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Reynolds Stress and deep counter-currents near the Gulf Stream

by Rory O. R. Y. Thompson[1]

ABSTRACT

Comparatively detailed current meter array observations of a dynamically interesting region of the ocean have been reported recently (Luyten, 1977; Thompson, 1977). This note will discuss a simple interpretation of the Reynolds stresses and mean currents reported, namely that the Reynolds stress convergences drive the countercurrents as well as the Gulf Stream.

It will be a basic assumption here that we should look for simplicity in data, rather than complications. Here we will look for what may be simple in observations of Reynolds stress between Site D (Thompson, 1977) and the Gulf Stream (Luyten, 1977).

For the Reynolds stress to be practically meaningful, it is necessary to choose a meaningful zonal direction—preferably a periodic one, so one can average out terms like $\partial p/\partial x$. We will then make the simplifying "ergodic" assumption that time average can be used in place of zonal averages; not because this is "true," but because it is useful. Then the mean zonal acceleration is given by

$$\frac{d\bar{u}}{dt} = f\bar{v} - \frac{\partial}{\partial y}(\overline{u'v'}) - \frac{\partial}{\partial z}(\overline{u'w'}) \, . \tag{1}$$

For quasi-geostrophic flow, the last term will be formally of order the Rossby number compared to the previous term, and is consequently negligible. Any mean meridional flow, $\bar{v}$, must be coupled to a mean vertical flow, $\bar{w}$, by continuity, hence advecting density surfaces. But the density surfaces are observed to be flat north of the Gulf Stream, so it is reasonable to assume $\bar{v} = 0$. (A vertical integral would make $\bar{v} = 0$ exact, but is not feasible with the given data.) With these assumptions, in a meaningful zonal direction, the Reynolds stress convergence

$$-\frac{\partial}{\partial y}(\overline{u'v'}) \tag{2}$$

will represent the acceleration of the mean flow. To maintain a steady state as (baro-

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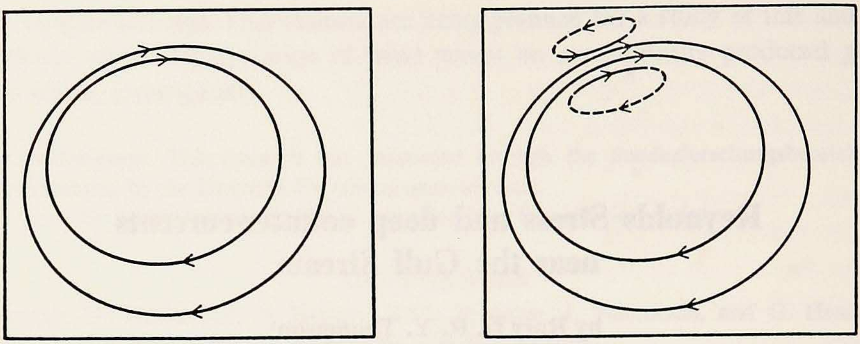

Figure 1. Schematic of streamlines of ocean circulation (a) without and (b) with an active region in the upper left that brings momentum from each side into the Stream.

clinic) energy is put in requires some kind of friction or advection away of the energy, which will not be considered here.

In the atmosphere, using x as East is sensible, because the atmosphere is periodic that way, and the jet stream runs essentially East. In the ocean, using x along an isobath makes more sense, because the restoring forces are mainly due to topography, because the isobaths close within the ocean (so $\partial p/\partial x$ can go out), and the Gulf Stream runs more nearly along isobaths than due East. Freeland, Rhines, and Rossby (1975) have found that perturbation flows over a large part of the deep Atlantic tend to be strongly influenced by the local topography; many people have noted how most deep mean currents tend to follow the topography. Luyten (1977) finds that the (rotationally invariant) kinetic energy shows no systematic difference between two meridional sections. Therefore, we will use coordinates along and upslope.

