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On Divergent Shelf Waves¹

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ABSTRACT

A dispersion relationship for divergent continental shelf waves on a shelf of exponential profile has been obtained. The dispersion curves calculated from this relationship are in good agreement with experimental results reported by Caldwell et al. (1972). In addition, a perturbation method has been used to obtain the first-order divergent correction to the dispersion curves of nondivergent shelf waves on a shelf of arbitrary profile. Numerical calculations by the perturbation method for the exponential profile show good agreement with other theoretical and experimental results.

1. *Introduction.* Observations on the nonhydrostatic response of sea level to atmospheric pressure systems by Hamon (1962, 1963, 1966) in Australia and by Mooers and Smith (1968) in Oregon have been attributed to topographical Rossby waves, otherwise known as continental-shelf waves. It has been suggested (Adams and Buchwald 1969) that these waves are generated by a periodic longshore component of the geostrophic wind. As will be seen below, the divergence parameter

$$\varepsilon = f^2 L^2 / gh_1 \quad (1.1)$$

plays an important role in theoretical models of shelf waves, where h_1 is the ocean depth, L is a typical shelf width, and f is the Coriolis parameter. The assumption $\varepsilon = 0$ implies zero horizontal divergence in the theory and, consequently, neglect of the vertical displacement in the equation of continuity. Theories based on taking $\varepsilon = 0$ have been considered by Robinson (1964) and Mysak (1967, 1968) in the case where wavelengths are long compared with the width of the continental margin and by Buchwald and Adams (1968) for all wavelengths.

For oceanographic applications, theories that assume $\varepsilon = 0$ are probably adequate. For instance, the parameters for the East Australian continental shelf give $\varepsilon \approx 10^{-3}$. However, in the laboratory experiments conducted by

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Caldwell et al. (1972), $\varepsilon = O(1)$, and theories based on $\varepsilon = 0$ do not agree with the experimental results. Given in the same paper is a more general theory that takes the divergence into account; the dispersion curves were found by numerical integration of an ordinary differential equation. The purpose of the present work has been to derive analytic expressions for the dispersion relations, in suitable cases, thus avoiding the necessity of numerical integration.

In order to write down the basic equation of shelf waves, we assume horizontal x^*, y^* coordinates; a shoreline at $x^* = 0$; a continental shelf in the strip $0 < x^* \leq L$, where the depth is $h^*(x^*)$; and, for $x^* \geq L$, an ocean of uniform depth, $h_1 = h^*(L)$. In terms of nondimensional coordinates, $x = x^*/L$, $y = y^*/L$, and of a nondimensional depth, $h = h^*/h_1$, the equation for the surface displacement, ζ , is (Caldwell et al.) for $0 < x \leq 1$,

$$(h\zeta)' + (\mu kh' - k^2 h - \delta)\zeta = 0; \quad (1.2)$$

here it is assumed that ζ represents a coastal wave that has implicit harmonic variation in both y and the time t of the form $\exp[i(ky + \omega t)]$; the primes are derivatives with respect to x , and

$$\mu = f/\omega; \quad \delta = \varepsilon(1 - \mu^{-2}). \quad (1.3a, b)$$

We assume that $h(0) \neq 0$, when the condition of zero normal velocity at the shoreline reduces to

$$\zeta' + \mu k \zeta = 0, \quad x = 0. \quad (1.4)$$

For the ocean, it is seen from (1.2) that if $h = 1$, the solution that vanishes as $x \rightarrow \infty$ is of the form

$$\zeta \sim \exp[-\lambda x],$$

where

$$\lambda = (k^2 + \delta)^{1/2}. \quad (1.5)$$

By invoking continuity of ζ and ζ' at $x = 1$, the condition

$$\zeta' + \lambda \zeta = 0, \quad x = 1, \quad (1.6)$$

is obtained. It can be shown that continuity of normal velocity at $x = 1$ implies that ζ' is continuous.

