Model of world ocean circulation: II. Thermally driven, two-layer

by George Veronis[1]

ABSTRACT

Oceanic motions are partly driven by thermal forces associated with differential heating of the surface waters between tropical and polar regions. In this study thermal driving is parameterized in a two-layer system by an assumed upwelling of fluid through the interface. The upwelling is steady and horizontally uniform wherever upper-layer water is present. With an assumed constant depth of the interface along the eastern boundary the transports of the upper and lower layers and the interface distribution are derived for the interior ocean. Mass conservation then yields the transports of the western boundary current and the interface depth along the coast. Sinks of upper-layer fluid correspond to thermal cooling in polar regions and, since the distribution and intensity of the latter process is not known very accurately, results are derived for different distributions and magnitudes of the sinks. Hence, various possibilities are explored. In particular, conditions under which separation of the western boundary current from the coast is possible are derived. The analysis shows that the solution to the problem is not unique unless the sinks are specified *a priori*. Nevertheless, the alternatives are informative and should help to interpret the results of the more general model driven by wind stress and thermal forces.

1. Introduction

Part I (Veronis, 1973) of this study reported an analysis aimed at deducing certain characteristics that are important for world ocean circulation. The most significant conclusions of the study had to do with the fact that separation of the main western boundary currents (Gulf Stream, Kuroshio, Agulhas, etc.) from their respective coasts is required by relatively simple integral constraints based on the structure of the wind stress. In particular, Ekman wind drift gives rise to an equatorward transport in mid-latitudes that is not balanced geostrophically. In a steady-state model for a basin closed to the north, mass balance requires that the western boundary current provide the return flow. The extra pressure head needed to return the Ekman wind drift geostrophically in the western boundary current forces the thermocline to surface at some latitude poleward of which the western boundary current must separate from the coast. An added feature in the separation process is

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that the amount of water forced into the northern boundary region by the cyclonic wind stress must be supplied by the western boundary current, thereby enhancing the separation mechanism. It was thus shown that the Norwegian, Alaskan and Cape Horn currents play important parts in the separation of the respective western boundary currents.

The present paper contains a discussion of certain features that may be expected when the ocean is driven by heating and cooling with no wind stress. Although no such circulation exists in the real oceans (wind stresses dominate oceanic response), there are certain aspects of the derived circulation that must be important for the actual circulation that is driven by both mechanical and thermal forces. Those features are isolated here.

Heating and cooling of oceanic waters involve the complications of turbulent vertical convection and it is not feasible to try to model those processes effectively in an analytical model, particularly one with a limited amount of vertical structure. Therefore, the process is parameterized in a simple fashion consistent with the following argument:

A typical vertical sounding shows that the temperature decreases with depth. Any turbulent mixing mechanism will lead to some vertical mixing of water near the surface and a consequent downward heat flux. Therefore, the water at subsurface levels will tend to heat up. In a steady-state model the temperature cannot increase so some process must be present to balance the downward flux of heat. The process invoked here is an upwelling of cold water from below. (Horizontal advection and diffusion can also be present but their effects are neglected in this study.) The actual mechanism responsible for the upwelling need not be (and probably is not) so simple as the one described above. All that we require is that there be an upwelling.

The model ocean is composed of two layers, each of constant density, with less dense water on top. Upwelling of lower-layer water into the upper layer is assumed to be horizontally uniform throughout the interior of the ocean basins wherever upper-layer water exists. The magnitude of the upwelling is not specified *a priori.* This imposed upwelling will drive a circulation because of the effect that vertical divergence has on the planetary vorticity. The circulation is closed by western boundary currents which absorb (or return) the required amount of water from (or to) the upwelling regions.

Somewhere there must be sinks[2] of upper-layer water to replenish the lower-layer water that upwells. The locations and magnitudes of these sinks are features that we would like to deduce as necessary consequences of the model because they can be

2. The word "sink" does not necessarily imply that upper-layer water flows downward. It is only necessary for upper-layer water to become lower-layer water and that may occur at or near the surface. One could think of both the sinking and upwelling processes simply as transfers of water from the warm-water sphere to the cold-water sphere or *vice versa.* Since the model is two-layer and since "sinking" occurs only when the thermocline surfaces (except at the northern boundary), the transfer can take place at the surface. In this sense the model differs from Stommel's (1957) abyssal circulation study.

only crudely estimated from observations. We would also like to obtain the optimal value of the upwelling velocity, a quantity which is much too small for direct observation.

Since the large scale ocean circulation is primarily wind-driven, it is not possible to use observational data as a guide for choosing the parameters of the present model. Therefore, we have confined our attention in this paper to obtaining general characteristics of thermally driven flows that must be important and present even when wind stress is included. Hence, we explore possible flows that may occur when sinks are specified to be present in certain locations. The necessary concomitant features will then serve as a guide to the combined wind- and thermally driven model.

After presenting the general equations for the model in the next section, we derive information about the structure of the interior. Then an argument based on mass balance leads to an expression for h_w^2, the square of the thermocline depth on the western boundary. When the thermocline surfaces (h_w^2 vanishes) the western boundary current may separate from the coast. The present analysis shows that more than one solution for the problem can exist when h_w vanishes; either the western boundary current stays along the coast and sinking occurs poleward of the surfacing latitude, or the current can separate and then flow due eastward. In the former case upper-layer water occupies the entire northern basin and there is a large amount of sinking. This situation implies relatively weak cooling over a broad area. The alternative solution leads to upper-layer water in a more limited area of the northern basin and a somewhat smaller amount of sinking. Such may be expected when cooling is intense because, even though the amount of sinking is less, it occurs at a lower latitude and limits the total amount of upper-layer water that is present.

The thermocline can surface in the southern hemisphere also. The simplest solution to the problem here is that surfacing is accompanied by separation because a simple exact balance is obtained between total upwelling and total sinking.

