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On the Elliptic Generating Region of a Tsunami¹

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ABSTRACT

The surface elevation is calculated for the three-dimensional motion of waves in a fluid of constant depth subject to a given bottom velocity. An example, modeling tsunami generation, with antisymmetric bottom velocity, is considered in detail. The amplitude of the wave front is found to decay much more rapidly than the main wave. The distribution of amplitude with wave number and with angular position is computed for some cases.

Introduction. A tsunami consists of a group of long water waves that are generated by earthquakes or other seismic events. These waves can propagate rapidly over large ocean distances and inundate coastal areas, causing great damage and loss of life. They occur most frequently in the earthquake-prone Pacific Fire Ring. The Tsunami Warning System now monitors seismic activity in the Pacific and warns such areas as Japan and Hawaii of possible tsunamis.

Observations indicate that a tsunami is usually initiated by large-scale seismic activity under the sea floor. The major part of the dislocation, accomplished quickly, is often followed by slow changes extending over a few minutes. Movements of the earth's surface due to a tsunamigenic earthquake are generally confined to an approximately elliptic region; see Wilson (1962) for a review of data and Hwang and Divoky (1970) for a more recent example. The major axis of the ellipse coincides approximately with the fault line, with the upthrust and downthrow occurring on opposite sides of the line. In some cases there is evidence that the elliptic disturbance region is subdivided by the axes into quadrants, with alternate upthrust and downthrow; see, for example, Suzuki (1970), who investigated the tsunami accompanying the 1968 Tokachi-oki earthquake.

A theoretical model to explain tsunami generation can be set up as a bound-

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ary-value problem, with a forcing term at the sea bed. Generally it is assumed that the model consists of a layer of homogeneous fluid of constant depth, with linearized conditions at the free surface and the sea bed. This gives a Cauchy-Poisson problem for the surface elevation; it can be formally solved by multiple Fourier transforms, as described by Stoker (1957), or by Green's functions, as used by Kajiura (1963). The formal solution for the amplitude from an arbitrary source is then expressed in terms of integrals; see, for example, Gazarian (1955), Kajiura (1963), Keller (1961), and Van Dorn (1965). But even for mathematically simple sea-bed motions, the integrals are complicated, and asymptotic estimates are usually employed. Thus the interpretation of results in practical situations is difficult. Most authors have assumed that the sea-bed disturbance is axially symmetric; this leads to some simplification, but it also restricts the results in the light of observations outlined above. An alternative approach is a numerical solution of the basic equations as used, for example, by Hwang and Divoky (1970) in their investigation of the 1964 Alaskan earthquake. As they have pointed out, a fundamental difficulty lies in obtaining reliable data on the time-and-space history of the sea-bed disturbance that initiates the tsunami.

In this paper an attempt is made to set up a simple but realistic three-dimensional model for tsunami generation. The time-and-space variation in the sea-bed disturbance fits the observed features described above; in particular, asymmetry is included in the model. The linearized boundary-value problem is formally solved by using multiple Fourier transforms; the asymptotic behavior of the resulting triple integrals is discussed in detail. This paper is an extension of the work of Braddock and van den Driessche (1971), in which asymmetry was found to be important in predicting the asymptotic-wave behavior.

The General Problem. The Cartesian coordinates, xyz , are taken with the fluid occupying an unlimited region in the x and y directions; the free surface of the fluid is at $z = h$ and the plane bottom is at $z = 0$. The fluid is assumed to be incompressible, inviscid, irrotational, and to be at rest for time $t < 0$. The velocity potential, $\varphi(t, x, y, z)$, is then required to satisfy

$$\begin{aligned} \nabla^2 \varphi &= 0, \quad t \in (-\infty, \infty), \quad x, y \in (-\infty, \infty), \quad z \in (0, h), \\ \varphi_{tt} + g\varphi_z &= 0 \quad \text{on} \quad z = h, \\ \varphi_z &= F(t, x, y) \quad \text{on} \quad z = 0, \end{aligned}$$

where $F(t, x, y)$ is the bottom disturbance that sets up the wave motion, g is the gravitational acceleration, and the subscripts denote partial derivatives. The surface displacement, $\zeta(t, x, y)$, is given, in terms of the velocity potential, by $\zeta(t, x, y) = -\varphi_t(t, x, y, h)/g$.