Given k , eq. (1.2) is a Sturm-Liouville equation for the eigenvalues $\nu = \mu k$. However, the eigenvalue appears in both the boundary conditions (1.4) and (1.5). Moreover, the form of (1.5) makes this a nonstandard Sturm-Liouville problem, but if $\varepsilon = 0$ and if we assume that then $\lambda = |k| > 0$, it is possible to extend the Sturm-Liouville theory to this case, on lines suggested by Friedman (1956), as follows:

Since $\delta = 0$, assume that $h(x)$ (1.2) can be rewritten as

$$\mathfrak{L}_0 \zeta - \nu \zeta = 0, \quad (1.7)$$

where

$$\mathfrak{L}_0 \equiv -\frac{1}{h'(x)} \left\{ \frac{d}{dx} \left(h \frac{d}{dx} \right) - hk^2 \right\}.$$

Let $z(x)$ be any twice-differentiable function in $[0, 1]$ and consider the space of two component vectors, $Z = (z(x), z(0))$. Define the scalar product of two vectors, Z_1, Z_2 , as

$$\langle Z_1, Z_2 \rangle = \int_0^1 z_1(x) z_2(x) h'(x) dx + z_1(0) z_2(0) h(0), \quad (1.8)$$

and consider the subspace of vectors such that

$$z'(1) + |k|z(1) = 0. \quad (1.9)$$

Let the operator, \mathfrak{L} , be defined by

$$\mathfrak{L} Z = \begin{pmatrix} \mathfrak{L}_0 z \\ -z'(0) \end{pmatrix}, \quad (1.10)$$

so that

$$(\mathfrak{L} - \nu) Z = 0, \quad (1.11)$$

together with the condition (1.9), is now a standard Sturm-Liouville eigenvalue problem, equivalent to (1.2) through (1.6) when $\varepsilon = 0$. It is easy to show that \mathfrak{L} is self-adjoint, and, by considering the scalar product

$$\langle Z, (\mathfrak{L} - \nu) Z \rangle = 0, \quad (1.12)$$

it is shown in the APPENDIX that, if $h'(x) \geq 0$, all $x \in [0, 1]$; then \mathfrak{L} is positive definite. It follows that, given k , there is an infinite set of positive eigenvalues, $\nu_n(k)$, for which (1.11) is satisfied, thus providing the dispersion curves for the possible modes of nondivergent shelf waves. A specific example of the solution to this problem has been given by Buchwald and Adams (1968) for the case $h(x) \sim \exp(2bx)$.

If $\varepsilon \neq 0$, then the eigenvalue problem is too complicated to admit any general conclusions. The specific case of a rectangular shelf has been analyzed by Munk et al. (1970), who plotted the dispersion curves for both shelf waves and Kelvin waves at the low-frequency ($\omega < f$) end of the spectrum and for trapped Kelvin and Poincaré waves at the high-frequency end ($\omega > f$).

In this paper we discuss two different methods of approach. In § 2 we have derived an explicit dispersion relationship for a shelf having exponential slope that is valid for all values of ε and μ . Numerical results are obtained for low frequencies that are in excellent agreement with both the experimental and computed results of Caldwell et al. over the whole range of wavelengths. In

§ 3, for an arbitrary shelf, a method used by Smith (1972) is extended to the case of the whole range of wavelengths and is used to obtain the first-order corrections to the dispersion curves for small values of ϵ . Here again, numerical computations for an exponential shelf give good agreement with both the exact and experimental results.

2. *Exponential Depth Variation.* Assume that the depth in the shelf region is given by

$$h = \exp [2b(x-1)] \quad (2.1)$$

so that

$$h_0 = h(0) = \exp(-2b).$$

The changes of variable

$$\zeta = h^{-1/2} \eta, \quad (2.2)$$

$$x = -(\log \xi)/b \quad (2.3)$$

transform (1.2) into a Bessel equation,

$$\xi(\xi\eta\xi)\xi + (\sigma^2 - \rho^2\xi^2)\eta = 0, \quad (2.4)$$

where

$$\sigma^2 = (2\mu kb - k^2 - b^2)/b^2, \quad (2.5a)$$

$$\rho^2 = \delta/h_0 b^2. \quad (2.5b)$$

The general solution of (2.4) may be written in the form

$$\eta = A_+ B(\rho\xi, i\sigma) + A_- B(\rho\xi, -i\sigma), \quad (2.6)$$

where the functions B can be expressed in the series form

$$B(\xi, i\sigma) = \xi^{i\sigma} \left[1 + \frac{(\xi/2)^2}{1!(i\sigma+1)} + \frac{(\xi/2)^4}{2!(i\sigma+1)(i\sigma+2)} + \dots \right]. \quad (2.7)$$

