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Observations on Short-period Internal Waves in Massachusetts Bay[1]

David Halpern[2]

Department of Meteorology
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

Temperature measurements made 9 km west of a prominent sill in Massachusetts Bay indicate that the seasonal thermocline heaves up and down in phase with periods of 6 to 8 minutes for about 2.5 hours during flood tide. A sudden rise in the general temperature level accompanies the onset of the short-period motion. The maximum vertical displacement occurs at 20 m below the surface. The distribution of the displacements agrees within 10% with the eigenfunction of the first mode of internal gravity waves; the eigenfunction was computed from measurements of density and velocity. Long-crested short-wavelength narrow surface bands parallel to the sill were measured concurrently with the high-frequency temperature oscillations. The high-frequency fluctuations occurring at regular intervals seem to be internal waves of mode one propagating in the same direction with respect to the moving medium. Their generation is not clearly understood.

Introduction. During the summer of 1966, a buoy instrumented for recording temperature was moored in Massachusetts Bay at 42°16.5′N, 70°24.5′W, a site referred to as Station T (Fig. 1). The water depth at this site, about 9 km west of a submarine sill (Stellwagen Bank), was 82 m. The crest of Stellwagen Bank is about 27 m below the surface, extends up to the bottom of the seasonal thermocline, and separates Massachusetts Bay from the Gulf of Maine. The principal tidal constituent in Boston Harbor is semidiurnal, with an amplitude of about 1.5 m; the times of high and low tidal water occur nearly simultaneously at coastal stations around Massachusetts Bay. The circulation in Massachusetts Bay is dominated by tidal currents. According to Bigelow (1927), the maximum flood tidal current runs northward along the eastern shore of Cape Cod but runs westward into Massachusetts Bay.

On two occasions during the summer, the temperature was measured with nine thermistors at 2-minute intervals for 5 days. Fig. 2 shows a portion of the

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2. Present address: Pacific Oceanographic Laboratories, National Oceanic and Atmospheric Administration, Seattle, Washington 98105, USA.

observations recorded from July 28 to August 2, 1966 at 11.1 m below the surface. Fig. 3 contains a vertical distribution of temperature recorded during an 18-hour period. An increase in temperature indicates a downward motion of the water. These diagrams illustrate an interesting phenomenon that was measured simultaneously at all depths. A rapid rise in temperature, which defined the onset of a group of short-period (6-to-8-minute) temperature fluctuations, was observed at regular intervals. These fluctuations persisted for about 2.5 hours. The average time interval between successive initiations of a group of high-frequency fluctuations was 12.4 hours, with a standard deviation of 0.55 hours. In the 2.5-hour interval prior to each abrupt rise in temperature, the mean temperature of the water was nearly constant and there was only a small amount of high-frequency variation. The average vertical displacement associated with the abrupt increase in temperature was about 10 m at 17.2 m below the surface. The coincidence between the onset of the disturbance and the sudden rise in the general temperature level suggests a local phenomenon of a frontal nature.

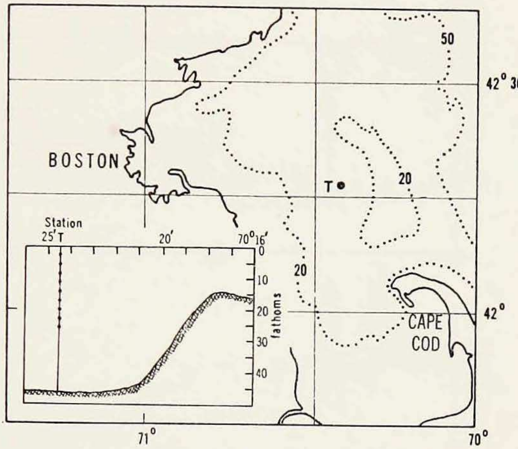

Figure 1. Location of St. T in Massachusetts Bay. Bottom contours are given in fathoms. Inset: vertical section along 42°16.5′N.

Data Acquisition. In the summer of 1967, two buoys were moored 100 m apart at St. T along a line perpendicular to Stellwagen Bank. At each buoy, temperature measurements, which had a precision and accuracy of 0.02°C and 0.05°C, respectively, were recorded at nine depths every 30 seconds. Data were obtained July 13–17 and July 25–28. On August 26, one of the buoys was placed 11 km east of Stellwagen Bank in order to obtain simultaneous temperature measurements on both sides of the sill; three days later it was missing and was never found. An attempt to measure the travel time of events recorded by the two systems was unsuccessful because tidal currents advected the high-frequency fluctuations, producing values of the travel times that were of the same magnitude as the experimental error involved in their measurement; viz., the travel times were of order $4\Delta t$ (120 seconds), and the uncertainty of the time origin of each temperature record was Δt.