The present model together with that of Part I prepares the way for the combined wind-driven, upwelling calculation. In the latter model it is possible to obtain more definitive results, particularly with respect to the locations and strengths of the sinking regions.

2. The model

The physical system to be analyzed below is based on the following simplifications and assumptions:

(i) The basin has a level bottom and is bounded at the northern latitude $\phi = \phi_n$ and at the longitudes $\lambda = 0$ and $\lambda = \lambda_e$. It is open to the south (see Fig. 1).

(ii) The ocean is composed of a layer of water with uniform density ρ_2 and

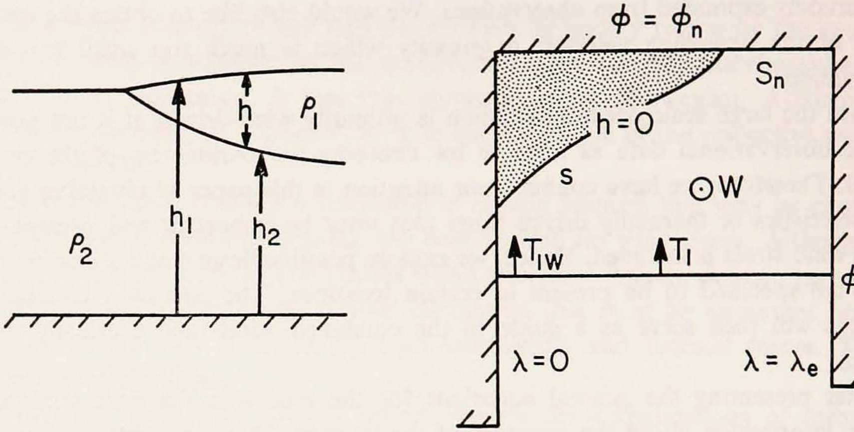

Figure 1. *Left* Two-layer quantities defined as shown. *Right* The ocean basin shown schematically with boundaries at $\phi = \phi_n$ (top), $\lambda = 0$ (left), $\lambda = \lambda_e$ (right). The basin is joined to the Southern Ocean in the south. Also shown schematically for the upper layer are: the (northward) transports T_1 in the interior and T_{1w} in the western boundary layer; total upwelling north of the latitude ϕ indicated by oncoming arrow and denoted by W; distributed sink, s, along curve where $h = 0$; and sink S_n along northern boundary.

height h_2 under a layer with uniform density ρ_1 and *thickness h* (see Fig. 1). The top surface of the ocean is at h_1 where

$$h_1 = h + h_2. \tag{1}$$

The interface may surface along some curve where $h_1 = h_2$ and $h = 0$. Thus there may be regions (shown stippled in Fig. 1) with water of density ρ_2 extending to the surface.

(iii) The fluid is inviscid everywhere (no viscous stresses anywhere).

(iv) Wherever upper-layer fluid exists there is upwelling of lower-layer fluid through the interface into the upper layer. The velocity at the base of the upper layer is w and, because of the density difference, the velocity at the top of the lower layer is $w\rho_1/\rho_2$. The heating mechanism is thus parameterized by this change of fluid from density ρ_2 to density ρ_1 as it crosses the interface.

(v) The cooling mechanism is parameterized by "sinking" of upper layer fluid, i.e., fluid of density ρ_1 is changed to fluid of density ρ_2. Such sinks may be expected to occur at the northern boundary (shown in Fig. 1 as S_n) and/or along the curve where $h = 0$ (shown in Fig. 1 as s).

(vi) The flow is hydrostatic.

(vii) The flow in the interior is steady and linear (hence geostrophic).

(viii) A western boundary layer is assumed to provide the return flow necessary for mass continuity across any latitude. The downstream velocity in the western boundary layer is geostrophically balanced but the dynamics of the cross-stream velocity are not specified. The western boundary layer is assumed to be very thin (infinitesimal compared to the ocean width) and the derived relations always involve an integral across the layer, i.e., the detailed dynamical balances are not used to obtain information within the boundary layer.

a. Dynamics of the interior. With these assumptions the interior flow is described by geostrophic, hydrostatic equations in each layer. If these are integrated over the depth of each layer, the following set of equations results (subscripts 1 and 2 identify upper- and lower-layer variables respectively):

$$2\Omega \sin\phi \, V_1 = \frac{gh}{a\cos\phi} \frac{\partial h_1}{\partial \lambda} \tag{2}$$

$$2\Omega \sin\phi \, U_1 = -\frac{gh}{a} \frac{\partial h_1}{\partial \phi} \tag{3}$$

$$\frac{\partial U_1}{\partial \lambda} + \frac{\partial (V_1 \cos\phi)}{\partial \phi} = a\, w \cos\phi \tag{4}$$

$$2\Omega \sin\phi \, V_2 = \frac{gh_2}{a\cos\phi} \frac{\partial}{\partial \lambda} \left(\frac{\rho_1}{\rho_2} h_1 + \frac{\Delta\rho}{\rho_2} h_2 \right) \tag{5}$$

$$2\Omega \sin\phi \, U_2 = -\frac{gh_2}{a} \frac{\partial}{\partial \phi} \left(\frac{\rho_1}{\rho_2} h_1 + \frac{\Delta\rho}{\rho_2} h_2 \right) \tag{6}$$

$$\frac{\partial U_2}{\partial \lambda} + \frac{\partial (V_2 \cos\phi)}{\partial \phi} = -\frac{\rho_1}{\rho_2} a\, w \cos\phi \tag{7}$$

where Ω, g, and a respectively are the earth's angular velocity, gravitational acceleration, and mean radius and U, V are the vertically integrated velocities. These equations are familiar ones in studies of two-layer systems but in (4) and (7) the usual continuity equations are augmented by the source and sink terms associated with the flux of fluid across the interface.

