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# Flux measurements on salt fingers at an interface

by Raymond W. Schmitt Jr.[1,2]

## ABSTRACT

A series of two-layer, heat-salt fingering experiments were performed in a one meter deep insulated tank. Repeated profiling of the temperature and conductivity with a small probe allowed the calculation of the vertical fluxes of heat and salt. Results indicate that:

1. The ratio of the density flux of heat to the density flux of salt (flux ratio = $\gamma$) is *not* a constant, contrary to the results of Turner (1967) and Linden (1971). Both short term variability (due to storage and release of heat by the finger interface), and a general trend of decreasing flux ratio with increasing stability ratio ($R = \alpha\Delta T/\beta\Delta S$) were observed. For $R < 2.5$, $\gamma \cong 0.7$, for $2.5 < R < 4$, $\gamma \cong 0.58$, in agreement with Turner and the time dependent model of Schmitt (1979), and for $R > 6$, $\gamma \cong 0.3$, closer to the estimate of Linden and the equilibrium model of Stern (1976).
2. The ratio of the salt flux to the product of viscosity and local density gradient due to temperature was found to be about one, in agreement with the collective instability model of Stern (1969).
3. The 4/3 power law dependence of the salt flux ($\beta F_s$) on the salinity difference across the interface ($\beta\Delta S$) is strongly supported; the least squares regression of log ($\beta F_s$) against log ($\beta\Delta S$) shows excellent agreement with the 4/3 slope over several ranges in $R$. When expressed as $\beta F_s = C\,(gK_T)^{1/3}\,(\beta\Delta S)^{4/3}$, with $C = 0.051$ for $R > 3.5$ and $C \rightarrow 0.1$ as $R \rightarrow 1$, the 4/3 law permits calculation of the flux to within 5-15%, and should be applicable to oceanic observations.

In addition, an asymmetrical entrainment effect was observed in which the lower layer grew at the expense of the upper layer, causing an upward migration of the interface. This effect is thought to be due to the variation of the thermal expansion coefficient with temperature and may be important in the movement of fingering interfaces in the ocean.

## 1. Introduction

We have known for some time now of the possibility of convection in a stably stratified ocean when the heat and salt density gradients act in an opposing sense. (Stommel, Arons, and Blanchard, 1956; Stern, 1960). When warm-salty water overlies colder, fresher water, the higher rate of heat conduction over salt diffusion can

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allow near thermal equilibrium to occur in adjacent water parcels. However, salinity differences and thus density differences still exist and drive small-scale (~ cm wide) vertical motions. The "fingers" diffuse heat laterally, but diffuse very little salt, and a convective vertical flux of salt occurs. A certain quantity of heat is also transferred in the vertical; the amount has been a matter of some controversy.

The heat and salt fluxes can be expressed as density fluxes (using the expansion coefficients of the equation of state), and their ratio indicates the fraction of the energy released by the falling out of salt that is used to raise the center of gravity of the temperature field. The value of this flux ratio ($\gamma$) is in dispute, with Turner (1967) finding experimentally a value of $0.56 \pm 0.02$ and Linden (1971) reporting a flux ratio of $0.12 \pm 0.02$. More recently, the flux maximization theory of Stern (1976) requires $\gamma = 0.25$, and Schmitt (1979) has pointed out that the fastest growing fingers in a region of constant vertical gradients have a flux ratio close to that reported by Turner. The attractive point about the fastest growing fingers is that the model also explains the high flux ratio (0.9) found by Stern and Turner (1969) and Lambert and Demenkow (1972) in the sugar-salt experiment. (The molecular diffusivities of sugar and salt differ by a factor of three, and fingers are formed when sugar solution, the slower diffusing, overlies salt water.)

The flux ratio is an important quantity to know because it sets the relative contributions of the salinity and temperature to the index of refraction changes between fingers. A flux ratio of about 0.6 can render the fingers invisible in the optical detection scheme of Williams (1975). Also the value of the heat flux itself is of interest, as the vertical salt flux due to fingers was found to be important in the large-scale salt balance of the Northeast Caribbean Sea by Lambert and Sturges (1977). If fingers account for a significant fraction of the vertical mixing in the main thermocline, the difference between the heat and salt transfer rates would have important consequences for thermocline theories which usually assume equal "eddy diffusivities" for heat and mass.

In order to resolve the experimental and theoretical questions about the flux ratio, as well as make more detailed tests of the models of the collective instability of fingers (Stern, 1969) and the 4/3-power law for the salt flux (Stern, 1976), I have undertaken a series of heat-salt fingering experiments. The experimental apparatus and techniques are described in Section 3; results are presented and discussed in Section 4 and summarized in Section 5. The following section is a short review of the experimental work of Turner (1967) and Linden (1971).

## 2. Previous experimental work

Turner (1967) was the first to attempt to quantify the salt and heat fluxes due to fingers. He used a 40 cm deep tank with stirring grids in the mixed layers above and below the fingering interface. Stirring was used to achieve uniformity of the mixed layers before and after each fingering period, 4 to 9 minutes long. The changes in the temperature and salinity of the mixed layers then gave estimates of the heat and salt fluxes. The heat flux was corrected for conduction of heat across the interface, to isolate that due to convective transfer by the fingers, by subtracting the heat flux found in heat run-down experiments (with no salt). This method should also have accounted for heat loss or gain through the tank sidewalls. The flux ratio was found to be 0.56 ± 0.02 with no apparent variation with the stability ratio, $\alpha \Delta T / \beta \Delta S = R$, ($\alpha = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}$, $\beta = \frac{1}{\rho}\frac{\partial \rho}{\partial S}$ and $\Delta T$, $\Delta S$ are the temperature and salinity differences across the interface). This result is now seen to be consistent with the time dependent model of Schmitt (1979). The stirring and the relatively short duration of the experiment probably enhanced the importance of the fastest growing fingers.