Note also that B can be expressed in terms of Bessel functions by

$$B(\xi, \alpha) = \frac{(2/i)^\alpha}{\Gamma(\alpha+1)} \mathcal{J}_\alpha(i\xi). \quad (2.8)$$

After transformation, the boundary conditions (1.4) and (1.6) reduce to

$$be^{-b}\eta_\xi + (b-\lambda)\eta = 0; \quad \xi = e^{-b}, \quad (2.9)$$

$$b\eta_\xi + (b-\mu k)\eta = 0; \quad \xi = 1, \quad (2.10)$$

since

$$1 \geq \xi \geq e^{-b} \quad \text{for} \quad 0 \leq x \leq 1.$$

Table I. Comparison of calculated results for the first mode of the 'wide shelf,' $h_0 = 0.06$, $b = 1.4067$, $\varepsilon = 1.03$. Values of $w = \omega/f$, calculated from (2.11), are given for various values of k . The figures in the w_0 column are the corresponding values for the nondivergent theory, $\varepsilon = 0$. The figures under w_1, w_1^* have been calculated by means of the perturbation and modified perturbation methods, respectively.

k	w	w_0	w_1	w_1^*
0.25	0.0622	0.1013	0.0479	0.0568
0.75	.1811	.2663	.1591	.1689
1.25	.2833	.3770	.2610	.2697
1.75	.3582	.4406	.3400	.3466
2.25	.4017	.4700	.3899	.3943
2.75	.4196	.4769	.4142	.4170
3.25	.4211	.4702	.4204	.4221
3.75	.4134	.4559	.4156	.4166
4.25	.4008	.4378	.4044	.4051
4.75	.3860	.4181	.3901	.3905
5.75	.3544	.3786	.3585	.3586
6.75	.3241	.3426	.3276	.3277
7.75	.2969	.3110	.2997	.2997
8.75	.2728	.2837	.2750	.2750
9.75	.2517	.2603	.2535	.2535

Substitution of (2.6) in (2.9), (2.10), elimination of $\mathcal{A}_+, \mathcal{A}_-$, and the use of a well-known addition formula for Bessel functions results in the frequency equation

$$H(\mu, k) = F(\sigma)G(-\sigma) - F(-\sigma)G(\sigma), \quad (2.11)$$

$$= 0,$$

where

$$F(\sigma) = (i\sigma b + b - \lambda)B(\rho e^{-b}, i\sigma) + {}^{1/2}\rho b e^{-b}(1 + i\sigma)B(\rho e^{-b}, 1 + i\sigma), \quad (2.12)$$

$$G(\sigma) = (i\sigma b e^b + b - \mu k)B(\rho, i\sigma) + {}^{1/2}\rho b(1 + i\sigma)B(\rho, 1 + i\sigma). \quad (2.13)$$

Solutions of (2.11) give the dispersion curves for continental-shelf waves, Kelvin waves, and trapped Poincaré waves. In this paper we confine the discussion to $\omega < f$, i.e. $\mu > 1$, for which only shelf waves and Kelvin waves are possible.

The series in (2.7) converges rapidly for the range of ξ considered, and, given k , we compute corresponding values of μ from (2.11), correct to four significant figures. For the sake of comparison, the computations have been based on data from Caldwell et al., in which $f = 2\pi$ rad./sec. and whose 'wide shelf' has a width of $L = 16$ cm, an 'ocean depth' of $h_1 = 10.0$ cm, and a shore depth of 0.6 cm, so that $h_0 = 0.06$, $b = 1.4067$, and $\varepsilon = 1.03$. In Table I, the second column gives the computed values, for a range of k , of

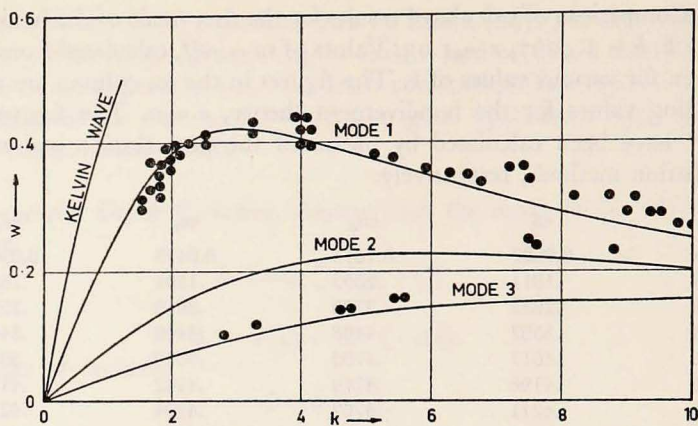


Figure 1. Dispersion curves of the Kelvin waves and the first three modes of shelf waves, calculated from (2.11) for the 'wide shelf,' $h_0 = 0.06$, $b = 1.4067$, and $\varepsilon = 1.03$. The dots are experimental results taken from Caldwell et al. (1972).