Cross-differentiating (2) and (3) and making use of (4) yields the planetary divergence relation

$$\beta V_1 = -fw - \frac{g}{a^2 \cos\phi} J(h, h_1) \tag{8}$$

where $f = 2\Omega \sin\phi$, $\beta = 2\Omega \cos\phi / a$ and the last term is the Jacobian, $J(h,h_1) =$

$$\frac{\partial h}{\partial \lambda} \frac{\partial h_1}{\partial \phi} - \frac{\partial h}{\partial \phi} \frac{\partial h_1}{\partial \lambda} .$$

A similar procedure with (5) to (7) gives

$$\beta V_2 = \frac{\rho_1}{\rho_2} \left[fw + \frac{g}{a^2 \cos\phi} J(h, h_1) \right], \tag{9}$$

where the definition (1) has been used to obtain $J(h, h_1) = -J(h_2, h_1)$. From (8) and (9) we obtain

$$\frac{\rho_1}{\rho_2} V_1 + V_2 = 0, \tag{10}$$

i.e., the total, north-south mass transport vanishes at each point.

Now the sum of (5) and (2) $\times \rho_1/\rho_2$ yields

$$2\Omega \sin\phi \left(\frac{\rho_1}{\rho_2} V_1 + V_2 \right) = \frac{g}{2a\cos\phi} \frac{\partial}{\partial\lambda} \left(\frac{\rho_1}{\rho_2} h_1^2 + \frac{\Delta\rho}{\rho_2} h_2^2 \right), \tag{11}$$

where (1) has been used to obtain h_1^2 on the right hand side. Since the left hand side vanishes, we obtain the result that $\frac{\rho_1}{\rho_2} h_1^2 + \frac{\Delta\rho}{\rho_2} h_2^2$ is constant in the interior of the ocean at each latitude. In particular,

$$\frac{\rho_1}{\rho_2} h_1^2 + \frac{\Delta\rho}{\rho_2} h_2^2 = \frac{\rho_1}{\rho_2} h_{1e}^2 + \frac{\Delta\rho}{\rho_2} h_{2e}^2 , \tag{12}$$

where subscript e denotes the value at λ_e.

The interface height h_2 can be eliminated from (1) and (12) to give h_1 in terms of h and the right hand side of (2) can then be rewritten in terms of h. A simpler procedure, however, is to rewrite (11) (with the left hand side vanishing) as

$$\frac{\rho_1}{\rho_2} h_1 \frac{\partial h_1}{\partial\lambda} = - \frac{\Delta\rho}{\rho_2} h_2 \frac{\partial h_2}{\partial\lambda} = - \frac{\Delta\rho}{\rho_2} (h_1-h) \frac{\partial}{\partial\lambda} (h_1-h). \tag{13}$$

Wherever h_1 appears as a coefficient, it can be replaced by H_1, the mean depth of the ocean, because $(h_1 - H_1)/H_1 < 10^{-3}$. Then with an error smaller than $\Delta\rho/\rho_2$ ($\sim 10^{-3}$) (13) can be written as

$$\frac{\partial h_1}{\partial\lambda} = \frac{\Delta\rho}{\rho_2} \left(1 - \frac{h}{H_1} \right) \frac{\partial h}{\partial\lambda} \tag{14}$$

so that (2) becomes

$$fV_1 = \frac{g'}{a\cos\phi} \frac{\partial}{\partial\lambda} \left(\frac{h^2}{2} - \frac{h^3}{3H_1} \right) \tag{15}$$

where $g' \equiv g\Delta\rho/\rho_2$.

Equation (14) can be integrated to yield

$$h_1 = h_{1e} + \frac{\Delta\rho}{\rho_2} \left[h - \frac{h^2}{2H_1} - \left(h_e - \frac{h_e^2}{2H_1} \right) \right] , \tag{16}$$

and, if h_{1e} and h_e are constant (independent of ϕ), we obtain

$$\frac{\partial h_1}{\partial\phi} = \frac{\Delta\rho}{\rho_2} \left(1 - \frac{h}{H_1} \right) \frac{\partial h}{\partial\phi} . \tag{17}$$

Thus the Jacobian, $J(h, h_1)$ vanishes and (8) takes the form

$$V_1 \cos\phi = - aw \sin\phi \tag{18}$$

Hence, (15) becomes

$$-\frac{a^2 w f \sin\phi}{g'} = \frac{\partial}{\partial\lambda}\left(\frac{h^2}{2} - \frac{h^3}{3H_1}\right). \tag{19}$$

Integrating from λ to λ_e yields

$$\frac{h^2}{2} - \frac{h^3}{3H_1} = \frac{h_e^2}{2} - \frac{h_e^3}{3H_1} + \frac{a^2 w f \sin\phi}{g'}(\lambda_e - \lambda). \tag{20}$$

where we have now taken w to be constant.

Thus, once w and h_e are specified, h is determined from (20), V_1 from (18), and U_1 from (3) and (17). Lower-layer quantities are determined in an analogous way.

Finally, we obtain a value of the total upper-layer transport, T_1, from the eastern edge to any longitude λ by integrating $aV_1 \cos\phi$ from λ to λ_e,

$$T_1 = -a^2 w \sin\phi\,(\lambda_e - \lambda) \tag{21}$$

Then (20) has the alternative form

$$\frac{h^2}{2} - \frac{h^3}{3H_1} = \frac{h_e^2}{2} - \frac{h_e^3}{3H_1} - \frac{f}{g'}T_1. \tag{22}$$

b. Transport balances. Consider the total amount of water flowing into the upper layer north of latitude ϕ in Fig. 1. The following quantities are involved:

T_1 is the transport in the interior taken positive northward.

T_{1w} is the transport in the western boundary current, also taken positive northward.

W is the total amount of water that enters the upper layer by upwelling through the interface. Its value is given by the product of the upwelling velocity, w, and the area north of ϕ over which h is positive.

S is the amount of upper-layer water that *sinks*, i.e., the amount of water of density ρ_1 that becomes water of density ρ_2. If S is negative, it corresponds to a *source* of upper-layer water. Both the magnitude of S and the locations where sinks exist are unspecified for the moment.

