The tide in coastal waters[1]

by Alfred C. Redfield[2]

ABSTRACT

The tide in many straits and embayments between New York and the Bay of Fundy may be described by theoretical equations based on the interference of a progressive wave entering at one end of the reach with, in the case of straits, a second wave entering at the opposite end or, in the case of embayments, a second wave arising from the reflection of this wave from a barrier at the head of the embayment. The constants which must be introduced into the equations are found by a method of nomographic analysis. Exceptions are found in a few cases in which the topography is complex or the reach is one of transition.

The purpose of the present study is to examine the utility of a method of analysis of the tide in confined waters which depends on the interference of waves moving along the channel in opposite directions and to characterize numerically the tidal systems present in some coastal waters between New York and the Bay of Fundy.

The tides in enclosed basins have been analyzed by a dynamic method introduced by Sterneck and Defant which depends on the application of fundamental laws of hydrodynamics to the dimensions of the basins (Proudman, 1953). It has been applied successfully to the tides in the Mediterranean and adjacent seas (Defant, 1960), to Lake Maracaibo (Redfield, 1961), to the Bay of Fundy (Duff, 1970), and to Long Island Sound (Swanson, 1971). The method requires adequate charts and is adapted to basins of relatively simply form, conditions frequently lacking in coastal waters.

A more generally applicable method is derived from the theory of seiches, which considers the motion in an enclosed basin to depend on the interference of a progressive wave and its reflectant from the ends of the basin (Redfield, 1950). When applied to a coastal embayment opening on an outer sea it is considered that the motion is a co-oscillation with that of the outer sea, produced by the interference of a free progressive wave entering from the outer sea and its reflection from the head of the embayment (Doodson and Warburg, 1941). The analysis depends on a comparison of the predicted relations of range and time of high water in natural channels with those in ideal channels. The method has been applied to the reflected

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co-oscillation of Long Island Sound, the Bay of Fundy and the Juan de Fuca-Georgia Strait system (Redfield, 1950) and to the tides of the continental shelf (Redfield, 1958). It has been adapted to the analysis of straits in the case of the Nantucket-Vineyard Sounds system (Redfield, 1953).

An ideal channel is one in which the cross-section is the same throughout its length. Nondimensional Cartesian nomograms can be constructed for ideal channels showing the relations of mean range, time of high water and the phase of the entering wave when it is subject to various degrees of attenuation. Points representing the predicted mean range and the predicted time of high water at positions along natural channels (suitably corrected for the units of measurement and the presence of harmonics) may be entered on these monograms. If they fall along a line for some value of the attenuation coefficient its value may be assigned to the natural channel and values for the phase difference between the positions may obtained. The basic assumption underlying this comparison is that while the dimensions of the channel may alter the rate of advance of the waves and thus distort its shape it does not alter the relation of elevation to time of high water at any position. The procedure is independent of the dimensions of the channel but indicates how its topography distorts the tidal wave. Its virtue is that it assigns numerical values to the attenuation which the wave undergoes in different channels and to the relative amplitudes of the two interfering waves.

1. Theory

The following notation is employed in developing the theory and its application:

The position of phase equality is where the two waves are in phase. It is the origin of distance, phase, and time.

- k is one wave length of 360°
- x is a fraction of one wave length, measured from the position of phase equality
- kx is the difference in phase from the position of phase equality
- σ is the speed number of the constituent. In the North Atlantic the tide is due chiefly to the M_2 constituent and $\sigma = 29°$ per hour
- t is time in hours measured from the time of high water at the position of phase equality
- σt is a time angle giving the difference in phase of any position at any time relative to that of the position of phase equality at high water
- σt_H is a time angle giving the difference in phase of any position at high water from that at the position of phase equality at high water
- σt_s is a time angle giving the difference in phase at any position at slack water from that at the position of phase equality at high water
- η_1 is the elevation at any time of the wave moving in the x direction
- η_2 is the elevation at any time of the wave moving in the $-x$ direction

η is the elevation of the combined waves at any position at any time

η_H is the elevation of the combined waves at any position when it is high water there

η_{Ho} is the elevation of the combined waves at the position of phase equality when it is high water there

A is the amplitude of the wave moving in the x direction at the position of phase equality

R is the ratio of the amplitude of the wave moving in the $-x$ direction to that moving in the x direction at the position of phase equality

RA is the amplitude of the wave moving in the $-x$ direction at the position of phase equality

μ is an attenuation coefficient expressing the loss in amplitude per wave length

$e^{\mu x}$ is the change in amplitude of a wave in traversing a fraction, x, of one wave length.

The theory of the tide in straits is developed by Redfield, 1953.[3] When two waves move along a channel of sufficient length there will be some position where they are in phase which may be taken as the position of phase equality and as the origin of phase and time. The elevation at any time of the wave moving in the x direction is

$$\eta_1 = A \cos(\sigma t - kx)\, e^{-\mu x}\,, \tag{1}$$

that at any time of the wave moving in the $-x$ direction is

$$\eta_2 = RA \cos(\sigma t + kx)\, e^{\mu x}\,. \tag{2}$$

The resulting elevation of the tide is the sum of that of the two waves, or

$$\eta = A \cos(\sigma t - kx)\; e^{-\mu x} + RA \cos(\sigma t + kx)\, e^{\mu x}\,.$$

letting $F = \dfrac{Re^{\mu x} - e^{-\mu x}}{Re^{\mu x} + e^{-\mu x}}$ the time angle of high water is given by

$$\sigma t_H = \tan^{-1}(-\tan kx \cdot F) \tag{3}$$

and the time angle of slack water is given by

$$\sigma t_S = \tan^{-1} \frac{F}{\tan kx} \tag{4}$$

At the time of high water at any position the elevation of the tide is

$$\eta_H = A\, \frac{(\cos kx - F \cdot \tan kx \cdot \sin kx)\, e^{-\mu x} + R\,(\cos kx + F \cdot \tan kx \sin kx)\, e^{\mu x}}{\sqrt{1 + F^2 \tan^2 kx}} \tag{5}$$

3. In this paper the ratio of the two waves at the position of phase equality is defined by $(1+f)$ which is here denoted more simply by R.

Table 1. Greenwich Lunitidal Intervals at Reference Stations. (Supplied by the National Oceanographic and Atmospheric Administration).

Station	Mean High Water Interval, Hours	Mean Low Water Interval, Hours
St. John, N.B.	3.32	9.57
Eastport, ME	3.32	9.73
Portland, ME	3.60	9.72
Boston, MA	3.75	9.93
Newport, RI	0.25	5.85
New London, CT	2.08	8.52
Bridgeport, CT	3.77	10.13
Willets Point, NY	4.18	10.73
The Battery, NY	0.98	7.32
Albany, NY	9.67	4.37
Sandy Hook, NJ	0.33	6.72

Mean Current Intervals—Hours

	Slack Flood Begins	Maximum Flood	Slack Ebb Begins	Maximum Ebb
Bay of Fundy Entrance	9.60	0.10	3.27	6.62
Portsmouth Harbor Entrance	11.93	1.77	5.30	8.08
Pollock Rip Channel	7.87	11.25	1.98	4.77
Cape Cod Canal—RR Bridge	8.25	11.07	2.22	4.67
The Race—Long Island Sound	10.82	1.07	4.28	7.08
Hell Gate—East River	9.03	11.87	2.63	5.58
The Narrows—New York Harbor	9.32	12.00	2.37	5.67

The tide in embayment where the motion is that of a reflected co-oscillation and that where it is due to a single progressive wave without modification are special cases of these equations. In the former $R = 1$, in the latter $R = 0$.

2. Data

The data employed are given in the tide and current tables of the U.S. Department of Commerce (1971) being predictions for 1972 and the Greenwich intervals of high water, low water and currents at the reference stations supplied by the National Ocean Survey and given in Table 1. To reduce these predicted intervals to a common system of time the differences between the individual positions and the reference stations have been added to the Greenwich intervals of the reference stations given in this table.

The predictions are based on observations which are affected by a number of factors which are not considered by the theory. These are the effects of constituents having speed numbers which differ from that of the semidiurnal lunar tide (K_1, O_1, and S_2), meteorological effects, geostrophic effects, hydraulic effects, and the presence of higher harmonics of M_2. The use of predictions of the mean tide largely eliminates the effects of the diurnal and the semidiurnal solar constituents and of the meteorological effects when they are based on measurements made over 29 days or its multiple. Significant geostrophic effects are only to be expected in wide passages. Hydraulic effects may be suspected from the topography, and when present they cause the Greenwich interval of slack water to differ from that expected in their absence.

The effects of higher harmonics of the lunar semidiurnal constituent are significant and must be eliminated before the predictions and theory are compared. The predictions indicate that at any position the duration of the rise of the tide differs from that of the fall, thus leading to an asymmetry of the tide curve which is attributed to the presence of harmonics. Theory requires that they be equal. The Greenwich high water interval may be corrected so as to bring about this equality by adding to the high water interval and subtracting from the low water interval a correction so that high and low waters follow at intervals of 6.21 hours. The corrected value of the high water interval is given by $\frac{1}{2}$(HWI + LWI, as predicted—6.21 hours). Similarly the corrected value of the slack water interval is given by $\frac{1}{2}$(SWI before ebb + SWI before flood, as predicted—6.21 hours).

Theoretically the mean range of the tide is not affected by the presence of the M_4 and M_8 harmonics because these have the same effect on the range at high water and low water whatever may be the relation of their phase to that of the M_2. This is not true of the M_6 constituent which will have some effect on the mean range. In the absence of harmonic analyses this effect cannot be allowed for. The predicted values for the mean range have not been corrected and may be subject to some small error for this reason.

3. Nomograms

Values of R and μ characteristic of individual passages and those of kx assigned to positions along each passage are conveniently found by nomographic analysis. Nomograms for any combination of R and μ are prepared from equations (3) and (5). For a series of values of kx the amplitude of tidal wave is estimated from equation (5) and its ratio to the amplitude at the position of phase equality determined. The logarithm of this ratio, $\log \eta_H/\eta_{Ho}$, is plotted as a coordinate. The time angle of high water, σt_H, for corresponding values of kx is given by equation (3) and is plotted as an abscissa. It is convenient to prepare a separate nomogram for each value of R on which a series of lines is drawn, each representing a value of μ. A second series of lines is drawn, each representing a value of kx. Examples of such nomograms are shown in Figure 1. In the special cases of reflected co-oscillations, in which $R = 1$, and of those in which there is no wave moving in the direction