Fourier transform analysis gives

$$\zeta(t, x, y) = -\frac{i}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(wt+kx+ly)} w \mathcal{F}(w, k, l)}{\cosh(mh)(w^2 - \sigma^2)} dw dk dl, \quad (1)$$

where

$$\mathcal{F}(w, k, l) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(wt+kx+ly)} F(t, x, y) dt dx dy,$$

$\sigma^2 = gm \tanh(mh)$, and $m = (k^2 + l^2)^{1/2}$. Further evaluation of this formal solution depends on the nature of $F(t, x, y)$.

Bottom Velocity Modeling Tsunami Generation. Most authors have considered only disturbances that are symmetrically distributed over the sea floor. Here the asymmetric case is considered in detail. The basic system of equations is linear. It is well known that any function can be represented as the sum of a symmetric and an asymmetric function. Thus a general disturbance can be represented by a superposition of these two special cases. Let the bottom disturbance $F(t, x, y) = T(t)X(x)Y(y)$, where

$$\begin{aligned} T(t) &= Lt e^{-\alpha t}, & t \geq 0, & \quad T(t) = 0, & \quad t < 0; \\ X(x) &= Mx e^{-\beta|x|}, \\ Y(y) &= Ny e^{-\gamma|y|}. \end{aligned}$$

Constants $L, M, N, \alpha, \beta, \gamma$ are real and positive and can be varied to give a variety of bottom velocities that are both time-and-space dependent. The disturbance grows rapidly for $t \in (0, 1/\alpha)$ and then decays slowly. Appreciable disturbance is confined to a region that is approximately elliptical, the first and third quadrants having a positive velocity, the second and fourth quadrants a negative velocity. This models the earthquake observations cited in the *Introduction*. The x -axis, which is the major axis of the ellipse for $\gamma > \beta$, is identified with the fault line.

The free-surface elevation given by (1) then becomes

$$\zeta(t, x, y) = \frac{iLMN}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(kx+ly)} \psi(k, l) \left[\int_{-\infty}^{\infty} \frac{e^{iwt} w dw}{(w^2 - \sigma^2)(\alpha + iw)^2} \right] dk dl,$$

where

$$\psi(k, l) = \frac{16\beta\gamma kl}{\cosh(mh)(\beta^2 + k^2)^2(\gamma^2 + l^2)^2}$$

a function that depends on the transforms of $X(x)$ and $Y(y)$. In the particular case of a rapid bottom disturbance, the time dependence of F can be modeled as a delta function and the w integral has singularities that are simple poles at $w = \pm \sigma$. The w integral in the present model has a double pole at $i\alpha$ in addition to the simple poles at $\pm \sigma$; it can be evaluated by using a contour in the

complex plane and by computing the residues. The surface displacement that results is

$$\begin{aligned}\zeta(t, x, y) &= 0, & t \leq 0, \\ \zeta(t, x, y) &= \zeta_T(t, x, y) + \zeta_P(t, x, y), & t > 0;\end{aligned}$$

here

$$\begin{aligned}\zeta_T(t, x, y) &= -\frac{LMN}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(kx+ly)} \psi(k, l) e^{-\alpha t} \left[\frac{\sigma^2 - \alpha^2}{(\alpha^2 + \sigma^2)^2} - \frac{\alpha t}{(\alpha^2 + \sigma^2)} \right] dk dl, \\ \zeta_P(t, x, y) &= -\frac{LMN}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(kx+ly)} \psi(k, l) \left[\frac{e^{i\sigma t}}{(\alpha + i\sigma)^2} + \frac{e^{-i\sigma t}}{(\alpha - i\sigma)^2} \right] dk dl.\end{aligned}$$

The particular form taken for $T(t)$ has given rise to the transient motion, ζ_T , which contains only exponentially damped terms. The poles at $\pm\sigma$, which give the dispersion relationship, give rise to the propagating motion, ζ_P .

The Transient Motion. ζ_T can be estimated by Lighthill's (1960) asymptotic method. Contributions to the asymptotic value come from the infinite number of purely imaginary zeros of $\alpha^2 + \sigma^2$ and from the double poles at $k = \pm i\beta$ and $l = \pm i\gamma$. However, as this motion is exponentially decaying in time, it soon becomes insignificant compared with ζ_P , and so it will not be fully discussed.