Then the steady-state transport balance for the upper layer requires

$$T_1 + T_{1w} + W - S = 0, \tag{23}$$

where the sign of S is negative because positive S is a sink of upper-layer water.

For the lower layer a completely analogous argument applies. However, because of the density difference the total amount of lower layer water that upwells through the thermocline is $W\rho_1/\rho_2$ and the amount received from sinks of upper layer water is $S\rho_1/\rho_2$. Hence,

$$T_2 + T_{2w} - \frac{\rho_1}{\rho_2}(W - S) = 0. \tag{24}$$

The product of (23) by ρ_1/ρ_2 added to (24) yields

$$\frac{\rho_1}{\rho_2} T_1 + T_2 + \frac{\rho_1}{\rho_2} T_{1w} + T_{2w} = 0. \tag{25}$$

Equation (10) states that the meridional interior transport from bottom to top vanishes at each point. Hence, the value integrated across the basin, i.e., $\rho_1/\rho_2\ T_1 + T_2$, also vanishes and (25) becomes

$$\frac{\rho_1}{\rho_2} T_{1w} + T_{2w} = 0. \tag{26}$$

c. Geostrophic downstream velocity in western boundary layer. Since the downstream velocity in the western boundary layer is geostrophic, the vertically integrated velocity for each layer is given by

$$fV_{1w} = gh \frac{\partial h_1}{\partial n} \tag{27}$$

$$fV_{2w} = gh_2 \frac{\partial}{\partial n}\left(\frac{\rho_1}{\rho_2} h_1 + \frac{\Delta\rho}{\rho_2} h_2\right), \tag{28}$$

where $\partial/\partial n$ is the normal derivative directed to the right of V_w. (If the boundary layer is along the coast, $\partial/\partial n \equiv (a \cos \phi)^{-1}\, \partial/\partial\lambda$.) Adding to (28) the product (27) $\times\ \rho_1/\rho_2$ yields

$$\frac{g}{2} \frac{\partial}{\partial n}\left(\frac{\rho_1}{\rho_2} h_1^2 + \frac{\Delta\rho}{\rho_2} h_2^2\right) = f\left(\frac{\rho_1}{\rho_2} V_{1w} + V_{2w}\right). \tag{29}$$

Integrating with respect to n from the left side of the boundary layer (denoted by subscript w) to the offshore edge, then gives

$$\frac{\rho_1}{\rho_2} h_{1w}^2 + \frac{\Delta\rho}{\rho_2} h_{2w}^2 = \frac{\rho_1}{\rho_2} h_1^2 + \frac{\Delta\rho}{\rho_2} h_2^2 - \frac{2f}{g}\left(\frac{\rho_1}{\rho_2} T_{1w} + T_{2w}\right) \tag{30}$$

where h_1 and h_2 on the right hand side are evaluated on the offshore side of the boundary layer. Because of (26), (30) reduces to

$$\frac{\rho_1}{\rho_2} h_{1w}^2 + \frac{\Delta\rho}{\rho_2} h_{2w}^2 = \frac{\rho_1}{\rho_2} h_1^2 + \frac{\Delta\rho}{\rho_2} h_2^2, \tag{31}$$

and it is clear from (12) that this combination of h_1 and h_2 is constant everywhere in the interior and on the western wall.

The last expression relates h_{1w} to h_{2w}. We would like to obtain an additional relation which allows an explicit determination of h_{1w} and h_{2w} (or h_w). To do so we return to (29) and use the same procedure as that leading to (14) to obtain

$$\frac{\partial h_1}{\partial n} = \frac{\Delta\rho}{\rho_2}\left(1 - \frac{h}{H_1}\right)\frac{\partial h}{\partial n} + \frac{f}{gH_1}\left(V_{2w} + \frac{\rho_1}{\rho_2}V_{1w}\right). \tag{32}$$

Substituting (32) into (27) yields

$$\frac{f}{g'}\, V_{1w} = \frac{\partial}{\partial n}\left(\frac{h^2}{2} - \frac{h^3}{3H_1}\right) + \frac{fh}{g'H_1}\left(V_{2w} + \frac{\rho_1}{\rho_2}V_{1w}\right) \tag{33}$$

If it were not for the last term, (33) could be integrated across the boundary layer to yield h_w in terms of h and the transport. Now we know from (26) that $T_{2w} + \rho_1/\rho_2\, T_{1w}$ vanishes and normally we would not expect $\int h(V_{2w} + \rho_1/\rho_2\, V_{1w})dn$ to vanish since h is monotonic. However, V_{2w} and V_1 are connected to V_2 and V_1 of the interior solution since the boundary layer transports are fed by (or feed into) the interior. In the latter region $V_2 + \rho_1/\rho_2\, V_1$ vanishes *at each point* (so does $U_2 + \rho_1/\rho_2\, U_1$ by continuity). Hence, since each column of fluid connecting the interior to the boundary layer has zero transport at the point of connection, it seems likely that $V_{2w} + \rho_1/\rho_2\, V_{1w}$ vanishes or is very small compared to V_{2w} and V_{1w} individually. In addition, the factor h/H_1 is also small. Hence, the last term can at best contribute only a small correction to the balance and we shall neglect it.[3] Then integrating (33) with respect to n yields

$$\frac{h_w^2}{2} - \frac{h_w^3}{3H_1} = \frac{h^2}{2} - \frac{h^3}{3H_1} - \frac{f}{g'}T_{1w}$$

$$= \frac{h^2}{2} - \frac{h^3}{3H_1} + \frac{f}{g'}(T_1 + W - S), \tag{34}$$

where we have used (23) and where h on the right hand side is evaluated on the offshore (right) side of the boundary layer. Making use of (22) then gives

$$\frac{h_w^2}{2} - \frac{h_w^3}{3H_1} = \frac{h_e^2}{2} - \frac{h_e^3}{3H_1} + \frac{f}{g'}(W - S) \tag{35}$$

which can be solved for h_w in terms of h_e, S and W. Of the latter quantities only h_e is known (by assumption). In general, S will not be known without additional information, (e.g., specification of the cooling mechanism). W depends on the area of the region occupied by upper-layer water. If there is no separation, W is given by the product of w and the area of the ocean north of ϕ. If the western boundary current separates, the longitude where h vanishes must be known and (35) then involves an implicit evaluation. With the limited information at our disposal we cannot obtain unequivocal results so we shall explore *possible* types of solutions.

