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A Numerical Tidal Model and its Application to Cook Inlet, Alaska¹

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ABSTRACT

A numerical scheme for predicting tides and tidal currents has been designed for rapid application to new situations with a minimum of effort. The model is two-dimensional and includes Coriolis and frictional terms. An application to Cook Inlet, Alaska, is described for a tide having a period of 12.42 hours and an amplitude of one half the mean tidal range. The tidal wave in the lower inlet suggests a progressive Kelvin wave while the upper inlet shows standing-wave characteristics. Currents and ranges predicted by the model show good agreement with the few available observations. The maximum error in the model was 8% for the tidal range predicted for Kenai.

Introduction. Analytical solutions to tidal problems in coastal waters have not yet been achieved except in very simple cases. But during the last 50 years a large number of numerical solutions have been developed for various purposes. Probably the first actual tidal model (as opposed to an analytical solution) was that developed by Blondel (1912) for the Red Sea. Similar models were developed by Sterneck (1915), Defant (1920), Lorentz (1926), Grace (1936), and Proudman (1953).

These early models were numerical solutions to the equations of motion and continuity without time dependence. It was not until digital computers became available that solutions involving time dependence became feasible. Following Hansen's (1952) time-dependent model for the North Sea, many numerical solutions to tidal problems appeared in the literature. These models and their mathematical principles have been reviewed by Heaps (1969) and Dronkers (1969) and will not be repeated here. Most of the models described to date deal with one specific coastal area, with the model usually tailored to the per-

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continent region. Consequently there is no model that is fairly widely applicable and easily used by those whose field is not primarily tidal hydraulics.

This paper presents a numerical tidal model that has been designed for wide applicability to coastal tidal problems without much additional work or re-programming.

Mathematical Basis. The equations of motion and continuity in two dimensions are the mathematical foundation of the model. They are written in Cartesian coordinates as follows:

$$\frac{\partial u}{\partial t} = fv - g \frac{\partial z}{\partial x} - k(u^2 + v^2)^{1/2} u/h, \quad (1)$$

$$\frac{\partial v}{\partial t} = -fu - g \frac{\partial z}{\partial y} - k(u^2 + v^2)^{1/2} v/h, \quad (2)$$

and

$$\frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} + \frac{\partial z}{\partial t} = 0. \quad (3)$$

The x , y , and z axes form a right-handed system, with z positive upward; u and v are the depth-mean velocity components along the x and y axes; f is the Coriolis parameter; h is the total water depth (positive); z is the height of the water surface above some convenient horizontal datum; k is a dimensionless friction coefficient related to the Chézy friction coefficient by the relationship $k = g/C^2$. In formulating the equations, it has been assumed that the water possesses constant density both vertically and horizontally and that the convective acceleration and the wind shear stress are negligible.

The x , y , t -grid chosen is that developed by Hansen (1952); its projection on the xy plane is shown in Fig. 1. Heights, Z , have been evaluated at even time steps; currents, U and V , have been evaluated at odd time steps. The grid network is thus "staggered in time and space." The locations of U , V , and Z are such that space derivatives may generally be evaluated directly, although some interpolation is necessary in the finite-difference form of the equation of continuity. The terms of the equations were written in finite-difference form, using the two-point central difference as follows:

$$\frac{\partial u^t}{\partial t} = \frac{U^{t+1} - U^{t-1}}{2\Delta t}, \quad (4)$$

$$\frac{\partial z_{m,n}}{\partial x} = \frac{Z_{m+1,n} - Z_{m-1,n}}{2l}. \quad (5)$$

The superscripts $t-1$, t , $t+1$ are successive time steps at intervals of Δt ; the subscripts $m-1$, m , $m+1$ or $n-1$, n , $n+1$ denote successive grid positions in the x and y directions at intervals of l .

