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The optimum wiggliness of tidal admittances

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ABSTRACT

Some numerical experiments with recent offshore tide measurements have examined various parameters involved in tidal prediction by the response method: the number of prediction weights, their lead (and lag) times, and the treatment of radiational tides. The optimum number of weights depends directly on the length of record and inversely on noise level in a tidal band; more weights degrade the prediction and generate an artificial wiggliness in the admittance.

1. Introduction

The incentive for the present study came from our tidal analyses of deep bottom pressures ($5^1/_2$ km depth) in the MODE area (Zetler *et al.*, 1975) and of shallow bottom pressures in Baltasound off Unst, Shetland Islands (SCOR Working Group 27, 1975). In the MODE case the purpose was to remove the tides with sufficient accuracy to reveal fluctuations of longer period and much smaller amplitude. The SCOR analysis was organized in conjunction with an international intercalibration exercise for bottom sensors (both shallow and deep) west of Brest, France. Here the concern was that variations in the results are related, not to differences in instrumentation, but rather to differences in analysis (they are apt to).

2. Reference series

We briefly review the response formalism. For any linear[2] system, an input function $x_m(t)$ and an output function $x_n(t)$ are related according to

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2. The formalism can be extended to weakly nonlinear systems.

$$x_n(t) = \int_{-\infty}^{\infty} x_m(t-\tau)\, w_{mn}(\tau)\, d\tau + \text{noise}(t) \tag{1}$$

where $w_{mn}(\tau)$ is the "impulse response" of the system, and its Fourier transform

$$Z_{mn}(f) = \int_{-\infty}^{\infty} w_{mn}(\tau)\, e^{-2\pi i f \tau}\, d\tau = R_{mn}(f)\, e^{i\varphi_{mn}(f)} \tag{2}$$

is the system admittance (coherent output/input) at frequency f. If input and output are alike, then evidently

$$w(\tau) = \delta(\tau), \quad \text{and} \quad Z(f) = 1, \tag{3}$$

where $\delta(\tau)$ is Dirac's delta-function. For further discussion of the response method we refer to Munk and Cartwright (1966) and to Cartwright *et al.* (1969).

The central concept is to select a reference (input) series of suitable length and quality, and also similar (in the sense of equation (3)) to the record to be analyzed. For the MODE record, the obvious choice is Bermuda. As an island station, it does not suffer an appreciable coastal distortion (unlike continental stations) and is therefore quite representative of open sea conditions. The separation by 700 km is moderate by tidal standards. For Baltasound, although the choice is not as clear-cut for the 15-day series as for the 29-day series, the tide potential is preferable to Lerwick (nearby) as reference.

Choice of reference is related to the treatment of nongravitational excitation, and will be discussed under "radiational tides".

3. Number of weights

Our first empirical attempt to determine an optimum number of weights in a response analysis is based on a MODE two-month bottom pressure series, EDIE MAY Pl, relative to Bermuda (Fig. 1). There is a systematic reduction in ratio of residual to recorded variance as additional weights are added to the analysis. The number of weights was experimental and goes far beyond what one might reasonably use routinely. The residual variance decreases first rapidly, and then slowly with an increasing number of weights. We surmise that the initial sharp decrease is related to a removal in the tidal contribution. Beyond the abrupt change in slope, any further weights presumably serve only to reduce the noise (continuum) portion of the record. Eventually the residual goes to zero as the number of weights equals the number of data points. Fig. 1 shows rather sharp changes in slope at 2 (complex) weights for the diurnal tides, and at 3 weights for the semidiurnal tides.

We suspect that going beyond these turning points does not improve a *future* tide prediction, inasmuch as the noise that is so removed is non-periodic and unrelated to the noise of future (or past) records. To test this, we have divided the two-month record into two one-month sections, A and B. "Self-prediction" consists of using section A to compute A weights and determine A residuals; similarly, B weights to determine B residuals. On the other hand, "prediction" refers to A residuals from B weights, or B residuals from A weights. The results (Fig. 2) resemble those in Fig. 1 in the case of self-prediction, showing first a sharp and then a gradual decrease of variance with increasing number of weights. But predictions indeed deteriorate beyond a critical number of weights, and this critical number corresponds to the abrupt change in slope of the self-prediction.

In the MODE analyses we used 2 weights for diurnal tides and 3 weights for semidiurnal. We attribute the ability to use a more refined solution for semidiurnals to two factors: the tidal lines are larger, and the sea level continuum is lower (it decreases with increasing frequency). Both factors contribute to a greater signal-to-noise ratio for semidiurnal tides.