One would like to know the flux ratio in the absence of any mechanical stirring in longer experiments. Linden's (1971) experiments were longer (> 1 hour), rundown, experiments and should have been less influenced by time dependent effects. However, he did not attempt to directly estimate the heat flux from the temperature changes in the mixed layers, because of the difficulties with the sidewall and conductive heat fluxes. He did use other methods which relied on the assumption that the fingers were described by a steady state balance of advection and diffusion within the fingers, and horizontal profiles of temperature to estimate $T'$ within the fingers. The value of $\gamma \cong 0.12 \pm .02$ also showed no variation with $R$. This number can be questioned because of the theoretical bias in its computation, as well as the possible underestimation of $T'$ due to the tendency of the boundary layer around a slowly moving small probe to low pass filter the true temperature structure. However, Linden has one plot showing the relationship between finger width and temperature gradient which should depend primarily on the flux ratio, almost independently of the growth rate of the fingers (Schmitt, 1979). This plot (Figure 3.13 of Linden's dissertation) suggests that the flux ratio was less than 0.56 but possibly a bit higher than 0.12, and would support Stern's (1976) value of 0.25. While admitting that the evidence is prejudiced by our own theoretical conception of the detailed finger dynamics, it does seem likely that Linden's longer rundown experiments achieved a lower flux ratio than Turner's short, stirred experiments, and we are thus left uncertain as to the proper flux ratio to apply to oceanic observations.

The experiments described in the next section were designed to make direct heat and salt flux measurements in long experiments, unaffected by mechanical stirring.

## 3. The experiments

Two aspects of the experimental problem seemed of particular importance; the tank design, and a small probe to continuously profile temperature and salinity (conductivity) in the vertical.

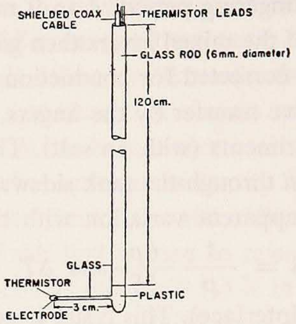

**Figure 1. The temperature-conductivity probe. The offset thermistor and electrode sampled undisturbed fluid on both up and down profiles.**

The tank was 19 cm × 19 cm × 98 cm deep (inner dimensions) constructed of Plexiglas.® It was contained inside another plastic tank (27 × 27 × 101 cm) which provided insulation from room temperature, either left as an air gap or by filling the gap with plastic foam chips. The depth of the tank was more than twice the depth of that used in the Turner and Linden experiments. This means that the rate of change of the temperature and salinity of the mixed layers is less likely to affect the fingers, and the assumption that the fingers pass through a series of quasi-equilibrium states will be better met. Also, the convection in the mixed layers will be less inhibited by the boundaries, making the convective stirring sufficiently intense that a good estimate of *T* and *S* within the layers may be made without mechanical stirring.

In order to adequately sample the temperature and salinity structure of the experiments, a special probe was constructed. A bead thermistor and a single platinum electrode (radius = 0.25 mm) were mounted at the end of a 3 cm long thin glass tube (r = 0.8 mm). This in turn was mounted perpendicular to the end of a 6 mm diameter glass tube, 120 cm long. This configuration, shown in Figure 1, allowed the probe to sample undisturbed fluid on both up and down profiles.

The thermistor output through a simple resistive network was amplified to give a 0-10 volt output in the range of 0-45°C. The probe was calibrated at 100 points against a quartz-crystal thermometer (Hewlett Packard 2801A) and a fourth degree polynomial fit to the data provided temperature as a function of voltage. The accuracy is better than ± 0.05°C and precision (limited only by the analog chart recorders used to log the data) is ± 0.04°C in the early experiments, ± 0.02°C when an expanded scale was used. The thermistor (Yellow Springs Inst. Co., #44018) had a one second time constant which was no disadvantage for the flux measurements reported here, but did not adequately sample the detailed structure of the fingering interface.

Conductivity was monitored by measuring the current from a 60 KHz constant amplitude oscillator, through the electrode and salt water to a stainless steel grounding strip in the tank. The probe is sensitive to the conductivity of the fluid within about 10 radii, about 2.5 mm. The high carrier frequency seemed to minimize drift problems common to small probes, with only a 3% change in cell constant over 4 months, probably due to formation of deposits around the electrode. The conductivity range 0-10 mmhos/cm corresponded to 0-10 volts output of the electronics with accuracy of about ± 0.05 mmhos/cm, and precision of ± 0.01 mmhos/cm, ± 0.005 mmhos/cm on an expanded recorder scale.

The probe was calibrated in various temperature and salinity samples, (seawater diluted with deionized water) with salinity being determined on an inductive salinometer calibrated with Standard Sea Water. Salinity was calculated from the conductivity and temperature using the fifth degree polynomial provided by Brown and Allentoft (1966) for salinities below 8‰. (The standard oceanographic formula becomes inaccurate below 3‰.) This allowed the calculation of salinity to an accuracy of ± 0.04‰, and a precision of ± 0.01‰.

Filtered sea water was used in these experiments because of the availability of the formula relating temperature and conductivity to salinity, and temperature and salinity to density. Turner (1967) used NaCl and Linden (1971) used sucrose and both obtained comparable values of the density flux. The large difference between the conductivity of heat and the diffusivities of dissolved salts seems to make small differences in diffusivities unimportant. Turner, Shirtcliffe and Brewer (1970) found evidence for different salt transports across double-diffusive interfaces, where the salt flux is diffusive. However, since the transfer of salt is convective in salt fingers, it seems unlikely that there is significant fractionation of dissolved species in the highly supercritical regimes of these experiments and typical oceanic conditions, although this has not been checked.