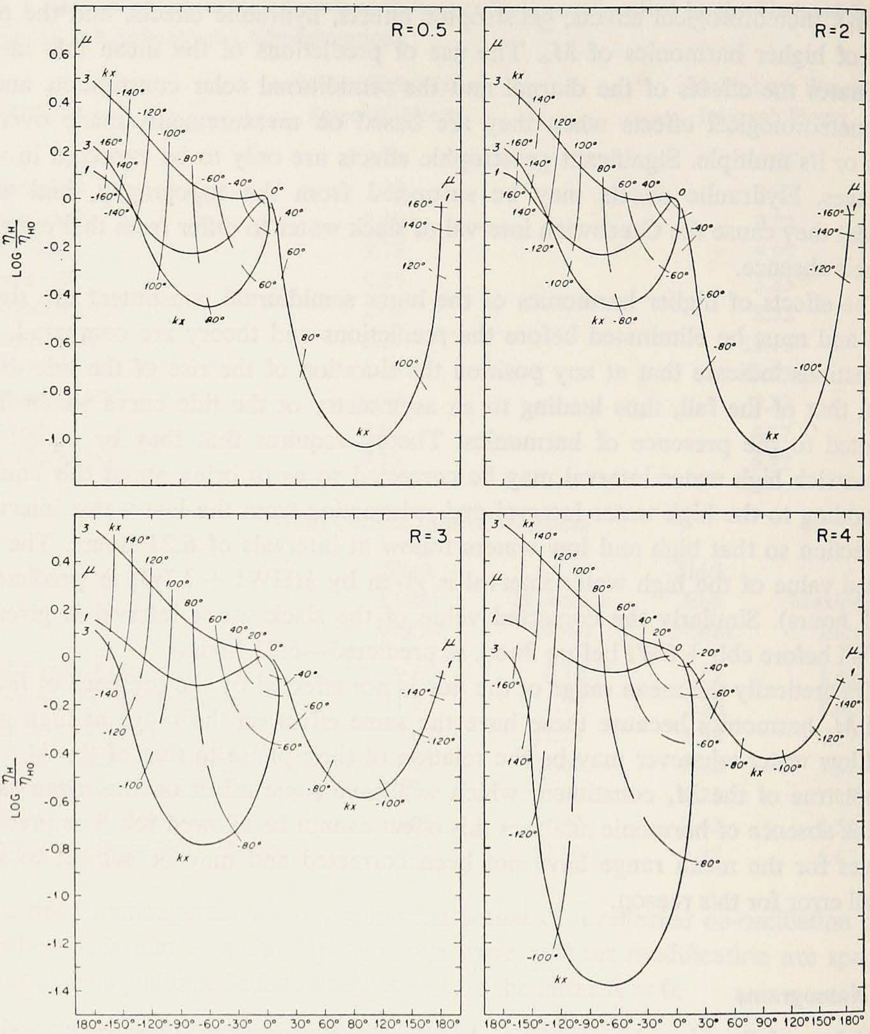

Figure 1. Nomograms for straits in which R=0.5, 2, 3, and 4 and μ=1 and 3 based on equations (3) and (5). For R=1 and R=0 see Figures 14 and 25 respectively.

opposite to that from which the tide enters, in which $R = 0$, a single nomogram shows the possible relations, as illustrated in Figures 14 and 25.

Figure 1 shows that the nomograms fall into two patterns. In both patterns if kx is positive σt_H is negative. In one pattern if kx is negative σt_H is always positive. This pattern has not been found in any of the passages examined. In the other pattern if kx is negative σt_H may be negative or may become positive as the position of phase equality is approached. Equation (3) shows that the change in sign occurs

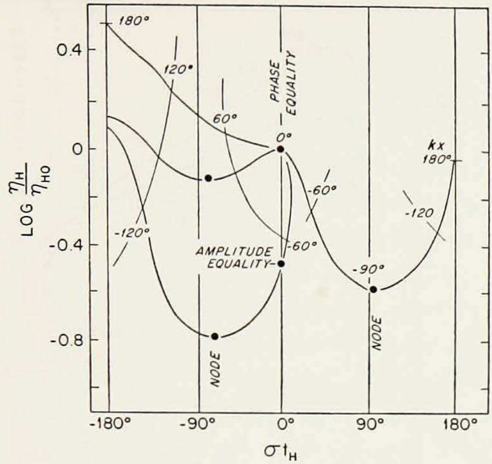

Figure 2. Diagrams of two patterns of nomogram for straits showing the characteristic singular points.

when $R = e^{-2\mu x}$. Negative values of σt_H occur when $R < e^{-2\mu x}$ and positive values when $R > e^{-2\mu x}$.

Figure 2 is a diagram of each of the two patterns of nomogram on which are shown three singular points which, if present, are characteristic of individual passages. They are:

1. The position of phase equality where σt_H, $\log \eta_H/\eta_{Ho}$ and $kx = 0$.
2. The position of amplitude equality where both waves are of the same amplitude and $\sigma t = 0$. At this position the sign of σt_H changes from negative to positive as kx increases. However, if $R = 1$ the position of amplitude equality coincides with that of phase equality and σt_H is never positive.
3. A node may occur where the mean range is minimal.

To determine the values of R which are applicable to the individual passages the data for positions along the passage are plotted, on the same scale as in the nomogram, expressing the mean range as its logarithm and the time of high water as the Greenwich high water interval (GHWI). A line may usually be drawn through these points indicating the relation of mean range to high water interval. By expressing the mean range as the logarithm of its value the *shape* of this line is independent of the actual value of the mean range. The shape of this line is then matched with the shape of some segment of a theoretical line calculated for given values of R and μ and corrections are then added to the predictions so as to bring the predicted line into correspondence with the segment of the theoretical curve. The values of these corrections are given by the logarithm of mean range and the Greenwich high water

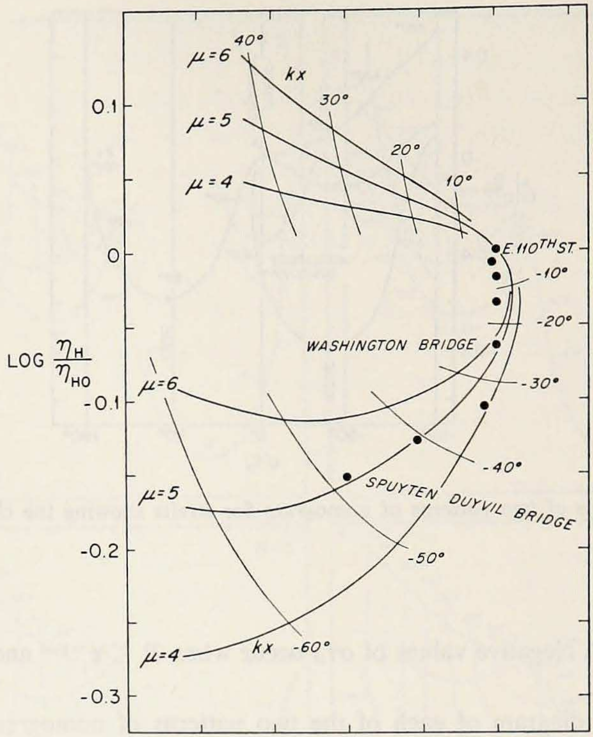

Figure 3. Nomogram for Harlem River. Lines show the theoretical value when $R=2$ for μ and kx as σt_H varies. Solid circles show the predicted values of log η_H/η_{Ho} and σt_H when $R=2$ and $\mu=5$ and indicate the value of kx at each position.

interval at the position of phase equality where log η_H/η_{Ho} and $\sigma t_H = 0$. If the predicted values, thus corrected, scatter along some theoretical line for R and μ these values are applied to the tide in the passage in question.

In considering the individual channels examples are given of the nomographic analysis of the Harlem River, a strait (Fig. 3) of Long Island Sound which may be treated either as a part of a strait or as a reflected co-oscillation (Fig. 14) and of the Lower Reach of the Hudson River in which the tide appears to be due to a single entering wave, uninfluenced by waves moving in the opposite direction (Fig. 25). In all other cases the values of R and μ have been obtained by nomographic analysis. In every case the validity of the analysis is checked by figures showing a comparison of the theoretical values obtained when these are calculated from equations (1), (3), (4), and (5) and the predicted values when the latter are corrected for the presence of harmonics.

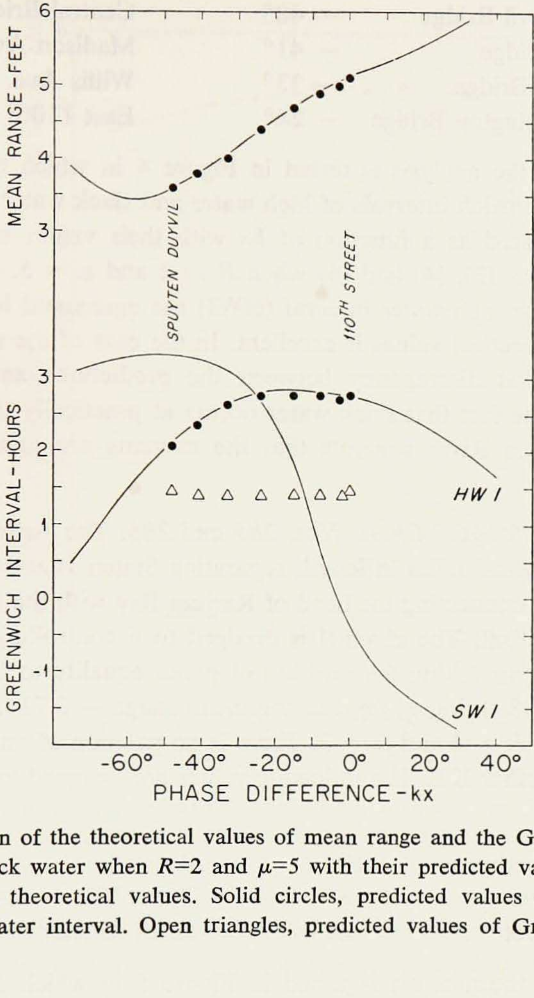

Figure 4. Comparison of the theoretical values of mean range and the Greenwich intervals of high water and slack water when $R=2$ and $\mu=5$ with their predicted values for the Harlem River. Solid lines, theoretical values. Solid circles, predicted values of mean range and Greenwich high water interval. Open triangles, predicted values of Greenwich slack water interval.

4. The tide in individual passages

The Harlem River. USC&GS[4] *Chart No. 274.* The Harlem River is a strait about 6.6 nautical miles in length separating Manhattan Island from the mainland of New York and connecting the Hudson River with the East River. The nomographic analysis is shown in Figure 3. If the position of phase equality is taken to be at East 110th Street $A = 0.85$ ft $\log \eta_H/\eta_{Ho} = \log$ mean range -0.63, $\sigma t_H =$ (GHWI $\times$ 29) $-80°$, when $R = 2$ and $\mu = 5$. The position of amplitude equality where $kx = -25°$ is near George Washington Bridge. There is no node within the Harlem River. The following values are assigned to positions along the passage:

4. The United States Coast and Geodetic Survey is now known as the National Ocean Survey, Dept. of Commerce.

Spuyten Duyvil Bridge	$-48°$	Central Bridge	$-15°$
Broadway Bridge	$-41°$	Madison Ave. Bridge	$-8°$
207th Street Bridge	$-33°$	Willis Ave. Bridge	$-3°$
George Washington Bridge	$-24°$	East 110th Street	$-0°$

The validity of the analysis is tested in Figure 4 in which the predicted mean range and the Greenwich intervals of high water and slack water, corrected for harmonics, are compared as a function of kx with their values calculated from the theoretical equations (3), (4) and (5) when $R = 2$ and $\mu = 5$. In the cases of the mean range and the high water interval (HWI) the agreement between the predictions and their theoretical values is excellent. In the case of the slack water interval (SWI) there is great discrepancy between the predictions and their theoretical values. This and the fact that slack water occurs at practically the same time in all parts of the Harlem River suggests that the currents are primarily hydraulic in nature.

The Arthur Kill. USC&GS Charts Nos. 285 and 286. The Arthur Kill is a narrow strait about 10 nautical miles in length separating Staten Island from the mainland of New Jersey and connecting the head of Raritan Bay with the junction of Newark Bay and Kill van Kull. The channel is dredged to a controlling depth of 35 feet. Nomographic analysis taking the position of phase equality to be at Carteret indicates that $A = 0.85$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range $- 0.714$, $\sigma t_H =$ (GHWI $\times$ 29) $- 24.6°$ when $R = 2$ and $\mu = 3$. There is no position of amplitude equality or node within the Arthur Kill. The following values are assigned to kx positions along its length:

Elizabethport	$-10°$	Rossville	$+9°$
Chelsea	$-6°$	Tottenville	$+20°$
Carteret	$0°$	Perth Amboy	$+15°$[5]

The validity of the analysis is tested in Figure 5 in which the predicted mean range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of kx with their values calculated from equations (3), (4), and (5) when $R = 2$ and $\mu = 3$. In the cases of the mean ranges and the high water interval (HWI) the agreement between the predictions and their theoretical values is excellent. In the case of the slack water interval (SWI) slack water occurs earlier than predicted in theory. This suggests the presence of a hydraulic component but the fact that slack water does not occur simultaneously in the Arthur Kill indicates that the wave current is also significant.