The particular idea to be looked at is essentially that of Thompson (1971): Consider a large-scale linear circulation as in Figure 1a, and suppose that in the upper left there is positive β (or equivalent topographic β) and baroclinic instability. Then Thompson (1971) argued that momentum ought to converge into the Stream from the surroundings. This momentum convergence is a force, causing the Stream to intensify, as in Figure 1b. The added mass-flux lines cannot simply end in mid-ocean, but most close, as the dashed lines. But the mid-ocean (Sverdrup) dynamics cannot carry additional mass flux, whereas the momentum divergence on each side of the Stream needs exactly this flux, since the push backward on each side must balance that forward on the Stream. Therefore postulated momentum flux into a baroclinic zone (for positive β) is expected to cause a nongeostrophic recirculation. Schmitz (1977) presents data which supports this suggestion that the down-stream increase in transport of the Gulf Stream could be driven by the eddy field.

Figure 2 here plots the Reynolds stress $\overline{u''v''}$ for coordinates rotated to the local topography, taken from Table II of Thompson (1977)). An upslope coordinate was

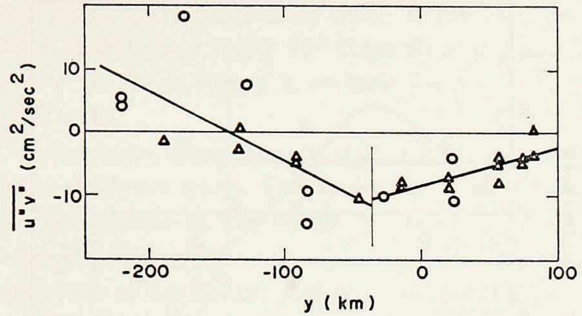

Figure 2. Reynolds stress estimate $\overline{u''v''}$ versus upslope distance in the Site D area. v'' is the local upslope, u'' the along-slope perturbation from the 8 month velocity average. The y coordinate is distance (in km) from the 3200 m contour to each mooring. The lines are least squares regression lines of Reynolds stress on y, for the two regimes separately. The triangles are from moorings along 70W; the circles from nearer 69W.

needed, but the bottom slope is only roughly planar, and the meridional sections are not perpendicular to the isobaths. The upslope coordinate chosen as apparently representative was distance from the 3200 m isobath. Since Thompson (1971) had hypothesized a region of Reynolds stress convergence [$(\overline{u''v''})_y < 0$] in the Gulf Stream region and divergence away from it, the data are divided into two regimes, as marked. The division corresponds approximately to the 3600 m isobath; it could have been moved say 40 km downslope, nearly to the 4000 m isobath with little effect on the results. The least square lines seem each to be reasonable fits; correlation is $r = 0.75$ for the shallow regime and $r = -0.69$ for the deep; these are both statistically significant, though it is doubtful if individual estimates are in the deep water (Luyten, 1977). The two lines match well at the join, as they must to be physically meaningful. The slope of the line in the shallower part of the array is $(\overline{u''v''})_y = 6 \times 10^{-7}$ cm/sec^2, or an acceleration of -1 cm/sec each 19 days. In the deeper part the estimate is -11×10^{-7} cm/sec^2, or 1 cm/sec in 10 days.

If the simplifying "ergodic" assumption is to be useful, there should be no great difference between the two meridional sections. Indeed, the circles and triangles in Figures 2 and 3 look to me much as if they were random samples from the same population. However, a referee expressed doubt about the lower section, so some statistical tests were made.

For the six observations along 70W (the triangles), the regression line for the Reynolds stress was found to be

$$\overline{u'v'} = -0.12y - 11.3 \tag{3}$$

with y in km and $\overline{u'v'}$ in cm^2/sec^2. The standard error in the slope is the standard deviation about the line divided by $\sqrt{n-2}$ times the standard deviation of y, or

