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# *A Comparison of Direct Measurements and G.E.K. Observations in the Florida Current Off Miami<sup>1</sup>*

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## ABSTRACT

Electrical measurements represent an indirect measure of the velocity structure of an ocean current. An interpretation of such measurements requires consideration of many influences, including the distribution of the vertically averaged velocity, of the bottom topography, and of the electrical-conductivity structure of the ocean and the sea bed. Two extensive sets of data from the Florida Current off Miami, Florida, are compared: G.E.K. fixes accumulated in the period 1952-1958 and free-instrument measurements obtained in the period 1964-1967. No significant difference in the mean is found between speeds obtained from G.E.K. observations and from the directly measured quantity: surface speed minus vertically averaged speed. Topographic effects are small, as are the net effects due to electrical conductivity variations in the sea and in the sea bed. It is shown that the transport of the Florida Current off Miami could be measured electrically with an expected uncertainty of about 10% of the mean.

*Introduction.* Numerous G.E.K. observations across a section of the Florida Current off Miami, Florida, were collected in the six-year period, 1952-1958; these measurements have been summarized by Chew (1967). In the period 1964-1967, a comparable number of free-instrument observations were made at essentially the same location; certain features of these results have been summarized by Schmitz and Richardson (1968). The two sets of data, though taken many years apart, provide the basis for a comparison of G.E.K. measurements with direct observations. Mean values derived from the two sets of data are compared.

The motionally induced electric field has been used to monitor the transport of the Florida Current (Wertheim 1954). We have used the G.E.K. and free-

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instrument data to discuss the interpretation of potential measurements, using submarine cables. In particular, it is found that off Miami such measurements would be less uncertain than those taken off Key West, Florida.

*Initial Considerations.* Let  $(u, v, w)$  denote velocity ( $\mathbf{v}$ ) components in a  $(x, y, z)$  Cartesian coordinate system (east, north, vertical-positive upward);  $t$  is the time coordinate. Attention has been focused on observations of  $v$ . The  $y$  direction is very nearly downstream. Vertical averages, denoted by an overbar, extend to the local bottom,  $D(x, y)$ , unless otherwise noted:

$$\bar{v} = \frac{1}{D} \int_{-D}^0 v dz. \quad (1)$$

Time averages are denoted by capital letters. In practice,

$$V(x, y, z) = \frac{1}{N} \sum_{i=1}^N v_i(x, y, z, t_i) \quad (2)$$

has been used to calculate temporal means for the series  $v_i$  collected at a given position and at times  $t_i$ . The subscript,  $s$ , is used to denote surface ( $z = 0$ ) values. Free-instrument observations have yielded  $v_s$  and  $\bar{v}$  for each station occupation;  $V_s$  and  $\bar{V}$  were calculated by (2) from several such samples. The volume transport of the current is

$$T = \int_{x_I}^{x_0} dx \int_{-D(x)}^0 v dz = \int_{x_I}^{x_0} \bar{v} D dx; \quad (3)$$

where  $(x_I, x_0)$  denote the position of the (inshore, offshore) edge of the Current.

Under conditions approximately appropriate to the region where the data are available, the electric-potential ( $\varphi$ ) gradient has been given by Sanford (1971):

$$\nabla \varphi = \mathbf{v}_H \times \mathbf{F} - \frac{1}{\sigma} \mathbf{J}, \quad (4)$$

where

$$\frac{\mathbf{J}}{\sigma} = \nabla \times \left\{ \int_z^0 F_z (\mathbf{v}_H - \bar{\mathbf{v}}_H) dz + \frac{\mathbf{k}Q}{2\pi} \right\}, \quad (5)$$

and

$$Q = \frac{1}{2\pi} \iint_{-\infty}^{\infty} [\nabla \cdot (F_z \bar{\mathbf{v}}_H)] \ln [(x - x')^2 + (y - y')^2]^{1/2} dx' dy',$$



