Tide-well Systems III: Improved Interpretation of Tide-well Records[1]

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ABSTRACT

Recent studies of the non-linear response of the conventional tide-well system have pointed out the deficiencies of the system when the tide records are to be used for scientific purposes requiring very accurate data. An experimental method of checking the response of such tide-wells, by means of a drainage test, is described. This provides the basis of a further method, by which the height of the water level in the well may be approximately corrected to give the height of the sea level outside the well. The application of these methods, to a recently advocated linear tide-well system, is also described.

1. Introduction. In a number of papers (e.g. Noye 1970, 1974a,b), the response of several types of tide-well systems has been examined in detail. For the conventional tide well, which has an orifice connecting the water in the well with the sea outside, it was shown that, for a narrow deep well, the governing differential equation has the form

$$dH_w/dt + C_1 |H_w|^{1/2} \operatorname{sgn}(H_w) = dh_0/dt, \tag{1.1}$$

where t is the time and $H_w(t)$ is the head response of the tide well to the fluctuations in sea level, $h_0(t)$.

$$H_w = h_0 - h_w, \tag{1.2}$$

where h_w is the height of water in the well, referred to the same datum as the sea level. C_1 is a constant, depending only on the dimensions of the tide well, viz.,

$$C_1 \simeq C_c (2g)^{1/2} A_p / A_w, \tag{1.3}$$

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where A_w is the uniform area of cross-section of the well, A_p is the area of the orifice, g is the acceleration due to gravity and C_c is the contraction coefficient of the orifice, generally taken to be $C_c \simeq 0.6$.

For the tide well with a long horizontal pipe connection near the sea floor, it was shown that, provided a parameter $N \geq 5$, the head response to the fluctuating sea level is given by the following relation, derived assuming steady Poiseuille pipe flow,

$$dH_w/dt + C_2 H_w = dh_0/dt, \tag{1.4}$$

where the tide-well constant, C_2, is given by

$$C_2 = gD_p^4/32 v L_p D_w^2. \tag{1.5}$$

Here v is the kinematic coefficient of viscosity of sea water, L_p is the length and D_p the uniform diameter of the pipe connection, D_w is the uniform diameter of the circular well and

$$N = \frac{128 v^2 L_p D_w^2}{g D_p^6}.$$

The response of the non-linear system described by equation (1.1) is dependent on the parameter C_1 (Noye, 1974a); the response of the linear system described by equation (1.4) depends on C_2 while, for $N < 5$, unsteady effects in the upper pipe flow produce a more complicated form for the response (Noye, 1974b).

In many tide-well installations, the value of the tide-well constant (C_1 or C_2) is not well known. For instance, the orifice of a conventional tide well might not be sharp edged or exactly circular, so that the contraction coefficient C_c might be unknown, or marine growth might have reduced its area after installation. Methods of determination of the tide-well constants and the response function from drainage tests, in the field or the laboratory, can overcome this problem.

For the conventional tide well, an experimental method for finding C_1 and hence the pseudo-response of the system is described in the next Section. For a linear system with $N \geq 5$, the quasi-steady theory is a good approximation and the method described in Section 2 may be used. For $N < 5$, a more general method is applicable. In addition, two cases which can occur if the well is tested in situ, when the water level outside the well does not remain constant during the tests, are described.

The response of conventional tide wells seems well established for two regimes of sea-level oscillations; there is negligible response to wind waves (apart from "set-down") and unit response with no lag to oscillations of tidal period. For oscillations such as tsunamis and harbour seiches, which have periods between these two regimes, the response varies according to the dimensions of the tide well and the amplitudes and periods of the oscillations. In Noye (1974a), it was shown that overall response functions could assist in corrections for time lag and attenuation in the case of tsunamis, for example, but there appeared to be no way of converting the energy density spectrum of the oscillations in the well into the spectrum of the oscillations outside. In fact, comparing the spectral analysis of a record from a conventional tide well with that of a tsunami recorder, such as a well with a small pipe connection or a pressure recorder on the sea floor, could show differences simply due to spurious oscillations in the tide-well record arising from non-linear effects at the orifice. These differences could be mistakenly attributed to the effect of different bottom topography on the incident waves. Recently, several such comparisons have been made; for instance, Hatori (1967), in a study of the effects of the continental shelf on a tsunami, compared records and their spectra from a tide well at Onagawa and a tsunami recorder at Enoshima. One way of making such a comparison is to convert, before analysis, the readings of water level in the conventional tide well to corresponding values of sea level. Such a method is described for a tide well with an orifice, a linear tide-well system, and a mixed system such as a tide well with a short tube connection in Section 3.