Each surface float consisted of a toroid and a tripod tower with two platforms. Since the total weight of the mooring cable was insufficient to stabilize

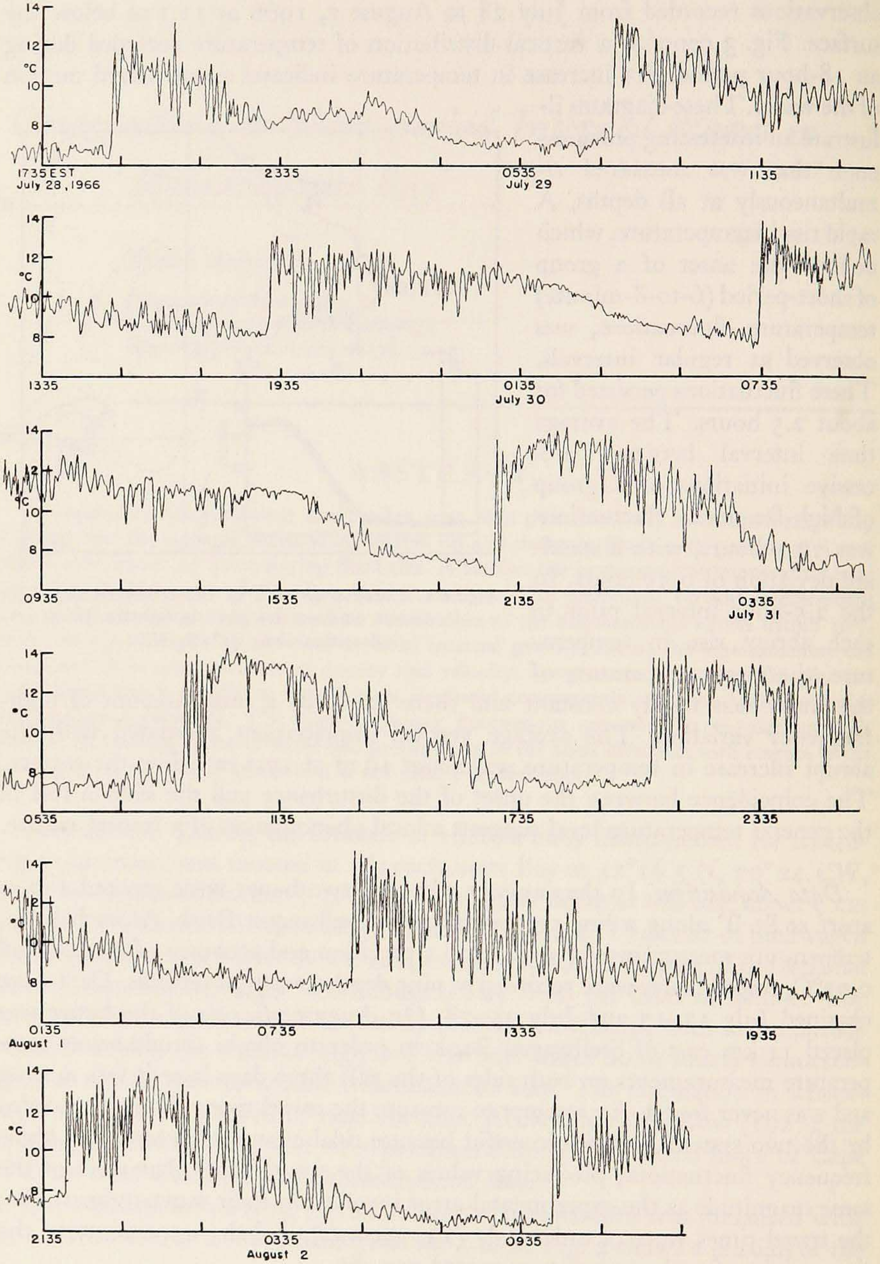

Figure 2. Temperature measurements at 11.1 m below the surface at St. T from July 28 to August 2, 1966.