The Propagating Motion.

$$\zeta_P(t, x, y) = -LMN(P_+ + P_-)/(8\pi^2),$$

where

$$P_{\mp} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\psi(k, l)}{(\alpha \mp i\sigma)^2} e^{i(\mp\sigma t + kx + ly)} dk dl. \quad (2)$$

By means of the change in variables, $x = r \cos \theta$, $y = r \sin \theta$, $r = (x^2 + y^2)^{1/2}$, $k = m \cos \eta$, $l = m \sin \eta$, $\chi = mh$, $\mu = (\chi \tanh \chi)^{1/2}$, and $\bar{h} = 1/h$, the above can be expressed in the form

$$P_{\mp} = \bar{h}^2 \int_0^{\infty} \frac{\chi e^{\mp i\mu (g\bar{h})^{1/2} t}}{(\alpha \mp i\mu (g\bar{h})^{1/2})^2} \int_0^{2\pi} \psi(\chi \bar{h} \cos \eta, \chi \bar{h} \sin \eta) e^{i\bar{h} r \chi \cos(\theta - \eta)} d\eta d\chi.$$

The integrals can now be estimated by applying the method of stationary phase twice; see, for example, Stoker (1957), Chaudhuri (1968), Nikitin et al. (1970), where the method has been used for other three-dimensional water-wave problems.

To evaluate the η integral, let $A(\eta) = r\bar{h}\chi \cos(\theta - \eta)$, and observe that the stationary points of $A(\eta)$ occur at $\eta = \theta$, $\theta + \pi$ for $\theta \in [0, \pi]$, and at $\eta = \theta$, $\theta - \pi$ for $\theta \in (\pi, 2\pi)$. The method of stationary phase gives

$$P_{\mp} \approx (2\pi \bar{h}^3/r)^{1/2} \int_{\chi_0}^{\infty} \frac{\chi^{1/2} e^{\mp i\mu (g\bar{h})^{1/2} t}}{(\alpha \mp i\mu (g\bar{h})^{1/2})^2} \left[\psi(\chi \bar{h} \cos \theta, \chi \bar{h} \sin \theta) e^{i(\bar{r}\bar{h}\chi - \pi/4)} + \right. \\ \left. + \psi(-\chi \bar{h} \cos \theta, -\chi \bar{h} \sin \theta) e^{i(-\bar{r}\bar{h}\chi + \pi/4)} \right] d\chi + o(\bar{r}\bar{h})^{-1}. \quad (3)$$

This approximation is valid for each value of χ_0 such that $r\bar{h}\chi_0 \gg 1$, except at the zeros of ψ , which are at $\eta = 0, \pi/2, \pi, 3\pi/2$, i.e., on the axes of the ellipse.

On the line $x = 0$,

$$P_{\mp}(x = 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\psi(k, l)}{(\alpha \mp i\sigma)^2} e^{i(\mp\sigma t + ly)} dk dl.$$

The integrand is an odd function of k , so $P_{\mp}(x = 0)$ is exactly zero. Thus, due to the assumed antisymmetric nature of the bottom-velocity profile, there is zero surface displacement for all time on $x = 0$. Similarly, by interchanging the order of integration, there is zero surface displacement on $y = 0$.

To evaluate P_- at points that are not on the axes, consider the phase terms in (3) and define

$$Q(\chi) = -\mu(g\bar{h})^{1/2} t + r\bar{h}\chi - \pi/4, \quad \text{and} \quad R(\chi) = -\mu(g\bar{h})^{1/2} t - r\bar{h}\chi + \pi/4.$$

The stationary points of $Q(\chi)$ occur when $\mu'(\chi)(g\bar{h})^{1/2} = r/t$; that is, when

$$(g\bar{h} \tanh \chi)^{1/2} (1 + \chi (\cosh \chi \sinh \chi)^{-1}) / (2\chi^{1/2}) = r/t. \quad (4)$$