3. This point cannot be justified completely without invoking the complete boundary layer dynamics (which I want to avoid). The main function of the boundary layer is to provide mass continuity for the system and the present argument leads to a good approximation to the desired relation.

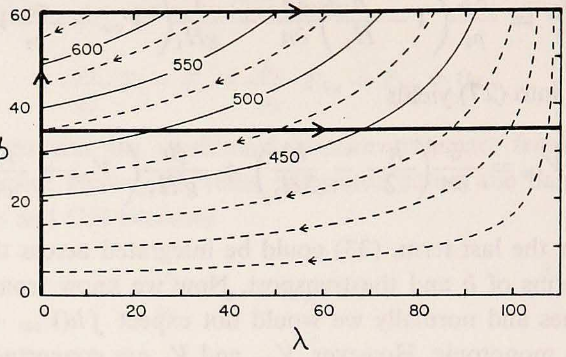

Figure 2. Solid curves are contours of h in meters for a northern hemisphere basin 110° wide and extending to 60°N. Dashed curves are trajectories for upper layer. Contours for lower-layer height, h_2, parallel those for h and for lower layer trajectories arrows are reversed. Trajectory and h fields are symmetric about the equator. Also shown schematically are the solution where the western boundary current stays along the coast and turns eastward along $\phi = \phi_n$ and the solution showing separation at 35°N (by assumption) and eastward extension (deduced) to $\lambda = \lambda_e$. The numerical values of the h contours were based on parametric values $h_e = 400$ m, $\Delta\rho/\rho_2 = 0.0015$, $w = 2 \times 10^{-5}$cm s^{-1} and $H_1 = 4000$ m.

3. Results for the interior

The interior solution is shown in Fig. 2. The solid curves are contours (as marked) for h when the parameters are $h_e = 400$ m, $w = 2 \times 10^{-5}$cm s^{-1}, and $\lambda_e = 110°$. Contours of h_2 are parallel to those for h since they are essentially given by $h_2 = H_1 - h$ where H_1 (= 4000 m) is the mean depth of the ocean. Horizontal trajectories for the vertically integrated velocities of the upper layer are indicated by the dashed curves and are constructed from the solution given by

$$U_1 = (\lambda - \lambda_e) 2aw\cos\phi, \qquad V_1 = - aw\tan\phi \tag{36}$$

Now, by definition,

$$U_1 = ha\cos\phi \frac{d\lambda}{dt}, \qquad V_1 = ha \frac{d\phi}{dt} \tag{37}$$

Then

$$\frac{U_1}{V_1} = \cos\phi \frac{d\lambda}{d\phi} = - 2(\lambda - \lambda_e) \cos^2\phi/\sin\phi$$

so that the trajectories are defined by

$$\lambda = \lambda_e (1 - \sin^2\phi_0/\sin^2\phi), \tag{38}$$

where ϕ_0 is the latitude of the trajectory at $\lambda = 0$. The trajectory paths are thus independent of the physical parameters. For the lower layer the trajectories are parallel to the ones shown but the direction of flow is reversed and the pattern is the same as that given by Stommel (1957).

The upper-layer trajectories are everywhere equatorward. Hence, fluid can reach the polar regions only via boundary currents. This point is very important because the entire burden of providing fluid that may sink in the polar regions rests on the western boundary currents. When wind stress is included, poleward flow can take place in the interior ocean as we showed in I.

If the western boundary current separates from the coast, the contours of h and the trajectories terminate at the separated current (shown as a broad eastward jet at 35° N). In the region occupied by lower-layer water only (shown stippled in Fig. 1) h vanishes and there is no flow in that part of the interior.

4. Possibilities for western boundary current

As mentioned before, the effects of heating and cooling must be known if explicit results for the transports are to be obtained. In the present model that means that the distribution of S must be given. Since our model is driven only by internal processes (no wind stress), observational data is not available and S cannot be determined explicitly. Hence, we list a number of possibilities and discuss the consequences of these.

a. No sinking in the northern hemisphere. If no water sinks north of the equator, S vanishes and the transport balance (23) becomes

$$T \equiv T_{1w} + T_1 = -W \tag{39}$$

where $T_1 = -a^2 w \lambda_e \sin\phi$, $T_{1w} = a^2 w \lambda_e (2\sin\phi - \sin\phi_n)$,

$$W = a^2 w (\sin\phi_n - \sin\phi)\lambda_e \tag{40}$$

and T denotes the total transport (by interior and boundary current) in the upper layer. Thus, the total transport vanishes at the northern boundary ϕ_n, and is negative for $\phi < \phi_n$ because the water entering the upper layer by upwelling north of ϕ must flow southward at ϕ.

From (35) and (40) we obtain

$$\frac{h_w^2}{2} - \frac{h_w^3}{3H_1} = \frac{h_e^2}{2} - \frac{h_e^3}{3H_1} + \frac{a^2 w f}{g'} (\sin\phi_n - \sin\phi)\lambda_e \tag{41}$$

and we see that h_w is larger than h_e everywhere in the northern hemisphere except at ϕ_n where $h_w = h_e$. This result is to be expected since the meridional component of flow is everywhere geostrophic and a net southward flow requires $h_w > h_e$ for $\phi > 0$.