where  $\mathbf{v}_H$  is the horizontal-current vector,  $\mathbf{F}$  is the geomagnetic-field vector with components  $(F_x, F_y, F_z)$ ,  $\mathbf{J}$  denotes the electric-current density,  $\sigma$  denotes the electrical conductivity, and  $\mathbf{k}$  is the unit vector in the vertical direction. A second approximation to  $\nabla\varphi$  allows for effects due to variations in the free-surface elevation and in the bottom topography. In this second approximation, the second term in eq. (5) is amplified (attenuated) by the ratio  $(H/D)$  of the spatially averaged depth to the local depth. Other second-order contributions to  $\nabla\varphi$  are not considered in this paper. The effects of the presence of lateral boundaries have not been considered, so that the approximation (4), (5) is best near the center of the Current. The essential effect of variable  $\sigma$  may be handled by replacing vertical averages with conductivity-weighted vertical averages. The results (4), (5) are generalizations of those of Longuet-Higgins et al. (1954).

In comparing the two sets of data, the cross stream or x component of (4) has been used:

$$\frac{\partial\varphi}{\partial x} = F_z \bar{v} - \frac{\partial Q}{\partial y}. \quad (6)$$

Equations (4)–(6) are applicable to potential measurements with fixed electrodes. If the measurement of potential is made with moving electrodes, then the e.m.f. generated by the motion of the electrodes in the Current is subtracted from (4) to obtain:

$$\nabla\varphi_G = -\frac{\mathbf{J}}{\sigma}; \quad (7)$$

$\varphi_G$  is the potential in the moving coordinates. If electrodes of separation,  $L$ , are being towed at the surface (G.E.K.), then the component of (7) of interest is

$$\frac{\Delta\varphi_G}{L} = -F_z(v_s - \bar{v}) - \frac{\partial Q}{\partial y}. \quad (8)$$

The abbreviation,  $v_G$ , is introduced for  $-\Delta\varphi_G/LF_z$ , so that (8) becomes

$$v_G = v_s - \bar{v} + \frac{1}{F_z} \frac{\partial Q}{\partial y}. \quad (9)$$

It is customary to interpret G.E.K. measurements in terms of a  $k$  factor—a dimensionless number defined by

$$k = \frac{-v_s L F_z}{\Delta\varphi_G} = \frac{v_s}{v_G}. \quad (10)$$

Knowledge of  $k$  permits estimates of  $v_s$  to be obtained from G.E.K. observations. As a matter of convenience, the  $k$  factor is generally considered to be

constant over a large region. Two possible reasons for the breakdown of the "constant"  $k$ -factor approach would be: (i) if the  $(x,t)$  variation in  $v_G$  were significantly different from the  $(x,t)$  variation in  $v_s$ , resulting in  $k$  being a strong function of  $(x,t)$ , (ii) if  $\partial Q/\partial y$  were of the order of  $F_z v_s$ . We will show that  $\partial Q/\partial y$  is typically, but not always, of secondary importance in the interpretation of the time-averaged potential measurements for the observational situation; but  $k$  has a strong  $(x,t)$  dependence. Although the G.E.K. and the direct measurements available can be compared only in the mean, the time dependence of  $k$  may be studied in terms of a "theoretical  $k$  factor", (neglecting  $\partial Q/\partial y$ )  $k^*$ , where

$$k^* = \frac{v_s}{v_s - \bar{v}}, \quad (11)$$

estimates of which may be formed from available free-instrument data. If the interpretation of the G.E.K. measurements is in terms of local and instantaneous  $(v_s - \bar{v})$ , it is necessary to obtain additional information in conjunction with the G.E.K. data. Since it is likely to be easier to obtain concurrent estimates of  $v_s$  than of  $\bar{v}$ , an effort has been made to determine the accuracy with which estimates of the latter can be made. Define

$$\bar{v}^* = v_s - v_G \quad (12)$$

as the electrical estimate of  $\bar{v}$ . Estimates of the utility of this scheme are made in the following for the mean fields. The data necessary to evaluate this program for instantaneous observations have not been available. Recently, however, one or two simultaneous G.E.K. and free-instrument profiles have been made of the Florida Current off Fort Pierce, Florida (Chew et al. 1971).