The function $\mu'(\chi)$ is a positive and strictly monotonic decreasing function on $(0, \infty)$ and has a maximum value of 1 at $\chi = 0$; see Gazarian (1955: fig. 1). Thus, for $r > t(g\bar{h})^{1/2}$ there is no stationary value of $Q(\chi)$. For $r < t(g\bar{h})^{1/2}$, there is a single positive root, $\chi = \chi_1$, of (4). There is no stationary value of $R(\chi)$ on $(0, \infty)$. Thus,

$$P_- \approx \left[\frac{2\pi \bar{h} (\chi_1 \bar{h}/r)^{1/2} \psi(\chi_1 \bar{h} \cos \theta, \chi_1 \bar{h} \sin \theta)}{(\alpha - i\mu(\chi_1)(g\bar{h})^{1/2})^2 (-\mu''(\chi_1)(g\bar{h})^{1/2} t)^{1/2}} \right] \exp \{ i(-\mu(\chi_1)(g\bar{h})^{1/2} t + \\ + r\bar{h}\chi_1) \} + o(\bar{r}\bar{h})^{-3/2},$$

where $r < t(g\bar{h})^{1/2}$, $r \gg h$, and $\chi_1 r \gg h$. Note that the denominator of P_- is zero for only purely imaginary χ ; thus this makes no contribution to the above asymptotic value. Similarly,

$$P_+ \approx \left[\frac{2\pi \bar{h} (\chi_1 \bar{h}/r)^{1/2} \psi(-\chi_1 \bar{h} \cos \theta, -\chi_1 \bar{h} \sin \theta)}{(\alpha + i\mu(\chi_1)(g\bar{h})^{1/2})^2 (-\mu''(\chi_1)(g\bar{h})^{1/2} t)^{1/2}} \right] \exp \{ i(\mu(\chi_1)(g\bar{h})^{1/2} t - \\ - r\bar{h}\chi_1) \} + o(\bar{r}\bar{h})^{-3/2},$$

where $r < t(g\bar{h})^{1/2}$, $r \gg h$, and $\chi_1 r \gg h$. The exponential terms in P_- and P_+ above represent sinusoids at wave number $m_1 = \chi_1 \bar{h}$ and amplitudes that are

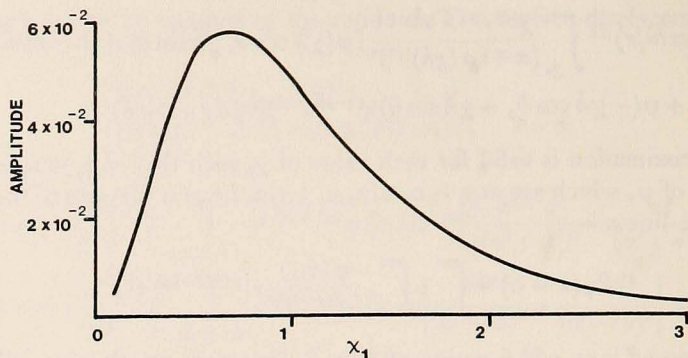


Figure 1. Wave amplitude as a function of χ_1 , $\theta = 70^\circ$, $\tau = 1.0$, $\beta h = 0.3$, $\gamma h = 3.0$.

given by the quantities in square brackets. By virtue of the stationary-phase condition (4), the stationary value corresponds to a definite value of r/t . The actual wave field, at a fixed time, t , thus consists of sets of water waves whose wave number, m_1 , decreases with increasing distance from the origin. The asymptotic values show that, for large time, the disturbance of the main wave train decays as $o(t^{-1})$, or as $o(r^{-1})$ by (4).

Now

$$(\alpha \mp i\mu(\chi_1)(g\bar{h})^{1/2})^{-2} = (\alpha^2 + \mu^2(\chi_1)g\bar{h})^{-1} \exp \left\{ i \arctan \frac{\pm 2\alpha\mu(\chi_1)(g\bar{h})^{1/2}}{\alpha^2 - \mu^2(\chi_1)g\bar{h}} \right\}.$$

Thus P_- and P_+ have the same amplitude, A , but differ in phase.

$$A \propto \left(\frac{\chi_1}{\mu'(\chi_1)\mu''(\chi_1)} \right)^{1/2} \frac{\beta\gamma h^2 \chi_1^2 \cos \theta \sin \theta}{\cosh \chi_1 ((\beta h)^2 + \chi_1^2 \cos^2 \theta)^2 ((\gamma h)^2 + \chi_1^2 \sin^2 \theta) (\tau^2 + \mu^2(\chi_1))},$$

where $\tau^2 = \alpha^2 h/g$. A plot of this amplitude as a function of χ_1 for fixed angle, $\theta = 70^\circ$, is shown in Fig. 1. Values of the parameters used, namely $\tau = 1$, $\beta h = 0.3$, $\gamma h = 3.0$, were obtained from actual earthquake data by estimating both the time required for the disturbance to reach its maximum and the position of the maximum disturbance. Small variations in the values of these parameters do not greatly affect the shape of the curve. The asymptotic approximation is not valid near $\chi_1 = 0$.

