrum only at very high wave numbers ( $k^* \gg 1$ ). An increase in the value of this parameter reduces the rate of increase in spectral density at very high wave numbers.

It appears that equation (2') can be used to simulate some of the observed features associated with temporal variations in the chlorophyll spectrum. The major point of this note has been, however, to suggest that temporal variations in the chlorophyll spectrum can be considerable, and that the examination of time dependent spectra could produce useful results concerning the relative importance of growth and physical mixing processes at different length scales.

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