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# Numerically Predicted Changes in the Circulation of the Gulf of Mexico Accompanying a Simulated Hurricane Passage[1]

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## ABSTRACT

To obtain a quasi-steady-state basic circulation pattern for the Gulf of Mexico, a barotropic prognostic numerical model, with no changes in input conditions and with sufficient friction, has been used. It has been found that a simulated hurricane that would theoretically pass across the Gulf of Mexico from the Yucatan Strait to a point just east of the Mississippi Delta would generate a two-centered cyclonic flow region in the western Gulf waters, with a remnant of the steady-state anticyclonic flow in the northwestern corner. The passage of such a hurricane would cause the loop current to extend into the region west of Florida, where a closed anticyclonic flow is generated. The planetary vorticity would cause a westward migration of the lows as well as a migration of the high from the Florida shelf into the loop current; subsequently, an anticyclonic eddy would break off from the loop and migrate westward. The friction and advection of vorticity through the Florida Strait dissipate the extra energy supplied by the storm; the flow would eventually return to the quasi-steady state.

*Introduction.* A circulation model that predicts major circulation features can be helpful in weather prediction; it can be useful also to marine industries such as those engaged in shipping, fishing, fossil-fuel production, defense, and mining. A model of a major ocean must necessarily be limited in resolution due to the great size of an oceanic basin and to practical limits on the core space in computers. The Gulf of Mexico, though small relative to the size of an ocean, has many of the characteristics of a large oceanic basin. The well-defined northerly flow at the Yucatan Strait, the easterly flow at the Florida Strait, the deep abyssal plains, the shallow shelves, and the available data on

circulation for the Gulf of Mexico make it a desirable basin for study. In a previously considered barotropic prognostic numerical circulation model of the Gulf of Mexico (Paskausky 1969), wind was neglected while the primary controls on the circulation were the bathymetry, the seasonal westward in-
tensification of flow in the Yucatan Strait, and, to a lesser extent, the variation in the total transport through the Yucatan Strait.

This report adds wind stress to the vorticity equation of the above model and then appraises the model by considering a hurricane that theoretically passes across the Gulf of Mexico in a north-north-westerly direction.
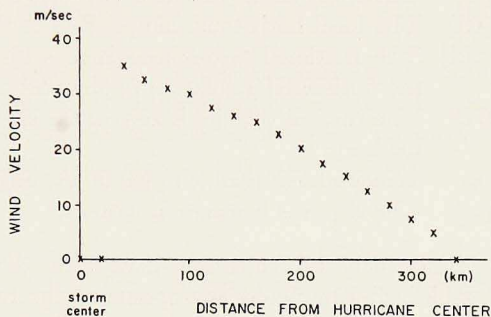


Figure 1. Wind velocity versus distance from center of eye of hurricane.

*Procedure.* The rectangular approximation of the Gulf of Mexico with this model is shown in Figs. 2–11. The Coriolis parameter was altered with simple trigonometric correction to compensate for the 14° rotation of the model coordinates from latitude and longitude.

A wind-stress term that is proportional to the wind velocity squared, as discussed by Sverdrup (1947) and Reid (1948), was added to the vorticity equation (1). The depth was held constant at 500 m to eliminate topographic effects. The wind field consisted of circular winds having a maximum velocity of 35 m/sec, an eye diameter of 60 km, and a radius of 330 km. The wind-velocity profile is shown in Fig. 1. It has been assumed that this theoretical hurricane followed the same path as frequent hurricanes in the Gulf of Mexico that enter the Gulf near the Yucatan Strait and leave it near the Mississippi Delta.

The model was run with constant depth until a quasi-steady state was reached; the only variations in the circulation pattern were the generation of a weak closed cyclonic circulation just west of Florida and the westward migration of the cyclonic circulation with period of 61 days. The hurricane was superimposed on this essentially steady-state condition, and any changes in the circulation pattern were due to the wind. The model is similar to that of Holland (1966) except that the topography is eliminated and the circulation is not in a steady state when the wind is acting on it.