*The Data.* The G.E.K. data were collected by several investigators; 623 fixes were made on 42 cruises during 1952-1958. A cruise normally consisted of an initial transect and a return transect across the Current; a few cruises were spent in the vicinity of a particular station  $(x,y)$  or station pair. The data were split by Chew (1967) into groups covering 20 zones across the Current. Each zone contains 31 observations; 3 observations were discarded. These zones tend to be narrow in the vicinity of the speed maximum and wider on the flanks of the Current. The temporal averages,  $V_G$ , for this data set are listed in Table I.

The free-instrument data were collected between August 1964 and May 1967;  $V_s$  and  $\bar{V}$  are listed in Table II. This program of cruises was similar to that used to collect the G.E.K. data (see, for example, Schmitz and Richardson 1966, 1968). The free-instrument data were taken along two sections (Table II) across the Current (Section III, Sts. 1-13; Section II, Sts. 14-26). The



Table I. G.E.K. Data. ( $\Delta\lambda, \bar{\lambda}, \bar{x}$ ) denote the longitude interval, average longitude, and average cross-stream position for a given zone. Mean values from the G.E.K. fixes are denoted by  $V_G$ .

Zone #	$\Delta\lambda$ (°W)	$\bar{\lambda}$ (°W)	$x$ (km)	$V_G$ (cm/s)
1.....	1.1	80° 04.9	8.3	43 ± 7
2.....	1.3	03.7	10.3	69 ± 9
3.....	2.2	01.9	13.3	62 ± 9
4.....	1.8	79° 59.7	17.1	83 ± 6
5.....	2.4	57.6	20.6	90 ± 8
6.....	1.8	55.4	24.3	105 ± 7
7.....	2.0	53.5	27.5	106 ± 5
8.....	0.1	52.5	29.2	85 ± 4
9.....	3.1	50.9	31.9	117 ± 6
10.....	2.1	48.2	36.5	117 ± 6
11.....	1.2	46.5	39.4	103 ± 3
12.....	1.9	45.0	41.9	106 ± 4
13.....	3.0	42.5	46.1	99 ± 6
14.....	3.0	39.5	51.2	84 ± 4
15.....	2.9	36.6	56.1	64 ± 7
16.....	3.0	33.5	61.3	64 ± 5
17.....	3.1	29.6	67.9	50 ± 5
18.....	2.5	26.8	72.7	38 ± 5
19.....	3.4	23.7	77.9	31 ± 6
20.....	2.3	20.7	83.0	23 ± 5

average number of time samples at each station is 24 for Sts. 1-13 (Table II) and 11 for Sts. 14-26. The data for Section III have been used principally in this paper while the data for Section II have been used only to form an estimate of  $Q$ .

All of the data have been referred to the ( $x, y$ ) coordinate system (longitude, latitude) used in collecting the free-instrument observations. The positional control exerted in obtaining the two data sets was different. The free-instrument data were collected within a few tens of meters about a given ( $x, y$ ). The G.E.K. data were collected at variable and much less accurately known ( $x, y$ ). The average difference in the G.E.K. positions from the line  $y = 0$  is 0.7 km; the standard deviation is 3.5 km. The averaging of the G.E.K. data in time over observations distributed across a longitudinal zone has led to an unknown contamination of time averages with space variability. The average is interpreted (Table I) to be applicable to the midpoint of the zone.

Estimates of error in calculating the mean values (presented in Tables I and II) were obtained by dividing the standard deviation of the samples in time by the square root of the number of samples. It has been assumed that the sampling error is the primary contribution to the overall observational error for the averages; the estimates of the instrument error (at 5 cm/s for the free-instrument observations and at a comparable magnitude for the G.E.K. data) divided

Table II. Free-instrument data for Sections II and III.  $(x, y)$  denote the (east, north) position for a given station.  $(V_s, \bar{V})$  are the mean surface and vertically averaged downstream ( $y$ ) velocity components.