2. Frequency Response Determined from Drainage Tests. The frequency response of a causal linear system can be obtained once the response $R(t)$ to any input $I(t)$ is known, provided $I(t) = 0$ for $t \leq 0$. The transfer function of the system is

$$F(s) = \frac{L[R(t)]}{L[I(t)]}, \tag{2.1}$$

where L is the Laplace transform operator, viz.

$$L[f(t)] = \int_0^\infty f(t) \exp(-st)\,dt.$$

The amplitude response of the system is then given by

$$\alpha(\omega) = |F(i\omega)|, \tag{2.2}$$

and the phase lag by

$$\varepsilon(\omega) = -\operatorname{Arg} F(i\omega), \tag{2.3}$$

where ω is the circular frequency.

If the level outside a well is constant and an excess head of water H_s inside is allowed to drain out, the fall in level inside is the response $R(t)$ to the step $I(t) = H_s U(t)$ outside, where $U(t)$ is the unit step function. If $f(t)$ is the residual head of water in the well after time t,

$$R(t) = H_s - f(t), \quad t > 0. \tag{2.4}$$

Then

$$F(s) = 1 - \frac{s}{H_s} L[f(t)] \tag{2.5}$$

and

$$\left.\begin{aligned} F(i\omega) &= 1 - \frac{i\omega}{H_s}\int_0^\infty \exp(-i\omega t) f(t)\,dt \\ &= H_s^{-1}\{H_s - \omega S(\omega) - i\omega C(\omega)\}, \end{aligned}\right\} \tag{2.6}$$

where C is the Fourier cosine transform of $f(t)$, viz.,

$$C(\omega) = \int_0^\infty f(t) \cos \omega t \, dt,$$

and S is the Fourier sine transform of $f(t)$, viz.,

$$S(\omega) = \int_0^\infty f(t) \sin \omega t \, dt.$$

Hence, the amplitude response of a tide well with a long pipe connection is given by

$$\alpha_2(\omega) = H_s^{-1}\{(H_s - \omega S)^2 + \omega^2 C^2\}^{1/2} \tag{2.7}$$

and its phase lag by

$$\varepsilon_2(\omega) = \arctan\{\omega C (H_s - \omega S)^{-1}\}. \tag{2.8}$$

Reading the values of $f(t)$ at equal time intervals as the well drains, until some time T when $f(t)$ is negligible, one can approximately compute C and S for various ω using

$$C(\omega) \simeq \int_0^T f(t) \cos \omega t \, dt$$

and

$$S(\omega) \simeq \int_0^T f(t) \sin \omega t \, dt.$$

The amplitude response α_2 and phase lag ε_2 can then be estimated for these values of ω.

a. *The Linear System with* $N \geq 5$. In this case, the process of computing the frequency response from a drainage test can be simplified considerably, since the system can be described by equation (1.4). Solving this equation, with

$$h_0(t) = I(t) = H_s, \quad t \geq 0, \tag{2.9}$$

and initial condition

$$h_w(0) = R(0) = 0, \tag{2.9a}$$

yields

$$h_w(t) = R(t) = H_s - H_s \exp(-t/T_0), \tag{2.10}$$

where $T_0 = 1/C_2$. Clearly, in this case the residual head of water in the well after time t is

$$f(t) = H_s \exp(-t/T_0). \tag{2.11}$$

After substitution in (2.5), this yields the response function

$$F(i\omega) = (1 + i\omega T_0)^{-1}, \tag{2.12}$$

from which is obtained the amplitude response

$$\alpha_2(\omega) = (1 + \omega^2 T_0^2)^{-1/2} \tag{2.13}$$

and the phase lag

$$\varepsilon_2(\omega) = \arctan(\omega T_0). \tag{2.14}$$

Once the time constant T_0 is found, the response of this system is known. In practice, two methods have been used to do this; the graph of the fall of the water level inside the well can be fitted by an exponential curve of the form

$$A \exp(-t/T_0) + B,$$

where A, B and T_0 are constants chosen to minimise the sum of the squared errors, or T_0 can be read from the drainage curve, since it is the time taken for the excess head of water inside the tide well to fall to $\exp(-1) = 0.37$ of its initial value.