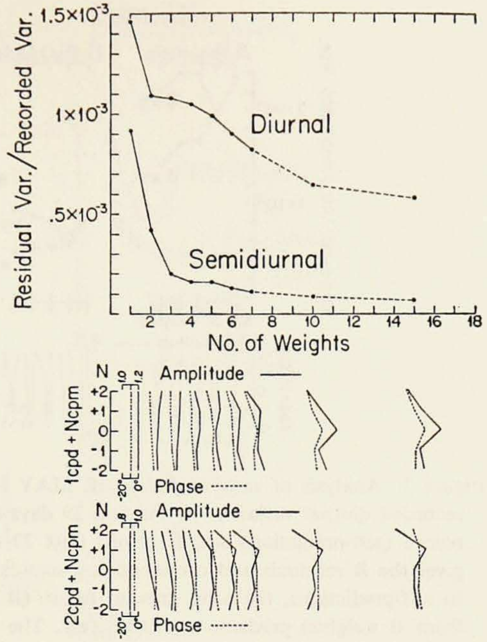

Figure 1. Analysis of residual variance for EDIE MAY Pl, two months' self-prediction. The upper part shows ratio of residual to recorded variance for 1 and 2 cycles per day (cpd) for 1, 2, . . . 7, 10 and 15 complex weights. The lower half shows the corresponding admittances relative to Bermuda reference, solid lines for amplitude and dashed lines for phase, for 1 cpd ± 0, 1, 2 cycles per month (cpm) and 2 cpd ± 0, 1, 2 cpm for the same numbers of weights.

a. Wiggliness of admittances. In a response analysis, the measured series is fitted in the least-square sense by a weighted sum of the reference series for various leads or lags. For example, a single complex weight (1 + 0i) for zero lag corresponds to identical series; 0 + 2i to a measured series in quadrature with the reference series and of twice its amplitude.

As noted previously, admittances are Fourier transforms of weights. The solid and dashed vertical lines on the left side of Fig. 1 for 1 cpd + N cpm show a diurnal amplitude of 1.18 and a phase lag of 8° obtained for a single (complex) weight. Munk and Cartwright (1966) recommend lag intervals of two days. For 2 complex weights (lags 0, 2 days) the admittance is now a smoothly varying function of frequency. For additional weights, the admittances become increasingly more wiggly, in part (one surmises) as a result of the noise content. The trick is to terminate when one's credo in the smoothness of oceanic admittances is violated. For diurnal tides in Fig. 1, one might be tempted to stop after 3 weights, one more than indicated by the inflection point in the residual plot. However, a comparison of A and B admit-

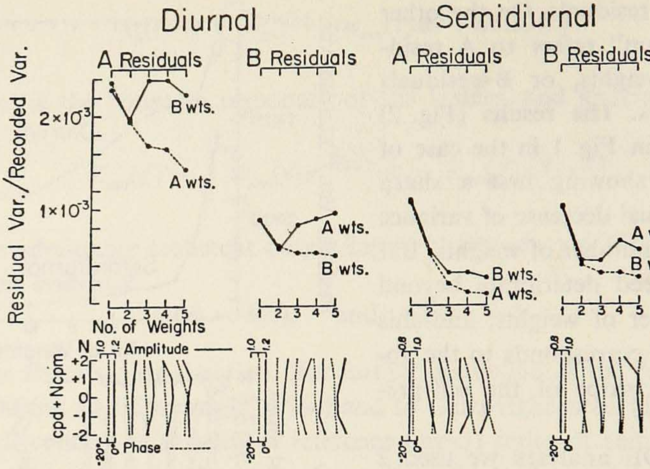

Figure 2. Analysis of variance for EDIE MAY Pl. The left upper panel gives the ratio residual/recorded diurnal variance for the first 29 days (record A), with prediction weights based on A record (self-prediction) and B record (last 29 days), respectively. The remaining upper panel gives the B residuals and corresponding semidiurnal ratios. In each case the dashed lines refer to self-predictions, the solid lines to future (B residuals from A weights) or past (A residuals from B weights) predictions, respectively. The lower panels give the corresponding amplitude ratios (solid) and phase lags (dashed) relative to Bermuda reference, as a function of frequency (1 cpd ± 0, 1, 2 cpm for diurnals, 2 cpd ± 0, 1, 2 cpm for semidiurnals), for 1, 2, . . . 5 complex weights. With increasing number of weights the self-prediction residuals (but not necessarily those for the future predictions) diminish, and the admittances become increasingly wiggly.

tances for diurnal tides in Fig. 2 shows comparable results for 1 and 2 weights, but significantly different admittances for 3 weights, consistent with the increase in future residuals at this point. For semidiurnals, a choice of 3 weights is in accord with the inflection point in the residuals and the match in A and B admittances.