The principal assumptions and/or restrictions used in developing and applying the model are:

(i)     The fluid is homogeneous and incompressible;
(ii)    The inflow and outflow are normal to the cross sections of the ports;

(iii)   Transport through the openings is constant ($30 \times 10^6$ m³/sec), but the flow distribution in the outflow can vary at each time step;
(iv)    A no-slip condition is used on all solid lateral boundaries;
(v)     Tidal forces are neglected;
(vi)    The stress at the sea floor is proportional to the velocity;
(vii)   The local horizontal components of the Earth's rotation are neglected;
(viii)  The frictional torque associated with lateral mixing is proportional to the horizontal Laplacian of the vorticity;
(ix)    The vertical velocity is zero at the sea surface and the bottom is a stream surface;
(x)     Winds of less than 5 m/sec at the edge of the hurricane are neglected;
(xi)    The depth is constant at 500 m.

The analytical equations have been developed and a numerical analog established. Parameters appropriate to the region and the grid size have been based on the earlier no-wind model (Paskausky 1969). The model has been used to predict the circulation with the previously described wind field.

*Theory.* Using the above assumptions and differentiation and integration of the momentum equation, the following barotropic prognostic equation for the vorticity has been obtained:

$$\frac{\partial \zeta}{\partial t} + \nabla_h \cdot (f + \zeta)\bar{V}_h + \sigma\zeta = K_h \nabla_h^2 \zeta + 2 C_D \varrho^1 \left( v_w \frac{\partial v_w}{\partial x} - u_w \frac{\partial u_w}{\partial y} \right), \qquad (1)$$

where $t$ is time, $\zeta$ is the vertical component of vorticity relative to the Earth, $f$ is the Coriolis parameter, $\bar{V}_h$ is the horizontal velocity vector, $K_h$ is the horizontal turbulent exchange coefficient, $\nabla_h$ is the horizontal gradient operator, $\sigma$ is a bottom-friction coefficient having dimensions of frequency, $C_D$ is the wind drag coefficient, $\varrho^1$ is the ratio of density of air to water, $u_w$ and $v_w$ are the $x$ and $y$ components of wind velocity, and $D$ is the depth at any point (constant in this model). In anticipation of future variable depth models, an Ekman boundary-layer approximation at the sea floor leads to:

$$\sigma = (K_v f/2) \frac{1}{2} D^{-1}, \qquad (2)$$

where $K_v$ is the vertical-exchange coefficient.
    Condition $i$ and the continuity equation for a typical column of fluid require that:

$$\nabla_h \cdot D\bar{V}_h = 0; \qquad (3)$$

a volume-transport stream function ($\psi$) can be introduced to satisfy (3) as follows:

$$D\bar{V}_h = \hat{k} \times \nabla_h \psi, \tag{4}$$

where $\hat{k}$ is the vertical unit vector. With depth independent of $x$ and $y$, the vorticity component $\zeta$ in terms of $\psi$ is given by:

$$\zeta = \nabla_h^2 (\psi/D). \tag{5}$$

The model is of fourth order spatially in the volume-transport stream function, $\psi$; so two conditions are required on all boundaries for a unique solution, given specific initial conditions. For consistency with the previously mentioned conditions and restrictions, the boundary conditions were taken as follows:

(i)   On the solid boundaries:
    a.  The stream function, $\psi$, is taken fixed (o or $30 \times 10^6$ m³ sec⁻¹). This defines the total transport across the Yucatan Strait as $30 \times 10^6$ m³ sec⁻¹.
    b.  The no-slip condition requires that $\partial\psi/\partial n = 0$.
(ii)  Input port:
    a.  The stream function is specified versus distance across the opening at each time step.
    b.  Inflow is perpendicular to the port cross section so that $\partial\psi/\partial n = 0$.
(iii) Exit port:
    a.  Vorticity is advected from the interior to the exit port in the following manner:

$$\zeta_{\text{port}}^{\text{new}} = \gamma \left( \zeta_{\text{interior}}^{\text{old}} + f_{\text{interior}} \right) + (1 - \gamma) \left( \zeta_{\text{port}}^{\text{old}} + f_{\text{port}} \right) - f_{\text{port}}, \tag{6}$$

    where $\gamma = u\Delta t/\Delta s$, $u$ is the horizontal velocity, $\Delta t$ is the time step, and $\Delta s$ is the grid interval.
    b.  As in ii-b above, $\partial\psi/\partial n = 0$.

