Journal of
MARINE RESEARCH

Volume 35, Number 1

Does the Sverdrup Relation account for the Mid-Atlantic circulation?

by Ants Leetmaa,[1] Pearn Niiler,[2] and Henry Stommel[3]

ABSTRACT

A comparison is made between the geostrophic transport across selected latitudes in the North Atlantic Ocean and the theoretical transport as derived from the Sverdrup relationship. In the subtropical gyre these agree to within the error estimates. At 32°N and 24°N the interior geostrophic transports compare well with the measured values in the Florida Straits off Jacksonville.

1. Geostrophic shear on Mid-Atlantic hydrographic sections

Estimates of geostrophic shear across selected latitudes of the North Atlantic Ocean are compared to theoretical transports based on the Sverdrup relationship and the new wind-stress tabulations of Bunker (Leetmaa and Bunker, 1976). The sections chosen are shown in Fig. 1 superposed upon a chart of dynamic topography (1500 db-100 db) in dynamic millimeters (Stommel, personal communication). The sections are also listed in Table 1; both temperature and salinity profiles have been published in the atlases of Fuglister (1960) and Worthington and Wright (1970). These sections cross both the subtropical and subpolar gyres, but avoid the complex region near 40°N where the Gulf Stream System extends out into mid-ocean in a series of loops and eddies, and the existing sections seem to be dominated by what are probably transient features of large amplitude.

1. Atlantic Oceanographic and Meteorological Laboratories, NOAA, 15 Rickenbacker Causeway, Miami, Florida, 33149, U.S.A.

2. Oregon State University, School of Oceanography, Corvallis, Oregon, 97331, U.S.A.

3. Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139, U.S.A.

Table 1. Sections used for geostrophic transports.

Lat	Ship	Date	Reference Stations	Longitude W	Separation (km)
59°N	*Erika*	March-	360	41°.55	1745
	Dan	April 1962	388	11°.03	
53°.5N	*Erika*	Feb.	188	15°.07	2351
	Dan	1962	221	50°.58	
32°N	*Atlantis*	Nov. 1954	5203	63°.05	4674
	Discovery II	Nov.-Dec. 1957	3646	13°.40	
24°N	*Discovery II*	Oct.	3593	20°.83	4817
		1957	3619	68°.33	
16°N	*Crawford*	Nov.	283	25°.68	3629
		1957	308	59°.70	
8°N	*Crawford*	May	—		
		1957	—		

Some degree of variability appears, of course, on all the sections as can be seen in Fig. 2 where the same relative dynamic topography shown in Fig. 1 is shown in greater detail, station by station. The observed points do not fall exactly upon a smooth line or curve, the deviations being due probably to the presence of low frequency eddies. Estimates of the total mean geostrophic shear across these sections therefore require some smoothing of the small scale features. We deem it inappropriate to sum up transports calculated between all individual stations in the manner of Wust (1957), because it puts too much weight on the stations at the extreme ends of the sections, where boundary phenomena may intervene. If the section is at a constant latitude, the total transport, of course, depends on the end stations. Instead we put more weight on mid-oceanic stations as follows.

2. The subtropical sections

Consider the data for 24°N displayed in Fig. 2. The observed points seem to lie close to a straight line, so that the slope of the dynamic topography (1500 db-100 db) is almost independent of longitude. Instead of choosing stations at the far ends of the section (which quite obviously would determine a slope unrepresentative of the mid-ocean regions), we choose two stations that define a line that visually fits the rest of the points better. These choices are shown in Fig. 2 by the larger circles: we call these "reference stations." This choice is subjective, and we will later introduce a more objective method as well. We first use these reference stations to compute the vertical distribution of geostrophic transport between the reference stations (for a discussion of geostrophic computations, see Sverdrup *et al.*, 1942). The ob-