The Cape Cod Canal. USC&GS Chart No. 251. The Cape Cod Canal is a passage

5. The data for Perth Amboy has been disregarded because the predicted time of high water is later than at Tottenville, whereas it should be earlier, Perth Amboy being nearer the position at which the tide appears to enter.

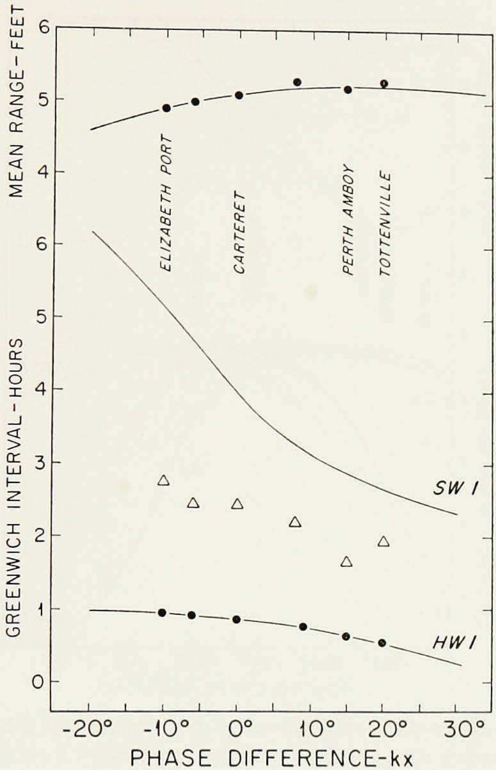

Figure 5. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when R=2 and μ=3 with their predicted values for the Arthur Kill. For explanation see Figure 4.

cut through the upland between Cape Cod Bay and the village of Buzzards Bay for a distance of about 6 nautical miles and to a width of about 500 ft. Beyond the village the canal is dredged across the shallow waters at the head of Buzzards Bay on which it opens at Abiels Ledge. The controlling depth is 35 ft. Nomographic analysis, taking the position of phase equality to be virtual and in Cape Cod Bay, indicates that $A = 1.58$ ft, $\log \eta_H/\eta_{No} = \log$ mean range $- 0.98$. $\sigma t_H = (\text{GHWI} \times 20) - 105°$, when $R = 2$ and $\mu = 3$. The position of amplitude equality where $kx = -40°$ is near Sagamore Bridge. A node is present at the Railroad Bridge, when $kx = -80°$. The following values are assigned to kx at positions on the Cape Cod canal:

Abiels Ledge	− 92°	East Entrance	− 20°
Railroad Bridge	− 80°		

The validity of the analysis is tested in Figure 6 in which the predicted mean range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of kx with their values calculated from equations (3),

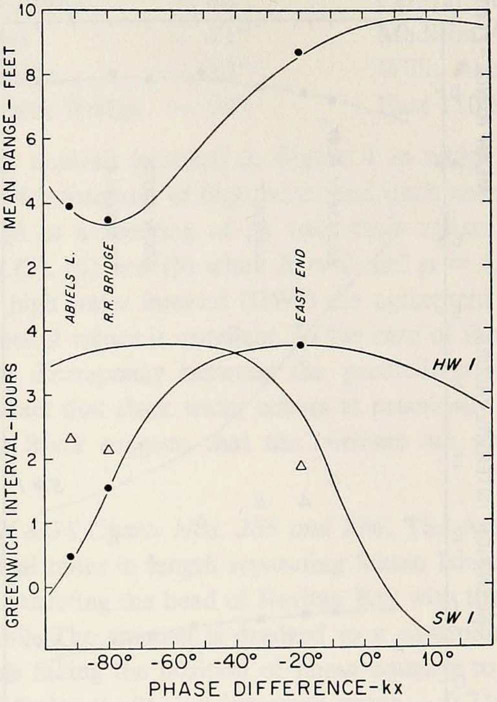

Figure 6. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=2$ and $\mu=3$ with their predicted values for the Cape Cod Canal. For explanation see Figure 4.

(4), and (5) when $R = 2$ and $\mu = 3$. In the cases of the mean range and the high water interval (HWI) the agreement is excellent. In the case of the slack water interval (SWI) slack water occurs earlier than predicted in theory and nearly simultaneously. This suggests the presence of a hydraulic current in the Cape Cod Canal. In a study made prior to the present widening and deepening of the canal Panish (1933) also concluded that the flow then approximated more nearly hydraulic than tidal conditions.

Quicks Hole. USC&GS Chart No. 249. Quicks Hole is a strait about 1 nautical mile in length connecting Buzzards Bay with Vineyard Sound. Nomographic analysis, taking the position of phase equality to be virtual and in Buzzards Bay indicates that $A = 0.6$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -0.556, $\sigma t_H = (\text{GHWI} \times 29) + 5°$ when $R = 2$ and $\mu = 2$. There is no position of amplitude equality or node in Quicks Hole. The following values are assigned to kx at positions in the strait:

North end	− 10°	South end	− 43°
Middle	− 30°		

The validity of the analysis is tested in Figure 7 in which the predicted mean

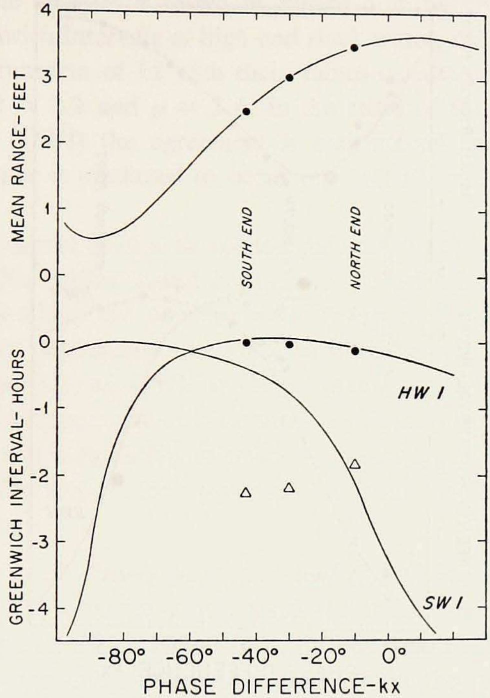

Figure 7. Comparisons of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=2$ and $\mu=2$ with their predicted values for Quicks Hole. For explanation see Figure 4.

range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of kx with their value calculated from equations (3), (4), and (5) when $R = 2$ and $\mu = 2$. In the cases of the mean range and high water interval (HWI) the agreement is excellent. In the case of the slack water interval (SWI) slack water occurs much earlier than in theory. This suggests that the current in Quicks Hole is predominantly hydraulic.

Nantucket and Vineyard Sounds. USC&GS Charts No. 1209 and 1210. Nantucket and Vineyard Sounds form a strait about 40 nautical miles in length separating Martha's Vineyard and Nantucket Islands from Cape Cod and connecting the continental shelf off Gay Head with the Gulf of Maine off Monomoy Point. Muskeget Channel which separates the islands appears to have little effect on the tides in the sounds except in the southeast part of Nantucket Sound. The theoretical equations for the tide in straits were developed to account for that in these sounds by Redfield (1953) who concluded that they could be accounted for by the interference of waves entering at opposite ends if it were assumed that $R = 2$ and $\mu = 3$.[6]

6. In Redfield (1953) the value of $f = 2$, given in the legend of Figures 4 and 5, is in error. It should have been $f+1$ (or R) $= 2$ as confirmed by recalculation from the original data.

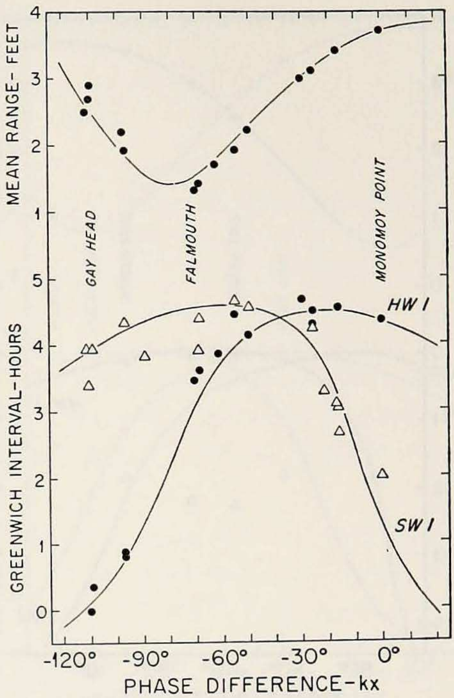

Figure 8. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when R=2.2 and μ=3.4 with their predicted values for Nantucket and Vineyard Sounds. For explanation see Figure 4.

Recalculated from data corrected for harmonics and using the Greenwich intervals of the reference stations given in Table 1 nomographic analysis indicates that the position of phase equality is near Monomoy Point, $A = 0.578$ ft, $\log \eta_H/\eta_{Ho} =$ log mean range -0.568, $\sigma t_H =$ (GHWI $\times$ 29) $-126°$, $R = 2.2$ and $\mu = 3.4$. The position of amplitude equality is where kx is about $-48°$, which is east of Succonnesset Point. A node occurs where kx is about $-75°$, between Falmouth Heights and Nobska Point. The following values are assigned to kx at positions along the sounds.

Quicks Hole—South side	− 110°	Succonnesset Point	− 55°
Gay Head	− 109°	Cape Pogue	− 50°
Menemsha Bight	− 109°	Nantucket	− 30°
Cedar Tree Neck	− 97°	Great Point	− 26°
Tarpaulin Cove	− 97°	Hyannis Port	− 26°
Nobska Point	− 88°	Dennis Port	− 17°
Falmouth Heights	− 70°	Wychmere Harbor	− 0°
West Chop	− 68°	Monomoy Point	− 0°
East Chop	− 62°		

The validity of the analysis is tested in Figure 8 in which the predicted mean range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of kx with their values calculated from equations (3), (4), and (5) when $R = 2.2$ and $\mu = 3.4$. In the cases of the mean range and the high water interval (HWI) the agreement is satisfactory except at Succonnesset Point where high water is predicted to occur one-half hour later than indicated by theory.

The slack water interval cannot be related precisely to the phase change in such a broad channel as Nantucket Sound because of the difficulty of relating the positions in mid-channel where the currents were measured to the shoreline positions where the phase was determined. At many positions slack water is predicted at about the slack water interval (SWI) expected in theory, indicating that hydraulic currents are absent in these parts of Nantucket and Vineyard Sounds. In a number of positions slack water is predicted as much as one-half hour before it is expected in theory. It is possible that at such positions the local topography has resulted in local hydraulic effects.

Hempstead Bay. USC&GS Charts Nos. 1215 and 120SC. Hempstead Bay is a strait separating Jones Beach, a barrier island, from the south shore of Long Island and connecting Jones Inlet with Fire Island Inlet via Great South Bay. At the west and south the passage is through salt marsh creeks but in the greater part it crosses South Oyster Bay where the depth at low water is for the most part less than 2 ft.