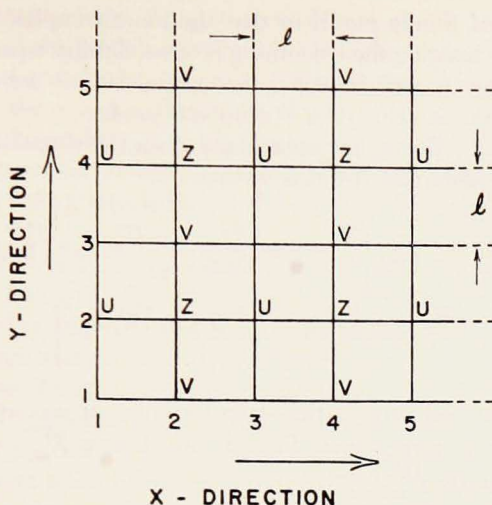


Figure 1. A corner of the two-dimensional grid showing the disposition of U , V , and Z points; l is the grid spacing.

The current $U_{m,n}$ is found from (1); in finite-difference form it is written:

$$U_{m,n}^{t+1} = \overline{U_{m,n}^{t-1}} + 2 \Delta t \left\{ fV_{m,n}^{t-1} - g \frac{(Z_{m+1,n}^t - Z_{m-1,n}^t)}{2l} - k \frac{\overline{U_{m,n}^{t-1}} (U_{m,n}^{2t-1} + V_{m,n}^{2t-1})^{1/2}}{H_{m,n}^t} \right\} \quad (6)$$

In the equations for U^{t+1} and V^{t+1} , U^t and V^t are required; they have been approximated by using the values calculated at time step $(t-1)$. $V_{m,n}$ has been interpolated from the four surrounding V values; thus $V_{3,2}$ in Fig. 1 was interpolated by using the four surrounding V values: $V_{2,1}$, $V_{4,1}$, $V_{2,3}$, and $V_{4,3}$. U values at V points have been similarly interpolated. The calculations do not call for U and V values at Z points or at unmarked points in Fig. 1. The terms with the overbar in (6) are smoothed terms to which a stability factor has been applied such that

$$\overline{U_{m,n}^{t-1}} = \alpha U_{m,n}^{t-1} + \frac{1-\alpha}{4} (U_{m+1,n+1}^{t-1} + U_{m-1,n-1}^{t-1} + U_{m-1,n+1}^{t-1} + U_{m+1,n-1}^{t-1}) \quad (7)$$

$$0 < \alpha < 1.$$

For this smoothed term we again used the interpolated values from the surrounding grid points. It has the advantage over the one applied by Yuen

(1967) in his Bay of Fundy model in that the more complex boundaries can be treated without changing the smoothing process. Similar equations apply for the V -velocity terms. It will be noted that interpolations were all linear for speed in computation. A value of $\alpha = 0.99$ was used.

The tidal elevation, $Z_{m,n}$, has been found from the equation of continuity (3), which in finite-difference form is written:

$$Z_{m,n}^{t+2} = \overline{Z_{m,n}^t} - 2\Delta t \left\{ \frac{(H_{m+1,n}^t U_{m+1,n}^{t+1} - H_{m-1,n}^t U_{m-1,n}^{t+1})}{2l} + \frac{(H_{m,n+1}^t V_{m,n+1}^{t+1} - H_{m,n-1}^t V_{m,n-1}^{t+1})}{2l} \right\} \quad (8)$$

H should have been evaluated at time-step $t+1$, but it was approximated by the value calculated at time-step t . The smoothed term $\overline{Z_{m,n}^t}$ was found in a way similar to the term $\overline{U_{m,n}^{t-1}}$ (7).

In the explicit formulation used here, the equations are written in such a way that only one unknown appears in any equation. All other parameters are known from previous time steps. Generally, explicit methods should fulfil a stability criterion of the type $l > \Delta t (bgh_{\max})^{1/2}$, where b is a constant that varies with the grid system and the finite-difference scheme. Leendertse (1967) has discussed the effect of the choice of Δt and l on the difference between the calculated wave and the physical wave. For this grid network, the constant b in the stability-criterion equation is equal to 2 (see Lax and Richtmeyer 1956); thus

$$\Delta t < \frac{l}{(2gh_{\max})^{1/2}} \quad (9)$$

is the stability criterion that must be observed.