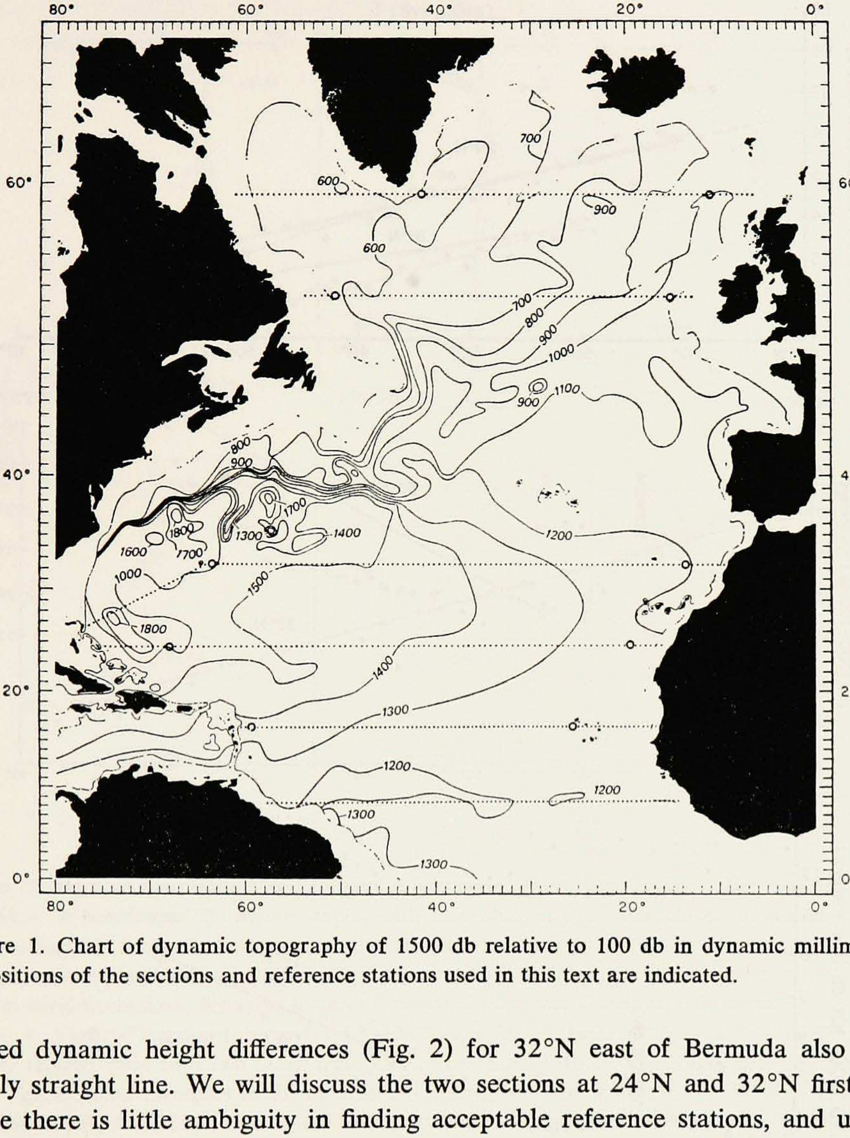

Figure 1. Chart of dynamic topography of 1500 db relative to 100 db in dynamic millimeters. Positions of the sections and reference stations used in this text are indicated.

served dynamic height differences (Fig. 2) for 32°N east of Bermuda also fit a nearly straight line. We will discuss the two sections at 24°N and 32°N first, because there is little ambiguity in finding acceptable reference stations, and uncertainties in the slope of the line thereby defined can hardly exceed 20% of what the true value is.

An example of a bad choice of reference stations is that made by Stommel (1956), before the *Discovery* stations to the east had been made. He chose in addition to *Atlantis* station 5203 (the present reference station near Bermuda), station 5210 (the low point at 50°41′W). Because this latter station now appears to be so unrepresentative of the points on the extended *Discovery* section, it is clear that Stommel over-estimated the slope by a factor of about 1.7, thus yielding an exces-