For the system to be linear during drainage tests, the flow in the pipe must be laminar. Therefore, the permissible magnitude of the step has an upper bound.

Womersley (1955) showed that, for a given pressure gradient which is sinusoidal in time,

$$Q = \eta Q_s, \tag{2.15}$$

where Q is the rate of flow through the pipe, Q_s is the corresponding flux assuming steady flow conditions and $0 \leq \eta \leq 1$ is an attentuation factor. Hence, at any instant during drainage,

$$V < V_s, \tag{2.16}$$

where V is the mean velocity in the pipe and V_s is the mean velocity of the steady flow assumed to occur for the same pressure gradient.

Using the very restrictive requirement that laminar flow occurs under unsteady conditions, provided that the Reynolds number never exceeds 1300 (Noye, 1974b), the inequality

$$D_p V v^{-1} < 1300 \tag{2.17}$$

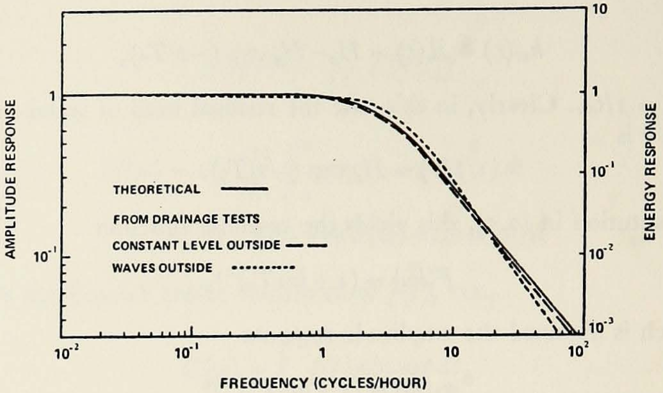

Figure 1. Theoretical and experimental values of amplitude and energy response of the water level indicator used in the Coorong experiment.

must be satisfied. However, for Poiseuille flow,

$$V_s = \left(\frac{gD_p^2}{32\,v\,L_p}\right) f(t), \tag{2.18}$$

where $f(t)$ is the excess head in the tide well. Using the relations (2.16) and (2.18) in (2.17), it follows that the flow must be laminar if, throughout draining, the step used satisfies

$$H_s < \frac{42000\,v^2 L_p}{gD_p^3}. \tag{2.19}$$

Applying this criterion to the water level indicator used in the Coorong experiment and described in Noye (1974b), the permissible head for the drainage test is found to be 5 cm. For this step, with the recording system set at a fast chart speed, values of the residual head $f(t)$ were obtained at half second intervals. The amplitude response was then derived by finding the time constant T_0 and applying (2.13). This experimentally deduced response is shown as a large-dashed line in Fig. 1; it is seen to be very close to the theoretically predicted response. Note that using (2.19) as a criterion for the maximum permissible step is very restrictive. When the step test was repeated for double the head, the same time constant was found.

Besides providing a means of checking the frequency response of a system in a laboratory before installation in the ocean, this method can be used later at the site to check whether marine growth or corrosion in the pipe has altered the response characteristics.

In the field, several problems might arise which relate to the constancy of the water level outside the well. Two cases which were treated in detail yielded the following results.

b. *Drainage Test in the Presence of Small Waves.* If waves of observed amplitude a and circular frequency ω_0 are superimposed on the step H_s and readings are taken positive downwards from the position of the top of the water in the well, commencing when the wave is at its lowest point, the input is

$$I(t) = H_s U(t) + a \cos(\omega_0 t). \tag{2.20}$$

For a tide well with negligible response to waves, the response is given by (2.4), viz.,

$$R(t) = H_s - f(t), \quad t > 0,$$

where $f(t)$ is the level of the water in the well above the original mean sea level. If $f(t)$ is again read at equal intervals of time until it is negligibe in value, C and S can be computed for various ω. The response function is

$$F(i\omega) = \frac{(\omega^2 - \omega_0^2)}{(H_s + a)\,\omega^2 - H_s\,\omega_0^2}\{H_s - \omega S - i\omega C\} \tag{2.21}$$