In summary, $\partial\psi/\partial n = 0$ at all boundary points; $\psi$ is specified at all boundary points excluding the exit port, at which $\psi$ is found by relaxation from a knowledge of $\zeta$ at the exit port as predicted from the simple outward advection relationship (6).

*Parameters and Numerical Method.* Numerical analogs for (1) and (5) were established by using a double step in time and space. The DuFort Frankel scheme (Richtmeyer and Morton 1967) was used for the lateral-friction term in (1). A grid spacing, $\Delta s$, of 20 km was chosen so that the numerical scheme would be stable for a reasonably large time step. The resulting grid dimensions are $81 \times 43$ in $x$ and $y$, respectively. The problem was initiated by defining a stream field, generating a vorticity field from it by using (5), and then using a single step in time in (1) to predict a new $\zeta$ field. The computing recipe was
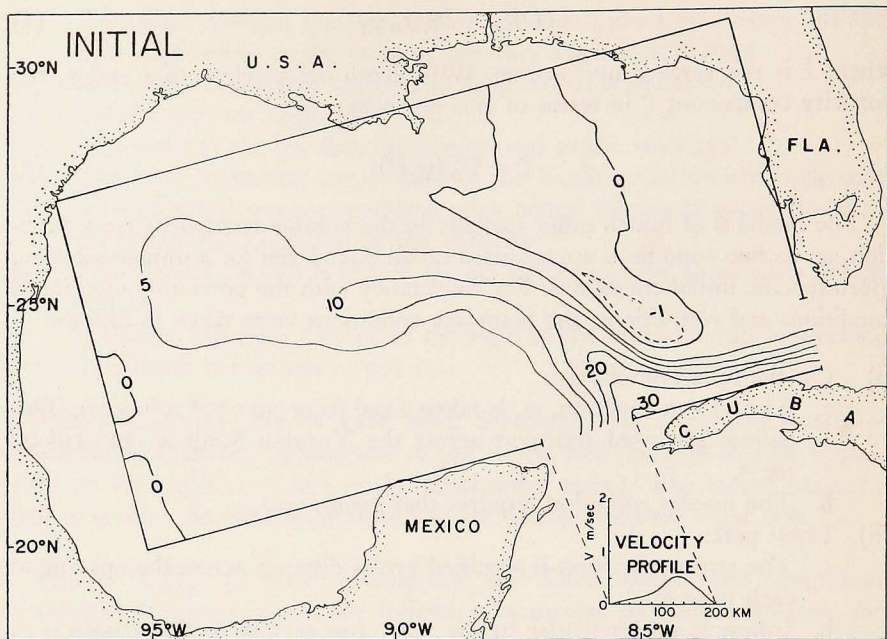
Figure 2. Initial conditions for both hurricane and no-wind models. Contour of stream function in intervals of $5 \times 10^6$ m³/sec. Negative contours are dashed and the $-1 \times 10^6$ m³/sec contour is shown and labeled. The velocity across the Yucatan Strait is shown.

to relax (5) to obtain a new $\psi$ field and then use the new $\psi$ field in (1) to predict the next $\zeta$ field. Significant computer time was saved by using Gauss-Seidel over-relaxation.

The choice of the constants $K_v$, $K_h$, and $\Delta t$ was based on earlier parametric studies with a no-wind model. A value of 22.5 cm² sec⁻¹ was chosen for $K_v$; this gave $\sigma = 5.12 \times 10^{-7}$ sec⁻¹ for the 500-m depth used in this model. Charney (1955) used a $K_v$ of 15 cm² sec⁻¹ while Arons and Stommel (1956) used about 1 cm² sec⁻¹ in studies of the planetary deep-water circulation.

On the basis of trial runs of the no-wind model over periods of 5 to 10 days with constant depth and with variable depth, and by varying only $K_h$, the value of $1 \times 10^6$ cm² sec⁻¹ was chosen and used for $K_h$ in this model.