Sta. no.	$x$ (km)	$y$ (km)	$V_s$ (cm/s)	$\bar{V}$ (cm/s)	$(V_s - \bar{V})$ (cm/s)
1.....	10	0	74 ± 15	44 ± 11	30 ± 7
2.....	15	0	125 ± 11	48 ± 6	77 ± 7
3.....	20	0	165 ± 9	66 ± 4	99 ± 7
4.....	25	0	192 ± 6	85 ± 3	107 ± 6
5.....	30	0	195 ± 5	101 ± 3	94 ± 5
6.....	35	0	190 ± 5	65 ± 3	125 ± 5
7.....	45	0	171 ± 4	76 ± 2	95 ± 3
8.....	55	0	151 ± 3	74 ± 2	77 ± 3
9.....	65	0	123 ± 4	67 ± 2	56 ± 3
10.....	70	0	110 ± 4	67 ± 3	43 ± 4
11.....	75	0	97 ± 5	68 ± 3	29 ± 4
12.....	80	0	85 ± 7	75 ± 3	10 ± 6
13.....	83	0	79 ± 7	83 ± 4	-4 ± 7
14.....	10	-25	114	79	
15.....	15	-25	136	55	
16.....	20	-25	157	77	
17.....	25	-25	169	75	
18.....	30	-25	169	53	
19.....	35	-25	167	65	
20.....	45	-25	151	73	
21.....	55	-25	132	64	
22.....	65	-25	108	56	
23.....	75	-25	81	47	
24.....	80	-25	62	42	
25.....	83	-25	50	44	
26.....	87	-25	39	43	

by the square root of the number of samples in time is an order of magnitude less than the estimated sampling errors.

It is also possible to utilize selected portions of the instantaneous free-instrument data that have been collected in the Florida Current off Miami in order to discuss the interpretation of the G.E.K. measurements in time and to discuss the interpretation of the potential measurements across a submarine cable. For this purpose, we have selected  $V_s$  and  $\bar{V}$  data, taken on 10 transects that were made across Sts. 1-13 during May-June 1965, from the data presented by Schmitz and Richardson (1966). In the following, the results of calculations made on these data are considered to be associated with cruises 1-10; in the original report, these cruises were referred to as Cross Sections (1,3,5,7,9,15, 17,19,21,23).

*Comparison of  $V_G$  with  $V_s$ .* The G.E.K. fixes ( $V_G$  in Table I), corrected as in (10) by a constant  $k = 1.7$ , are compared with  $V_s$  data (Table II) in



Fig. 1(A). This value of  $k$  is that used historically in the Florida Current off Miami (Hela and Wagner 1954). Although the agreement between the two data sets is reasonable in the vicinity of the core of the Current, there are significant discrepancies. The bimodal nature of the G.E.K. data near the current axis is not a feature of the free-instrument data; moreover, the two sets of data diverge considerably toward the east of the maximum in  $V_s$ . It is well known (vide Pillsbury 1890) that the vertical shear varies from  $V_s \sim 2-3 \bar{V}$  on the western side to  $V_s \sim \bar{V}$  on the eastern side of the Current. There is a systematic difference of  $-13$  cm/s between the two data sets; the standard deviation is 21 cm/s. The standard observational error is 7 cm/s in  $V_s$  and 6 cm/s in  $V_G$ , yielding a relative observational error of 9 cm/s. Thus the typical difference between the two data sets, 34 cm/s, is a factor of four larger than the relative observational error.

The systematic difference may be removed by another choice of  $k$ ; the resulting typical difference between the two data sets would be somewhat larger than twice the relative observational error. Although the time of sampling for the two data sets is different, it is thought that the Florida Current is stable in the mean to within a few percent over these time scales (Schmitz and Richardson 1968).

*Comparison of  $V_G$  with  $V_s - \bar{V}$ .*  $V_G$  and  $V_s - \bar{V}$  values are compared in Fig. 1(B). The standard deviation of  $V_G$  from  $V_s - \bar{V}$  is 11 cm/s; there is no systematic difference. The relative observational error is 8 cm/s. The interesting bimodal character of the G.E.K. data is clearly a result of this type of  $x$  variability in the  $V_s - \bar{V}$  distribution. The contrast between (A) and (B) in Fig. 1 in the reproduction of the shape of the free-instrument profiles by the G.E.K. data is an indication of the distortion in profile that is introduced by using a constant  $k$ . In the free-instrument data, the bimodal feature is due to a peak in  $\bar{V}$  in the vicinity of the surface-speed maximum; it is thought that the large value of  $\bar{V}$  is associated with the abrupt emergence of the topographic feature known as the Miami Terrace.