The tank was mounted on an elevating table, capable of moving the tank vertically, with the probe held rigidly from above. The table is very solidly constructed and vibrations were minimal. The vertical position was sensed by a weighted string passing over a 10 cm circumference pulley attached to a 10 turn potentiometer. This allowed vertical profiles of temperature or conductivity to be recorded on an $x$-$y$ plotter. The table had reversing switches at the ends of its run, allowing operation in a "yo-yo" mode, with a variable speed of up to 2 cm/sec.

The experiments were set up by introducing cold, deionized water into the bottom of the tank (to a depth of 45 cm), suspending a thin plastic baffle on the surface of the cold water, then pouring warm water on top. Allowing about an hour for thermal equilibrium with the tank to become established, a quantity of warm filtered seawater ($S = 32‰$), sufficient to give the desired $\Delta S$, was thoroughly stirred into the upper layer. The baffle was then slowly tilted along a diagonal and withdrawn from above. This generally produced satisfactory results, although when the stability was

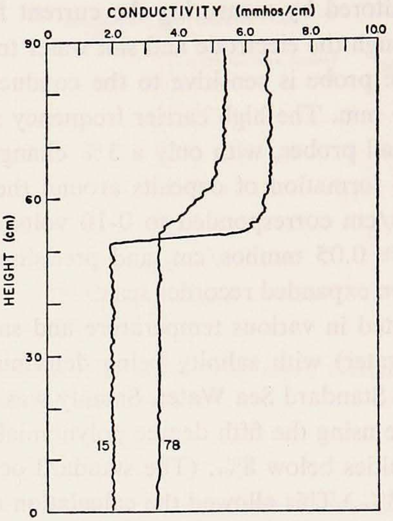

Figure 2. Sample profiles of conductivity, from 2/22/77, at 15 and 78 minutes from the initiation of the experiment. The layers are well mixed, and the interface grows from a few cm to almost 10 cm thick in this time.

low ($R \cong 1$), the removal of the baffle could cause large amplitude mixing. The lowest density ratio achieved was $R = 1.3$; usual starting conditions in a successful run had $R$ between 1.6 and 2.0.

Salt fingers formed immediately on the interface between the two layers and the probe sampled first one layer then the other as it "yo-yoed." Temperature ($T$) and conductivity ($C$) were continuously recorded with time on strip chart recorders, and vertical profiles of $T$ and $C$ recorded every 10-15 minutes, depending on the rate at which the layer depths were changing. The vertical profiles yielded the interface and layer thicknesses; the time series of $T$ and $C$ were digitized to obtain a single value of the $T$ and $C$ of each layer at the times of interface crossings. The variations due to active mixing within the layers (see Fig. 2) were averaged by eye during the digitizing process. This variability was the largest contribution to the uncertainty of the flux calculations near the beginning of each experiment; the digitizing level was the limiting factor in the later part of a run.

Table 1 shows the initial conditions, duration, and the number of points sampled for each of the experiments described here. Only one experiment, that of 3/16/77, had an initial stability ratio less than 1.6; this run displayed a pronounced entrainment effect, in which one layer grew at the expense of the other layer—something not noted in previous salt finger experiments. Other experiments displayed limited entrainment when the stability ratio was low, during the early part of the run. The entrainment was always asymmetric with the lower layer growing at the

Table 1. Initial conditions of the experiments, as determined from the first profiles of $T$ and $C$.

| Experiment | $T_1$ (°C) | $S_1$ (‰) | $T_2$ | $S_2$ | $R$ Initial | $R$ Final | Duration (min.) | No. of Data Points |
|---|---|---|---|---|---|---|---|---|
| 1/28 | 31.27 | 3.96 | 11.63 | 0.50 | 1.66 | 4.36 | 60 | 41 |
| 2/11 | 28.75 | 3.33 | 14.55 | 0.79 | 1.67 | 8.08 | 116 | 55 |
| 2/15 | 34.35 | 4.80 | 9.53 | 0.59 | 1.76 | 6.42 | 93 | 52 |
| 2/22 | 31.62 | 3.40 | 11.58 | 0.43 | 1.99 | 8.03 | 103 | 45 |
| 3/7 | 27.78 | 3.21 | 14.07 | 1.23 | 1.99 | 7.07 | 105 | 58 |
| 3/14 | 28.93 | 3.20 | 14.55 | 0.97 | 1.92 | 19.45 | 192 | 81 |
| 3/16 | 28.18 | 4.66 | 15.08 | 1.63 | 1.30 | 9.63 | 93 | 54 |
| 3/26 | 30.52 | 2.77 | 11.49 | 0.04 | 1.99 | 15.76 | 192 | 118 |
| 5/12 | 28.56 | 0.83 | 12.32 | 0.00 | 5.36 | 34.87 | 181 | 38 |

expense of the upper. One might expect that an initial difference in the layer depths might cause the larger to grow at the expense of the smaller, because the turbulent velocities are less inhibited by the proximity of boundaries. Such an effect is described by Linden (1976) for the double-diffusive experiment (heating a stable salt gradient from below). But when an initial imbalance in the layer depths was introduced, with the lower layer 40 cm thick and the upper 50 cm thick, the results were the same, it was still the lower layer which grew. The initial disturbance of removing the baffle was small compared to the convective turbulence caused by the fingers and is not likely to be the cause of the asymmetry. The most likely explanation lies in the variation of the thermodynamic properties of water with temperature, especially the changes in the thermal expansion coefficient ($\alpha$), which changes by a factor of 3 from 10°C to 30°C at these salinities. Thus, if the mean temperature gradient is constant across the interface, the stability will be lowest at the lower edge due to the decrease in $\alpha$, making disruption of the fingers more likely. (Also, the ratio of the thermal conductivity to the diffusivity of salt ($K_T/K_S$) changes, from 200 at 5°C to 80 at 30°C (Caldwell, 1974), but this is not likely to be important in these highly supercritical experiments.)