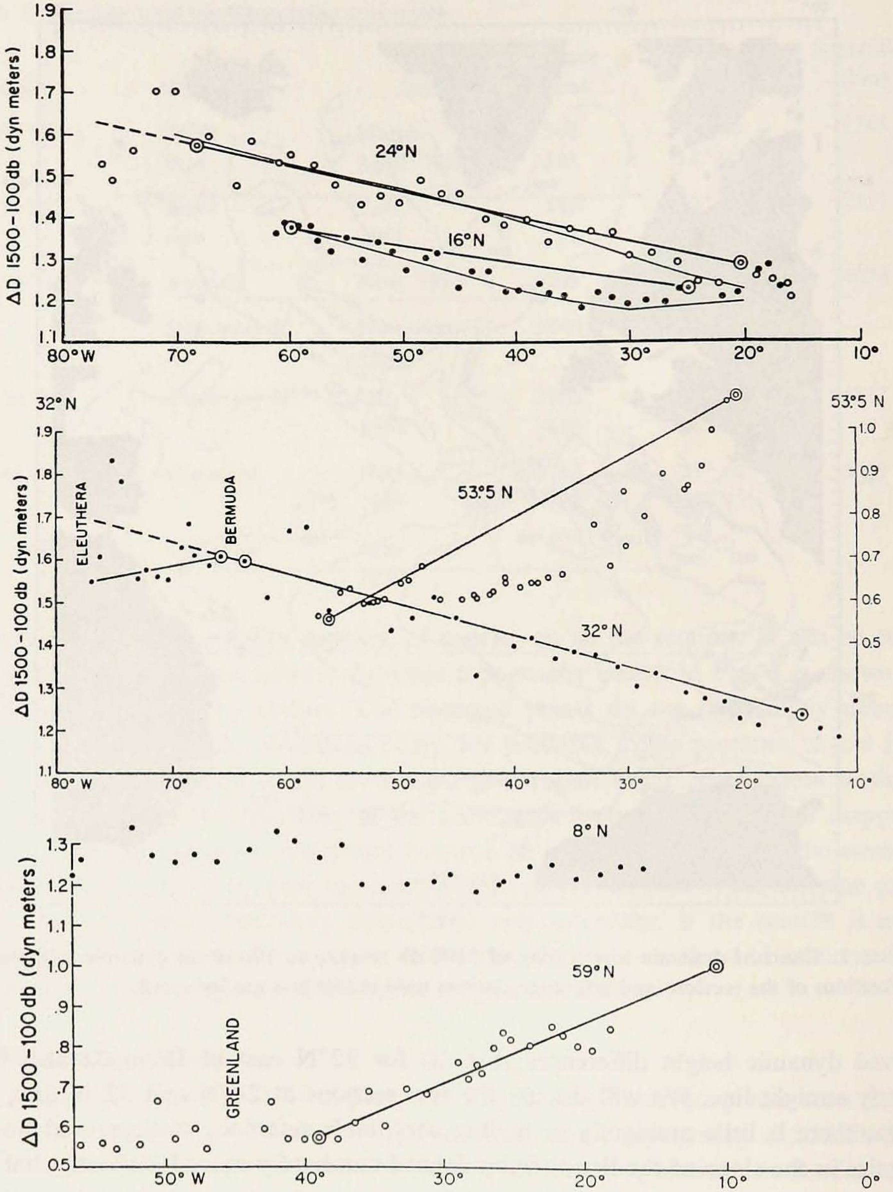

Figure 2. Dynamic height (1500 db-100 db) plotted for individual stations vs longitude on each of the east-west sections discussed. Reference stations circled. Light lines in 16°N and 24° N section display dynamic height field computed from the curl of the wind stress.

sive geostrophic shear, excessive northward abyssal flows, excessive upwelling at mid-depth, and excessive vertical eddy coefficients.