*Estimates of the Influence of  $Q$ .* Estimates of  $Q$  (5) and  $[(1/F_z)(\partial Q/\partial y)]$  (9) have been formed by using the free-instrument data (Table II) at two sections ( $y = 0$  and  $y = -25$  km) across the Current. The estimates of  $[(1/F_z)(\partial Q/\partial y)]$  are of the order of 5 cm/s, which is less than the standard deviation of  $V_G$  from  $V_s - \bar{V}$  (8 cm/s). The largest estimates have been associated with regions of large bottom slope near the edges of the Current and near the Miami Terrace (located at approximately  $x = 30$  for Section III). The influence due to  $Q$  is not significant and will be ignored hereafter.

*Comparison of  $\bar{V}^*$  with  $\bar{V}$ .* The difference between the G.E.K. observations and the free-instrument surface velocities is a measure of  $\bar{V}$ . These dif-



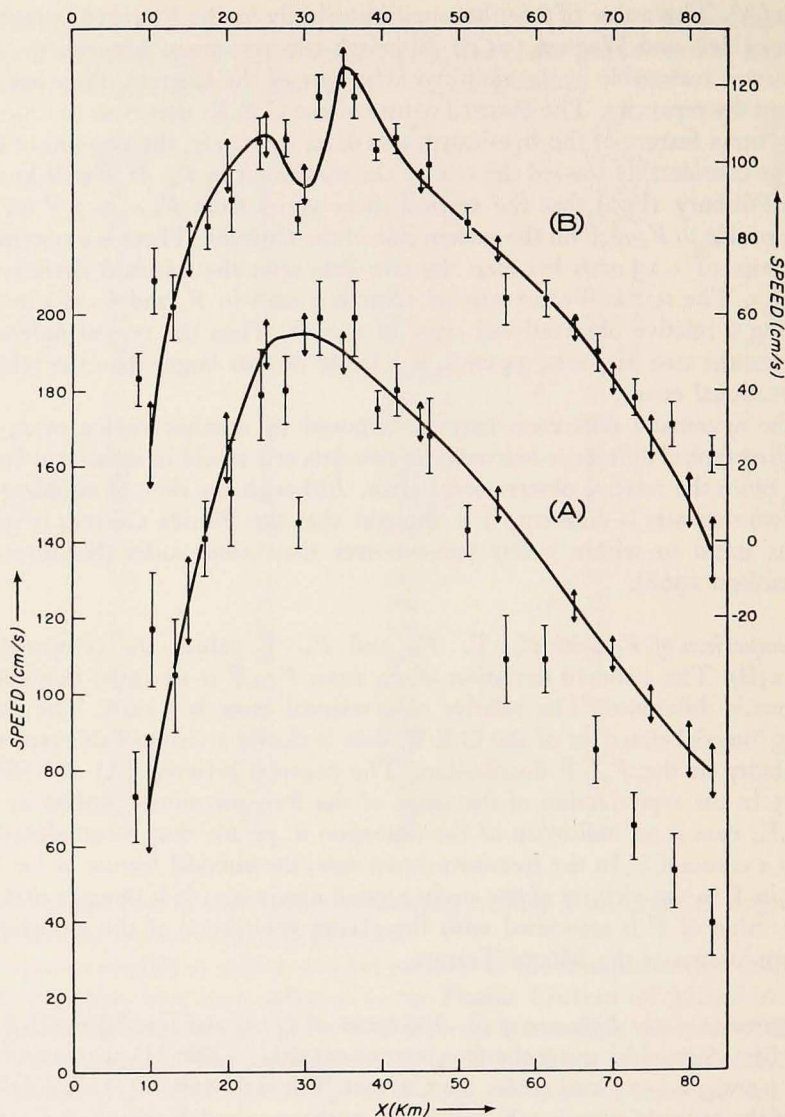


Figure 1. Comparison of G.E.K. observations (squares) with free-instrument data (solid curve). The brackets denote error bounds.  $x$  denotes cross-stream distance. (A) Comparison with surface speeds; the ordinate scale is on the left. (B) Comparison with the difference between surface speed and vertically averaged speed; the ordinate scale is on the right.