Others have reported on the linear growth of the fingering interface with time, as well as other details of the fingering experiments (Linden, 1973); I will concentrate on calculated quantities, such as the ratio of the heat and salt fluxes, and their dependence on the stability ratio, $R$.

The density flux due to salt, $\beta F_s$, was computed by differencing the time series of salinity for each layer, and averaging the two flux estimates. That is

$$\beta F_s = \beta \left[ \left( -H_1 \frac{\delta S_1}{\delta t} + H_2 \frac{\delta S_2}{\delta t} \right) /2 \right]$$

$\beta$ was computed at the mean temperature and salinity of the experiment, and $H_1$ and $H_2$ are the upper and lower layer depths as determined from a quadratic fit of the layer thickness data, as a function of time, read from the profiles. $\delta S_1$, $\delta S_2$ are the salinity changes within each layer over the time interval $\delta t$, which was generally 10-15 minutes. This provided better stability than first differencing the data at the 1-5 minute sample interval, and still gave adequate resolution of changes in computed quantities with $R$. The values of $\beta\Delta S$ and $R$ paired with each flux estimate were the averages of these quantities at the two differenced points.

This estimate of the flux neglects the effects due to the growth and migration of the interface; that is, terms like $S_1 \left(\frac{\delta H_1}{\delta t}\right)$. This is justified because such terms are more than an order of magnitude smaller than those above, due to the large layer depths. They are, in any case, difficult to estimate, since the uncertainty in the layer depths as read from the vertical profiles ($\pm$ 1 cm) is as large as the typical change in $H$ over the differencing interval. These neglected terms did, however, contribute to the variations in the heat flux calculation, because $\alpha\Delta T > \beta\Delta S$ and $\alpha F_T < \beta F_s$.

The heat flux was computed using a similar relationship with additional corrections to account for heat losses through the side walls and the pure conduction of heat across the interface. The correction for the tank heat losses was established in heat rundown experiments. The heat loss or gain in a layer is proportional to the temperature difference between the water and room temperature. Since the mean temperature of each experiment was close to room temperature (~ 20°C), the tank correction was determined as being proportional to $\alpha\Delta T$. A tank correction constant was established for each of two experimental configurations, with and without plastic foam insulation in the air gap between the inner and outer tanks. The heat conduction across the interface was taken as being proportional to $K_T \left(\frac{\alpha\Delta T}{h}\right)$ where $h$ is the interface thickness. When these corrections were subtracted from the observed heat flux, the remaining flux was due only to the salt fingers. This procedure is not unlike the technique of Turner (1967) except that the two effects (sidewall heat losses and conduction across the interface) have been separated, and many (30-100) flux estimates were made for each experiment instead of just one. The correction due to the tank was small relative to the finger flux at low $R$, becoming larger than the finger flux at high $R$. The conductive correction ranged from 1/10 to 1/3 of the finger heat flux during an experiment.

## 4. Results

The ratio of the density flux due to heat to that due to salt is of primary interest and I will discuss that data first. All heat fluxes were normalized by the salt fluxes for plotting against the stability ratio, $\alpha\Delta T/\beta\Delta S$, as in Turner (1967). The raw data show considerable scatter and Figure 3 is a seven point running average of the flux ratio from the experiment of 3/26/77. Both short term variability and a general decrease in $\gamma$ with increasing $R$ can be seen. Some of the variability is due to the

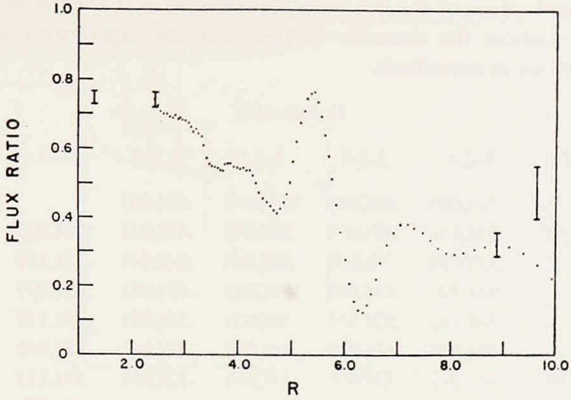

Figure 3. The flux ratio of experiment 3/26/77 with 7-point running average applied. The precision of the measurement is indicated by the error bars on the data points, and the uncertainty in the accuracy, largely due to the tank-sidewall heat losses, is indicated by the error bars at the side.

experimental uncertainty (primarily in the heat flux) but there is a certain amount of real variation that has not been previously noted; Turner and Linden both reported the flux ratio to be constant (although different). The other experiments showed similar short term variations, generally with a greater scatter than Turner's (1967) data. This may be due to the storage and release of heat in the interface as its thickness varies, or to changes in the width (modal structure) of the fingers, as they adjust to the changing layer concentrations in the rundown experiments. Perhaps the stirring of Turner's experiments returned the interface to a standard thickness that removed variations caused by the adjustment of the fingers to the changing layer concentrations. (Turner in more recent heat-salt experiments found similar variations in the flux ratio, with values in the same range, and suggests that internal waves on the interface may be the cause. Personal communication, 1978). The variations seen in Figure 3 occurred on time scales of tens of minutes whereas internal waves at the interface had periods of tens of seconds.