On the whole, we can expect to make reasonably good estimates of the ocean

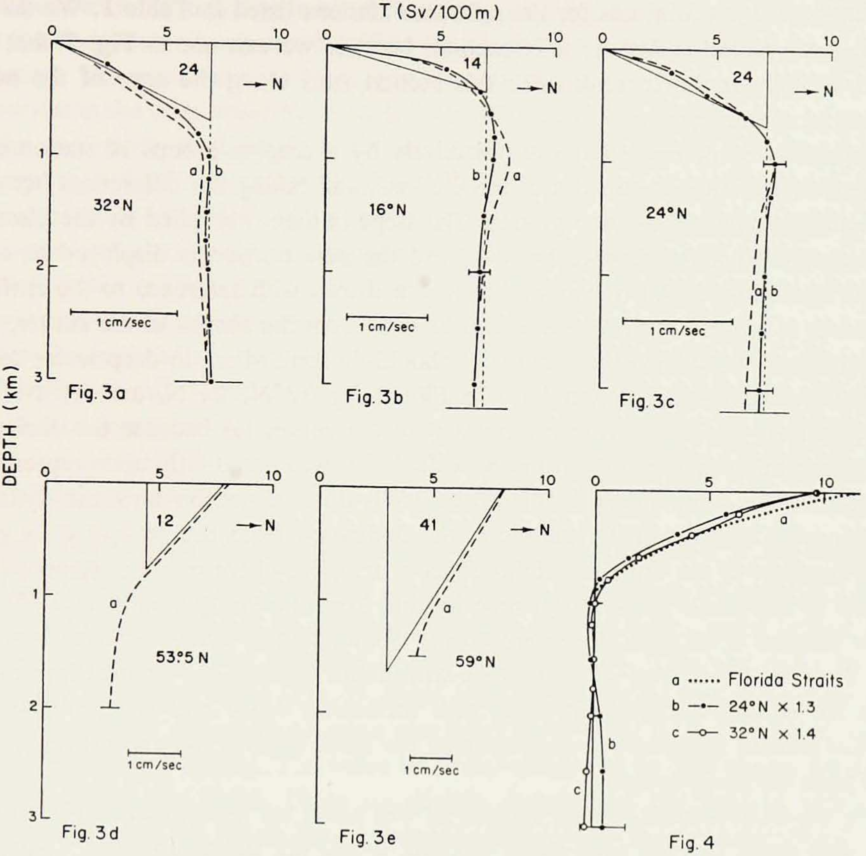

Figure 3. Vertical distribution of geostrophic transport on various sections, between reference stations in Sverdrups/100 meters depth, with zero abitrarily set at surface. (a) curves/dashed are computed from the two reference stations alone, (b) solid curves are computed from slopes determined by group means. Triangles have area corresponding to transport computed from wind-stress data, for same length of section. Error bars shown on (b) curves.

Figure 4. Vertical transport curve for Florida Straits (a) off Jacksonville—from directly measured velocity data with two other transport curves superposed: (b) extrapolated transport for 24°N (expanded transport to fill up whole ocean width), and (c) same for 32°N. The zeroes of (b) and (c) have been shifted arbitrarily to make the curves fit best by eye. Sample error bars shown at 3000 m on (b).

scale vertical distribution of geostrophic transport between reference stations on 24°N and 32°N, but only to depths above major topographic features of the Mid-Atlantic Ridge (this limits us to the upper 3200 meters).

Curves of the vertical distribution of transport (in Sverdrups/100 meters) for 24°N, 32°N, and for 16°N are given as dashed curves in Fig. 3. Note that at 16°N the dynamic topography slope depends on longitude. These were computed using the NODC station printouts for the reference stations listed in Table 1. We do not attempt a calculation for the 8°N section, because we can see in Fig. 2 that the east-west topography is weak since this section runs along the core of the north equatorial countercurrent.

Slopes can be determined more objectively by averaging groups of station data at the western and eastern sides of the sections, and taking the differences between the mean positions of the two groups. The slope is then multiplied by the distance between the original reference stations, and the new transports displayed as solid (b) curves in Fig. 3. All transports have been drawn with reference to the surface. However, it is much more likely, as can be seen from the shapes of the curves, that the true zero of the baroclinic transport should lie somewhere in deep water as indicated by the vertical dotted lines in Fig. 3 for 32°N, 24°N, and 16°N. The curves (b) are probably better estimates than the curves (a) because the choice of reference stations made in the (a) curve calculation was based only upon representativeness in the depth interval 100 db to 1500 db and not on how the dynamic height varies with depth. Furthermore, the derivation from the group-means gives some measure of the error-bars (chosen as $\pm\ \sigma/\sqrt{N}$, where σ is the standard deviation of individual stations' dynamic heights from the group-means, and N =(6) is the number of stations in a group-mean). Some of these error bars are shown in Fig. 3 on the (b) curves. They increase with depth because the vertical integration from the surface accumulates noise with increasing depth and are probably too large as shown, because the residuals from the fitted sloping line are noticeably less than from the group-means.

For the 24°N section, a least squares fit was made for the dynamic height variation with depth (i.e. for each depth interval, 0-100 db, 0-200 db, etc.). Using the fit at each depth, least-squares stations were determined at 70°W and 20°W. The transport between these stations from 0 db to 800 db was 24.7 Sverdrups. For the (a) and (b) curves the same transports are 23.2 and 24.0 Sverdrups respectively. Clearly all three techniques give about the same transport in this depth range.