Computer Program. The finite-difference equations have been evaluated for the grid network by means of a computer program written in Fortran IV. The program is given in Mungall and Matthews (1970). Fortran was chosen because it is widely accepted as the scientific programming language and is almost universally available.

The flexibility of this model hinges upon the use of an integer-control matrix. This matrix, consisting of input data that are used to control a program operation, obviates the need for major alterations in a program for different inlets. Once specified, the matrix controls the allocation of depth and tidal height during the input and output phase, the computation of the current and tidal height during routine calculations, and the calculation of the mean range, the phase of tide, the current ellipses, and other parameters during the output stage. Details are available in Matthews and Mungall (1970). Basically, the

matrix is composed of the integers 1, 2, and 3, which are codes that define the boundaries of the model. Provision is made for complex inlets that include islands and land-and-sea or sea-and-sea boundaries. The program scans the matrix in both the x and y directions; in this way it eliminates special computations for different boundary configurations involving, for example, bays or peninsulas. This model should be compared with other typical models, such as that of Heaps (1969), which has 22 different special computations.

Experimental Procedure. In applying the program to an inlet, the following procedure was followed. A grid spacing and a time interval were chosen to satisfy the stability criterion (9). The grid spacing was determined from the dimension of the inlet to be analyzed. The number of time intervals chosen would usually be 360 or 720, which would allow theoretical tidal-phase resolution of 2° or 1° , respectively. The 360 intervals give a 2° resolution because there are two time intervals between successive height calculations.

In fitting a grid to an inlet, we found that a slide projector was most useful during preliminary investigations into the grid spacing. A slide with a grid ruled on it was projected onto a chart of the inlet. The grid position, the size, and the orientation were then adjusted until the best fit was obtained. The inlet profile and the grid outline were then traced, and the depths of the basin were read off at grid points. The boundaries must be aligned so that the V points fall along one set of parallel boundaries, with the U points at the perpendicular boundaries. The chosen boundary, after being coded in integers, then formed the integer-control matrix. For example, a code digit 1 indicated a land-and-sea boundary and had the effect during the program operation of setting U or $V = 0$ at the boundary point, the condition for no flow across the boundary. Depth data were supplied, and the initial values of height and current were set equal to zero throughout the inlet. To run the model, time variations in tidal elevations at the sea-and-sea boundaries were supplied.

Application and Results. The model was applied to Cook Inlet, Alaska. The Inlet, from Homer to Anchorage, is about 200 km (110 n miles) long. It is relatively shallow, the greatest depth being only 137 m or 75 fathoms (Fig. 2). The salinity is almost constant from surface to bottom and thus fulfills the assumption of vertical homogeneity.

The shallowness of the inlet and its general topography give rise to many shallow-water harmonics and overtides. Zetler and Cummings (1967) employed 140 tidal constituents for a description of the Anchorage tides. In order to investigate, as economically as possible, the overall tidal regime in Cook Inlet, we used a sinusoidal wave of period 12.42 hours and an amplitude of one half the mean tidal range; this approximation appears reasonable. The formzahl $F = (K_1 + O_1)/(\mathcal{M}_2 + S_2) = 0.24$ at Anchorage and 0.30 at Seldovia, thus indicating the presence of predominantly semidiurnal tides throughout the