and the required amplitude response and phase lag are

$$\alpha_2 = \frac{(\omega^2 - \omega_0^2)}{(H_s + a)\,\omega^2 - H_s\,\omega_0^2}\{(H_s - \omega S)^2 + \omega^2 C^2\}^{1/2}, \tag{2.22}$$

$$\varepsilon_2 = \arctan\{\omega C (H_s - \omega S)^{-1}\}. \tag{2.23}$$

This method was applied to the Coorong water level indicator, by placing it in the reservoir of the apparatus described in Noye (1974b) and simulating the effect of 12 second waves with an amplitude of 1.7 cm. The step selected was six centimetres above the mean level and S and C were computed from the drainage curve $f(t)$; the result is shown as a small-dashed line in Fig. 1. This is again very close to the theoretically computed curve.

c. *Drainage Test at Place of Large Tidal Range.* If the time taken by a well to drain is reasonably long at a location where there is a big tidal range, it might become necessary to allow for a steady rate of change in water level outside the well. For an excess head H_s at the start of drainage and an observed constant rate of fall γ in tide height, the input is

$$I(t) = H_s U(t) + \gamma t. \tag{2.24}$$

The response $R(t)$ will converge to a line $H_s - E + \gamma t$ for large t, where E is a constant, and can be written in the form

$$R(t) = H_s - E + \gamma t - f(t), \quad t > 0. \tag{2.25}$$

Again, values of $f(t)$ can be read, until such time that they become negligible, and C and S calculated for various values of ω. The response function is

$$F(i\omega) = \frac{(\gamma + \omega^2 C) + i\omega(H_s - E - \omega S)}{\gamma + i\omega H_s}, \tag{2.26}$$

from which is obtained the amplitude response and phase lag

$$\alpha_2 = \left(\frac{(\gamma + \omega^2 C)^2 + \omega^2 (H_s - E - \omega S)^2}{\gamma^2 + \omega^2 H_s^2}\right)^{1/2}, \tag{2.27}$$

$$\varepsilon_2 = \arctan\left(\frac{\omega H_s}{\gamma}\right) - \arctan\left(\frac{\omega(H_s - E - \omega S)}{\gamma + \omega^2 C}\right). \tag{2.28}$$

Tests have shown that these methods work well in practice. However, the following procedure avoids the use of the more complicated transfer functions (2.21) or (2.26). Some time before low tide, on a calm day, the pipe is closed. At low tide, when the rate of change of water level is negligible, the pipe is reopened and the fall in level in the well recorded, using a more sensitive recorder than available on most tide wells, with a higher pen-to-float movement ratio and a much faster chart speed. The necessary condition on the head, given by (2.19), is determined using approximate values of the pipe diameter and length. Application of the transfer function (2.6) then yields the required amplitude response and phase lag.

d. *The Tide Well with an Orifice.* With the same notation as that used to calculate the theoretical step response of the linear tide-well system, we can investigate the use of drainage tests on conventional tide wells. Substitution of (2.9), with initial condition (2.9a), in equation (1.1) gives the theoretical step response of this system, viz.

$$\left.\begin{aligned} R(t) &= H_s - (\sqrt{H_s} - 1/2\, C_1 t)^2, &&\text{for} \quad t \leq Td, \\ &= H_s \quad , &&\text{for} \quad t > Td, \end{aligned}\right\} \tag{2.29}$$

where

$$Td = 2\sqrt{H_s}/C_1. \tag{2.30}$$

Td is the time required for the well to drain completely.

Because the conventional tide-well system is not linear, the concepts of a transfer function and a frequency response function, used previously in this section, no longer apply. However, the results of such a drainage test can be used in the following way.

C_1 is determined from the initial step H_s and the corresponding drain time Td, using the relation

$$C_1 = 2\sqrt{H_s}/Td. \tag{2.31}$$

The value of C_1 should remain invariant over a number of drainage tests with different values of the step H_s. This experimentally determined value of C_1 can be used to check the theoretical value obtained using the tide-well dimensions in relation (1.3).

Estimates of the "pseudo" frequency response functions, both the amplitude response and the phase lag, can then be deduced by substituting the value of C_1 into the appropriate formulae given in Noye (1974a).

In practice, it is difficult to determine exactly when the well has drained completely. However, by minimising squared errors the value of Td can be found by fitting a parabolic curve of the form

$$\left.\begin{array}{rl} H_s(t-Td)^2+B, & T\leq Td \\ B, & T>Td \end{array}\right\}$$

to the graph of the fall of the water level inside the well.