The jumps in the admittances with increased number of weights occur at those frequencies where the tidal lines are weak. Admittances at the strong tidal lines[3] do not change significantly with increasing number of weights. Again, this is in accord with signal-to-noise ratio being the important consideration.

b. Long series. Given a ten-year series of hourly heights at Bermuda, we computed a combined analysis for three 355-day series, equally spaced in a period of lunar perigee (8.85 Julian years). For the principal gravitational input series, G_2^1 and G_2^2, we tried 1, 2, 3, 4, 5, 7 and 10 lags to provide comparable numbers of complex weights for these series. The three series were then self-predicted and mean residuals obtained for the tidal bands. Using these weights, we also prepared future predictions for two other 355-day series, spaced midway between the analyzed first and second, and second and third, series, respectively. The tidal residual variances were computed

3. O_1 at 1 cpd − 1 cpm, K_1 and P_1 at 1 cpd + 1 cpm for the diurnals; N_2 at 2 cpd − 1 cpm, M_2 at 2 cpd, S_2 and K_2 at 2 cpd + 2 cpm for the semidiurnals.

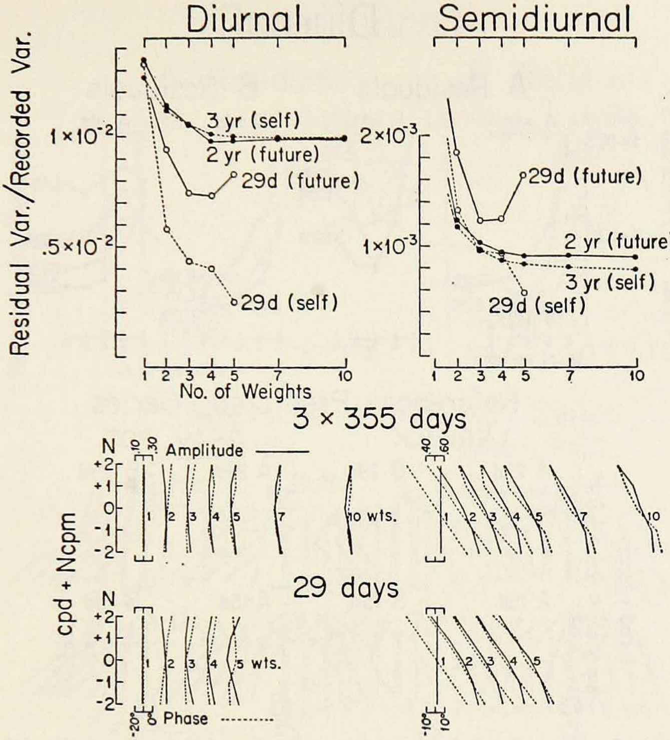

Figure 3. Analysis of residual variance for a long series at Bermuda. The upper part shows ratio of residual to recorded variance (self-prediction) for 1 and 2 cpd for 1, 2, . . . 5, 7 and 10 complex weights for a combined 3×355-day analysis and, for comparison purposes, 1, 2, . . . 5 weights for a 29-day analysis. Comparable values are furnished for future predictions for a 2×355-day series and a 29-day series. The lower half shows the corresponding admittances relative to the reference series of gravitational potential, solid lines for amplitude and dashed lines for phase, for 1 cpd ± 0, 1, 2 cpm and 2 cpd ± 0, 1, 2 cpm for the same numbers of weights.

on a combined basis for these two years. Finally, a similar test was made with two 29-day Bermuda series, analyzing the first to predict both, and obtaining residuals for each. For the 29-day series, the tests were terminated at 5 weights.

The results for the 29-day series are similar to the MODE results, with cutoffs indicated at 3 weights (Fig. 3). The long-series plots are quite different in that the self-prediction (3-year) and future prediction (2-year) give nearly the same residuals up to the 10-weight limit. There is little (if any) improvement after 4 weights (we have used 5 weights in our MODE analyses). However, unlike the short-series plots, there is no significant deterioration in future predictions in going to a larger number of weights.