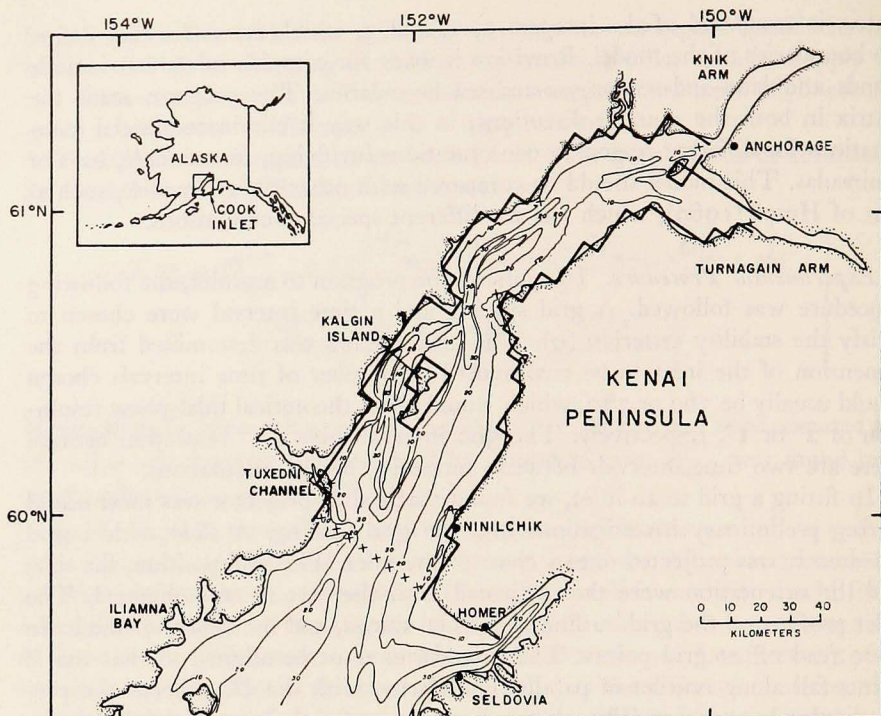


Figure 2. The grid boundary superimposed on an outline of Cook Inlet, Alaska. Depth contours in fathoms. \times denotes points for which the time variation of the tidal elevations was specified.

modeled inlet. Also, during the first two tidal cycles of the model operation, the actual tidal elevations were forced on the model at several points in order to speed up the convergence. The saving in computer time justified the mathematical overspecification that this entailed.

In Fig. 2, the grid outline chosen for Cook Inlet is superimposed on the actual outline. Beyond the North Foreland, there are extensive tidal flats, and representing these in a fixed-boundary model poses a problem. In our solution, the boundary was fixed midway between the low-water and high-water marks, and the mean-water level over the included mudflat area was arbitrarily set at 7.3 m (4 fathoms). The input points shown in Fig. 2 are the points for which tidal elevations were specified. These were derived from tidal data supplied by the U.S. Coast and Geodetic Survey. The input data at the sea entrance were linear interpolations of values for Iliamna Bay and Tuxedni Channel, Ninilchik, and Seldovia. The inlets representing Knik Arm (on which Anchorage stands) and Turnagain Arm were left as sea-and-sea boundaries. This was done because the grid spacing is too large to accommodate the narrow arms, and the model cannot handle the mudflats. It was realized that