Nomographic analysis has been made for positions at and between Neds Creek and Babylon, a distance of 10.7 nautical miles. Taking the position of phase equality to be near Amityville, $A = 0.5$ ft, $\log \eta_H \eta_{H1} =$ log mean range -0.79, $\sigma t_H =$ (GHW1 $\times$ 29 $-$ 90.5°, $R = 0.2$ and $\mu = 7$. The position of amplitude equality is between Amityville and Babylon where $kx = 40°$. There is no node within Hempstead Bay. The following values are assigned to positions within this strait:

Neds Creek	− 52°	Biltmore Shores	− 13°
Deep Meadow Creek	− 46°	Amityville	− 0°
Bellmore	− 44°	Gilgo Head	+ 8°
Cuba Island	− 43°	Babylon	+ 50°
Green Island	− 33°		

The validity of the analysis is tested in Figure 9 in which the predicted mean range and the Greenwich high water interval, corrected for harmonics, are plotted as a function of kx against curves calculated from equations (3) and (5). In both cases the agreement is satisfactory in view of the irregular form of the passage. The current tables give no predictions for positions in Hempstead Bay.

The East River. USC&GS Chart No. 1213. The East River is a strait about 13.6 nautical miles in length separating Long Island from Manhattan Island and the

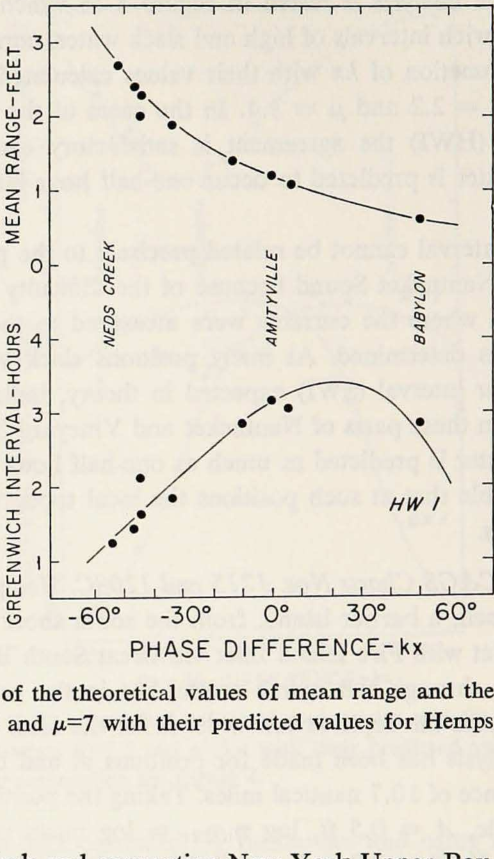

Figure 9. Comparison of the theoretical values of mean range and the Greenwich high water interval when R=0.2 and μ=7 with their predicted values for Hempstead Bay. For explanation see Figure 4.

mainland of New York and connecting New York Upper Bay at Governors Island with the head of Long Island Sound at Throgs Neck. Marmer (1935) considered that the tidal movement in the East River was primarily hydraulic in character.

When the log of the mean range at positions along the East River as predicted in the tide tables is plotted against their Greenwich high water interval the points do not fall along a line similar to any of the lines calculated in theory for a strait. See Figure 10. The East River cannot be treated as a single strait, but may be analyzed if divided into three reaches each of which is considered separately.

East River—Lower Reach. The Lower Reach extends for 6.6 nautical miles from Governors Island to Hallets Point. The passage is deep, narrow and regular. Nomographic analysis indicates that the position of phase equality is virtual, in as much as such water does not actually exist, being east of Hallets Point where A = 1.97 ft, log η_H/η_{Ho} = log mean range −0.235, σt_H = (GHWI × 29) −118° when R = 2 and μ = 3. The position of amplitude equality is also virtual but there is a node near Williamsburg Bridge. The following values of kx are assigned to positions on the Lower Reach:

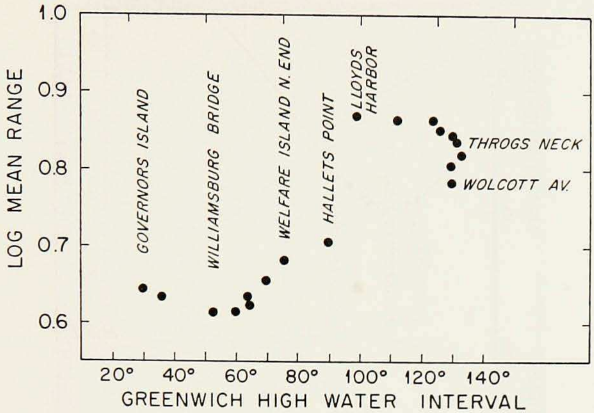

Figure 10. The log of the mean range at positions in the East River plotted against their Greenwich high water intervals according to the conventions used in constructing the nomograms for straits.

Governors Island	– 90°	East 41st Street, NYC	– 72°
Brooklyn Bridge	– 85°	37th Ave., Long Is. City	– 68°
Williamsburg Bridge	– 76°	Welfare Island, North End	– 63°
East 19th Street, NYC	– 75°	Hallets Point	– 53°
East 27th Street, NYC	– 74°		

The validity of the analysis is tested in Figure 11 in which the predicted mean ranges and the Greenwich high and slack water intervals, corrected for harmonics, are compared as a function of kx with lines calculated from equations (3), (4) and (5) when $R = 2$ and $\mu = 3$. In the cases of the mean range and the high water interval (HWI) the agreement is satisfactory. In the case of the slack water interval (SWI) slack water is predicted to occur about 1.5 hours earlier than is expected by theory. This indicates that hydraulic currents are present in the Lower Reach of the East River.

East River—Middle Reach. The Middle Reach extends for 2.2 nautical miles from Hallets Point to North Brother Island. The passage is relatively shallow and of irregular topography. Nomographic analysis indicates that the position of phase equality is virtual, being east of North Brother Island where $A = 1.61$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -0.985, $\sigma t_H = (\text{GHWI} \times 29) - 137°$ when $R = 2$ and $\mu = 4$. The position of amplitude equality is also virtual but a node occurs near Hallets Point. The following values are assigned to kx at positions on the Middle Reach.

Hallets Point	– 70°	Lawrence Point	– 47°
Walcott Avenue	– 49°	North Brother Island	– 44°

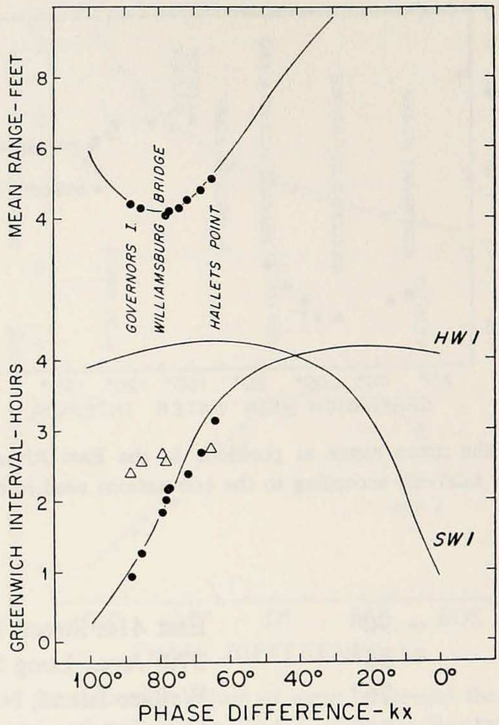

Figure 11. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=2$ and $\mu=3$ with their predicted value in the Lower Reach of the East River. For explanation see Figure 4.

The validity of the analysis is tested in Figure 12 in which the mean range and the predicted intervals of Greenwich high and slack water, corrected for harmonics, are compared as a function of kx with lines calculated from equations (3), (4) and (5) when $R = 2$ and $\mu = 4$. In the cases of the mean range and the high water interval (HWI) the agreement is good. In the case of the slack water interval (SWI) slack water is predicted to occur about 2 hours earlier than is expected by theory. This indicates that strong hydraulic currents dominate the flow in the Middle Reach of the East River. Because of the very strong currents which occur in the lower part of this reach the region is known as Hell Gate.

East River—Upper Reach. The Upper Reach extends for about 4.8 nautical miles from North Brother Island to Throgs Neck where it opens on the head of Long Island Sound. Nomographic analysis indicates that the position of phase equality is at Throgs Neck where $A = 1.167$ ft, log η_H/η_{Ho} = log mean range -0.845, σt_H = (GHWI $\times$ 29) $-130°$ when $R = 2$ and $\mu = 3.6$. There is no position of amplitude equality or node in this reach. The following values of kx are assigned to positions in the Upper Reach of the East River and beyond:

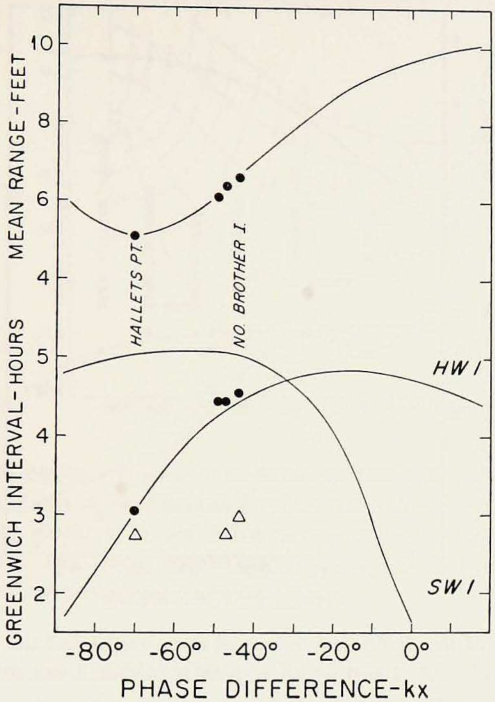

Figure 12. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=2$ and $\mu=4$ with their predicted values in the Middle Reach of the East River. For explanation see Figure 4.

North Brother Island	− 13°	Execution Rocks	+ 15°
Hunts Point	− 5°	Great Captain Harbor	+ 30°
Throgs Neck	− 0°	Lloyds Harbor entrance	+ 42°

The validity of the analysis is tested in Figure 13 in which the predicted mean range and the Greenwich intervals of high water and slack water, corrected for harmonics, are compared as a function of kx with lines calculated from equations (3), (4) and (5) when $R = 2$ and $\mu = 3.6$. In the cases of the mean range and high water interval (HWI) the agreement is excellent. In the case of the slack water interval (SWI) slack water occurs in the Upper Reach (at North Brother Island, Throgs Neck and between) at nearly the time indicated by the theoretical equation. There is no evidence from the nomographic analysis that hydraulic currents are significant in the Upper Reach of the East River. This is contrary to the statement of Marmer that implied that the tidal movement in the entire East River is hydraulic in character.

The Long Island Sound System. USC&GS Charts Nos. 1211, 1212, 1213. Long

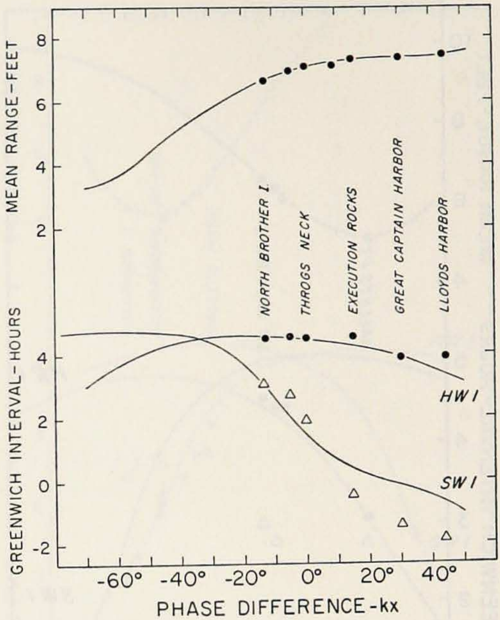

Figure 13. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when R=2 and μ=3.6 with their predicted values in the Upper Reach of the East River. For explanation see Figure 4.