In the southern hemisphere the last term of (41) is negative and, since southward geostrophic transport requires that the thermocline be deeper on the left, we have

$h_w < h_e$. In this region h_w can vanish. The condition for vanishing h_w is that the right hand side of (41) vanish, i.e., that W corresponds to the geostrophic transport given by $g'(h_e^2/2 - h_e^3/3H_1)/f$. If w is sufficiently large or h_e sufficiently small, this condition can easily be realized.

When h_w vanishes, the relation (35) cannot be satisfied and some modification must be made. The situation is similar to that described in paper I where the vanishing of h_w was related to the separation of the western boundary current from the coast. The determining feature is the amount of geostrophic transport in the western boundary current, the maximum amount of which occurs when h_w vanishes. For the wind-driven model, separation was determined at least in part by the Ekman wind drift at each latitude. The transport due to the Ekman wind drift had to be geostrophically balanced in the western boundary layer and at mid-latitudes the interface was forced to surface. In the present case the surfacing of the thermocline is related to the amount of upwelling and the sources or sinks north of the latitude in question. When there is no source or sink, the upwelled water must flow southward in the western boundary layer and this tends to depress the thermocline in the northern hemisphere and to raise it in the southern hemisphere. This asymmetry in the effect on h_w is associated explicitly with the upwelling and gives rise to a qualitatively different behavior in the southern hemisphere. Also, the upwelling leads to an effective north-south scale that is much larger than that imposed by the wind-stress. The latter two features are very important for the world ocean circulation.

It should be observed that even though there may be no sink of upper-layer water north of the equator there *must* be a sink someplace to replenish the deep water that upwells. In the present case the sink would have to be either in the southern hemisphere of the basin or in one of the other basins connected to the present one by the Southern Ocean.

We shall return to the effect of vanishing h_w after considering the question of separation of boundary currents.

b. Sinking at ϕ_n but no surfacing of thermocline. This situation is only quantitatively different from the one just considered. With $S = S_n \neq 0$ equation (35) evaluated at ϕ_n is

$$\frac{h_w^2}{2} - \frac{h_w^3}{3H_1} = \frac{h_e^2}{2} - \frac{h_e^3}{3H_1} - \frac{f}{g'} S_n \tag{42}$$

The condition that the thermocline *not* surface is

$$S_n < \frac{g'}{f}\left(\frac{h_e^2}{2} - \frac{h_e^3}{3H_1}\right) \tag{43}$$

i.e., only a limited amount of water can sink at ϕ_n. Thus, some of the water carried north by the western boundary current sinks and the rest flows eastward along the northern boundary to provide the water necessary for the southward flow in the interior.

For $\phi < \phi_n$ (35) becomes

$$\frac{h_w^2}{2} - \frac{h_w^3}{3H_1} = \frac{h_e^2}{2} - \frac{h_e^3}{3H_1} + \frac{f}{g'}\left[a^2w(\sin\phi_n - \sin\phi)\lambda_e - S_n\right]. \quad (44)$$

Hence, in the southern hemisphere the latitude at which h_w vanishes is farther to the south than for case *a*. This result is to be expected since the western boundary current is required to carry less of the upwelled water southward because of the sink at ϕ_n.

c. Surfacing of thermocline but no separation. When S_n exceeds the value on the right hand side of (43), h_w^2 as determined by (42) is negative. Since negative h_w^2 has no physical meaning, the analysis must be modified. There are several alternate paths to follow, most of which involve an implicit evaluation for w. However, a simple approach that avoids the implicit evaluation is the following:

From (23) we can write

$$T = T_1 + T_{1w} = S - W \quad (45)$$

where T is again the total meridional transport of upper-layer water. At ϕ_n W must vanish, so

$$T = S, \quad (46)$$

i.e., the *total* (via interior plus boundary layer) transport of upper-layer water into the northern boundary region is equal to the amount that sinks. Now because of geostrophy the maximum transport, T_{max}, that can exist at ϕ_n is that obtained when h_w vanishes. Hence, from (35) evaluated at ϕ_n,

$$T = T_{\max} = S_n = \frac{g'}{f_n}\left(\frac{h_e^2}{2} - \frac{h_e^3}{3H_1}\right), \quad (47)$$

where S_n is the sink of upper-layer water at the northern boundary and $f_n = f(\phi_n)$.

Now suppose that S_n corresponds to all of the water that upwells north of some latitude ϕ_*. Then $W(\phi_*) = S_n$ and from (40)

$$W(\phi_*) = S_n = a^2w(\sin\phi_n - \sin\phi_*)\lambda_e. \quad (48)$$

If g' and h_e are known, (47) and (48) can be equated to yield the following expression for w required for the foregoing balances

$$w = \frac{g'(h_e^2/2 - h_e^3/3H_1)}{f_n a^2(\sin\phi_n - \sin\phi_*)\lambda_e}. \quad (49)$$

If there is no additional sink of water in the northern hemisphere, h_w vanishes at ϕ_n and has finite values for $\phi < \phi_n$ south to some latitude in the southern hemisphere where h_w can again vanish because of the excess amount of upwelled water that must flow southward in the western boundary current. South of that latitude h_w^2 would be negative and we have the same interpretational problem as in case *a*.

Another possibility in this case is that the thermocline surfaces at some latitude $\phi_0 < \phi_n$ and does not separate for $\phi > \phi_0$. Then h_w will vanish at ϕ_0 and at that latitude (35) yields

$$S = S_0 + S_n = \frac{g'}{f_0}\left(\frac{h_e^2}{2} - \frac{h_e^3}{3H_1}\right) + W \tag{50}$$

where we have included the possibility of an additional sink, denoted by S_0. S_n is still given by (47) and W by (40) evaluated at ϕ_0. Hence,

$$s = g'\left(\frac{1}{f_0} - \frac{1}{f_n}\right)\left(\frac{h_e^2}{2} - \frac{h_e^3}{3H_1}\right) + a^2w(\sin\phi_n - \sin\phi_0)\lambda_e. \tag{51}$$

Both terms in (51) are positive and we see that there must be a sink along the western boundary from ϕ_0 to ϕ_n. South of ϕ_0, with S_n given by (47) and S_0 by (51), h_w^2 will again be positive south to the latitude in the southern hemisphere where h_w^2 again vanishes.