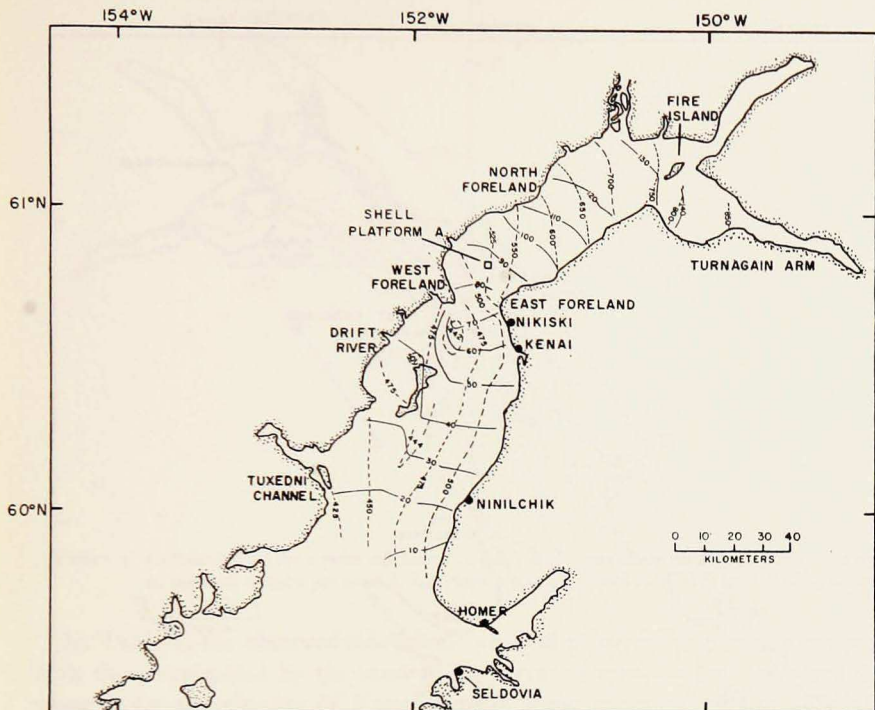


Figure 3. Corange (dotted lines) and cotidal (solid lines) contours predicted for Cook Inlet by the numerical model. Ranges are shown in centimeters and phases in degrees based on 0° at Seldovia.

the specification of sinusoidally varying tidal elevations would cause the tides in the upper part of the inlet to show less distortion than that which actually occurs, but the errors incurred are small.

In this analysis, a grid of 65×29 points was utilized, resulting in a grid spacing of 3.052 km. A time interval of 62.10 seconds was used; this satisfied the stability criterion. For the friction coefficient, k , a value of 0.003 was used throughout the computation. Five full tidal cycles were computed, and each cycle required about 20 minutes of computer time on an IBM 360/40 computer. Every value calculated during the last cycle was stored on magnetic tape. An additional 15-minute analysis of the output tape was then carried out.

The corange and cotidal lines predicted by the model are shown in Fig. 3. In lower Cook Inlet, these lines are orthogonal, with the corange lines lying approximately along the longitudinal axis of the Inlet, suggesting a progressive Kelvin wave. Thus, tidal friction does not appear to be dominant in lower Cook Inlet. That these lines are not orthogonal in the upper Inlet demonstrates the importance of tidal friction in these shoaling waters.

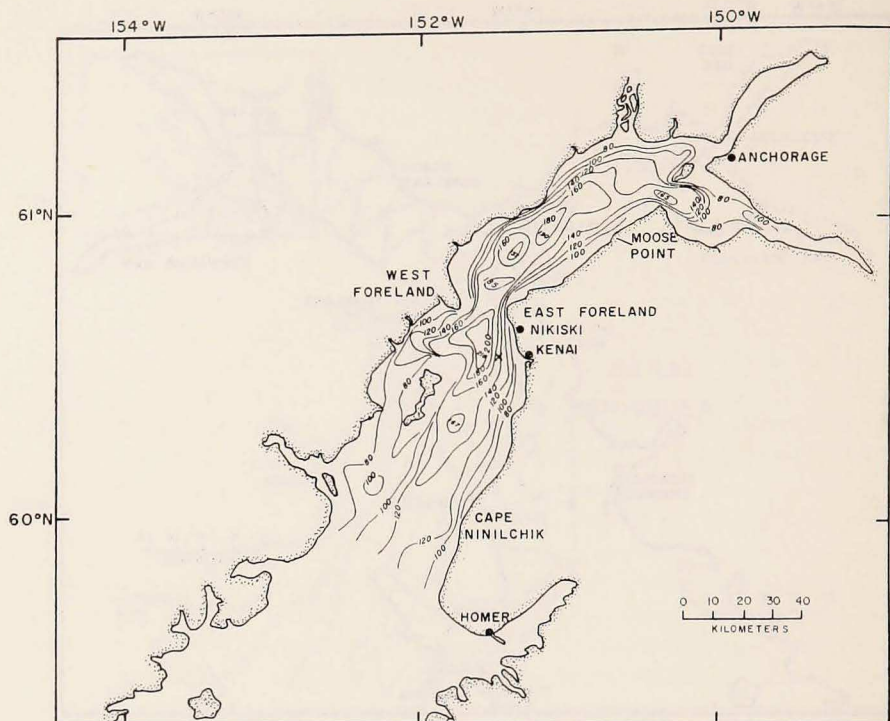


Figure 4. Maximum tidal-current contours predicted for Cook Inlet. Contour values are given in centimeters per second. \times marks the point for which the current ellipse in Fig. 5 was obtained.