Tidal admittances for the 29-day series exhibit an increased wiggliness beyond 4 weights. The long series also shows a wiggliness for 4 or more weights but, unlike the short series, the wiggliness does *not* increase beyond that point. This implies that

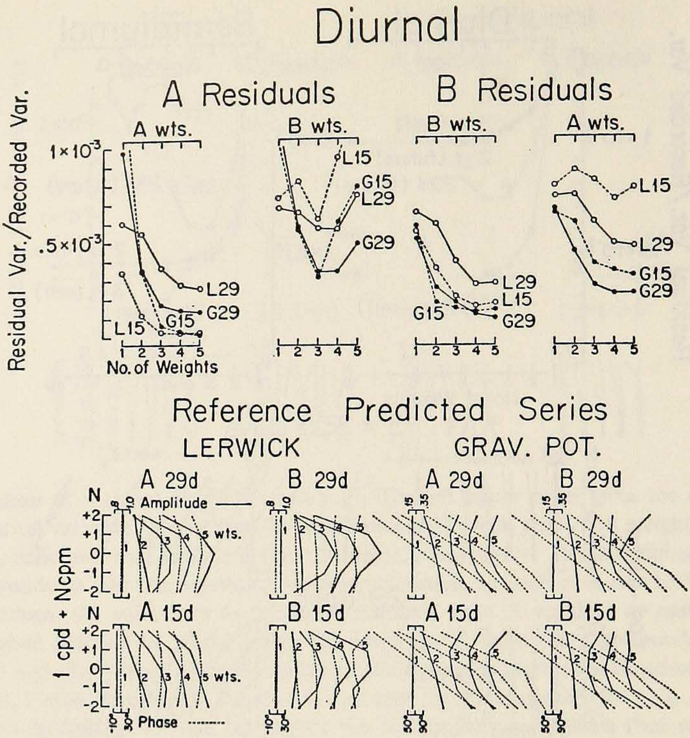

Figure 4. Analysis of variance for Baltasound test series A and B for 1 cpd. The left upper panel shows residual variance from an analysis of the A series (self-prediction) for 29 days using a Lerwick complex harmonic prediction (L29) as reference, the same for 15 days (L15), a gravitational potential prediction for 29 days as reference (G29), and the same for 15 days (G15). The next panel shows A residuals using B weights for prediction (future) and the last two panels show comparable B residuals using B weights (self-prediction) and A weights (future). Solid lines denote 29-day analysis, dashed lines 15-day analysis, open circles Lerwick as reference, and closed circles gravitational potential as reference. The lower panels give the corresponding amplitude ratios (solid) and phase lags (dashed) for the indicated reference series and length of record as a function of frequency (1 cpd ± 0, 1, 2 cpm) for 1, 2, ... 5 complex weights.

the wiggliness is real (though not adequately portrayed by points plotted 1 cpm apart). There must be some cutoff in number of weight beyond which the continuum has an impact, but this seems to be beyond 10 weights.

c. Baltasound. Similar plots were prepared for 15- and 29-day series, and for the gravitational potential and Lerwick as input series (Figs. 4 and 5). The "future" residuals for both 1 and 2 cpd in the 29-day analysis indicate clearly the advantage in choosing a grav reference and 3 weights. The turning points on the self-prediction residuals in general conform to our choice. Admittances are reasonably smooth and alike up to but not beyond 3 weights.

For the 15-day analysis, a decision is more difficult. For Lerwick as reference we