Island Sound is actually a part of a strait separating Long Island from the mainland of New York and Connecticut and is so treated. Earlier investigators, however, have treated it as a closed embayment to which the contribution of the East River tide was negligible. They have placed the position of reflection where $R = 1$ near its western end.

Le Lachur and Sammons (1932) took Throgs Neck as the position of reflection and obtained as a crude approximation that the distance between that position and the Race represented one quarter of the wave length of a semidiurnal standing wave, having a node at the Race (quoted by Swanson (1971)). Swanson, using the Defant method of analysis, concluded that this was essentially correct. On the other hand Redfield (1950) placed the position of reflection between Glen Cove and Eatons Point and using a method of analysis based on the interference of an entering wave and its reflection considered that the node exists near Montauk Point where $kx = -88°$ and that at the Race (Little Gull Island) the phase relative to the position of reflection was $-79°$. The effect of shifting the assumed position of reflection from Throgs Neck to Glen Cove is to increase the value of kx by about $10°$ at each position.

Mathematically the results are indistinguishable whether the tide in Long Island Sound is treated as a reflected co-oscillation having some designated position of

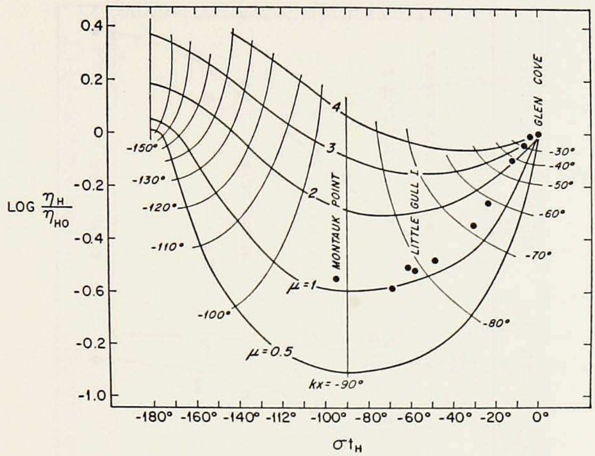

Figure 14. Nomogram for the special case of straits in which R=1, which also applies to refected co-ocillations. Lines show theoretical values when R=1 for μ and kx as σt_H varies. Solid circles show the predicted values of log η_H/η_{Ho} and σt_H at positions along the system and indicate that R=1 and μ=1.2 approximately in the Long Island Sound System when Glen Cove is taken as the position of phase equality.

reflection or as a part of a strait having an antinode at this position. Treatment as a part of a strait is preferred as taking better account of the geography and because it explains the continued decrease in the mean range east of the Race.

The Long Island Sound system has been re-examined using values for the reference stations given in Table 1 and correcting for harmonics. Nomographic analysis (Fig. 14) indicates that the position of phase equality is at Glen Cove where A = 1.825 ft, log η_H/η_{Ho} = log mean range −0.863, σt_H = (GHWI × 29) −119° when R = 1 and μ = 1.2. The position of amplitude equality is at the position of phase equality and a node is present near Montauk Point. The following values of kx are assigned to positions in the system:

Montauk Point	− 92°	Horton Point	− 61°
Montauk Harbor	− 88°	Herod Point	− 41°
Promised Land	− 82°	Stratford Shoals	− 28°
Little Gull Island	− 81°	Port Jefferson	− 28°
Three Mile Harbor	− 78°	Glen Cove	− 0°
Truman Beach	− 69°		

The validity of the analysis is tested in Figure 15 in which the predicted mean range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of kx with lines calculated from equations (3), (4) and (5) when R = 1 and μ = 1.2. In the cases of the mean range and the high water interval (HWI), the agreement is good. In the case of the slack water interval (SWI)

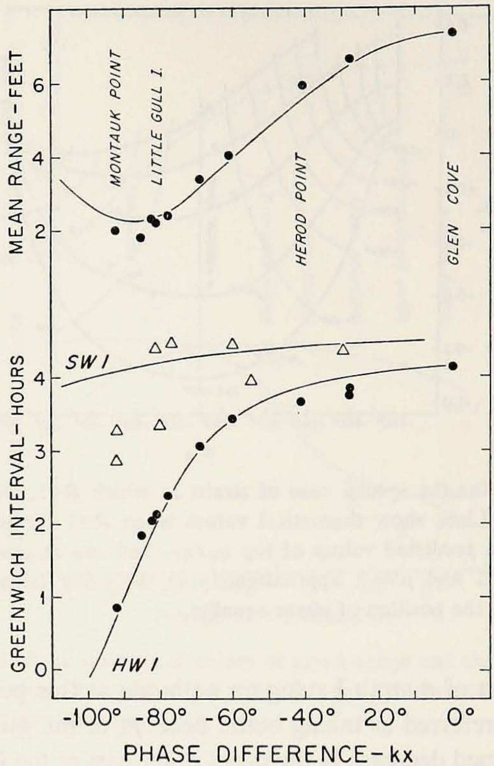

Figure 15. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=1$ and $\mu=1.2$ with their predicted values in the Long Island Sound System. For explanation see Figure 4.

the predictions within the sound scatter about the theoretical estimates by small amounts which may be attributed to errors. At the Race and near Montauk Point the predicted slack water is about one hour earlier than expected in theory indicating that local hydraulic effects may be produced by the local topography.

The Peconic Bay System. USC&GS Charts Nos. 363 or 1212. The Peconic Bay system is a tributary of the Long Island system about 13 nautical miles in length. It opens onto this system via Gardiners Bay by two narrow passages some 6 nautical miles in length on either side of Shelter Island.

Nomographic analysis indicates that when South Jamesport is taken as the position of reflection (and of phase and amplitude equality) $A = 0.675$ ft, $\log \eta_H/\eta_{N_0} = \log$ mean range -0.431, $\sigma t_H = (\text{GHWI} \times 29) - 189°$, when $R = 1$ and $\mu = 4$. There is no node present. The following values of kx are assigned to positions in the Peconic Bay system.

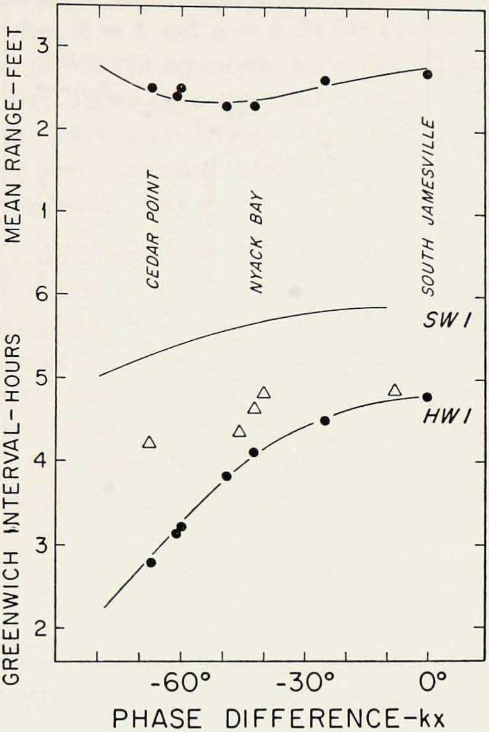

Figure 16. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=1$ and $\mu=4$ with their predicted values in the Peconic Bay system. For explanation see Figure 4.

Cedar Point	− 67°	Noyak Bay	− 42°
Greenport	− 61°	New Suffolk	− 25°
Sag Harbor	− 60°	South Jamesport	− 0°
Southold	− 49°		

The validity of the analysis is tested in Figure 16 in which the predicted mean range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of *kx* with lines calculated from the theoretical equations (3), (4) and (5) when $R = 1$ and $\mu = 4$. In the cases of the mean range and the high water interval (HWI) the agreement is excellent. In the case of the Slack Water interval (SWI) the predicted slack water occurs about one hour earlier than expected in theory. This indicates that significant hydraulic currents are present in the narrow passages on either side of Shelter Island which also influences the time of slack water in the large bays which they feed.

Jamaica Bay. USC&GS Chart No. 542. Jamaica Bay occupies an indentation of

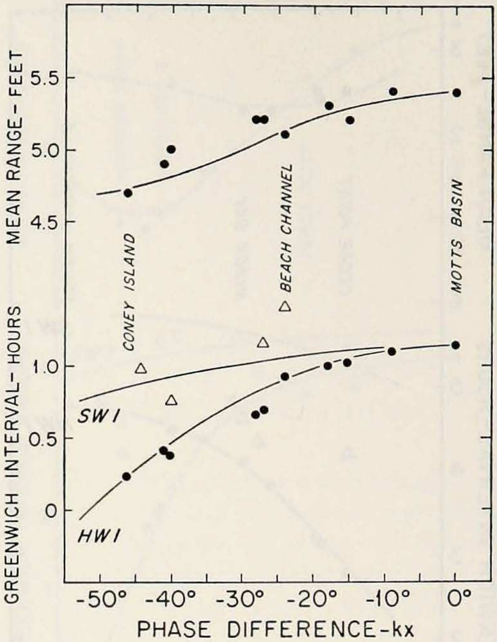

Figure 17. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=1$ and $\mu=4$ with their predicted values in Jamaica Bay. For explanation see Figure 4.

the southwest coast of Long Island which is separated from the ocean by Rockaway Beach, a barrier beach. It connects with New York Lower Bay at the southwest. The center of the bay is occupied by salt marsh and other islands separated by minor channels. The principal access of the tide to the inner parts of the bay is by larger channels along its margin.

Nomographic analysis indicates that when the tide at Motts Basin is taken to represent that at the position of reflection (and of phase equality and amplitude equality) $A = 1.35$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -0.732, $\sigma t_H = (\text{GHWI} \times 29) -33.1°$ when $R = 1$ and $\mu = 4$. There is no node in Jamaica Bay. The following values of kx are assigned to positions at which the mean range is predicted.

Coney Island	− 46°	Beach Channel	− 24°
Plum Beach Channel	− 41°	JFK International Airport	− 18°
Barren Island	− 40°	Grassy Bay	− 15°
Mills Basin	− 23°	Norton Point	− 9°
Canarsie	− 27°	Motts Basin	− 0°

The validity of the analysis is tested in Figure 17 in which the predicted mean range and Greenwich high water and slack water intervals, corrected for harmonics, are compared as a function of kx, with lines calculated from the theoretical equations (3), (4) and (5) when $R = 1$ and $\mu = 4$. In the cases of the mean range and the high water interval (HWI) the agreement is satisfactory when it is considered how greatly Jamaica Bay differs from the ideal channel on which the theory is based. In the case of the slack water interval (SWI) the predicted intervals scatter about the line indicating their theoretical values. There is no evidence that hydraulic currents are significant in Jamaica Bay or its approaches.

Great South Bay. USC&GS Charts Nos. 1214 and 1215 or 120C. Great South Bay is a shallow, open embayment about 21 nautical miles in length lying between the south shore of Long Island and Fire Island, a barrier island. It opens onto the continental shelf at its western end through Fire Island Inlet. Minor connections with the ocean occur through Jones Inlet via Hempstead Bay and through Mariches Inlet via Mariches Bay which are neglected in the present analysis.