Table I. Observed and computed phase differences (degrees) and mean tidal ranges (cms).

Location*	Source**	Observed		Computed		Computed less observed	
		Phase†	Mean range	Phase†	Mean range	Phase	Mean range
Fire Island	HC	131	724	134	757	3	33
North Foreland	TT	113	558	112	585	-1	27
Shell Platform A.....	HC	86	504	84	525	-2	21
East Foreland.....	TT	79	549	80	550	1	1
Nikiski.....	HC	72	523	70	520	-2	-3
Chinulne Point (Kenai)	HC	60	547	58	501	-2	-46
Drift River.....	HC	53	489	40	490	-13	1
Ninilchik.....	TT	24	509	23	515	-1	6
Tuxedni Channel	TT	21	430	22	425	1	-5

* See Fig. 3.

** TT indicates values derived from standard tables (U.S.C. & G.S. 1968).

HC indicates values derived from harmonic constants furnished by U.S. Coast and Geodetic Survey, E. S. S. A., Rockville, Maryland.

† Phases relative to Seldovia.

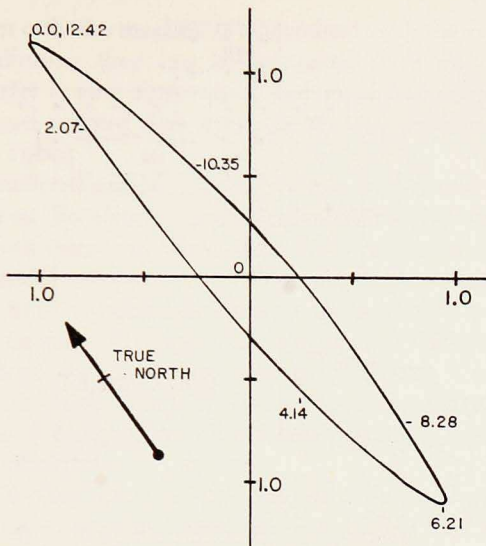


Figure 5. Current ellipse for a point off Kenai in Cook Inlet (marked \times in Fig. 4). Axes are marked in units of meters per second. Indicated times are hours following high tide at Seldovia.

In Table I, the observed tidal elevations and phase differences are compared with those computed by the model. The greatest error in the computed tidal range is for the vicinity of Kenai, with a mean range of 5.01 m computed. This compares with an observed value of 5.47 m, giving a difference of about 8%. This probably results largely from the complex geometry of the model in the vicinity of Kenai (Fig. 3). Errors of 4–5% are indicated for the North Foreland and for Fire Island. These deviations are tolerable in view of the difficulty of representing the extensive mudflats in this area.

The maximum error in phase differences is one of 13° for Drift River. It is now known that this resulted from a value that was some 10° too small for Tuxedni Channel; the erroneous quantity was taken from the U.S. Coast and Geodetic Survey Tide Tables for 1969. This probably accounts for the crowding of the cotidal lines south of the West Foreland. The results for beyond the North Foreland should be accepted with caution because of the known inaccuracies of the model. However, the predictions of the tidal elevations do appear to be close to the mean tidal ranges throughout the Inlet.

An analysis of the output tape was made for maximum currents predicted by the model. Fig. 4 shows the contours of these currents. The highest values occur between the West Foreland and Kenai. These contours represent maximum current values irrespective of time during the tidal cycle or direction. The currents throughout the tidal cycle show a pronounced ellipticity, as illustrated in Fig. 5. The location, off Kenai, for which this current ellipse was computed, is marked on Fig. 4. Many of the predicted ellipses show