In determining the time step, $\Delta t$, I have used the stability criterion that $\Delta t$ must be less than a critical value proportional to $\Delta s/c$; here $c$ is the velocity of the phenomena under consideration. With $c = 3$ m/sec and $\Delta s = 20$ km, a time step of less than 2 hr was necessary. The nonlinear terms required too many approximations for a rigorous stability analysis to be meaningful, so a time step ($\Delta t = 6800$ sec) that did not lead to computational instability in 500 model days of running was found by trial and error. For convenience in moving the wind field, the time step used in this study was 5400 sec (1.5 hr).
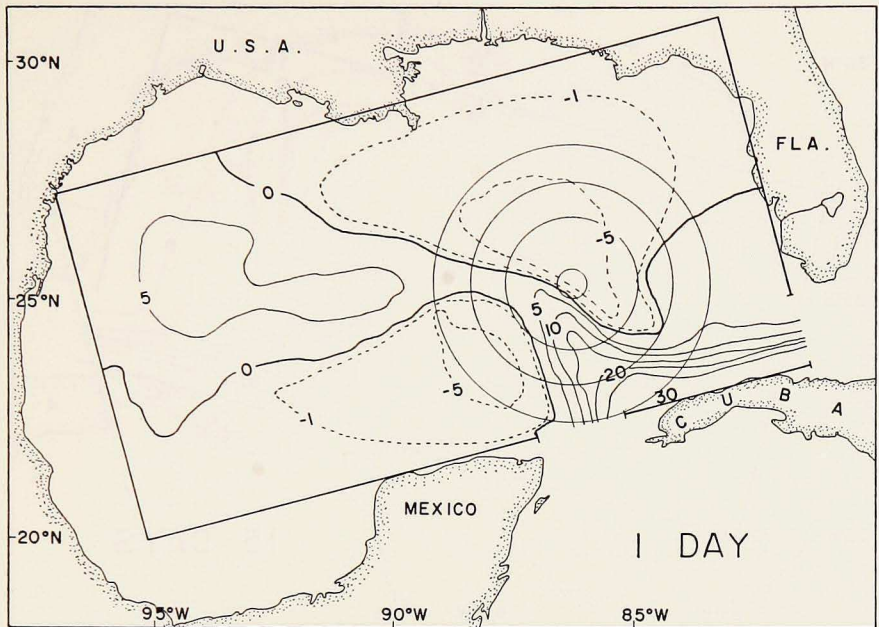
Figure 3. Hurricane model. Contour of stream function in intervals of 5×10⁶ m3/sec. Negative con-
tours are dashed and the − 1×10⁶ m3/sec contour is shown. Wind is cyclonic and contoured
in intervals of 5 m/sec from 5 to 35 m/sec. Depth constant at 500 m.

*Results.* The quasi-steady-state circulation used as the initial conditions is
primarily a loop of current similar to that described by Leipper (1970) with
a tongue of current extending into the western Gulf of Mexico, as in Fig. 2.
As a hurricane passes from the Yucatan Strait to the northern boundary of
the Gulf, it cuts off the tongue, leaving a weak closed anticyclonic circulation
(high) in the northwestern corner of the Gulf. The cyclonic winds generate
one cyclonic circulation (low) just to the north and east of the loop current
and another just to the west of the loop current above the Yucatan Peninsula
(labeled Mexico in the figures). The wind also causes a flow upward from the
loop into the eastern Gulf west of Florida. As the hurricane leaves the Gulf
it generates a weak low on the line of advance at the northern shore, where
the original flow was very weak. These disturbances to the quasi-steady state
are shown in Figs. 3–5, which also show contours of the wind in intervals of
10 m/sec (starting at 5 m/sec) superimposed on contours of the volume-trans-
port stream function in intervals of 5 × 10⁶ m3/sec.