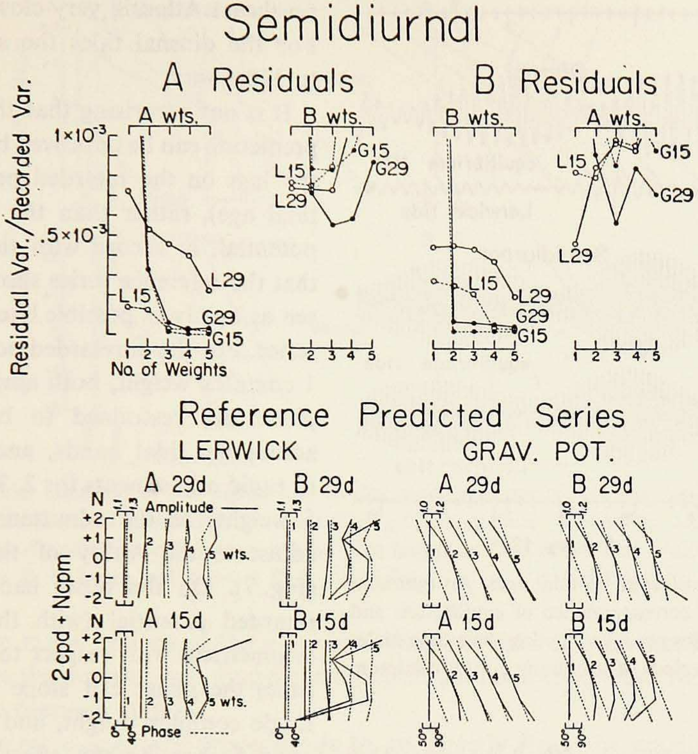

Figure 5. Same as Figure 4 for semidiurnal tides.

would use 1 weight; for grav, 3 weights again seem indicated. There is a significantly smaller variance using the grav prediction as reference. Admittances generally support a cutoff at 3 weights in the grav 15-day plots. The difficulty in making a decision for the 15-day analysis matches our intuition that as the available length of series becomes shorter, there is an increased desirability to match the data to a well-established nearby reference.

4. The retarded potential

In the past application of the response method, the lag times τ (Equ. 1) have been taken symmetrical (plus and minus) with respect to the prediction time.[4] Thus, the prediction is written as a sum of weighted future and past values of the reference series. At most European ports, the *modulation* in the tides lags the tide potential (Fig. 6). For example, the semidiurnal spring tide at Lerwick lags equilibrium spring tide by 36 hours. This is known as the *age* of the tides, and can be associated with a rapid change in phase across the semidiurnal admittances: age = 0.984 (S_2°–M_2°) hours. Garrett and Munk (1971) have attributed the effect to a resonance of the

4. This does not, of course, imply that $w(+\tau) = w(-\tau)$.

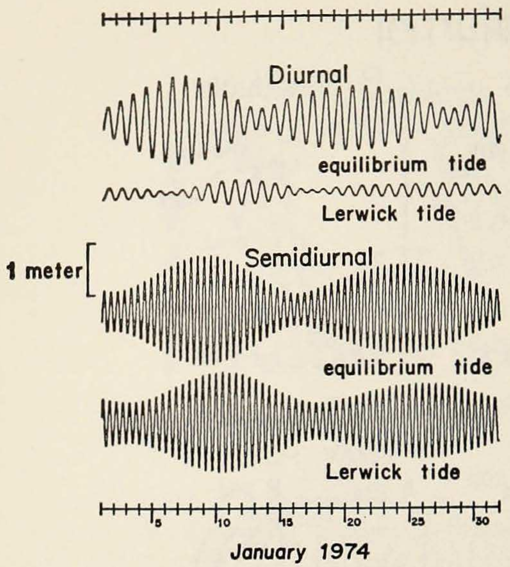

Figure 6. Illustration of tidal ages. An optimum one-to-one correspondence of equilibrium and Lerwick tides requires aligning their time scales to match periods of maximum and/or minimum ranges.

northeast Atlantic very close to 2 cpd. For the diurnal tides the age is close to 120 hours.

It is not surprising that the response prediction can be improved by centering the lags on the retarded potential (by tidal age), rather than the unretarded potential, in accord with the principle that the reference series should be chosen as nearly as possible like the output series. For the unretarded potential and 1 complex weight, both amplitude and phase are restrained to be constant across the tidal bands, and this leads to rapid adjustments for 2, 3, 4, or even 5 weights as the admittance seeks to adjust to the reality of the situation (Fig. 7). On the other hand, for the retarded potential (with the lags unsymmetrical with respect to prediction time) the phase can slope for even a single complex weight, and an acceptable fit is achieved with 3 weights. Figs. 4 and 5 showing the effect of weight numbers on residual were drawn for the retarded potential.

5. Radiational tides

The last, but not least, complexity associated with tidal prediction is the generation of tidal oscillations by nongravitational effects. Examples are the diurnal variations in the land-sea breeze regime, the semidiurnal atmospheric pressure fluctuations, etc. The nongravitational effects are too large to be ignored. The response procedure is to introduce, in addition to the gravitational potential $x_p(t)$, a "radiational potential"

$$x_p'(t) \sim \cos\,(\text{zenith angle}) \quad \text{in daytime}$$
$$= 0 \quad \text{in nighttime,}$$

in the expectation that the nongravitational oscillations are in some vital way connected to the variable radiant flux. Equation (1) is modified to include an additional term $\int_{-\infty}^{\infty} x_m'(t-\tau)\; w_{mn}'(\tau)\,d\tau$. The impulse responses w and w' are disentangled by least-squares, taking advantage of the difference in the spectral line structure of x_p and x_p' (basically because the Earth is transparent to gravity and opaque to radiation).