The present situation can be interpreted as follows: Suppose that cooling in the northern hemisphere is sufficiently intense to cause the thermocline to surface at ϕ_0. Then in order that (35) be satisfied with $h_w = 0$ at ϕ_0 it is necessary that $S = S_n + S_0$ as given above. North of ϕ_0, f will be larger, W will be smaller and (35) can be satisfied with $h_w = 0$ only if S decreases. North of ϕ_0, S is given by $S = s + S_n$ where

$$s = g'\left(\frac{1}{f} - \frac{1}{f_n}\right)\left(\frac{h_e^2}{2} - \frac{h_e^3}{3H_1}\right) + a^2w(\sin\phi_n - \sin\phi)\lambda_e. \tag{52}$$

At ϕ_0, $s = S_0$ and at ϕ_n, $s = 0$. There are two contributions to the sinking along the western boundary. One (the first term in (52)) is due to the variation of f. Since h_e is fixed and h_w vanishes, the amount of northward transport that can be balanced geostrophically, given by $g'/f\ (h_e^2/2 - h_e^3/3H_1)$, must decrease with increasing latitude. (This is analogous to trying to push a fixed amount of water through a channel with decreasing cross-sectional area. Some water will spill over the sides of the channel and corresponds here to part of the sink.) The remaining contribution to the sink is the second term in (52) (which is simply W). The sink must take care of the upwelling in the region between ϕ_n and the latitude in question.

Numerical values for the present case were calculated with $\phi_n = 60°$N, $\lambda_e = 110°$, $h_e = 400$ m, $\Delta\rho/\rho_2 = .0015$, $H_1 = 4000$ m, $\phi_0 = 35°$N and $\phi_* = 0°$. Thus, all of the water that upwells north of the equator sinks at ϕ_n. Then (49) gives the value $w = 1.29 \times 10^{-5}$cm s^{-1}, and (47) yields $S_n = 8.69 \times 10^6$ m^3 s^{-1}. The values of h_w for $0° < \phi < 35°$ and s for $35° < \phi < 60°$ are shown at 5° intervals in Table 1. Total upwelling north of the equator is the same as S_n (by assumption).

Table 1. Values of $h_w(\phi)$ for $0 < \phi < 35°N$ and $s(\phi)$ for $35°N < \phi < 60°N$.

ϕ	$h_w(m)$	ϕ	$s(10^6 m^3 s^{-1})$
0°	400.	35°	7.36
5°	380.	40°	5.26
10°	354.	45°	3.55
15°	322.	50°	2.14
20°	283.	55°	0.97
25°	233.	60°	0
30°	165.		
35°	0.		

Total sinking along the western boundary is 7.36×10^6 m^3 s^{-1}. Hence, our stipulation that the thermocline surface at 35°N and not separate from the boundary north of the latitude requires additional sinking almost equal to that at ϕ_n. The total amount of water that sinks is $S_0 + S_n = 1.61 \times 10^7$ m^3 s^{-1}, so that there must be northward flow in the western boundary layer from nearly 60°S carrying the upwelled water into the northern hemisphere to sink along the western boundary north of 35°N and along 60°N. In this example the thermocline would not surface in the southern hemisphere and the upper layer would connect to one of the adjoining basins via the Southern Ocean.

The approach taken here was to equate the value of S_n to the total upwelling between some prescribed latitude, ϕ_*, and the northern boundary, thereby deducing w. Then specifying the southernmost latitude, ϕ_0, of surfacing of the thermocline determines the distribution of the sink along the western boundary from ϕ_0 to ϕ_n. Finally, the total amount of water that sinks yields the southern hemisphere latitude (if any) at which h_w can vanish.

There are alternatives. One could specify w from the onset. Then the latitude ϕ_* would be determined. It would still be necessary to choose ϕ_0 and the remaining procedure would be the same as above. Or one might specify the total sinking in the north and carry out an implicit calculation to determine w. The value of S_n is given by (47) and the remainder of the sink would have to occur along the western boundary.

d. Separation of the western boundary current. In paper I it was shown that the western boundary current separates from the coast at the latitude where h_w vanishes. The path of the separated current was determined by the Ekman wind drift and the amount of water impinging on the northern boundary. Since both of the latter quantities are functions of the wind stress, so is the path of the separated current. In the present case the thermocline comes to the surface because the water that sinks north of the surfacing latitude is transported geostrophically in the western boundary current. There is no spatially continuous mechanism comparable to the

Ekman wind drift of the earlier study. However, as we shall see, the sinking in the north causes separation, and in this sense, it plays the same role as the wind stress does in the earlier study.

Now suppose that h_w vanishes at the western boundary. The transport balance (23) with $T = T_1 + T_{1w}$ is

$$T = S - W \tag{53}$$

If we take S as constant, we note that T must increase northward because W decreases northward. Since the flow is geostrophic, T can also be written in terms of the thermocline depth and, since h_w vanishes, we have

$$T = g'(h_e^2/2 - h_e^3/3H_1)/f \tag{54}$$

Thus, as ϕ increases T must decrease since f increases. The only way to make (53) and (54) compatible is to require that h_w vanish across the entire basin at the latitude at which it vanishes at the western boundary. Thus, upper layer water cannot penetrate beyond the surfacing latitude. Furthermore, since W vanishes at that latitude, S is also given by (54). This sinking of upper layer water takes place at the surfacing latitude but at the eastern end of the basin.