The tidal current in Fire Island Inlet, which has an average maximum velocity of 2.4 knots, is undoubtedly hydraulic. Between Fire Island Breakwater and Democrat Point the mean range falls from 4.1 to 2.6 ft while there is no change in the high water interval. East of Democrat Point there appears to be a region of transition in which the current due to the entering wave becomes stronger relative to the hydraulic effect, as evidenced by the increasing high water interval, until the inlet opens onto the bay proper at Fire Island Light where the mean range is 0.7 ft.

The tide in Great South Bay results from the inflow through Fire Island Inlet as occurring at Fire Island Light and its reflection from the head of the bay at Ballport and other parts of the Long Island shore. Since the mean range within the bay varies only between 0.6 and 0.8 ft and is predicted to only 0.1 ft great precision cannot be expected in the analysis.

The nomographic analysis indicates that when the tide at Bellport is taken to represent the position of reflection (and of phase equality and amplitude equality) $A = 0.2$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -0.0969, $\sigma t_H =$ (GHWI $\times$ 29) $-128°$, when $R = 1$ and $\mu = 3$. The following values of kx are assigned to positions within Great South Bay:

Fire Island Light	– 89°	Point O'Woods	– 57°
West Fire Island	– 68°	Sayville	– 44°
Oak Beach	– 65°	Patchogue	– 35°
Babylon	– 62°	Bellport	– 0°
Bay Shore	– 60°		

The validity of the analysis is tested in Figure 18 in which the predicted values of mean range and the interval of high water, corrected for harmonics, are compared as a function of kx with lines calculated from the theoretical equations (3) and (5) when $R = 1$ and $\mu = 3$. The agreement is satisfactory when the limited precision of the analysis is considered.

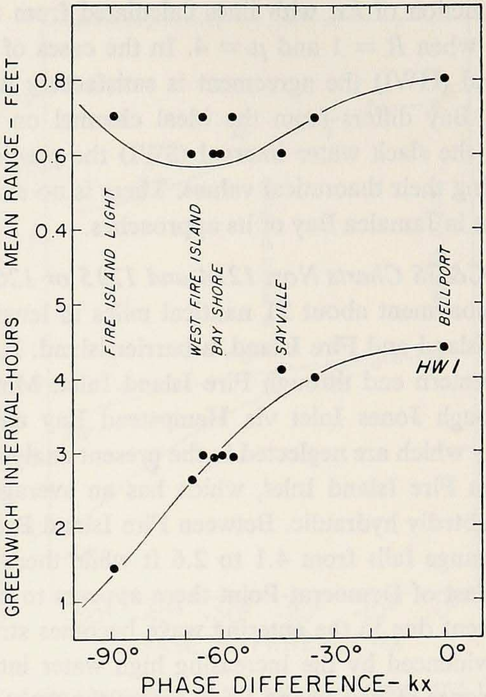

Figure 18. Comparison of the theoretical values of mean range and the Greenwich interval of high water when $R=1$ and $\mu=3$ and their predicted values at positions within Great South Bay. For explanation see Figure 4.

The current tables give no information on the currents in Great South Bay.

Buzzards Bay. USC&GS Chart No. 249. Buzzards Bay lies on the southeastern shore of Massachusetts between it and Cape Cod and the Elizabeth Islands, being about 20 nautical miles in length. It opens onto the continental shelf at Cuttyhunk Island near Penikese Island. The northwest shore is broken by rather large embayments at New Bedford, Mattapoisett and Marion. At the head of the bay reflection is interrupted by openings into the Wareham River and Cape Cod Canal making the position of reflection difficult to determine.

Nomographic analysis indicates that when the tide at Great Hill is taken to represent the position of reflection (and of phase equality and amplitude equality) $A = 1.025$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -0.612 ft, $\sigma t_H =$ (GHWI $\times$ 29) $-6.7°$, when $R = 1$ and $\mu = 3$. There is no node within Buzzards Bay. The following values of kx are assigned to positions within the bay:[7]

7. No estimates of kx are given for West Falmouth Harbor and Bird Island. High water is predicted at the former to occur later than at the position of reflection. The mean range is predicted at the latter to be slightly greater than at the position of reflection. Both are theoretically impossible when $R = 1$.

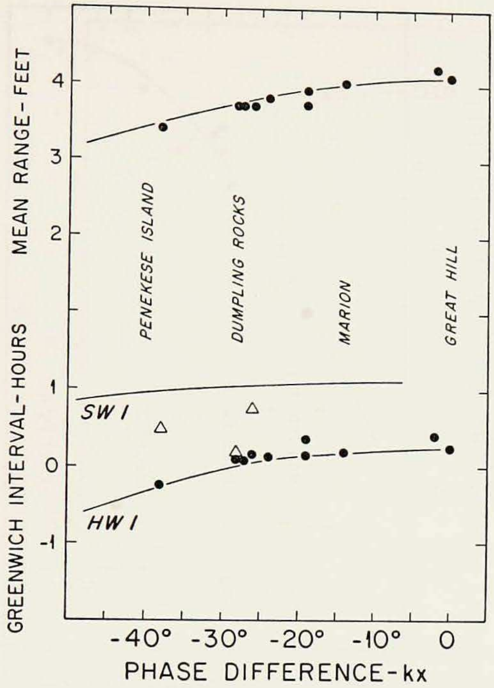

Figure 19. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=1$ and $\mu=3$ with their predicted values in Buzzards Bay. For explanation see Figure 4.

Penikese Island	− 38°	Barlows Landing	− 19°
Dumpling Rocks	− 28°	Mattapoisett	− 19°
Clarks Point	− 27°	Marion	− 14°
West Island	− 26°	Great Hill	− 0°
Kettle Cove	− 24°		

The validity of the analysis is tested in Figure 19 in which the predicted mean ranges and the Greenwich high water and slack water intervals, corrected for harmonics, are compared as a function of kx with lines calculated from the theoretical equations (3), (4) and (5) when $R = 1$ and $\mu = 3$. In the cases of the mean range and high water interval (HWI) the agreement is good. In the case of the slack water interval slack water is predicted to occur about one-half hour earlier than is expected in theory. This suggests that small hydraulic currents may be present in Buzzards Bay although the entrance does not appear to be sufficiently constricted to produce a significant effect of this sort.

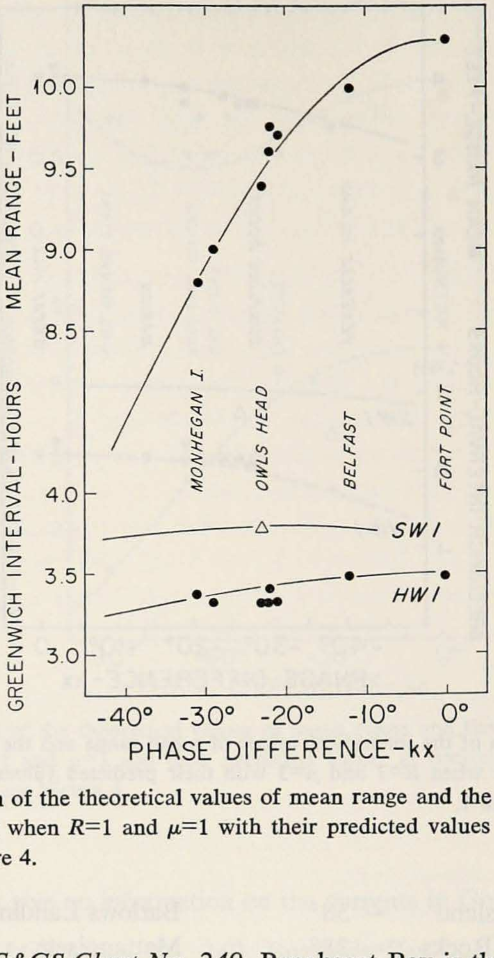

Figure 20. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when $R=1$ and $\mu=1$ with their predicted values in Penobscot Bay. For explanation see Figure 4.

Penobscot Bay. USC&GS Chart No. 249. Penobscot Bay is the largest of the bays which indent the coast of Maine, extending from its head at Fort Point to the outlying island of Monhegan for a distance of 47 nautical miles. Its form is irregular, being interrupted by the Penobscot River at Fort Point, by Eggamoggin Reach and Fox Island Thorofare on the east and by open waters between Vinalhaven and Matinicus Islands. Long Island occupies much of the central part of the bay. Like other bays on the Maine coast Penobscot Bay is relatively deep compared with the bays of the southern coast of New England, being 100 ft deep near its head and more than 350 ft deep in places.

Nomographic analysis indicates that if the tide at Fort Point is taken to represent the position of reflection (and of phase equality and amplitude equality) $A = 2.575$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -1.01, $\sigma t_H = (\text{GHWI} \times 29) - 101°$, when $R = 1$ and $\mu = 1$. There is no node within Penobscot Bay. The following values of kx are assigned to positions within the bay and its approaches:

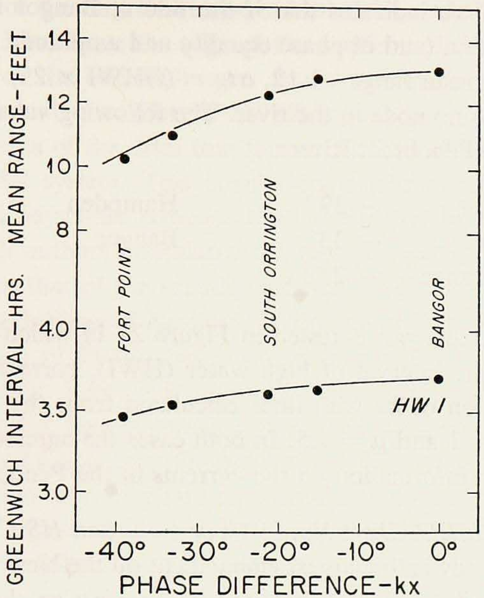

Figure 21. Comparison of the theoretical values of mean range and the Greenwich interval of high water when $R=1$ and $\mu=1.5$ with their predicted values in Penobscot River. For explanation see Figure 4.

Monhegan Island	− 31°	Camden	− 22°
Matinicus Island	− 29°	Rockland	− 21°
Owls Head	− 23°	Belfast	− 12°
Pulpit Harbor	− 22°	Fort Point	− 0°

The validity of the analysis is tested in Figure 20 in which the predicted mean range and the Greenwich intervals of high and slack water, corrected for harmonics, are compared as a function of kx with lines calculated from the theoretical equations (3), (4) and (5) when $R = 1$ and $\mu = 1$. In the cases of the mean range and the high water interval (HWI), the agreement is satisfactory. This is also true of the single determination of the slack water interval (SWI) at Owls Head. It indicates that hydraulic currents do not occur there.

Penobscot River. USC&GS Chart No. 311. The Penobscot River is blocked by a dam at Bangor, below which it extends for 22 nautical miles to enter Penobscot Bay at Fort Point. The dam checks and reflects the advancing tide so that in its lower reach the tide in the Penobscot River is in reflected co-oscillation with the tide at the head of Penobscot Bay. The river is relatively deep, the depths increasing from 15 ft at Bangor to 70 or 80 ft at Bucksport. It is uncomplicated by branching except below Bucksport where Verona Island divides the channel.

Nomographic analysis indicates that if the tide at Bangor is taken to represent the position of reflection (and of phase equality and amplitude equality) $A = 3.275$ ft, $\log \eta_H/\eta_{Ho} =$ log mean range -1.12, $\sigma t_H =$ (GHWI $\times$ 29) $-104°$, when $R = 1$ and $\mu = 1.5$. There is no node in the river. The following values of kx are assigned to positions along the Penobscot River:

Fort Point	− 39°	Hampden	− 15°
Bucksport	− 33°	Bangor	− 0°
South Orange	− 21°		

The validity of the analysis is tested in Figure 21 in which the predicted values of mean range and the interval of high water (HWI), corrected for harmonics, is compared as a function of kx with lines calculated from the theoretical equations (3) and (5) when $R = 1$ and $\mu = 1.5$. In both cases the agreement is excellent. The current tables give no information on the currents in the Penobscot River.