In order to examine the general trend of the data the average of the raw data on several intervals in $R$ was taken for each experiment. The results are contained in Table 2, with the standard deviations in parentheses. The last row has the ensemble average of the means of the various experiments in each interval. To compare with the theories these mean values have been plotted in Figure 4. The average of all the data in Table 2 is 0.56 (± 0.17) in agreement with Turner's value (0.56 ± 0.02) but the downward trend and the short-term variability must be explained.

Three regimes can be distinguished in the mean flux ratio plot. For $R < 2.5$ the flux ratio tends to be $\sim 0.7$, slightly higher than required by the maximum growth rate model of Schmitt (1979). This is probably due to the thinness of the interface

Table 2. The averaged values of the flux ratio for intervals in $R$ for the different experiments. The bottom row contains the ensemble average over the experiments in each $R$ interval; standard deviations are in parenthesis.

| | Range of $R$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Experiment | <2 | 2-2.5 | 2.5-3 | 3-3.5 | 3.5-4 | 4-5 | 5-6 | 6-10 |
| 1/28 | .66(.10) | .62(.06) | .60(.04) | .55(.04) | .63(.05) | | | |
| 2/11 | .77(.08) | .53(.11) | .48(.07) | .34(.08) | .48(.01) | .31(.04) | .28(.02) | .22(.04) |
| 2/15 | | .78(.09) | .78(.04) | .68(.06) | .64(.04) | .55(.15) | | |
| 2/22 | | .96(.09) | .85(.07) | .80(.08) | .67(.05) | .61(.07) | .59(.04) | .53(.04) |
| 3/7 | | .54(.10) | .52(.05) | .46(.05) | .56(.05) | .36(.11) | .18(.06) | |
| 3/14 | | .69(.05) | .60(.08) | .41(.05) | .58(.04) | .58(.04) | .36(.13) | .10(.07) |
| 3/16 | .74(.06) | .56(.09) | .47(.05) | .51(.09) | .52(.03) | .54(.11) | .83(.18) | .47(.17) |
| 3/26 | | .75(.04) | .69(.04) | .64(.04) | .53(.06) | .48(.07) | .62(.18) | .27(.11) |
| 5/12 | | | | | | | | .36(.03) |
| Averages | .72(.06) | .68(.15) | .62(.14) | .55(.15) | .58(.07) | .49(.11) | .48(.24) | .33(.16) |

in the early part of each experiment. The larger scale turbulence of the mixed layers may inject convective elements into the interface that are wider than the fastest growing fingers, and thus have a higher flux ratio, but still have an appreciable growth rate. Of course, the unbounded linear theory cannot be expected to apply on thin interfaces; and the increasing flux ratio as $R \rightarrow 1$ may indicate a somewhat different finger regime.

When $2.5 < R < 4$, the flux ratio is about 0.58, in agreement with the data of Turner (1967) and the model of the fastest growing fingers. For $R > 4$ the flux ratio begins to drop, and is about 0.3 for $R > 6$. The large changes in $\gamma$ for $5 < R < 6$ in Figure 3 were also present to some extent in other experiments, as evidenced by the increased standard deviations in this range in Figure 4. This may mean that a significant change in the modal structure of the fingers takes place at $R \cong 5$, perhaps an adjustment from a regime dominated by the fastest growing fingers to an equilibrium regime. The data for $R > 6$ do agree with the flux maximization theory of Stern (1976), which requires a flux ratio of 0.25. This may also indicate that the low flux ratios estimated by Linden (1971) were representative of the fingers at high $R$ only. This downward trend may not have appeared in Turner's experiments because the stirring in the mixed layers before and after the fingering period placed a selective advantage on the quick response of the fastest growing fingers, preventing an approach to equilibrium flow. One possible explanation of the decreasing flux ratio is that the fingers are slowly adjusting to equilibrium, after being initially "locked in" to the fastest growing modes.

In order to distinguish any effects of initial conditions on the flux ratio, two experiments were started somewhat differently. In experiment 3/7/77 a weak salinity difference was introduced and allowed to form fingers. After an hour, when the

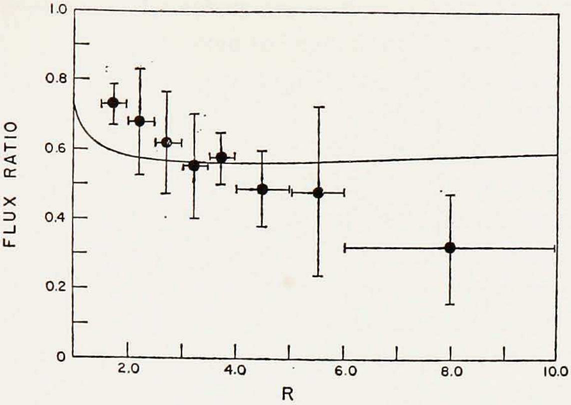

Figure 4. The averaged flux ratios from Table 2. The range of $R$ is indicated. The solid curve represents the flux ratio of the fastest growing fingers (Schmitt 1979).

finger interface was moderately thick (> 10 cm) and $R$ was high (~ 20), a porous float was used to introduce more warm salt water, bringing $R$ down to about 2. The interface thinned and the flux ratio was found to be slightly lower (by ~ 0.1) than in the other experiments started by removing the thin baffle; but it was not as low as required by the theory of Stern (1976) until $R$ exceeded 4 (Table 2). In the experiment of 5/12/77 a high stability ratio was used with the baffle technique to see if the thinness of the interface would raise the flux ratio. It apparently did not; the flux ratio was less than 0.5 for the first measurements. The interface also rapidly thickened to approximately the values realized in the other experiments which had rundown from initially low $R$.