ω/f , correct to four significant figures. The third column gives, for reference, corresponding values of ω/f , computed for $\varepsilon = 0$. The other two columns refer to § 3. Fig. 1 illustrates the Kelvin wave and the first three modes of shelf waves. The heavy dots represent the results of model experiments by Caldwell et al.; the agreement is satisfactory except for short wavelengths, where the effects of surface and bottom curvature in the laboratory model become important.² Table II gives similar results for the case $L = 8$ cm, $\varepsilon = 0.258$, with h_0 and b remaining at 0.06 and 1.4067, respectively. Fig. 2 gives the dispersion curve for the first mode of the 'narrow shelf', with $H_0 = 0.245$, $L = 8$ cm, and $b = 0.7034$, compared with the experimental results. Except in the short wavelength range, solutions of (2.11) are again in satisfactory qualitative agreement with the experimental results of Caldwell et al.

3. *Arbitrary Depth Variation.* In § 2 we found that, in the case of exponential depth variation, it is possible to find a closed solution to the differential equation. However, this is not possible in the general case, so that in this section we use a perturbation method to obtain corrections to the nondivergent theory for small values of ε . Smith (1972) has considered a similar theory, but he limited his discussion to long waves, of typical wavelength L/ε , when non-divergent waves are not dispersive. As has been seen, however, it is highly desirable to have a theory that includes the dispersive region, and in this sec-

2. I am indebted to Dr. M. S. Longuet-Higgins for pointing out that the depth measurements given by Caldwell et al. are parallel to the axis of rotation and not normal to the surface, yielding a depth profile that is not exactly exponential. Comparisons, therefore, of this theory with the measured and calculated results of Caldwell et al. should be regarded as qualitative only. No quantitative conclusions regarding the various sets of results should be drawn.

Table II. Comparison of calculated results for the first mode of the 'wide shelf', $h_0 = 0.06$, $b = 1.4067$, $\varepsilon = 0.258$. Values of $w = \omega/f$, calculated from (2.11), are given for various values of k . The figures in the w_0 column are the corresponding values for the nondivergent theory, $\varepsilon = 0$. The figures under w_1, w_1^* have been calculated by means of the perturbation and modified perturbation methods, respectively.

k	w	w_0	w_1	w_1^*
0.25	.0855	.1013	.0792	.0829
0.75	.2372	.2663	.2279	.2312
1.25	.3487	.3770	.3393	.3422
1.75	.4175	.4406	.4103	.4124
2.25	.4515	.4700	.4470	.4484
2.75	.4617	.4769	.4595	.4603
3.25	.4573	.4702	.4567	.4572
3.75	.4474	.4559	.4451	.4454
4.25	.4281	.4378	.4289	.4291
4.75	.4096	.4181	.4107	.4108
5.75	.3722	.3786	.3734	.3734
6.75	.3377	.3426	.3387	.3387
7.75	.3073	.3110	.3081	.3081
8.75	.2809	.2837	.2815	.2815
9.75	.2380	.2603	.2586	.2586

tion a perturbation method is developed that gives good results over the whole range of wavelengths.