The Bay of Fundy. USHO Chart No. 0940 or Canadian HS Charts Nos. 4010 and 4016. The Bay of Fundy is the largest embayment on the New England coast, being about 120 nautical miles in length between its opening on the Gulf of Maine between Brier Island and Grand Manan and the head of the Chegnects Channel. Near its head the bay is divided into two passages, the Chegnects Channel which continues as the Petitcodiac River and that leading to the Minas Basin which joins the bay east of Ile Haute, and is here treated as a tributary to the bay.

The great range of tide in the Bay of Fundy has been explained by its shoaling and narrowing toward its head and by a close correspondence between the natural period of oscillation of the bay to one quarter of the period of the lunar semidiurnal tide, a condition in which resonance may be expected to increase the tidal amplitude (Marmer, 1926). Redfield (1950) pointed out that the tide in Long Island Sound, in which the maximum range is 7.4 ft, and that in the Bay of Fundy, in which the maximum range is about 34 feet, are augmented about equally in their passage up the embayments. The form of the basin has little effect on the augmentation except as it determines the differences in phase along the passage. Using a method of analysis similar to that employed in this paper Redfield concluded that taking the tide at Hopewell Cape in the Chegnects Channel to represent that at the position of reflection the phase difference at the opening on the Gulf of Maine at Grand Manan was about $-62°$ and that the position where the natural period of oscillation equaled one quarter of the period of the semidiurnal tide lay somewhere beyond Lower East Pubnies. The analysis indicated that $R = 1$ and $\mu = 1$.

Interest in the exploitation of the energy of the tide in the Bay of Fundy has led to a number of studies employing the method of analysis of Defant to determine the effect of barriers installed near the head of the bay (McLellan, 1958; Rao, 1968; Yuen, 1969; Greenberg, 1969; Godin, 1969; Duff, 1970). Duff concluded that the tidal system of the bay should be extended beyond the mouth of the bay to a position about 15 km from the edge of the continental shelf where the natural period of oscillation of the enclosed water would be equal to one quarter of the period of the semidiurnal tide. Garrett (1972) concluded from the ratios of the semi-diurnal constituents of the tides that the whole Gulf of Maine and the Bay of Fundy is a single tidal system. This conclusion is confirmed by a mathematical model, based on hydrodynamic considerations calculated by Greenberg (personal communication). Both authors considered the resonant period of the system to be somewhat greater than that of the semidiurnal tide. Their conclusion is not incompatible with the earlier findings of Redfield (1950) and of Duff (1970).

The tide in the Bay of Fundy has been re-examined using the values of the Greenwich intervals at St. John, N.B. given in Table 1 and correcting the high and slack water intervals for the asymmetry attributed to harmonics. Nomographic analysis when Hopewell Cape, near the head of the Chegnacts Channel, is taken as the position of reflection (and of phase and amplitude equality) indicates that $A =$ 8.3 ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -1.52, $\sigma t_H =$ (GHWI $\times$ 29) $-110.5°$ when $R = 1$ and $\mu = 1$. The following values of kx are assigned to positions within the passage:

Lower East Pubnico	− 85°	St. John	− 53°
Yarmouth Harbor	− 81°	Digby Pier, Annapolis Basin	− 53°
Weymouth, St. Mary Bay	− 70°	Quaco	− 46°
Outer Wood Island	− 69°	Port George	− 44°
Cutler, Little River	− 66°	Ile Haute	− 42°
North Head, Grand Manan	− 60°	Spicer Cove, Chegnects Bay	− 34°
Ile Etang Harbor	− 59°	Herring Cove	− 30°
Eastport	− 59°	Joggins Wharf	− 10°
Lepreau Bay	− 58°	Hopewell Cape	− 0°

The validity of the analysis is tested in Figure 22 in which the predicted mean range and high and slack water intervals, corrected for harmonics, are compared as a function of kx with lines calculated from the theoretical equations (3), (4) and (5) when $R = 1$ and $\mu = 1$. In the case of the mean range the agreement is excellent except for Yarmouth Harbor and Weymouth where rotation of the earth may increase the range and for Port George and Ile Haute where the tide in the Minas Basin may also increase the range. In the case of the high water interval, HW1, the agreement is satisfactory. In the case of the slack water interval, SW1, in three positions the interval predicted is close to that indicated by theory. Although at two positions near the mouth of the bay slack water is predicted to occur earlier than

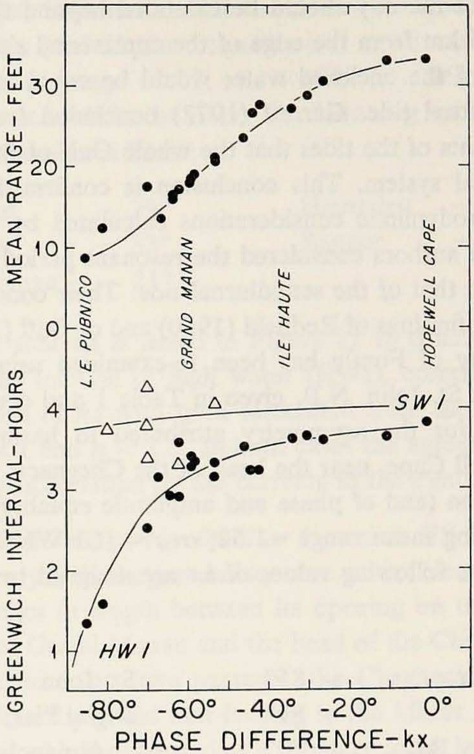

Figure 22. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when R=1 and μ=1 with their predicted values in the Bay of Fundy. For explanation see Figure 4.

indicated by theory the variation in the predictions and the lack of obvious constriction of the topography does not allow one to conclude that hydraulic effects are significant in the Bay of Fundy.

Minas Basin. US HO Chart No. 0940 or Canadian HS Chart No. 4010. The Minas Basin is a shallow tributary of the Bay of Fundy with which it is connected east of Ile Haute by a deeper channel. The head of the basin at Burntcoat Head is about 50 nautical miles from Ile Haute. The mean range of tide in the Minas Basin is not only the greatest in the Bay of Fundy system but in the entire world, being 38.4 ft at Burntcoat Head.

Nomographic analysis when the tide at Burntcoat Head is taken to represent the position of reflection (and of phase and amplitude equality) indicates that $A = 8.35$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -1.58, $\sigma t_H =$ (GHWI $\times$ 29) $-1.32°$, when $R =$ 1 and $\mu = 3$. There is no node in the Minas Basin. The following values of kx are assigned to positions where measurements are available:

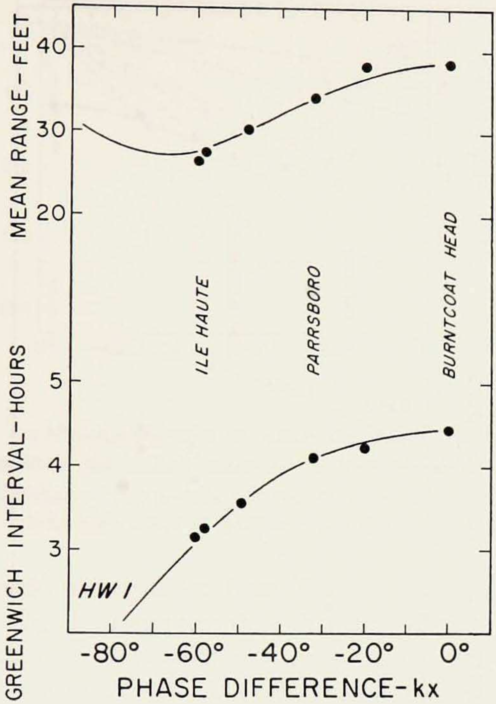

Figure 23. Comparison of the theoretical values of mean range and the Greenwich interval of high water when $R=1$ and $\mu=3$ with predicted values in Minas Basin. For explanation see Figure 4.

Port George	− 60°	Parrsboro	− 32°
Ile Haute	− 58°	Horton Bluff, Avon River	− 20°
Sencer Island	− 49°	Burntcoat Head	− 0°

The validity of the analysis is tested in Figure 23 in which the predicted mean range and the Greenwich interval of high water, corrected for harmonics, are compared as a function of kx with lines calculated from the theoretical equations (3) and (5) when $R = 1$ and $\mu = 3$. In both cases the agreement is good. The current tables give no information on the currents in Minas Basin.

The Hudson River. USC&GS Charts Nos. 745 to 748 and 382 to 384. The Hudson River is tidal from the Battery where it opens on New York Upper Bay to Troy where the advance of the tide is blocked by a dam, a distance of about 122 nautical miles. The crest of high water requires about 9.27 hours, about three quarters of the period of the semidiurnal tide, to traverse this distance. The predicted mean range is 4.5 ft at the Battery, falls to a minimum of 2.7 ft at West Point, then increases to 4.1 ft at Catskill. Above Catskill it decreases to a second minimum of

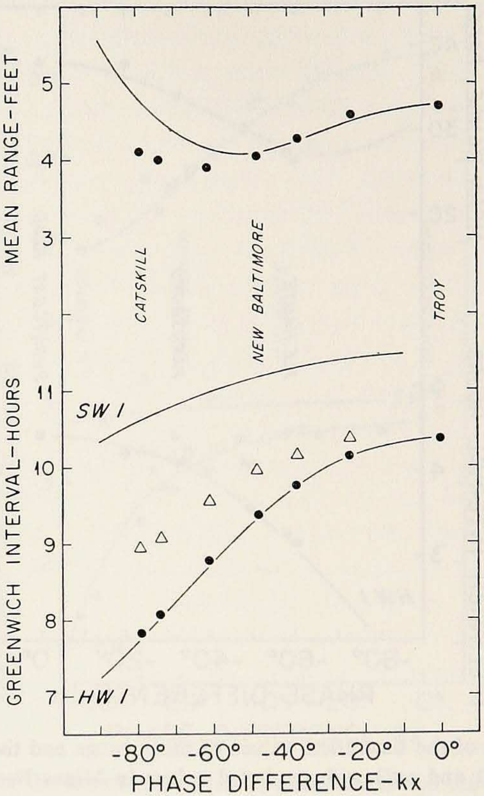

Figure 24. Comparison of the theoretical values of mean range and the Greenwich interval of high water and slack water when $R=1$ and $\mu=4$ with the predicted values in the Upper Reach of the Hudson River. For explanation see Figure 4.

3.9 ft at Coxaxies, then rises to a maximum of 4.7 ft at Troy. The range at Troy is actually greater than that at the mouth of the river, which may be accounted for by the conversion of the kinetic energy due to the current of the entering tide to potential energy when its movement is checked by the dam at Troy.

It has not been possible to treat the river as a whole as a single tidal system. However by dividing it into sections which are considered separately it is possible to make a theoretical analysis of the Upper Reach in which the tide behaves as a reflected co-oscillation for about 17 miles below the dam at Troy and of the Lower Reach in which it behaves as a single progressive wave, unmodified significantly by any reflected waves from upstream, for about 28 nautical miles above the river mouth at the Battery. Between these two sections for a distance of about 77 nautical miles is a region of transition, the Middle Reach, for which it has not been possible to find a satisfactory theoretical explanation.