The present solution, consisting of a current that surfaces at ϕ_0 and travels due east to λ_e, is an alternative to the one obtained in section *c* where sinking takes place along the western boundary north of ϕ_0 and the current does not separate. If we use the same numerical values as in section *c*, we find that in the present case the net sink is 1.32×10^7 m^3 s^{-1} as compared to 1.6×10^7 m^3 s^{-1} when the boundary current does not separate. Which of the two is the "correct" solution cannot be determined from the limited information available in the present model. Case *c* leads to more sinking but upper-layer water occupies a broader region. The calculation of this section has upper-layer water extending north only to 35°N. Poleward of that latitude the ocean is made up of lower-layer water only. In this sense the "cooling" process is more effective even though the net sinking is smaller.

Graphically the results for the case with separation are also shown in Fig. 2. The dark band at 35°N is the eastward, upper-layer jet that separates at 35°N and travels from $\lambda = 0$ to $\lambda = \lambda_e$. Upper-layer water terminates abruptly at 35°N, with $h = 0$ on the northern side of the jet. The trajectories and h contours in the interior north of 35°N do not apply to this solution with separation.

e. Separation in the southern hemisphere. We found earlier that h_w can vanish at some latitude in the southern hemisphere. For case *a*, particularly, all of the upwelled water must flow to the south in the western boundary layer causing h_w to vanish at some latitude, ϕ_s, in the southern hemisphere. In this region, too, it is possible that there can be a distributed sink along the western boundary poleward

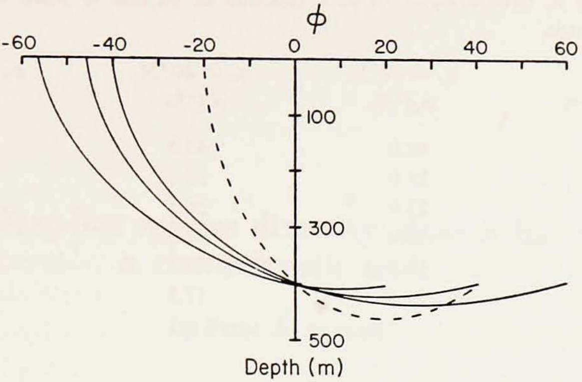

Figure 3. Values of h_w along the western boundary when $S_n = 0$ for the cases where $\phi_n = 20°, 40°$ and $60°$ with $w = 10^{-5}$cm s^{-1} and the case (dashed curve) where $\phi_n = 40°$ with $w = 3 \times 10^{-5}$cm s^{-1}. In each case h_w vanishes at some latitude in the southern hemisphere. Decreasing ϕ_n or w pushes the surfacing latitude southward.

of ϕ_s and the situation could be analogous to case *c*. However, the results indicate a much neater balance if the western boundary current separates. Thus, when h_w vanishes, equation (41) yields

$$-\frac{g'}{f_s}\left(\frac{h_e^2}{2} - \frac{h_e^3}{3H_1}\right) = W \tag{55}$$

The negative sign indicates that h_w vanishes in the *southern* hemisphere (f must be negative). After separation the jet extends to the eastern boundary where it sinks. The transport of that jet is also given by (55), i.e., it equals exactly the amount of water that has upwelled north of ϕ_s. Therefore, a complete transport balance is achieved. South of ϕ_s the ocean contains only lower-layer water.

The latitude of separation in the southern hemisphere depends on the various parameters of the model, especially on the magnitude of w and the area of upwelling. Several cases have been calculated with $h_e = 400$ m, $\Delta\rho/\rho_2 = 0.0015$ and $\lambda_e = 110°$. In all cases, it is assumed that there is no sinking north of the equator (case *a*). The latitude ϕ_s then depends on ϕ_n and w. In Fig. 3 the values of h_w are plotted as a function of latitude for $w = 10^{-5}$cm s^{-1} and $\phi_n = 60°, 40°$ and $20°$. Also shown is the case with $w = 3 \times 10^{-5}$cm s^{-1} and $\phi_n = 40°$ where the increased amount of upwelling leads to thermocline surfacing at a much decreased latitude, i.e., upper-layer water occupies a much more limited region.

Table 2 lists ϕ_s for all of the cases calculated. The table shows how the total amount of upper-layer water is decreased by increasing w and also how decreasing ϕ_n pushes ϕ_s southward.

Table 2. Values of ϕ_s (where $h_w = 0$) as a function of ϕ_n and w when there is no sinking north of the equator.

w (10^{-5}cm s^{-1})	$\phi_n = 60°N$ ϕ_s(°S)	$\phi_n = 40°N$ ϕ_s(°S)	$\phi_n = 20°N$ ϕ_s(°S)
1.0	40.0	45.5	56.0
1.5	29.0	33.0	41.0
2.0	23.0	26.5	33.5
2.5	19.0	22.5	28.5
3.0	16.5	19.5	25.0
3.5	14.5	17.5	22.5

5. Remarks

As mentioned earlier it is not possible to obtain exact results with this parameterized form of heating and cooling unless the effective cooling (value of S in the different regions) is specified. We have considered only *types* of behavior possible within the framework of the model.

There are three reasons for the decision to consider only possibilities rather than to try to obtain more definitive results. The first is that the oceans are driven by wind stresses as well as by thermal differences and there are no observational data to guide us in the choice of pertinent parameters for this restricted model. The second is that the form of the interior solution is qualitatively different when wind stresses are included. In particular, upper-layer water is transported poleward by cyclonic wind-stress curl in contrast to the southward interior flow forced by the upwelling. Poleward interior transport provides a certain amount of water that can sink; indeed, perhaps all of the poleward flow of upper-layer water that eventually sinks may be *via* the interior. The third reason is that the region occupied by lower-layer water only can be directly driven by the wind in contrast to the present model where that region must be quiescent. Hence, discussion of source and sink strengths is best postponed until the combined wind-stress, upwelling model is analyzed. The present calculations supplement those of paper I and were made primarily to facilitate the interpretation of the results for the combined model.

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