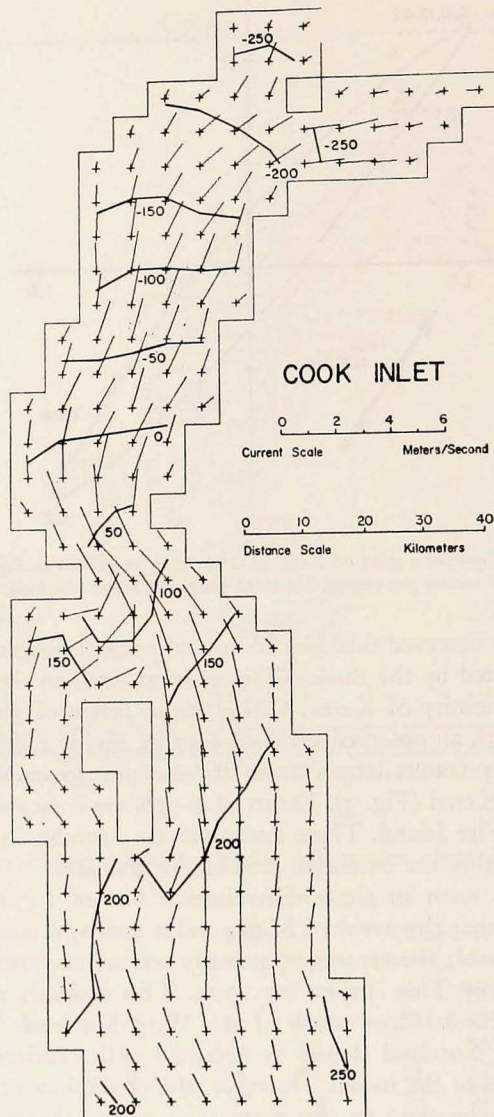


Figure 6. Current vectors predicted by the model for the time of high tide at Seldovia.

a counterclockwise rotation of the currents. Since a sinusoidal boundary condition was assumed for this model, the ebb and flow vectors are nearly equal and opposite in direction.

Because of the distortion of the tidal wave due to shallow water, the actual wave is not sinusoidal. The ebb and flood tides are of unequal duration. Our

currents represent only the maxima as determined with the assumption of mean-range tidal conditions; they can be compared with only average maxima. Further, the model is two dimensional and yields only depth-mean currents. The only observed current data available are the predicted surface currents (U.S.C. & G.S. 1969).

The model predicted maximum depth-mean currents of 2.1 m/sec between the East and West Forelands. The current tables give an average value of 1.96 m/sec for both maximum flood and ebb currents. For the waters off Cape Ninilchik, the model predicts currents of 0.96 m/sec compared with currents of 1.1 m/sec for flood currents and of 0.72 m/sec for ebb currents. Off Moose Point there are currents of 1.44 m/sec compared with tabulated currents of 1.49 m/sec for flood tides and 1.33 m/sec for ebb tides. The model is in close agreement with observed surface currents, and it seems likely that the shallow water responds as a whole to the tidal motion and does not have much variability with depth.

One feature of great value in the program is its production of a magnetic tape containing all of the data from the final cycle. The tape output can be preserved for a detailed analysis of any feature of the tides or tidal currents at some later time. By means of an off-line plotter, the program automatically plots the model boundary, the Z points, the current vectors, and the lines of equal tidal height for any one of the intervals during the last cycle. An example, for the time of high tide at Seldovia, is shown in Fig. 6. The current vectors are predominantly directed up the Inlet at this instant. This results from a gradual change in the time difference between the maximum inward current and the maximum tidal height with progression up the Inlet. The phase difference at Homer is some 35° while at Anchorage it has increased to about 90° .

Conclusions. We have developed a tidal model that is easy for the layman to use and is easily adapted to different estuarine situations. It has given satisfactory results for Cook Inlet, Alaska, which has very large tidal ranges and currents. The model affords an economical means of determining the tidal-current distribution in an inlet for which good tidal-height data are available. Where the effects of friction are relatively small, it can be used to obtain the overall distribution of the more important harmonic constituents throughout an estuary. Alternatively, where frictional effects are large, the mean ranges and currents can be found, as in the example given for Cook Inlet. This model should prove particularly useful in assessing the impact of engineering structures, such as barrages in a tidal estuary. We plan to make use of the integer-control matrix to alter the grid boundaries to simulate mudflat flooding.

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