ferences are hybrid quantities that depend on both data series. In order to emphasize the special character of this estimate of  $\bar{V}$ , we have denoted it as  $\bar{V}^*$  (12).  $\bar{V}$  and  $\bar{V}^*$  values are compared in Fig. 2. There is a systematic dif-

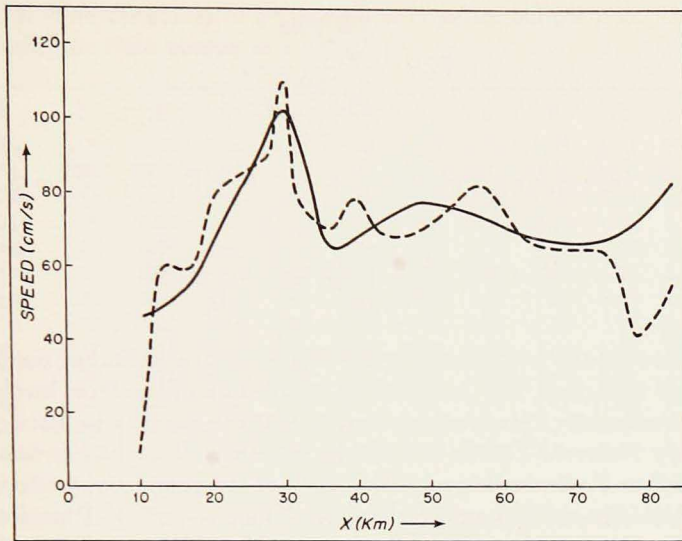


Figure 2. Comparison of the electrical estimate of vertically averaged speed (dashed line) with free-instrument observations (solid line).  $x$  denotes cross-stream distance.

ference of  $-2$  cm/s and a standard difference of  $13$  cm/s between  $\bar{V}^*$  and  $\bar{V}$ . The observational-error estimates are  $9$  cm/s for  $\bar{V}^*$  and  $4$  cm/s for  $\bar{V}$ : a relative error of  $10$  cm/s.

The average values of  $\bar{V}$  and  $\bar{V}^*$  across the Stream ( $10 \leq x \leq 83$ ) are  $72$  cm/s and  $69$  cm/s, respectively. The total volume transports (3) associated with  $\bar{V}$  and  $\bar{V}^*$  are  $32.9$  and  $31.7 \times 10^6$  m<sup>3</sup>s<sup>-1</sup>, respectively. The random differences of  $\bar{V}^*$  and  $\bar{V}$  are attenuated by a factor of  $3$  in the cross-stream average.

*Influence of Electrical-conductivity Variations in the Sea and Sea Floor.* Earlier in the paper it was stated after (5) that the influence of variable electrical conductivity,  $\sigma$ , could be handled in the manner of Longuet-Higgins et al. (1954). In general, the conductivity of the water and of the sea bed does affect the electrical response of an ocean current. So far we have ignored such effects and have found that  $V_G$  and  $V_s - \bar{V}$ , as well as  $\bar{V}^*$  and  $\bar{V}$ , do not differ systematically. The expected influence of including  $\sigma$  in the water and the sea bed is to systematically alter  $V_G$  and  $\bar{V}^*$ .

Consider an electrical conductivity model consisting of: a channel of depth,  $D(x)$ ; conductivity  $\sigma_1(x, z)$  overlying a sea bed of thickness,  $H_s(x) - D(x)$ ; conductivity  $\sigma_2(x, z)$ . The material below  $Z = -H_s$  has zero electrical conductivity. The conductivity-weighted  $\bar{V}$  is



$$\bar{V}_c = \frac{\int_{-D}^0 \sigma_1 V dz}{D \bar{\sigma}_1 + \int_{-H_s}^0 \sigma_2 dz}.$$

$$\text{Let } \sigma_1 = \bar{\sigma}_1 + \sigma'_1, \quad \bar{\sigma}'_1 = 0, \quad \text{and } V = \bar{V} + V', \quad \bar{V}' = 0.$$