A recent survey of tide gauges around the Australian coast, conducted by the Flinders University of South Australia (Easton, 1968) showed by drainage tests that in several cases the value of the tide-well constant C_1 differed by as much as 50% from the theoretical value. In each case, it was found that marine growth had partially closed the orifice.

3. Correction of Tide-Well Records. A tide record consists of values h_w of the water level inside the tide well. At tidal frequencies it is practically the same as the external sea level h_0, but at higher frequencies the difference

$$H_w = h_0 - h_w$$

might not be negligible.

Substitution into (1.1) gives, for the conventional tide well,

$$H_w = \left(\frac{1}{C_1}\frac{dh_w}{dt}\right)^2 \operatorname{sgn}\left(\frac{dh_w}{dt}\right) \tag{3.1}$$

and, for the linear tide well with $N \geq 5$,

$$H_w = \frac{1}{C_2}\frac{dh_w}{dt}. \tag{3.2}$$

Then the true tide height can be obtained from the tide-well record h_w, by the relation

$$h_0 = h_w + H_w. \tag{3.3}$$

This method has application to records obtained from both types of tide-well systems. For the conventional tide well it is a simple way of converting the original data into the true sea-level record, which can then be processed by spectral-density methods. For the linear tide well with $N \geq 5$ any correction of the energy-density spectrum for instrumental response becomes unnecessary. Thus, the possibility of large errors which can occur at high frequencies where the energy-density must be divided by $(\alpha_2)^2$ and α_2 is very small is avoided.

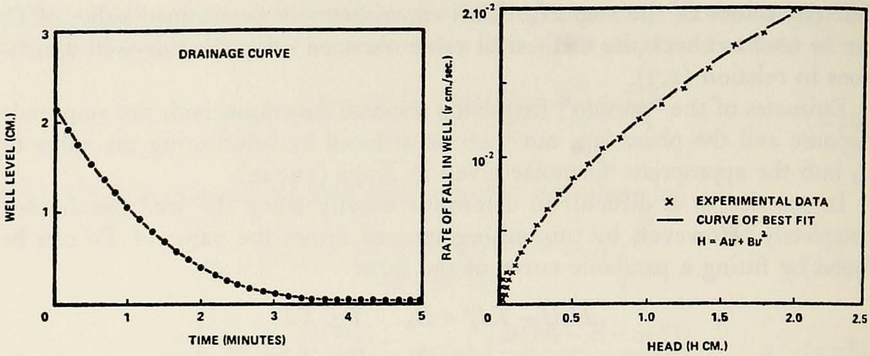

Figure 2. Drainage curve for short-tube tide-well system, and resulting parabolic law $H = Av + Bv^2$, $A = 32.4$, $B = 3360$.

When computing the correction H_w, it is most important that the value dh_w/dt be a good estimate of the rate of change of h_w, particularly for the conventional tide well when this rate of change is subsequently squared. Hence, the record must be digitised at sufficiently small intervals of time. For instance, when extracting information about a seiche superimposed on a tidal record, the interval of digitisation should be at least one-tenth of the seiche period and the correction should be applied to the record before removal of the predicted tide.

In many tide wells, such as those with short pipe connections, the head H_w overcomes energy losses proportional to the velocity of influx V and some losses proportional to V^2. Since, by continuity,

$$V = \frac{A_w}{A_p}\frac{dh_w}{dt}, \tag{3.4}$$

it follows that in such cases

$$H_w = G\frac{dh_w}{dt} + D\left(\frac{dh_w}{dt}\right)^2 \operatorname{sgn}\left(\frac{dh_w}{dt}\right), \tag{3.5}$$

where G and D are constants. The true sea-level record can then be obtained from (3.3) once G and D are known.

Results from a drainage test yield a set of corresponding values of H_w and dh_w/dt and estimates of G and D can be obtained by finding a relation of the form (3.5) with minimal sum of squared errors. The range of values of H_w and dh_w/dt used during the test must be comparable with those which occur during the rise and fall of the tides. Typical results for a short tube are shown in Fig. 2, in which both the drainage curve and the graph of the head H_w against the rate of fall of the surface in the well $v = dh_w/dt$ are given. The tide well tested was an adaptation of the Coorong water level instrument with the pipe connection shortened so that $L_p \simeq 15\, D_p$. This clearly contravenes the criterion for negligible end effects in the pipe flow, viz. $L_p > 100\, D_p$.