After the wind has diminished, the lows weaken, migrate westward, and
eventually vanish. The high west of Florida migrates to the loop and enhances
the formation of an anticyclonic eddy north of the Yucatan Strait. This eddy
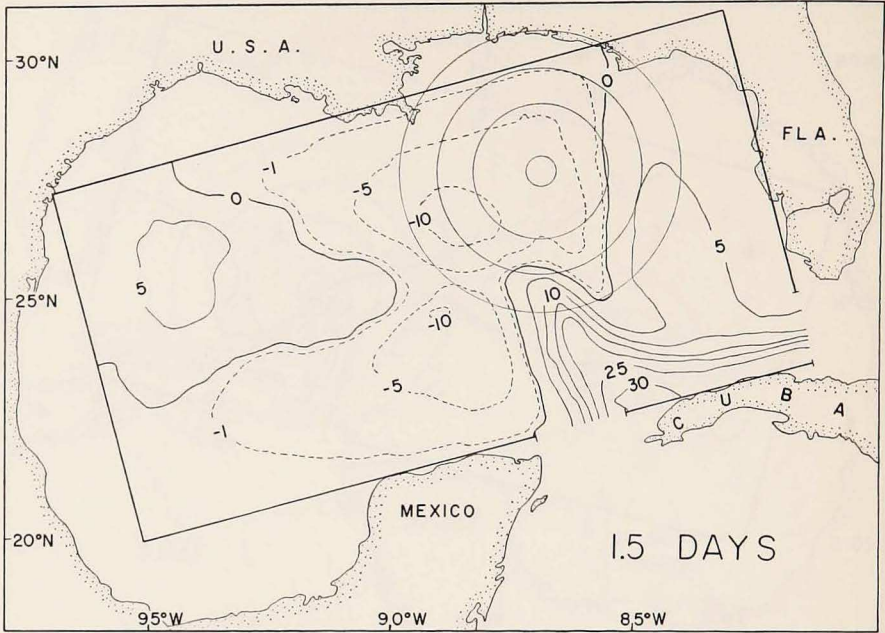migrates westward, weakens, and dies out. The tongue of current in the west-

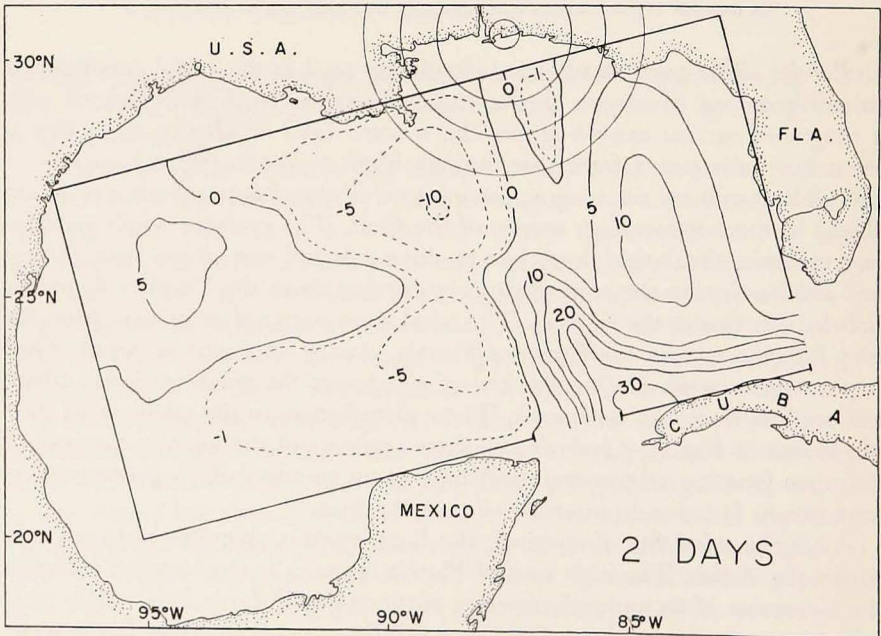Figure 4. Hurricane model. Wind and stream contouring the same as in Fig. 2.