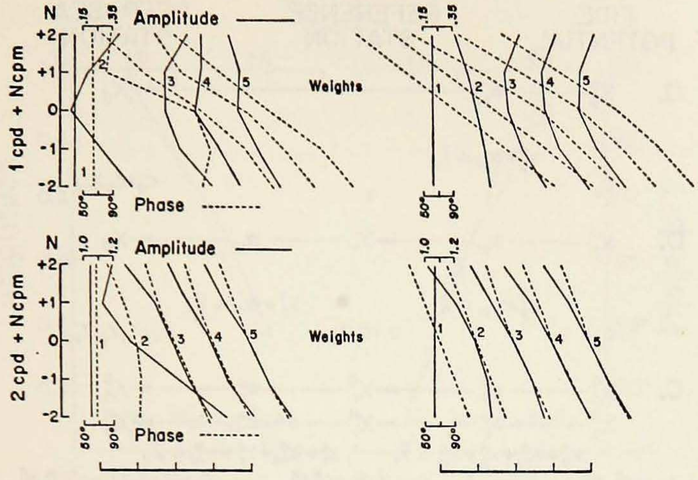

Figure 7. Tidal admittances for Baltasound 29-day series referred to the gravitational potential (left) and the potential retarded by tidal age (right), 120 hours and 36 hours for 1 cpd and 2 cpd, respectively. Amplitude admittances are solid lines and phase lags are dashed. Values are plotted for 1 cpd ± 0, 1, 2 cpm and 2 cpd ± 0, 1, 2 cpm for 1, 2, ... 5 comples weights.

There are many options in the way the radiational tides can be handled (Fig. 8). In each case the purpose is to analyze in an optimum manner an observed time series $\tilde{x}_d(t)$ at the deep-sea station.

a. The most elementary procedure is to go directly from the gravitational tide potential x_p^G to the deep-sea record $\tilde{x}_d$, with the weights w_{pd} determined by

$$x_p^G * w_{pd} = x_d \doteq \tilde{x}_d$$

with the convolution * equivalent to equation (1), and the symbol $\doteq$ designating a least-square fit of the predicted deep-sea tide x_d to the observed deep-sea tide $\tilde{x}_d$. Classical analysis consists of evaluating the amplitude H and epoch G of the principal tidal constituents. Let $C_p = H_p e^i G_p$ refer to the (complex) amplitude of some constituent (M_2, say) in the tide-producing potential, and C_d at the deep-sea station. Then

$$C_d = C_p Z_{pd}. \tag{4}$$

Option (a) is the only choice if no suitable reference station is available. The trouble is that for the relatively short deep-sea records now available, one or several months, there is inadequate resolution to separate gravitational from radiational effects in the deep-sea record. Hence w_{pd} and Z_{pd} will be a mix of gravitational and radiational effects and not easily interpretable.

b. This is an intermediary solution, assuming a suitable reference station of moderate length (over a year) is available. The reference series $\tilde{x}_r$ is not of sufficient length for a definitive separation of gravitational and radiational effects, but can