3. Comparison to transports from curl of wind-stress data

Estimates of the total geostrophic transport, as computed from observed wind-data, can be computed between the reference stations as differences between Sverdrup transport and Ekman transport, using the tabulations of Leetmaa and Bunker (1976). We find them to be: 12 Sverdrups (16°N), 25 Sverdrups (24°N), and 27 Sverdrups (32°N). These transports do not contain any information about the distribution of transport with depth, and they represent only an area on the diagrams in Fig. 3. We assume that the depth distribution is triangular, with fixed area, and fit suitably shaped triangles of proper area to the curves in Fig. 3. The fits shown for 16°N, 24°N and 32°N are quite good: within the likely errors and consistent with the idea that most of the transport is indeed in the upper ocean and that the deep water has a small (within the error bars indistinguishable from zero) transport. The error bars are large enough, however, to admit uncertainty of ± 10 Sverdrups in the deep water between 1000 and 3000 decibars.

The uncertainty in the transport as computed from the curl of the stress can be as large as 30% (Leetmaa and Bunker, 1976). An error of about 5% results from underestimation of the winds by merchant vessels. Choice of a drag coefficient in the stress computation introduces an uncertainty of 10-20%. An additional error of 10% arises in the computation of the curl of the wind stress because of inadequate spatial resolution of the wind field.

Can the longitudinal structure that exists in some of the sections in Fig. 2 be explained by similar structure in the curl of the stress? The geostrophic transport, as obtained from the curl of the stress, was computed over areas 2° of latitude by 5° of longitude. If the vertical distribution of geostrophic transport is reasonably represented by a triangle (Fig. 3), the transport is related to the dynamic height change at the surface, ΔD, by the relation, $T = \frac{gH}{2f} \Delta D$ where H is the height of the triangle. Computations from the wind data give the average transport for every 5° of longitude. A ΔD can be computed for each of these and a zonal section constructed which shows the variation of ΔD with longitude. This was done for the sections at 24°N and 16°N (Fig. 2). Since only changes in the dynamic height field can be computed, the absolute value was fixed to lie close to the observed values at one end of each section. The value of H was chosen to be 500 meters for 16°N and 800 meters for 24°N. These are slightly deeper than the level to which the transport triangles are drawn but correspond more closely to our choice of the level of no motion. These choices of H make the computed and observed range of dynamic height variations agree. As can be seen, the computed shape of the dynamic height field fits the observed points remarkably well for both sections (Fig. 2). A similar computation for the 53°N section had poor agreement.

4. The subpolar sections

Turning now to the two sections that cross the subpolar grye at 53.5°N and 59°N, we see in Fig. 2 that there is a very marked dependence of slope with longitude. However, the curves are reasonably well defined, and we can choose reference stations, as shown, to embrace the major mid-ocean difference of dynamic topographic level across the sections. The transport curves (a) for these two sections are also shown in Fig. 3. No attempt to use group-means seemed worthwhile for these sections. When the wind-derived transport triangles are fitted to these curves, there is no indication of approach to zero in the deep water. Evidently there is a complicated story to tell here. The geostrophic shear extends to the bottom, and in this respect this area is more like the western boundary regions of the subtropical gyres.

5. Comparison to measured transports in the Florida Straits

Finally we compare the interior transports to the directly observed measured values in the Florida Straits off Jacksonville at 30°N (Richardson, Schmitz, and Niiler, 1969). We suppose by continuity that the upper ocean transport southward across the full width of the subtropical ocean (excluding the Florida Straits) must correspond to a return flow at similar depths through the Florida Straits. Of course, the water does not *have* to return at the same depths, because vertical velocities and water-mass modifications must occur to some extent after water has passed southward across a section, but this may actually be minimal. Certainly deviations from liquid water conservation due to net evaporation over the Atlantic will be undetectable in our crude budgets.