Then

$$\bar{V}_c \doteq \bar{V} + \frac{1}{D \bar{\sigma}_1} \left\{ \int_{-D}^0 \sigma'_1 V' dz - \bar{V} \int_{-H_s}^0 \sigma_2 dz \right\}.$$

Over the section the term involving  $\sigma'_1 V'$  represents an additional positive contribution of 10% of  $\bar{V}$ . We can make no calculations of the term involving  $\sigma_2$ , since  $\sigma_2$  is unknown. However, we do note that the quantity in brackets is not significantly nonzero. This is because  $\bar{V}_c$  influences  $\bar{V}^*$ , which is found to be nearly equal to  $\bar{V}$ . Hence it appears that both of the conductivity effects in the sea water and the sea bed are nearly equal (order of 10% of  $\bar{V}$ ) and opposite.

Although the influence of the bottom conductivity is small off Miami, it can not be assumed that this is the case elsewhere, even in the same region. Recent electrical measurements on a submarine cable collected by us as well as observations by Chew et al. (1971) further north between Florida and Grand Bahama Island indicate that the bottom is highly conducting. In this region it has been found that  $\bar{V}_c \simeq 1/2 \bar{V}$ .

*Synopsis of the G.E.K. Comparison with Mean Values.* In the previous sections, it has been demonstrated that, in the mean,  $V_G$  is an estimate of  $V_s - \bar{V}$  essentially to within the relative observational error. The interpretation of  $V_G$  in terms of  $V_s$  with the use of a constant- $k$  factor leads to typical errors of at least 20 cm/s. Furthermore, this interpretation leads to fundamental discrepancies in the shape of the surface-speed profile, yielding a biaxial structure near the Current axis and very low speeds or a counter-current near the eastern edge of the Current. The agreement between  $\bar{V}^*$  and  $\bar{V}$  is nearly within the relative observational error; associated volume transports agree to within this error.

*The Mean Electrical Depth,  $H_e$ .* Equation (6) is applicable to measurements with fixed electrodes. If the electrodes are at the edges of a channel of width  $w$ , then

$$\Delta\varphi = F_z \int_0^w \bar{v} dx - \int_0^w \frac{\partial Q}{\partial y} dx. \quad (13)$$

Equation (13) is intended to be used to obtain transport estimates from the potential measurements. Since the influence of  $\partial Q/\partial y$  is essentially of second

Table III. The variability of  $k^*$ .  $x$  denotes cross-stream position. Each cruise represents one time sample at a given  $x$ .

$x$ (km)	Cruise number											Std. dev.
	1	2	3	4	5	6	7	8	9	10	Av.	
10	1.20	2.12	3.28	7.35	0.27	2.80	2.73	1.13	1.93	1.73	2.45	1.84
15	1.51	1.86	2.54	1.91	1.35	-	1.45	1.41	2.06	1.90	1.78	0.37
20	1.74	1.94	1.82	1.84	1.98	1.84	1.71	1.48	1.99	1.92	1.83	0.15
25	1.71	1.88	1.88	1.89	2.11	1.85	1.76	1.64	2.18	1.98	1.89	0.16
30	1.86	2.44	2.36	2.09	1.90	2.00	1.85	1.73	2.62	2.49	2.13	0.30
35	1.41	1.68	1.64	1.48	1.29	1.48	1.32	1.35	1.71	1.62	1.50	0.15
45	1.69	1.70	1.80	1.85	1.81	1.57	1.76	1.85	1.81	1.70	1.75	0.08
55	1.78	2.80	2.02	2.26	2.60	2.09	2.48	2.38	1.88	1.79	2.21	0.34
65	1.83	2.21	2.21	3.03	2.71	2.82	3.89	2.65	1.80	2.73	2.59	0.59
70	1.70	3.06	2.27	3.08	2.76	2.55	7.00	2.88	2.04	3.81	3.12	1.41
75	1.85	4.09	2.60	5.00	4.13	2.21	9.91	4.42	2.42	13.67	5.03	3.63
80	4.17	-40.00	3.93	-5.38	15.00	2.49	10.45	4.17	7.11	-7.10	-5.2	14.55
83	-12.57	-48.50	4.25	-3.05	-3.48	3.00	4.10	-17.50	-6.33	-1.50	-8.16	15.03