Assume, in (1.2), that k is positive and fixed and that there is a formal regular perturbation expansion;

$$\zeta = \zeta_0 + \varepsilon \zeta_1 + \dots \quad (3.1)$$

for the surface elevation, and

$$\mu = \mu_0 + \varepsilon \mu_1 + \dots \quad (3.2)$$

for the eigenvalues μ . Equating powers of ε in (1.2) and in the boundary conditions (1.4) and (1.5), the coefficient of ε^0 is

$$(h \zeta'_0)' + (\mu_0 k h' - k^2 h) \zeta_0 = 0, \quad (3.3)$$

$$\zeta'_0 + \mu_0 k \zeta_0 = 0, \quad x = 0, \quad (3.4)$$

$$\zeta'_0 + k \zeta_0 = 0, \quad x = 1. \quad (3.5)$$

Similarly, the coefficient of ε is

$$(h \zeta'_1)' + (\mu_0 k h' - k^2 h) \zeta_1 = (1 - \mu_0^{-2}) \zeta_0 - \mu_1 k h' \zeta_0, \quad (3.6)$$

$$\zeta'_1 + \mu_0 k \zeta_1 = -\mu_1 k \zeta_0, \quad x = 0, \quad (3.7)$$

$$\zeta'_1 + k \zeta_1 = -(1 - \mu_0^{-2}) \zeta_0 / 2k, \quad x = 1. \quad (3.8)$$

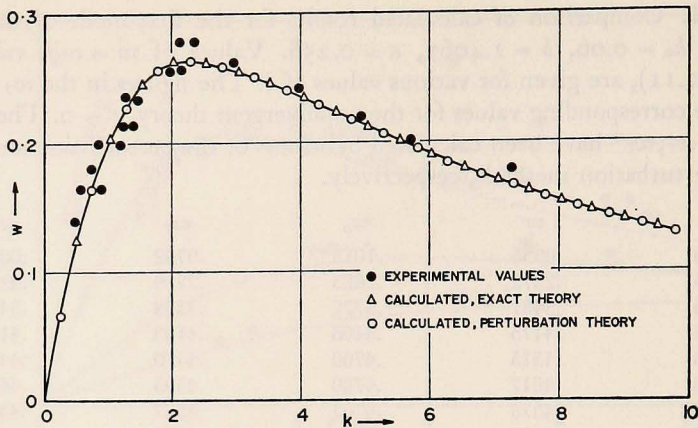


Figure 2. Dispersion curve for the first mode for the 'narrow shelf,' $h_0 = 0.245$, $b = 0.7034$, $\varepsilon = 0.258$. On the curve, the triangles indicate calculations from the 'exact' equation (2.11), the circles from the perturbation equation (3.13); the heavy dots are experimental results.

The problem of obtaining ζ_0 and μ_0 from (3.3) to (3.5) is the same as the Sturm-Liouville problem set out in (1.10). Hence there is a set of distinct positive and increasing eigenvalues, $\mu_0^{(n)}(k)$, for each of which (3.3) has a non-trivial solution, $\zeta_0^{(n)}$. Let us suppose that, for a given k , μ_0 is one of the eigenvalues and ζ_0 is the corresponding eigenfunction. Then (3.6) to (3.8) are equations for the first-order corrections, ζ_1 and μ_1 , to the eigenfunctions and the eigenvalue, respectively. Noting that the left-hand side of (3.6) is identical to (3.3), we can obtain μ_1 immediately by multiplying (3.6) by ζ_0 and integrating over the interval $[0, 1]$. After some calculations, we obtain:

$$\mu_1 k \left\{ \int_0^1 h' \zeta_0^2 dx + [h \zeta_0^2]_{x=0} \right\} = (1 - \mu_0^{-2}) \left\{ \int_0^1 \zeta_0^2 dx + \frac{1}{2k} [\zeta_0^2]_{x=1} \right\}. \quad (3.9)$$

The perturbation gives good results if the term in δ in (1.2) is much smaller than the maximum of $\{\mu kh', k^2 h\}$. Since h is of the order of unity for most of the range, there is no problem for large k . Thus, for short wavelengths the perturbation succeeds even for moderate values of ε . On the other hand, for small k , it is seen that, if we ignore $k^2 h$ in (3.3) and let the condition (3.5) be $\zeta_0' = 0$ at $x = 1$, then $\mu_0 k$ tends to a fixed eigenvalue in each mode, as $k \rightarrow 0$. This means, of course, that long nondivergent waves are not dispersive, as was found by Robinson (1964) and Mysak (1967) in a specific case. Here it implies that, in (3.3), $\mu_0 kh'$ is fixed in each mode for small k , so that, for sufficiently small ε , the perturbation works. Further, if, as in the numerical examples considered later, $h' \gg h$, then the perturbation may succeed for moderate values of ε , even for $\varepsilon \approx 1.0$, for all values of k , except in the immediate neighborhood of $k = 0$.