Figure 5. Hurricane model. Wind and stream contouring the same as in Fig. 2.
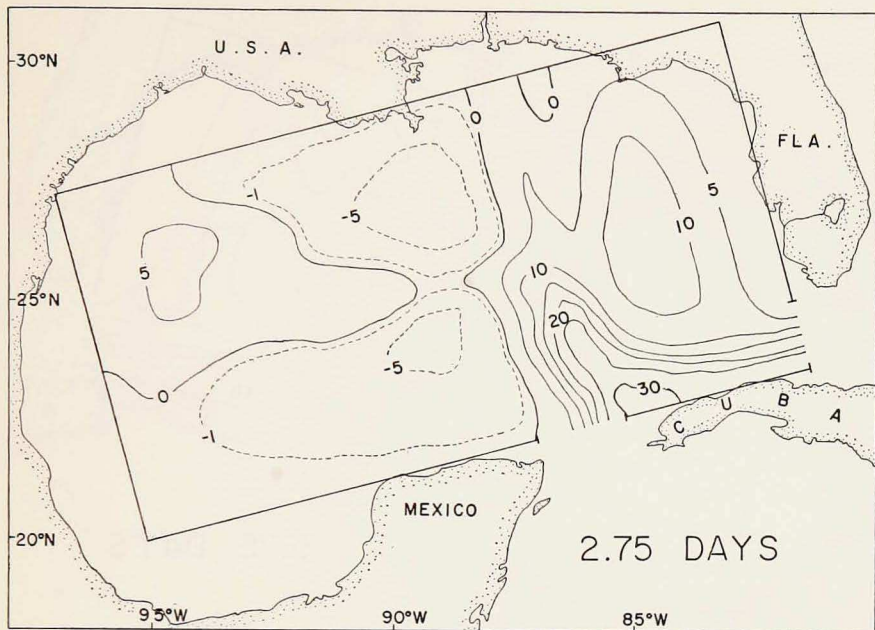
Figure 6. Hurricane model. Wind o. Contour of stream function in intervals of $5 \times 10^6$ m³/sec. Negative contours are dashed and the $-1 \times 10^6$ m³/sec contour is shown.
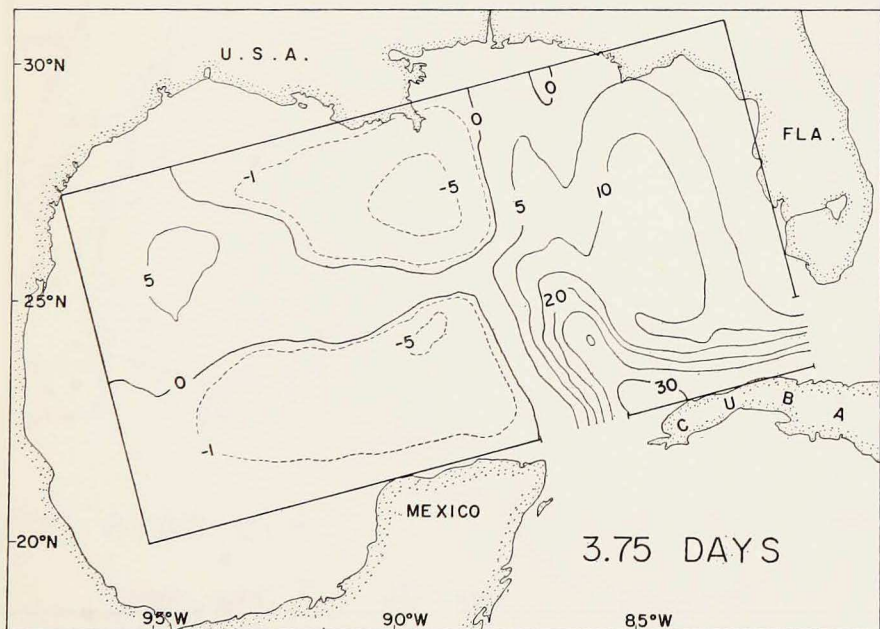


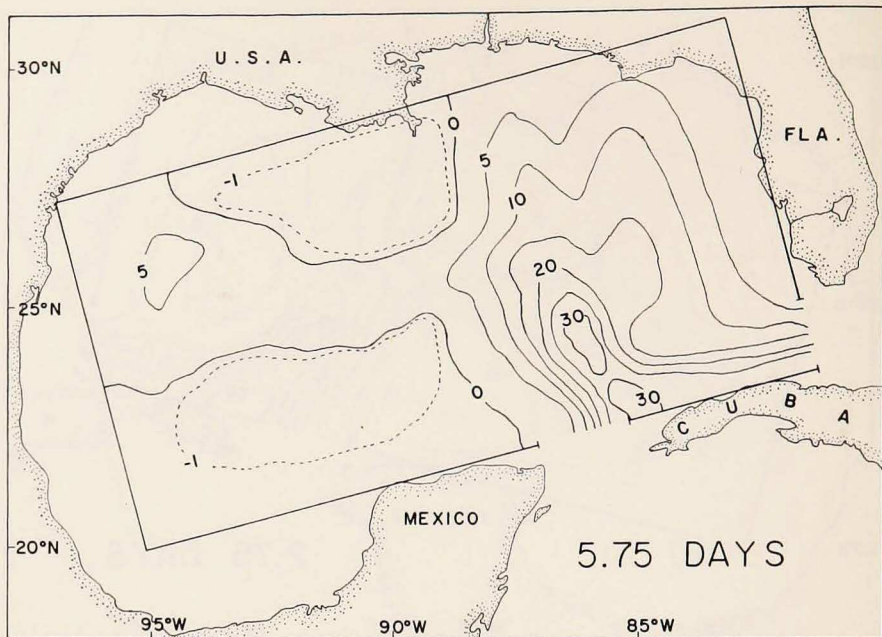Figure 7. Hurricane model. Wind o. Stream contouring the same as in Fig. 5.