Fig. 2 shows a plot of the amplitude as a function of the angle for fixed $\chi_1 = 1$, with $\tau = 1$, $\beta h = 0.03$, $\gamma h = 0.3$. The shape of the curve varies as the eccentricity of the approximately elliptic area is varied. The $\cos \theta \sin \theta$ term changes the sign of the amplitude, hence the phase of the wave, at $\theta = \pi/2$; this is indicated by the broken line. At $\theta = 0, \pi/2, \pi$, and $3\pi/2$, the amplitude is identically zero and the above approximations are not valid. In fact, the amplitude curve starts from zero at $\theta = 0$, increases rapidly to a maximum,

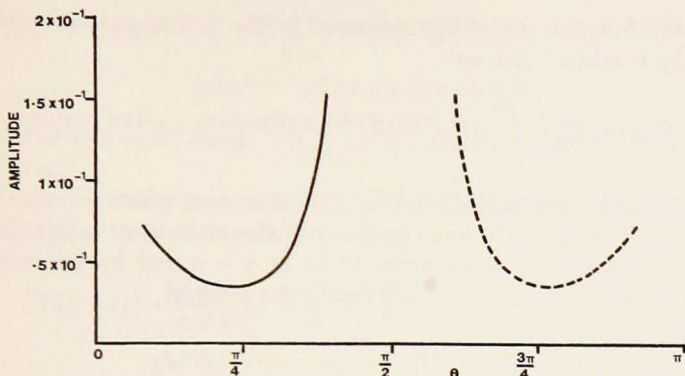


Figure 2. Wave amplitude as a function of angle, $\chi_1 = 1.0$, $\tau = 1.0$, $\beta h = 0.03$, $\gamma h = 0.3$.

and then closely approximates the curve shown in Fig. 2. Near $\theta = \pi/2$, the amplitude rises to a maximum and then drops to zero.

The above approximations to (3) are not valid when $\mu''(\chi)$ is zero; this occurs at $\chi = 0$, where the amplitude function, $\psi(0)$, is also zero. The above asymptotic results are thus not correct near the wave front, which is at the long-wave length limit. This is also the case in the corresponding two-dimensional problem in the Oxz plane; Braddock and van den Driessche (1971) have shown that, for large t with $\psi(0) = 0$, the wave front decays faster than the main wave. This is in contrast to the case $\psi(0) \neq 0$, when the wave front decays more slowly than the main wave.

In the three-dimensional problem, when the amplitude, ψ , is independent of angle, the η integral can be evaluated in terms of a Bessel function. This is equivalent to assuming an axially symmetric bottom velocity, formulating the problem in polar coordinates, and taking a Fourier-Bessel transform. Gazarian (1955) and Kajiuira (1963) have discussed the modifications that are necessary at the wave front for a symmetrical disturbance. Gazarian found that the amplitude of the first arriving sea wave is in general not the maximum and has shown that this remains true when $\psi(0) \neq 0$ although the disturbance is not necessarily symmetrical.

For the problem under discussion here, the asymptotic result near the wave front is derived by assuming χ to be small, such that $\beta, \gamma \gg \chi h$, $1 \gg \chi$. The η integral in P_{\mp} can be approximated by

$$\begin{aligned} & 8\chi^2 \bar{h}^2 / (\beta^3 \gamma^3) \int_0^{2\pi} \sin 2\eta e^{i r \bar{h} \chi \cos(\theta - \eta)} d\eta \\ & = -16\pi \chi^2 \bar{h}^2 / (\beta^3 \gamma^3) \sin 2\theta \mathcal{J}_2(r \bar{h} \chi), \end{aligned}$$

using Erdelyi et al. (1953, II, 7.12[2]).