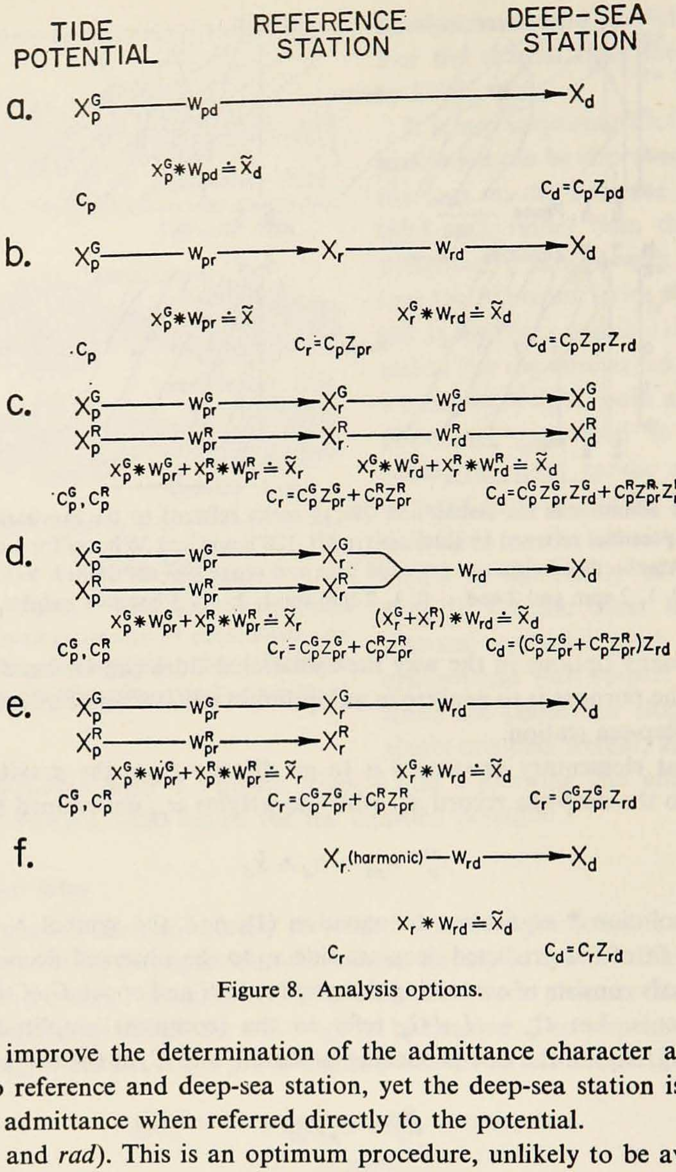

Figure 8. Analysis options.

be used to improve the determination of the admittance character assumed to be common to reference and deep-sea station, yet the deep-sea station is too short to resolve the admittance when referred directly to the potential.

c. (*Grav* and *rad*). This is an optimum procedure, unlikely to be available in the foreseeable future. Not only is there a good nearby reference series long enough (> 9 years) to separate the gravitational and radiational contributions, but the deep-sea series is of moderate length (over a year). Gravitational and radiational harmonic constants are computed separately from the appropriate admittances and then combined vectorially into grav-rad harmonic constants.

d. (*Grav* + *rad*). A long reference series is available (> 9 years), but the deep-sea series is short (several months). The rationale in summing the gravitational and radiational predictions for the reference station is that the radiational contributions

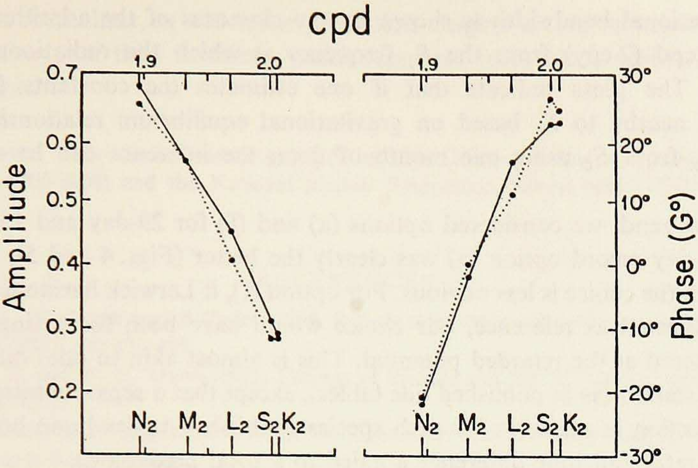

Figure 9. Amplitude and phase admittances for Bermuda referred to the gravitational potential. The solid lines are grav admittances from (grav and rad) response analysis; the dotted lines are admittances from traditional analysis. The response analysis was for three 355-day series over a nine-year period. The traditional analysis (1 year, 1934, IHB Spec. Pub. 26, # 600) furnishes a lumped (vector) sum of grav and rad admittances.

are comparable at both areas and in the same proportion to the gravitational contributions.

e. (*Grav*). Again a long reference series is available, but the deep-sea station is referred only to the gravitational component of the reference station.

f. (*Traditional*). The harmonic constants for a reference station are available and can be used for a harmonic tide prediction to serve as reference. This is the classical method which does not distinguish between gravitational and radiational effects.