Figure 8. Hurricane model. Wind 0. Stream contouring the same as in Fig. 5.



Figure 9. Hurricane model. Wind 0. Stream contouring the same as in Fig. 5.

Figure 10. Hurricane model. Wind 0. Stream contouring the same as in Fig. 5.



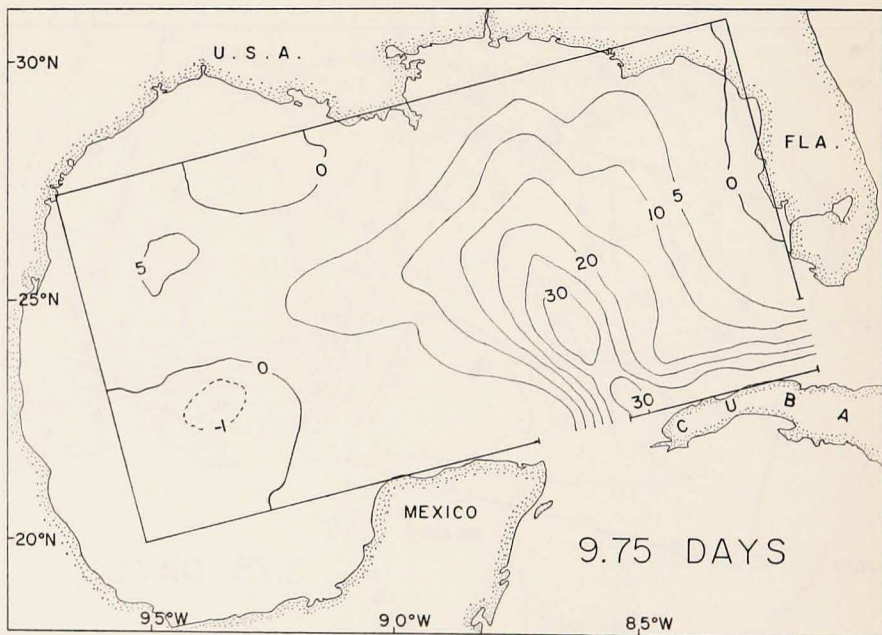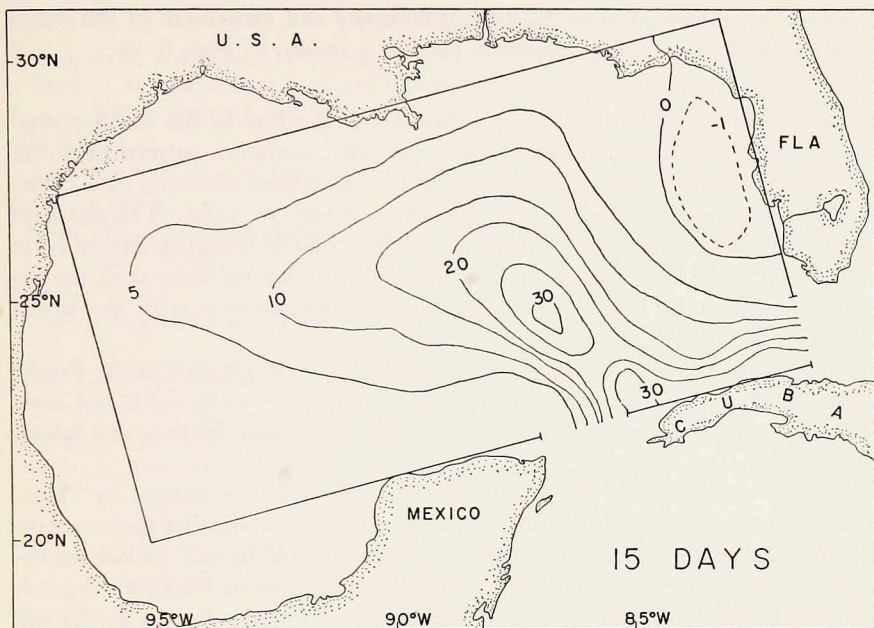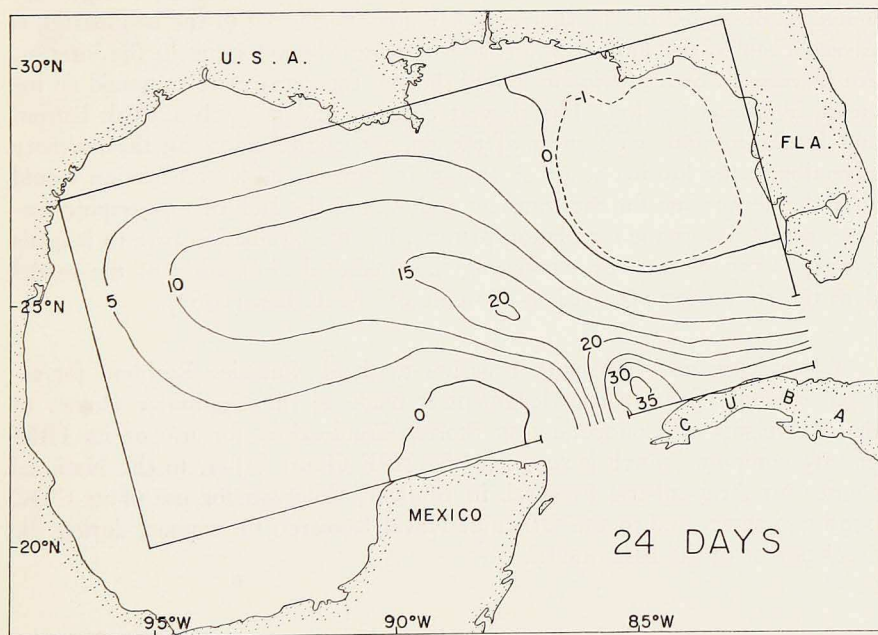Figure 11. Hurricane model. Wind 0. Stream contouring the same as in Fig. 5.