The Bessel function is now approximated by the first term in its asymptotic series as $r\bar{h}\chi$ is taken large, so

$$P_{\mp} \approx P(2/(\pi r\bar{h}))^{1/2} \int_0^{\infty} \chi^{5/2} e^{\mp i\mu(g\bar{h})^{1/2}t} (e^{i(r\bar{h}\chi - 5/4\pi)} + e^{-i(r\bar{h}\chi - 5/4\pi)}) d\chi,$$

where $P = -8\pi\bar{h}^4 \sin 2\theta / (\beta^3 \gamma^3 \alpha^2)$. When the stationary-phase point is at $\chi \neq 0$, the previous argument again leads to decay of the main wave as $o(t^{-1})$. Now, by forcing the stationary-phase point to be at $\chi = 0$ and by expanding the exponent in a series about $\chi = 0$, we obtain the integral

$$P_- \approx P(2/\pi r\bar{h})^{1/2} \int_0^{\infty} \chi^{5/2} e^{i(g\bar{h})^{1/2}t} \chi^{9/6} d\chi.$$

Watson's lemma, used to estimate the integral for large t , gives $P_- = o(t^{-5/3})$; P_+ has a similar asymptotic value; note that P_{\mp} are zero on $x = 0$ or on $y = 0$, in agreement with a previous result.

The predicted rapid decay of the wave front depends on the exact nature of the bottom velocity, but, for any antisymmetric disturbance, the wave front decays much more rapidly than the main wave. This is in contrast to the results for a symmetrical disturbance when the wave front decays as $o(t^{-1})$, although Gazarian (1955) has found that, at the instant of arrival of the leading front, the amplitude is approximately one half that of the main wave.

An alternative model. In Chaudhuri's (1968) discussion of surface waves excited by an initial surface impulse and by an elevation across arbitrary regions, one of the examples considered is an elliptic region. The same function can be used in the present problem to model a symmetrical bottom disturbance that is confined to an exactly elliptic region. Let

$$X(x)Y(y) = \frac{I\nu}{\pi ab} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^{\nu-1} \quad \text{inside} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

$$= 0 \quad \text{outside} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where I, ν are constants with $\text{Re } \nu > 0$. In this case, the amplitude function $\psi(k, l) = K \mathcal{Y}_{\nu}[(k^2 a^2 + l^2 b^2)^{1/2}] (k^2 a^2 + l^2 b^2)^{-\nu/2} / \cosh mh$, where $K = -2^{\nu} I \nu \Gamma(\nu) / (MN)$ (Erdelyi et al. 1954, I, 1.3[8], 1.13[50]). By changing to polar coordinates and by using the method of stationary phase, the main wave is found to decay as $o(t^{-1})$; this result holds for all angles and large distances from the disturbance. Chaudhuri has not considered the wave front, but, in his example as well as in the present case, the asymptotic approximations are not valid near the wave front where $\chi = 0$. However, by using the series expansion for the Bessel function

$$\psi(\chi \bar{h} \cos \eta, \chi \bar{h} \sin \eta) = K \sum_{n=0}^{\infty} (-1)^n (a^2 \chi^2 \bar{h}^2 \cos^2 \eta + b^2 \chi^2 \bar{h}^2 \sin^2 \eta)^n / \\ (n! 2^{2n+\nu} \Gamma(n+\nu+1) \cosh mh),$$

so $\psi \neq 0$ at the wave front. Thus, by Watson's lemma the wave front also decays as $o(t^{-1})$.

An antisymmetrical bottom disturbance confined to an exactly elliptic region can be modeled in a similar way by taking

$$X(x)Y(y) = \frac{I \nu xy}{\pi a^3 b^3} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^{\nu-1} \quad \text{inside} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \\ = 0 \quad \text{outside} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Then

$$\psi(k, l) = -Kkl \mathcal{J}_{\nu+2} [(k^2 a^2 + l^2 b^2)^{1/2}] (k^2 a^2 + l^2 b^2)^{-\nu/2-1} / \cosh mh,$$

(Erdelyi et al. 1954, I, 2.3[9], 2.13[51]). Proceeding as before, the main wave decays as $o(t^{-1})$ for $x, y \neq 0$; if either x or y is zero, the main wave is zero. At the wave front, $\psi = 0$, hence the asymptotic results must be modified by expanding the Bessel function in a power series:

$$\psi(\chi \bar{h} \cos \eta, \chi \bar{h} \sin \eta) = -K \chi^2 \bar{h}^2 \sin 2\theta \sum_{n=0}^{\infty} (a^2 \chi^2 \bar{h}^2 \cos^2 \theta + b^2 \chi^2 \bar{h}^2 \sin^2 \theta)^n / \\ (n! 2^{2n+\nu+3} \Gamma(n+\nu+3) \cosh mh).$$

Thus the wave front decays as $o(t^{-5/3})$, which is in agreement with the previous result for the antisymmetric model.

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