We need transport estimates for the full width of the oceans, but we cannot use hydrographic data near either coast because of the higher order dynamics there, and because of the large amplitude eddies that prevail near the Gulf Stream. Evidence of enhanced eddy activity near the western ends of the 24°N and 32°N sections can be seen in Fig. 2. We therefore extrapolate the results for these sections to cover the whole width of the ocean by expanding the (a) geostrophic transport curves uniformly with depth by the factor 1.4 for 32°N and 1.3 for 24°N. These expansion factors are calculated by dividing the full ocean width by the distance between the reference stations. These two expanded curves are superposed upon the measured (Jacksonville) Florida Straits transports in Fig. 4. The comparison appears to be remarkably good. This indicates that the water does not change much in depth as it goes around the grye from the midocean into the Florida Straits. The distribution of transport with depth in the upper 1000 meters is practically the same in all three profiles; deeper transports differ indistinguishably from zero within the limits of error.

To the geostrophic transports at 32°N and 24°N should, of course, be added the Ekman transports. At 32°N this would add about two Sverdrups to the transport shown in Fig. 4. At 24°N this subtracts about three Sverdrups. Neither value changes the curves significantly. The results of the 32°N section should perhaps be compared with the Richardson, *et al.*, 1969 section at Cape Fear (33°N). The measured transport at this section was 53 Sverdrups compared to 37 Sverdrups at the Jacksonville section. However, as was noted in Richardson *et al.*, 1969, this increase was uniform with depth (barotropic). Thus the agreement for the baroclinic transport as shown in Fig. 4 still holds.

The total transports at 30°N and 33°N as estimated from the wind stress data are 35 and 34 Sverdrups respectively. At 30°N the comparison with the measured transport is remarkably good; at 33°N the calculated and measured values are beginning to diverge.

6. Conclusion

In conclusion we believe that there is a good reason to suppose that the interior transport of the subtropical gyre of the North Atlantic is essentially consonant with the Sverdrup curl-of-the-wind-stress relationship. Our faith in a simple linear dynamic for this portion of the mean oceanic circulation is therefore strengthened.

Our conclusion, together with the results of Niiler *et al.* (1971), partly lays to rest a myth concerning discrepancies between theoretical and observed transport that began with Munk (1950) and has persisted through the years in the thinking of most of us. It has recently appeared in exaggerated form in the essay by Gill (1971). It states that observed transports exceed theoretical transports by a factor of between about 2 and 5.

This myth came about because the "observed" transports used were values obtained from measurements in the strong poleward portions of the Gulf Stream's flow while substantial return flows in a wide western boundary region near the Stream were ignored. The Stream and associated return flow are parts of a partly closed gyre of very large transport which includes substantial deep water flows (Worthington, in press). But these exotica do not extend very far into the ocean from the western coasts. Evidently the net meridional flows of the closed gyres in the western boundary regions are much less than those associated with the identifiable Gulf Stream itself, thus the discrepancies largely disappear. We believe that in the subtropics there is no demonstrable discrepancy of the kind mentioned by the above authors.

REFERENCES

Fuglister, F. C. 1960. Atlantic Ocean Atlas, Woods Hole, 209 pp.

Gill, A. E. 1971. Ocean models. Phil. Trans. Roy. Soc., London, *A 270*, 391–414.

Leetmaa, A. and Andrew Bunker. 1976. Update charts of the mean wind stress and its curl (in preparation).

Munk, W. 1950. On the wind-driven circulation. J. Met., *7*, 79–93.

Niiler, P. P., E. Simco, and R. Larue. 1971. A two-layer model of the north Atlantic thermocline. J. of Phys. Oceanogr., *1*, 257–277.

Richardson, W. S., W. J. Schmitz, and P. Niiler. 1969. The velocity structure of the Florida Current from the Florida Straits to Cape Fear. Deep-Sea Res., *16*, 225–231.

Stommel, H. 1956. On the determination of the depth of no meridional motion. Deep-Sea Res., *3*, 273–278.

Sverdrup, H. V., M. W. Johnson, and R. H. Fleming. 1942. The oceans, their physics, chemistry, and general biology. New York, Prentice-Hall, 1087 pp.

Worthington, L. V. (in press). On the North Atlantic circulation. Johns Hopkins Oceanographic Studies.

Worthington, L. V. and R. Wright. 1970. North Atlantic Atlas, Woods Hole.

Wust, G. 1957. Stromgeschwindigkeiten zur Statik und Dynamik des Atlantischen Ozeans. Wiss. Ergebn. d. Deutschen Atlantischen expedition METEOR 1925-27. Bd VI Teil 2, Lieferung *6*, 261–420.

Received: *November, 1976.*