ern Gulf is re-formed after 24 days. The decay and movement of the wind-caused disturbance of the circulation pattern is shown in Figs. 6–11.

*Discussion and Conclusions.* The formation of the low in the north-central waters of the Gulf of Mexico and the general circulation pattern predicted after a hurricane has passed are similar to the circulation observed by Leipper (1967) following the passage of "Hurricane Hilda" in 1964. The development of the low in the waters north of the Yucatan Peninsula seems to be in conflict with the wind, but a high would involve a strong shear with the inflow portion of the loop current and would be inconsistent with the lateral exchange included in the model.

The westward migration of the highs and lows is an illustration of the Rossby wave phenomenon. The decay of the lows as they move to the west and away from the support of the loop current is due to bottom friction and lateral friction.

Leipper observed that the major circulation disturbances created by "Hurricane Hilda" were within the upper 100 m. If it is assumed that the constant-depth model represents a barotropic layer, 100 m would be too shallow for the general circulation and would damp the motion because of the depth-dependence of the bottom friction. The choice of a 500-m depth for the model was a compromise selection to keep bottom friction reasonable while still approximating an upper layer. Since there are no shallow-shelf areas, energy that would be dissipated on the shelves is laterally transferred to the boundaries, is advected out of the Florida Strait, and increases the velocities in the interior. An increase in bottom friction would shorten the decay time as would an increase in lateral friction. The slowest decay would be with a weak bottom and lateral friction, with most of the energy carried away by the vorticity advection at the Florida Strait. A change in bottom and lateral friction would alter the decay time for the transient eddies, and the lack of topographic features would distort the circulation pattern; but the circulation pattern is qualitative, as can be seen by comparing the predicted circulation of the model with the circulation observed by Nowlin and McLellan (1967).

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