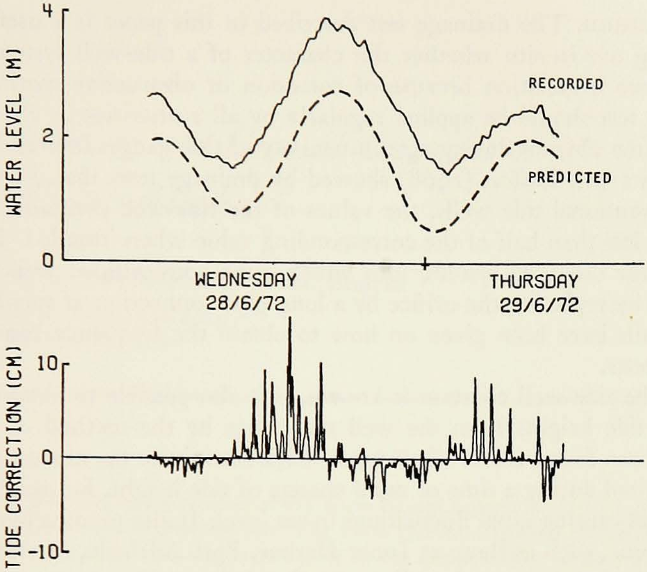

Figure 3. Tide-well record from Inner Harbor, Port Adelaide during storm of June 28–29, 1972, and correction to be added to give sea level outside well.

a. *An Example.* On June 28, 1972, the passage of a low pressure system across the Great Australian Bight, associated with persistent westerly winds over St. Vincent Gulf in South Australia, produced abnormally high tides at Adelaide. A high spring tide of 2.8 m was predicted for 1756 hours that evening, but the meteorological effect, which raised the sea level by an amount which varied between 0.6 and 1.2 m, produced a recorded maximum level of 3.95 m at Inner Harbor, Port Adelaide at 1712 hours. This was a significant event, since water came within 15 cm of the top of a nearby embankment which protects a large housing estate on the low-lying coastal plains. However, an observer noted that the water level on a tide staff less than 20 metres from the tide gauge reached a maximum of a little over 4.00 metres at approximately 1705 hours.

Figure 3 shows the predicted (astronomical) tide and the recorded tide for the evening of June 28 and the early morning of June 29, 1972. The water in this well is connected to the sea through a set of orifices at the end of a conical surge pipe. Using a value for the effective tide-well constant, C_1, of $10^{-3}\mathrm{m}^{1/2}\text{-sec}^{-1}$ during inflow and $2\times 10^{-3}\mathrm{m}^{1/2}\text{-sec}^{-1}$ during outflow, the correction, which must be added to the recorded tide height to give the sea level, has been computed. This is also shown in Figure 3. The greatest difference between the levels inside and outside the well was over 10 cm; this occurred at times of greatest rate of increase of the level in the tide well. These calculations show that the maximum tide height occurred just prior to the maximum recorded level and was greater than that recorded. Therefore, the water was closer to the top of the embankment than officially recognised.

4. Conclusion. The drainage test described in this paper is a useful method for finding out in situ whether the character of a tide-well system has been altered since installation because of corrosion or obstructive marine growth. That this test should be applied regularly by all authorities in charge of tide wells became obvious during a recent survey of tide gauges located around the Australian coast. Easton (1968) showed by drainage tests that, in the case of some conventional tide wells, the values of the tide-well constant C_1 had decreased to less than half of the corresponding value when installed. In the case of the linear tide-well system, into which some conventional wells are being converted by replacing the orifice by a long pipe connection of suitable dimensions, details have been given on how to obtain the frequency response from drainage tests.

Once the tide-well constant is known, it is also possible to obtain corrected values of tide heights from the well recordings by the method of Section 3. This has been found useful when values of tide height to the nearest centimetre were required during a time of rapid change of tide height, for example, when a storm was causing rapid fluctuations in sea level. It also permits past tide-well records, some, such as those at Inner Harbor, Port Adelaide, extending nearly a century into the past, to be adjusted before being used for accurate scientific analysis.

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