order in the Florida Current off Miami, the influence of the cross-stream integral of  $\partial Q/\partial y$  is not significant. Define

$$H_e = \frac{\int_0^w \bar{v} D dx}{\int_0^w \bar{v} dx} = \frac{T}{\int_0^w \bar{v} dx} \quad (14)$$

so that

$$\Delta \varphi = \frac{TF_z}{H_e} \quad (15)$$

The major difficulty in using (15) to relate transport to submarine-cable potential measurements is that  $H_e$  is not constant but is dependent on (i) the distribution of the Current over the variable bottom depth and (ii) the electrical conductivity of the sea and bottom. In the case of Wertheim's (1954) observations, it is possible that (e.g., Schmitz and Richardson 1968) the observed voltage variations were induced, not by fluctuations in transport but rather by variations in potential due to meandering or by lateral motion of the Current. If the channel were of uniform depth, then  $H_e$  would equal this uniform depth. In general,  $H_e$  will be different from the average depth, depending on how  $\bar{V}$  is distributed relative to the depth.

An estimate of the mean value of  $H_e$  off Miami was computed from the mean free-instrument data (Table II). The horizontal integral of  $\bar{V}$  across the Current is  $53.2 \times 10^3 \text{ m}^2/\text{s}$  while the total transport is  $32.9 \times 10^6 \text{ m}^3/\text{s}$ ; the mean value for  $H_e$  is 614 m. This value of  $H_e$  permits the conversion, in the mean, of a potential measurement into a total-transport estimate.



The relative contributions to the potential difference and to the transport across the Miami Terrace ( $10 \leq x \leq 30$ ) are different. In the mean, 26% of the total cross-stream potential is generated from  $x = 10$  km to  $x = 30$  km while 13% of the total transport occurs over the same interval.

*The Time Variability of  $k$  and  $H_e$ .* We have studied the time variability of  $k$  in terms of a theoretical  $k$  factor,  $k^*$ , as defined by (11). The data used in this study are results derived from 10 free-instrument sections, previously mentioned. At the same time, it is possible to examine the time variability of  $H_e$ .

Table IV. Variability of the "electrical depth" of the Current ( $H_e$ ) and the difference between observed ( $T$ ) and electrical estimates of total transport ( $T_e$ ) over 10 free-instrument transects across the Current.

Cruise no.	$H_e$ (m)	$T - T_e$ ( $10^6 \text{ m}^3 \text{ s}^{-1}$ )
1	638	1.2
2	593	-1.2
3	585	-1.9
4	608	-0.3
5	695	3.4
6	574	-2.4
7	667	3.0
8	667	2.5
9	602	-0.6
10	609	-0.3

The calculated values of  $k^*$  are listed in Table III. The standard deviation of  $k^*(t)$  varies from a few percent to a factor of 30 about the mean  $k^*$ , depending strongly on the cross-stream position.

The calculated values of  $H_e(t)$  are listed in Table IV. The average  $H_e$  for these 10 sections is 624 m, and the standard deviation is 39 m. Note that the average  $H_e$ , using the entire data set (the equivalent of 24 sections), is 614 m.

*Transport Measurements, Using  $H_e$ .* We want to examine the feasibility of using an average  $H_e$  to calibrate the time-dependent potential measurements in terms of transport,  $T$ . We have formed estimates (denoted  $T_e$ ) of the electrical transport, using a mean  $H_e$  (614 m) for each of the 10 sections. The differences,  $T - T_e$ , between the measured transport ( $T$ ) and  $T_e$  are listed in Table IV. The average difference is  $0.3 \times 10^6 \text{ m}^3 \text{ s}^{-1}$ , and the standard deviation of the differences is  $2 \times 10^6 \text{ m}^3 \text{ s}^{-1}$ .

These calculations suggest that a submarine cable from Miami to Bimini could be used to monitor the transport of the Florida Current with an uncertainty of about 10% of the mean. The expected uncertainty off Miami is much smaller than off Key West because the channel is narrower and the lateral motions of the Current are much smaller off Miami.

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