The Upper Reach of the Hudson River. The tide in the Upper Reach may be con-

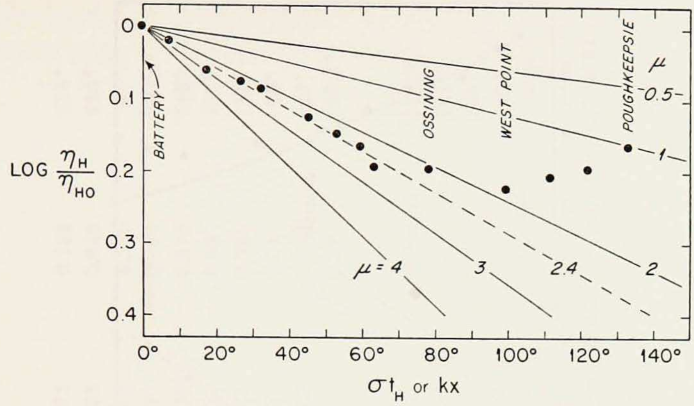

Figure 25. Nomogram for the special case in which the tide is due to a single progressive wave and R=0. Lines show theoretical values when R=0 for μ or kx as σ_{tH} varies. Solid circles show predicted values of log η_H/η_{H_0} and σ_{tH} or kx at positions along the Lower Reach of the Hudson River, taking high water at the Battery as the origin of time and phase.

sidered to be a reflected co-oscillation from Troy to New Baltimore below which the predicted mean range becomes less than that indicated by theory. Its length is about 17 nautical miles. Nomographic analysis when the tide at Troy is taken to represent that at the position of reflection (and the position of phase and amplitude equality) indicates that $A = 1.175$ ft, log $\eta_H/\eta_{H_0} =$ log mean range -0.672, $\sigma t_H =$ (GHWI $\times$ 29) $-300°$ when $R = 1$ and $\mu = 4$. There is no node in this reach. The following values of kx are assigned to positions in the Upper Reach:

New Baltimore	− 48°	Albany	− 23°
Castleton on Hudson	− 37°	Troy	− 0°

The validity of the analysis is tested in Figure 24 in which the predicted mean range and the Greenwich intervals of high and slack water corrected for harmonics are compared as a function of kx with lines calculated from the theoretical equations (3), (4) and (5) when $R = 1$ and $\mu = 4$. In the cases of the mean range and the high water interval (HWI) the agreement at New Baltimore and above is excellent. In the case of the slack water interval (SWI) slack water is predicted to occur about one hour earlier than expected in theory. This may be attributed to the river flow, slack water occurring when the current due to the advancing tidal wave is equal and opposite to the river flow, not when it has fallen to zero.

The Lower Reach of the Hudson River. The tide in the Lower Reach may be considered to be due to a single progressive wave entering the river at the Battery uninfluenced by reflection from upstream for about 28 nautical miles. It is of interest in showing that one of the two waves assumed by the theory of the tide in straits and bays actually exists in nature.

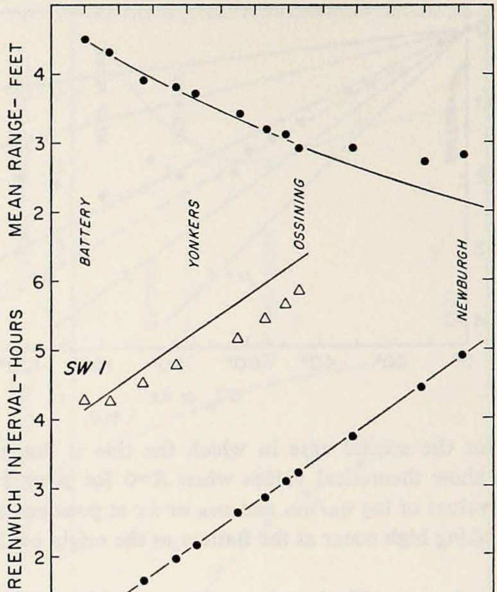

Figure 26. Comparison of the theoretical values of mean range and the Greenwich intervals of high and slack water when R=0 and μ=2.4 with the predicted values in the Lower Reach of the Hudson River. For explanation see Figure 4.

Nomographic analysis, Figure 25, when the tide at the Battery is taken as the origin of time and distance indicates that $A = 2.25$ ft, $\log \eta_H/\eta_{Ho} = \log$ mean range -0.635, $\sigma t_H =$ (GHWI $\times$ 29) $-30.5°$ when $R = 0$ and $\mu = 2.4$. Above Ossining the predicted values depart from the theoretical line for $\mu = 2.4$ indicating that the tide is no longer uninfluenced by waves from upstream. The following values of kx are assigned to positions on the Lower Reach:

Battery	0°	Yonkers	32.2°
Chelsea Docks	7.8°	Dobbs Ferry	45.8°
George Washington Bridge	17.4°	Tarrytown	52.8°
Spuyten Duyvil	26.7°	Ossining	59.5°

The validity of the analysis is tested in Figure 26 in which the predicted values of the mean range and the intervals of high and slack water, corrected for harmonics, are compared as a function of kx with lines calculated from the theoretical equation 1 when $\mu = 2.4$. In the cases of the mean range and the high water interval (HWI) the agreement is excellent between the Battery and Ossining. In the case of the slack water interval (SWI) the agreement is good at the Battery but slack

Table 2. Characteristics of Individual Passages.

Passage	R	μ	Position of Phase Equality	Mean range, ft	A ft	Log mean range $-\log \eta_H/\eta_{H_0}$	(GHWI×29) $-dt_H$
Straits							
Nantucket-Vineyard Sound	2.2	3.0	Monomoy Pt.	3.7	0.578	0.568	126°
East River—Upper Reach	2	3.6	Throgs Neck	7.0	0.875	0.845	130°
Quicks Hole	2	2	In Buzzards Bay	3.6	0.60†	0.556	−5°
Arthur Kill	2	3	Carteret	5.1	0.85	0.714	24.6°
East River—Lower Reach	2	3	East of Hallets Pt	11.8	1.97†	0.235	118°
Cape Cod Canal	2	3	In Cape Cod Bay	9.5	1.58†	0.98	105°
East River—Middle Reach	2	4	East of North Brother I	9.65	1.61†	0.985	137°
Harlem River	2	5	East 110th St	5.1	0.85	0.63	80°
Long Island Sound System	1	1.2	Glen Cove	7.3	1.825	0.863	119°
Hempstead Bay	0.2	7	Amityville	1.2	0.5	0.79	90.5°
Embayments							
Bay of Fundy	1	1	Hopewell Cape	33.2	8.3	1.52	110.5°
Penobscot Bay	1	1	Fort Point	10.3	2.575	1.01	101°
Penobscot River	1	1.5	Bangor	13.1	3.275	1.12	104°
Minas Basin	1	3	Burntcoat Harbor	33.4	8.35	1.58	132°
Buzzards Bay	1	3	Great Hill	4.1	1.025	0.612	6.7°
Great South Bay	1	3	Bellport	0.8	0.2	0.097	128°
Peconic Bay System	1	4	South Jamesport	2.7	0.675	0.431	139°
Jamaica Bay	1	4	Motts Basin	5.4	1.35	0.732	33.1°
Hudson River—Upper Reach	1	4	Troy	4.7	1.175	0.672	300°
Single Progressive Wave							
Hudson River—Lower Reach	0	2.4	Battery*	4.5	2.25	0.635	30.5°

† Position of phase equality is virtual.
* There is no position of phase equality when $R = 0$. The Battery is taken as the origin of distance, time, and phase.

water is predicted to occur progressively earlier than indicated by theory as the river is ascended. This may be attributed to the relative strengths of the tidal current and the river flow in this part of the river.

5. Discussion

The characteristics assigned by nomographic analysis to the individual passages are collected in Table 2, re-arranged for comparison and discussion. The matching of the predicted values to the theoretical curves for various values of R and μ and the designation of the position of phase equality is a subjective matter but when the values obtained in the 20 cases considered are introduced into the equations, which are developed on the theory that the observed tide depends on the interference of waves moving in opposite directions along the passage, the mean range and the high water interval at the various positions agree with those predicted about as precisely as the predictions are given in the tide tables. It may be noted that although the comparison is made between the Greenwich high water intervals corrected for harmonics and the theoretical values, if the correction were applied to the theoretical values and compared with the uncorrected values given in the tide tables the agreement would be the same.

In a few cases the nomographic analysis has not given satisfactory results. These are where the complexity of the topography is great, as in Narragansett Bay, for this reason and for insufficient data, as in Woods Hole, and where there is a region of transition as in the Middle Reach of the Hudson River. Also it may be noted from Figure 1 that the lines indicating the values of μ converge toward the position of phase equality so that if kx is less than about 30° they are not separated sufficiently to give reliable values of μ in view of possible small errors in the prediction of the time of high water. In consideration of these limitations it may be concluded that if the topography is not too complex or is not a region of transition and if the channel is sufficiently long so that at one end the phase differs from that at the position of phase equality by at least 30° the method of nomographic analysis gives values of R, μ, and kx which when introduced into the theoretical equations reproduce closely the predicted mean ranges and high water intervals.

The values of R in the cases of reflected co-oscillations and when a single wave is responsible for the tide, in which $R = 1$ and $R = 0$ respectively, are fixed by definition. In the case of straits they are variable, $R = 2$ and $R = 0.2$ being the extremes observed. They depend not only on the relative amplitude of the waves entering at opposite ends of the strait, but also on the attenuation of these waves within the passage before reaching the position of phase quality.

The values of μ vary between 1.2 and 7 in the case of straits and in the other cases fall between these values. The value of μ as here defined depends not only on the attenuation due to friction of water moving over the bottom but also on losses of energy due to reflection from irregularities of the shore line which reduce the amplitude of the waves as they move along the passage. While μ is small in the larger passages such as Long Island Sound and the Bay of Fundy, it is also small in the Penobscot Bay and River. Apparently its value depends on the ratio of the wetted surface, where friction and reflection occur, to the area of cross section of the passage, which is the seat of the energy of the wave. Because of the difficulty of measuring these parameters in irregular channels this relation has not been examined quantitatively.

A single value of μ is attributed to each passage. This appears to be unrealistic since the depth varies along each and in closed embayments usually decreases as the position of reflection is approached. It results from the definition of μ and its use in equations (1) and (2) as a function of the phase changes along the passage. In selecting the line characteristic of the value of μ for each passage weight has been given to these positions in which kx is greater than 30°. The validity of this procedure has been shown by introducing the values of R and μ so obtained into the theoretical equations and has led to a satisfactory agreement between theory and predictions. The values of R and μ given in Table 2 yield values of the mean range and time of high water which are at least *empirically* correct within the precision with which the predictions are made.

A striking feature of this coast is that the range of tide is greater at positions north of Cape Cod than it is on the southern coast of New England. This is explained by the consideration that in crossing the continental shelf the tide behaves as a co-oscillation reflected from the outer coast and consequently is greater the more distant the coast is from the deep water of the ocean (Redfield, 1958). North of Cape Cod the ocean tide must traverse the Gulf of Maine and consequently is greater. The value of A which is the elevation at high water of the wave moving in the x direction when it reaches the position of phase equality reflects this effect but is also modified in moving from the entrance to that position. In those passages which open in the Gulf of Maine (the Bay of Fundy including the Minas Basin and the Penobscot River and Bay) A is greater than 2.5 ft. In those passages which open on the south coast A is less than this.

Hydraulic currents are suggested when the slack water interval is less than indicated by theory. This is clearly the case in both the Lower and Middle Reach of the East River, the channels leading to Peconic Bay, and in the Cape Cod Canal, the Harlem River, the Arthur Kill, and Quicks Hole. The latter group are all constricted passages which separate bodies of water in which the range, high water interval, or both differ. The river flow also causes slack water to occur earlier than calculated in the case of the Hudson River.

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