These experiments indicate that the fingers are only partially influenced by different initial conditions (a difference in $\gamma$ of about 0.1) and that the flux ratio and interface thickness are largely determined by the present values of $\alpha\Delta T$, $\beta\Delta S$.

The tendency for the flux ratio to vary about the general trend in Figure 3 on a time scale of 15-20 minutes is partly due to variations in the interface thickness, but may also be evidence of a short-term hysteresis effect. The adjustment in the modal structure of the fingers to changing $\alpha\Delta T/\beta\Delta S$ conditions may take place in jumps; the wavelength established at one stability ratio may be retained for a time while $R$ increases, then suddenly change when a different wavelength would have a faster growth rate. The variability in flux ratio may just represent the active adjustment to the fastest growing fingers. It should also be noted that the growth rate maximum is rather broad with respect to the flux ratio; a wide range of $\gamma$'s have growth rates approaching that of the fastest growing and the growth rate cannot be considered a very strong constraint on the flux ratio (Fig. 1 of Schmitt, 1979). In addition, while the decrease in the flux ratio at higher stability ratio can be taken as

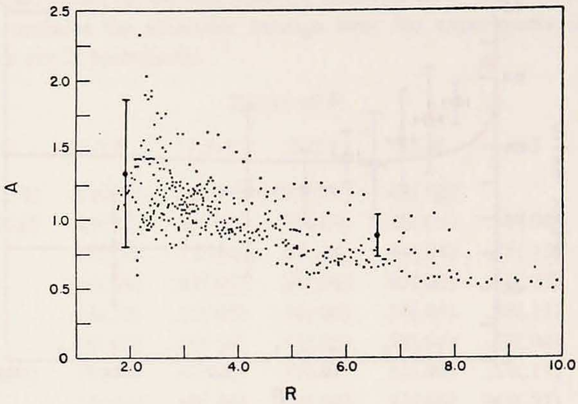

Figure 5. The salt density flux scaled with the viscosity times the local density gradient due to temperature, $A = \beta F_s / \nu \alpha \bar{T}_z$.

evidence for the maximum buoyancy flux model of Stern (1976), fingers with a flux ratio of 0.3 have a growth rate only about 15% less than the maximum at these stability ratios. A determination of whether the fingers were in an equilibrium or growing state would require detailed $T$ and $S$ profiles in the interface, difficult to achieve with the analog recording system and the one-second response time of the thermistor.

Another quantity of interest is the ratio of the salt buoyancy flux to the density gradient due to temperature times the viscosity; $\beta F_s / \nu \alpha \bar{T}_z = A$. This nondimensional group was suggested as being of order one by Stern (1969) in his collective instability model. The model predicts that the fingers would become unstable to internal wave disturbances when $A$ exceeded a critical value near one. This constitutes a mechanism by which the fingers can be considered self-limiting and has been used as a closure condition in the model of Stern (1976).

Linden (1973) calculated this quantity and found $A$ to vary from 0.2 to $\sim 2.0$, with some indication of a decrease at lower salt fluxes.

In this case $A$ was estimated by using $\alpha \Delta T / h$ to approximate the temperature gradient for each flux measurement. These unaveraged data have been plotted against stability ratio for all experiments in Figure 5. The main experimental errors are associated with the uncertainty in $h$, ± 1 cm, which is comparable to the interface thickness near the beginning of each experiment. This is probably the main contribution to the scatter at low $R$. At larger $R$ the data show less scatter and a gradual decrease in $A$. The estimates of high $A$ at low $R$ come from the experiment of 3/16/77 which was affected by entrainment. This may have kept the interface thinner than appeared on the profiles, and $h$ may have been over-estimated. The slight downward trend of $A$ with increasing $R$ is consistent with Linden's observation of a decrease in $A$ with lower fluxes because a high $R$ corresponds to a small

$\Delta S$ and thus a small flux. The values are in the same range as those reported by Linden, although none were observed to be quite as low as he reported. Perhaps his smaller tank size was becoming a limiting factor on the fluxes in the later stages of the experiments. The variation in $A$ with $R$ is slight and Linden's contention that $A$ is an adequate criterion for the break-up of the fingers only when $R \to 1$ is not really supported; at any rate a variation in the critical amplitude of this nondimensional group with $R$ is not inconsistent with the collective instability model. The result that $A$ is of order unity emphasizes that the flux due to fingers is an order of magnitude larger than the conductive flux of heat, and more than three orders of magnitude greater than the molecular diffusion of salt across the thin interfaces.

Perhaps the most important experimental result is the dependence of the salt flux upon the salinity difference $\Delta S$. Turner (1967) argued that the flux should depend on the haline Rayleigh number and become independent of the layer depths if the layers were deep enough. This suggests a salt flux that depends on the 4/3 power of the salinity difference, $\beta F_s \sim (\beta \Delta S)^{4/3}$. Turner scaled his measured fluxes with the "solid plane" flux that would result if the salinity difference were imposed at a solid boundary, with the molecular diffusivity of salt providing the flux through the laminar boundary layer. The coefficient of the 4/3 law derived from his data shows a roughly linear four-fold decrease over the range of $R$ of 2 to 10, with the largest fluxes realized at low $R$.