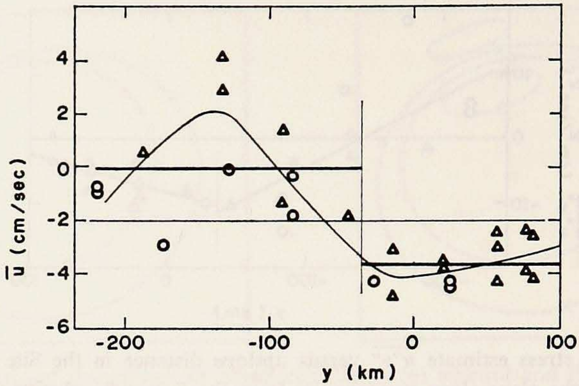

Figure 3. Mean along-slope velocity component $\bar{u}$ versus upslope distance in the Site D area. The y coordinate is distance (in km) from the 3200 m contour to each mooring. The light curves are a free-hand fit; the heavy lines are the averages for the two regimes. The triangles are from moorings along 70W; the circles from nearer 69W.

0.042. For the six observations along 69°20′W (the circles), the regression line is

$$\overline{u'v'} = -0.24y - 17.7\,, \tag{4}$$

with standard error in the slope of 0.13. One can test if the slopes are significantly different with a t-test with $n_1 + n_2 - 4 = 8$ degrees of freedom:

$$t = \frac{-0.12 - (-0.24)}{\sqrt{(0.042)^2 + (0.133)^2}} = 0.85\,; \tag{5}$$

the 2-tail 5% significance level is 2.31, so the difference is not significant. The difference between the intercepts is even less significant. For the mean velocities of Figure 3, the mean for the triangles in the lower section was 1.0 cm/sec with a standard deviation of 2.3 cm/sec, for the circles −1.1 and 1.1. A t-test with 10 degrees of freedom is then

$$t = \frac{1.00 - (-1.13)}{\sqrt{(2.34)^2 + (1.05)^2}} = 0.83\,,$$

with a two-tail 5% significance level of 2.23. Therefore, there seems no need to distinguish the two meridional sections.

Figure 3 shows the mean velocity component along the local isobath (from Thompson, 1977, Table II), plotted versus the same upslope distance y. The free-hand curve seems a reasonable representation of a weak (bottom) Gulf Stream and counter-current. The only really bad fit is mooring 5352 with $\bar{u} = -2.9$ at $y = -140$. Note this point is also anomalous in Figure 2. This local isobath was taken at 70° from East, due to a local twist in the topography (see Figure 20 of Thompson, 1977); it may well be that this twist is too much of a strain on the simple model, or that in fact the topographic chart is in error. If one decided that the general trend of the 4400 m isobath was really 10° from East, this would give $\bar{u} = +0.7$ cm/sec, and also $\overline{u''v''} = 7.5$ in Figure 2, so both figures would look better. How- they do not look bad as is.

The other 11 of the mean along-slope velocities in the deeper water are above all 14 of those in the shallower water. This is certainly significant, and fits well with division into the same regimes as the Reynolds stress. The heavy horizontal lines represent the average velocities of the two regimes.

Within the framework of this simple analysis then, the Reynolds stress is taking momentum away from the shallow end, where the velocity is negative, and putting it into the Gulf Stream end, where the velocity is relatively positive. That is, the Reynolds stress is doing work on the mean flow, to accelerate it to the observed magnitude in a time comparable to, and probably shorter than, the time a particle takes to flow through the area from the Grand Banks to Cape Hatteras. There is also a comparable positive Reynolds stress convergence under the Gulf Stream. This pattern of Reynolds stress is precisely what Thompson (1971) proposed. It also matches the general pattern observed by Schmitz (1977).

I conclude that more physical meaning can be gotten from the data in a proper coordinate system. Further, there is evidence that the mechanism proposed by Thompson (1971) is in fact important near the Gulf Stream. It appears that Reynolds stress convergence drives the Gulf Stream and its counter-currents, with the resultant recirculation much increasing the transport of the Gulf Stream above the Sverdrup (linear) balance.

Acknowledgment. I thank Jim Luyten for useful comments.

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Received: *27 July, 1977;* revised: *10 July, 1978.*