When k is very small, the right-hand side of (3.8) is large, and the method breaks down. However, the method can be patched up in the following way.

In (1.6), let

$$\lambda = \lambda_0 + \varepsilon^2 \lambda_2 + \dots, \quad (3.10)$$

where

$$\lambda_0 = [k^2 + \varepsilon(1 - \mu_0^{-2})]^{1/2}, \quad (3.11)$$

and replace (3.5) and (3.8) with

$$\zeta_0' + \lambda_0 \zeta_0 = 0; \quad \zeta_1' + \lambda_2 \zeta_1 = 0, \quad x = 1, \quad (3.12a, b)$$

respectively. Then μ_0 is the eigenvalue for (3.3), (3.4), (3.12a), and, using the same procedure as before, the amended expression for μ_1 is

$$\mu_1^* k \left\{ \int_0^1 h' \zeta_0^2 dx + [h \zeta_0^2] \right\} = (1 - \mu_0^{-2}) \int_0^1 \zeta_0^2 dx \quad (3.13)$$

instead of (3.10).

In order to test the accuracy of the perturbation method, we choose the profile in (2.1) so that approximate results may be compared with the exact ones determined in § 2. Buchwald and Adams (1968) have given the details of the solution of (3.3), which reduces to a differential equation with constant coefficients. The resulting dispersion relation for μ_0 is

$$2kb\mu_0 = m^2 + k^2 + b^2, \quad (3.14)$$

where

$$(k+b) \tan m + m = 0 \quad (3.15)$$

and where

$$\zeta_0 = Ae^{-bx} [(k\mu_0 - b) \sin mx - m \cos mx] \quad (3.16)$$

is the corresponding eigenfunction.

These values are substituted in (3.9), and an expression for μ_1 is then obtained. Results corresponding to the first mode in Fig. 1 are presented in Table I. Appropriate numerical values of the constants are $f = 2\pi$ rad./sec, $L = 16$ cm, $h_0 = 0.06$, $b = 1.4067$, and $\varepsilon = 1.03$. In Table I, the first column gives values of k , the second column the corresponding exact values of $w = \omega/f = 1/\mu$, which have been computed by the method discussed in § 2. The third column gives the appropriate value of $w_0 = 1/\mu_0$, the fourth column the corresponding $w_1 = 1/\mu_1$. The fifth column gives $w_1^* = 1/\mu_1^*$, where μ_1^* has been computed from (3.13), with (3.14) and (3.16) remaining unaltered but with (3.15) replaced by

$$(\lambda_0 + b) \tan m + m = 0.$$

Noting that in this example $h' = 2bh$, good results for moderate values of ε would be expected. This is borne out in Table I, where it may be seen that

the perturbation gives good results, particularly for $k > 1$. For smaller values of k , w_1^* is more accurate than w_1 , as is to be expected. Table II shows the results corresponding to $L = 8$ cm, so that $\varepsilon = 0.258$, the other constants being kept unaltered; the agreement between the perturbation solution and the exact solution is excellent, and w_1^* is slightly closer to w than is w_1 . In Fig. 2, alternate points on the curve were computed by the methods discussed in § 2 and § 3. Here again, $\varepsilon = 0.258$, and it is not possible on the graph to distinguish between the results of the two methods.

Acknowledgments. The computations were carried out on the University of New South Wales IBM 360/50 computer. Thanks are due Miss J. Gibson for assistance in testing and running the programs.

APPENDIX

If $h'(x) \geq 0$, for all $x \in [0,1]$ and $h' \neq 0$ for some interval in $[0,1]$, then, from (1.8),

$$Z^2 = \langle Z, Z \rangle = \int_0^1 \{z(x)\}^2 h'(x) dx + h(0) \{z(0)\}^2 > 0$$

for all real $z(x)$. Also,

$$\begin{aligned} \langle Z, \mathcal{L}Z \rangle &= - \int_0^1 z(x) [\{hz'(x)\}' - hk^2 z(x)] dx - h(0)z(0)z'(0), \\ &= \int_0^1 [z'^2 + k^2 z^2] h dx + |k| \{z(1)\}^2, \\ &> 0 \end{aligned}$$

for all $z(x) \neq 0$ in the interval. Thus, the operator is positive definite, and the eigenvalues $\nu_n = fk/\omega_n$ are all positive.

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