Linden (1973) has reported on flux measurements with sugar and heat and again used the "solid plane" flux to normalize the finger fluxes. He directly compares his data with Turner's by scaling with the diffusivity of salt, with the result that the actual density fluxes are the same. However, the "solid plane" scaling witth $(K_s)^{2/3}$ clearly cannot be correct, or the factor of 3 difference in the diffusivities of sugar and salt would have caused a factor of 2 difference between the density fluxes. It is the diffusion of heat that is causing the salt flux, and a more detailed model of a salt finger interface, due to Stern (1976), which emphasizes the role of the heat conduction, is to be preferred. This gives a 4/3 power law with a coefficient dependent on $(gK_T)^{1/3}$; that is $\beta F_s = C(gK_T)^{1/3} (\beta \Delta S)^{4/3}$, and Stern has estimated an upper bound for $C$ for an equilibrium finger model subject to several constraints.

Before adopting either of the above 4/3-power laws to scale the data, it seems prudent to examine how well the 4/3 law applies. This can be seen in Figure 6, a log-log plot of the salt flux ($\beta F_s$) against the salinity difference ($\beta \Delta S$). The salt flux data show less scatter and short-term variability than the flux ratio data since the salt flux is not as sensitive to the neglected terms in the flux estimation and requires no correction terms. Reliable estimates are available for $R$ as large as 30. Figure 6 shows excellent agreement with a 4/3 slope over most of the range of data, and lends a great deal of credibility to the 4/3 power law. Since Turner and Linden found the coefficient of the flux law to be a function of the stability ratio, $R$, specific tests of the best fit slope of the regression of log $\beta F_s$ against log $\beta \Delta S$ can be applied

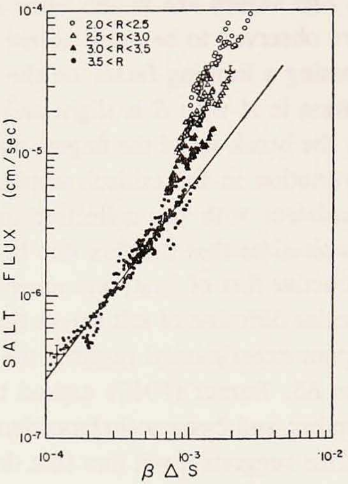

Figure 6. A log-log plot of $\beta F_s$ against $\beta\Delta S$. The line represents $\beta F_s = 0.05\ (gK_T)^{1/3}\ (\beta\Delta S)^{4/3}$, appropriate for $R > 3.5$, these data are indicated by (●). The fluxes in the other ranges have been offset upwards by 1/6 (▲), 1/3 (△), and 1/2 (○) of a decade, in order to distinguish the slope of the data in each interval.

only over short intervals in $R$. This has been done for all data except that of 3/16/77, which was strongly affected by entrainment. The correlation coefficient, regression coefficient and the standard errors are given in Table 3. The true number of degrees of freedom for these estimates is probably somewhat less than the number of data points, because data from any one experiment is correlated with itself. But it is at least 6 (the number of experiments minus one) for the first few groups and probably about 20 for $R > 3.5$. Even with fewer degrees of freedom, all correlation coefficients are significant at the 0.01 level. The regression coefficients show excellent agreement with the 4/3 power-law. For $R > 3.5$, 96% of the variance in (log) $\beta F_s$ can be attributed to the 4/3 dependence on (log) $\beta\Delta S$. This extremely

Table 3. The number of points, correlation coefficient, and regression coefficient with its standard error, for the regression of log ($\beta F_s$) against log ($\beta\Delta S$). The data of 3/16/77 were excluded.

Regression of log ($\beta F_s$) against log ($\beta\Delta S$)

| Range of $R$ | No. of data points | Correlation coefficient | Regression coefficient | Standard error |
|---|---|---|---|---|
| 2-2.5 | 57 | 0.94 | 1.37 | 0.07 |
| 2.5-3.0 | 60 | 0.92 | 1.31 | 0.08 |
| 3.0-3.5 | 52 | 0.91 | 1.24 | 0.08 |
| 3.5 < R | 227 | 0.98 | 1.33 | 0.02 |

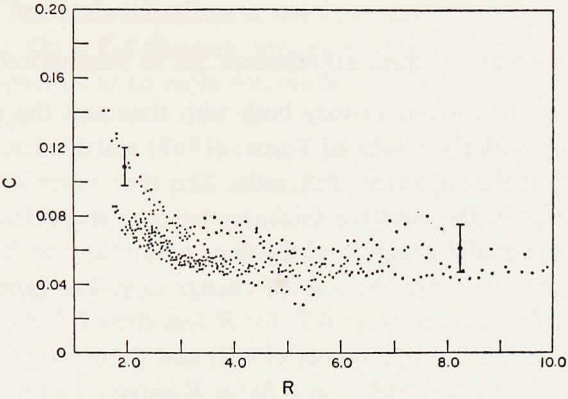

Figure 7. C, the coefficient of the 4/3 power law, obtained by dividing $\beta F_s$ by $(gK_T)^{1/3}$ $(\beta \Delta S)^{4/3}$.

good agreement can be partially ascribed to the very high haline Raleigh numbers achieved in these experiments ($Ra_s \cong 10^{13}$).