Both options (d) and (e) were used to analyze the relatively short MODE records, using a long Bermuda reference series. If the Bermuda radiational tides were largely related to a local phenomenon, such as island onshore-offshore breezes, then (e) would give the better results. If, however, Bermuda and the MODE area are subject to a coherent radiational effect, such as the global barometric tides, or if the radiational contribution (whatever its source) is subject to a modification in common with the gravitational component ($w^R_{rd} \approx w^G_{rd}$), then (d) would seem the better. (For a very long deep-sea record, option (c) would have been the obvious choice.) We found that option (d) gave somewhat lower residuals and much smoother admittances than (e). In the latter case there were sharp anomalies in admittance at solar frequencies.

Some of these effects can be demonstrated with the Bermuda record (Fig. 9). For the present discussion we regard Bermuda as the deep-sea station and the tide potential as the reference series, thus referring only to the right-hand portion of Fig. 8. The grav and rad option leads to smooth grav admittances, whereas traditional analysis shows a discontinuity centered at the solar frequency S_2. The narrowness of the radiational bandwidth is shown by the closeness of the admittances at K_2, only .0055 cpd (2 cpy) from the S_2 frequency at which the radiational tides are maximum. The plots indicate that if one estimates the constants for a tidal constituent nearby to S_2 based on gravitational equilibrium relationships, as for example K_2 from S_2 using one month of data, the inference can be significantly in error.

For Baltasound, we considered options (a) and (f) for 29-day and 15-day series. For the 29-day record option (a) was clearly the better (Figs. 4 and 5), but for the 15-day series the choice is less obvious. For option (f), if Lerwick harmonic prediction had been selected as reference, our choice would have been for a single complex weight centered at the retarded potential. This is almost akin to tidal differences on a reference station (as in published tide tables), except that a separate ratio and phase (time) correction is available for each species (published tables lump both together in a single ratio and time difference applied to a total prediction).

As a supplementary exercise, we repeated the 29-day analyses using response weights for Lerwick derived from six years of data. With these we followed option (d), referring the test data to a summed complex prediction for Lerwick. The residual variances obtained were significantly smaller than those previously obtained by options (a) and (f)).

6. Discussion

This paper deals with a series of numerical experiments, with no attempt at statistical controls. It will have occurred to the reader that the device of splitting records to distinguish between self-prediction and (real) prediction is not required if the results are compared to adequate statistical models, and further, that residual variance and admittance wiggliness are clearly related and can be combined in a single diagnostic for optimum procedures.

We wish to refer to some recent progress in developing statistical models of tide analysis. McMurtree and Webb (1974) have effectively displayed complex admittances of Australian ports as a series of Argand diagrams. For smoothing purposes they expand the admittances in a power series of frequency f (Munk and Cartwright use a power series of $z = \exp 2\pi i f t$ and z^{-1}), and examine the significance of any added parameters. A new feature is the inclusion of a pole term $(f - f_1)^{-1}$ if there are resonances near the tidal band.

Lambert (1974) has applied the response method to tidal gravity and tilt data, with a careful analysis of the significance of any additional parameters. Employing the t-distribution, the null-hypothesis is tested: that there is no significant adjustment in the admittances when the number of lags is increased. If the hypothesis is found to be false, then the number of lags is increased. Lambert finds that a small number of complex weights (≤ 5) accurately and concisely describe the Earth tide, even in regions of complex ocean tide perturbations.

Finally, Groves and Reynolds (1974) have suggested a marked improvement over the response formalism, in which the prediction weights are written as a converging series of orthogonal tidal functions which represent successively more wiggliness in the admittance.

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REFERENCES

Cartwright, D., W. Munk and B. Zetler. 1969. Pelagic tidal measurements—a suggested procedure for analysis. EUS, *50*: 472–477.

Garrett, C. J. R. and W. H. Munk. 1971. The age of the tide and the "Q" of the oceans. Deep-Sea Res., *18*: 493–503.

Groves, G. W. and R. W. Reynolds. 1974. An orthogonalized convolution method of tide prediction. Submitted to J. Geophys. Res.

Lambert, A. 1974. Earth tide analysis and prediction by the response method. J. Geophys. Res. 79, 32: 4952–4960.

McMurtree, R. and D. J. Webb. 1974. Tidal response functions around Australia from harmonic constants. In press, Australian J. of Marine and Freshwater Res.

Munk, W. H. and D. E. Cartwright. 1966. Tidal spectroscopy and prediction. Phil. Trans. Roy. Soc., A, *259*: 533–581.

Zetler, B., W. Munk, H. Mofjeld, W. Brown and F. Dormer. 1975. MODE tides. In press, J. Phys. Oceanogr.

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