Given this strong evidence for a 4/3 power law, I have adopted the scaling suggested by Stern (1976), and all salt fluxes were divided by $(gK_T)^{1/3}$ $(\beta \Delta S)^{4/3}$. The plot of $C$ as a function of $R$ is contained in Figure 7. $C$ shows a variation similar to that found by Turner and Linden with the highest fluxes found at low $R$. $C$ is close to 0.1 for $R < 2$ in agreement with the theory of Stern (1976) and the data of Turner (1967). However, as $R$ increases, the coefficient shows less of a drop than is found in Turner's data with $C$ staying rather constant at about 0.05 for $R$ above 3.5, whereas Turner's data shows a nearly linear drop with $R$ and would give a $C$ of about 0.03 at $R = 10$. Linden's data show a relatively constant coefficient for $R > 6$ that would correspond to a $C$ of about 0.04, in reasonable agreement with the present data. Slightly higher fluxes may have been achieved in the later stages of these experiments, possibly due to the greater depth of the mixed layers. It appears that the coefficient of the power law becomes constant for $R > 3.5$, rather than decreasing linearly as Turner's data would suggest or as $(R)^{-1/3}$ as Linden proposes. The average values of $C$ over various ranges in $R$ have been computed and are contained in Table 4.

Table 4. The average coefficient of the 4/3 power law (C), obtained by dividing $\beta F_s$ by $(gK_T)^{1/3}$ $(\beta \Delta S)^{4/3}$, for various ranges in $R$.

| | <2 | 2-2.5 | 2.5-3 | 3-3.5 | 3.5-4 |
|---|---|---|---|---|---|
| All data | .097(.026) | .076(.013) | .064(.009) | .058(.008) | .055(.009) |
| excluding 3/16 | .078(.006) | .073(.009) | .063(.009) | .057(.007) | .054(.007) |

| | 4-5 | 5-6 | 6-10 | 10< |
|---|---|---|---|---|
| All data | .054(.010) | .049(.009) | .054(.009) | .049(.008) |
| excluding 3/16 | .052(.009) | .049(.009) | .053(.008) | .049(.008) |

## 5. Conclusions

The three major results of these experiments can be summarized as follows:

1. The flux ratio was found to vary both with time and the stability ratio, $R$; which is in contrast with the results of Turner (1967) and Linden (1971) who both estimated a constant (but different) flux ratio. The time variations may be due to periodic adjustments in the interface thickness (storage and release of heat in the interface) and in the modal structure of the fingers, to changing $R$. Varying the initial conditions seemed to cause about a 0.1 change in $\gamma$. The general trend was for high flux ratios to be realized, $\gamma \cong 0.7$, for $R$ less than 2.5, for $\gamma \cong 0.58$ when $2.5 < R < 4$, in agreement with Turner (1967) and the theory of Schmitt (1979), and for low flux ratios to be found ($\gamma \cong 0.3$) for $R$ greater than 6, closer to the data of Linden (1971) and the theory of Stern (1976). The value of $\gamma \cong 0.7 \pm 0.13$ for $R < 2.5$ is of most importance to oceanographers, because at low $R$ the fingers have large growth rates and are most likely to contribute to the mixing processes. The oceanic observations of Magnell (1976) are in agreement with a high flux ratio at low $R$.

2. The ratio of the salt flux to the product of viscosity and local density gradient due to temperature, was found to be about one. That is, $\beta F_s/\nu\alpha\bar{T}_z \cong 1$ with a slow decrease with increasing $R$. Thus, the salt flux is an order of magnitude greater than the conductive heat flux across the thin interface, and three orders of magnitude greater than the molecular diffusion of salt. This relation supports the collective instability model of Stern (1969), which provides a mechanism for the regulation of the salt finger interface thickness.

3. The evidence for the dependence of the salt flux on the 4/3 power of the salinity difference is very strong; the least squares regression of log $(\beta F_s)$ against log $(\beta\Delta S)$ agrees quite well with a 4/3 slope. Thus, one can determine the salt flux from the relation $\beta F_s = C \cdot (gK_T)^{1/3} \cdot (\beta\Delta S)^{4/3}$ with $C = 0.051$ (within 5%) for $R > 3.5$ and $C \rightarrow 0.1$ as $R \rightarrow 1$. This lends support to the recent studies (Lambert and Sturges, 1977; Schmitt and Evans, 1978) which use this flux law to suggest that the flux due to fingers is an important term in the salt budget of the main thermocline. This seems plausible because the flux law indicates that a salinity step of only 0.036‰ would have a salt density flux equal to the surface input of salt due to evaporation (about $10^{-7}$ cm/sec) and steps of such magnitude are not uncommon in the ocean. Thus the usual assumption of equal "eddy diffusivities" for heat and mass in thermocline modeling may be quite inadequate.

The large growth rate of fingers at low stability ratios (Schmitt, 1979) makes the salt fingers especially likely to affect isopycnal intrusions where water masses interleave at their boundaries. The salt flux being greater than the heat flux, warm, salty intrusions should cool, freshen and become lighter; cold fresh ones should warm, increase in salinity and become denser. Just such water mass transformations have been observed in fine scale intrusions at the Antarctic Polar Front by Joyce, Zenk, and Toole (1978). On a *T-S* diagram, the point representing the core of the intrusion will cross isopycnals at an angle determined by the flux ratio. It is interesting to note that according to Wüst (1978) the core of the Antarctic Intermediate Water crosses isopycnals, gaining more salt than heat and becoming denser, with an apparent flux ratio of 0.62 ± 0.11, as it travels northward in the Atlantic. This is in the proper sense for salt fingering from the warm salty water above, suggesting that parts of the thermocline may be maintained by a combination of lateral processes and double-diffusive vertical mixing rather than the perfect mixing—upwelling balance usually assumed.

Another effect of potential interest to oceanographers was the asymmetrical entrainment in which the lower layer grew at the expense of the upper. This effect is thought to be due to the variation of the thermal expansion coefficient with temperature. It is not known whether this could occur in the ocean, where the temperature changes across the interfaces would be much smaller, thus making the higher order terms in the equation of state less important. However, the observed upward migration of the interface should be kept in mind in studies of thermo-haline layering caused by oceanic salt fingers.

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