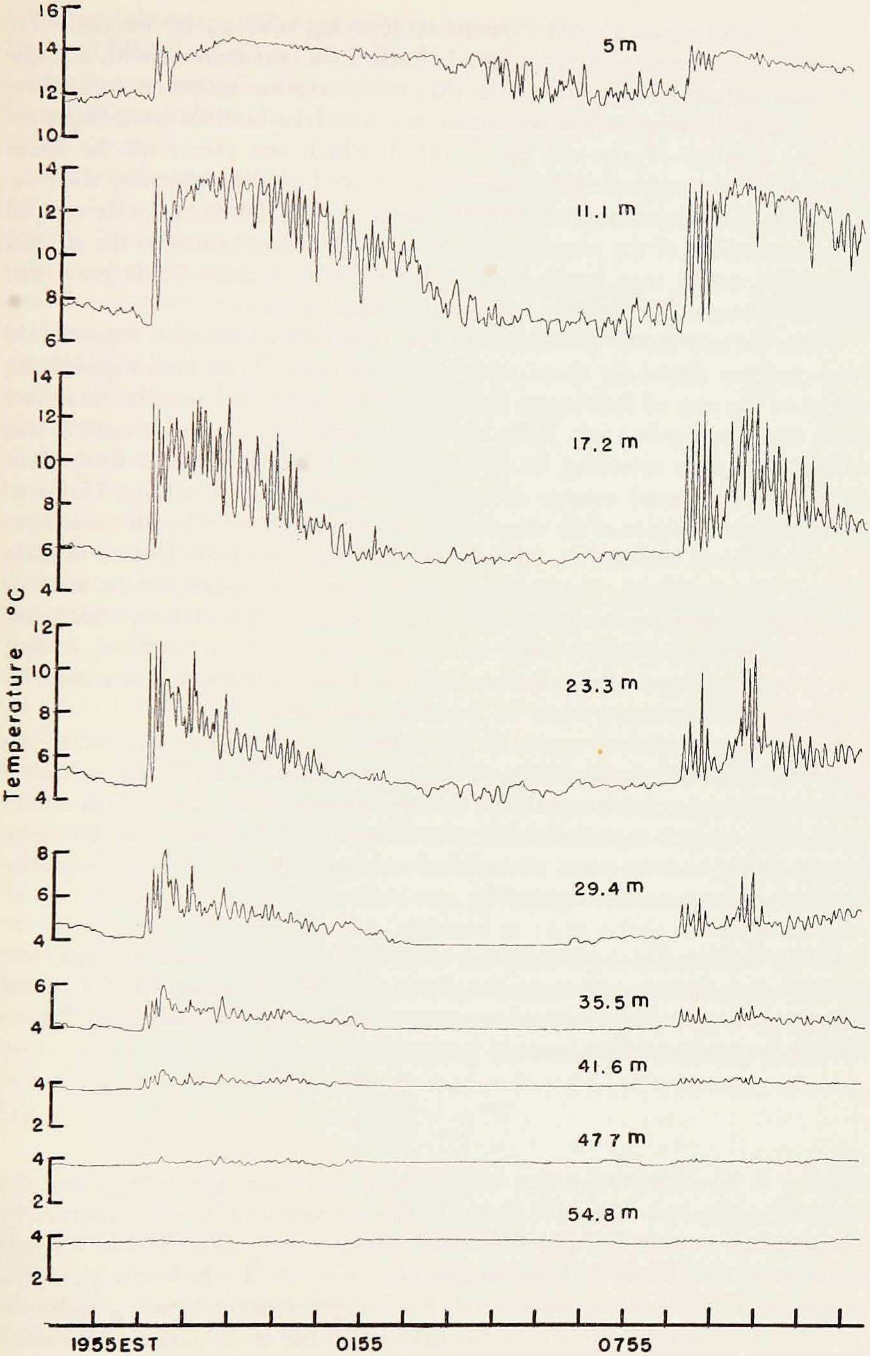

Figure 3. Thermistor measurements at St. T. The depth of the measurements is shown above each curve.

the buoy, which had a net buoyancy of 2600 kg, nine 22-kg weights were attached to the bottom of each toroid. Each buoy was moored with a single 7.93-mm galvanized aircraft cable. A multiconductor neoprene cable containing nine thermistors was connected to a Geodyne internal-recording temperature digitizer (Perry and Smith 1965), which was placed on the lower platform of each tripod. The sensors were spaced at 5-m intervals, with the uppermost thermistor at 5 m below the surface. The thermistor cable escaped dynamic tensions of the mooring line by being loosely attached to the aircraft cable with swivel snap hooks. A 9.5-mm galvanized chain bridle prevented tensions between the digitizer and the thermistor cable.

Three current meters (Richardson et al. 1963) were suspended from a third buoy that was placed for about three days each at St. T, on Stellwagen Bank, and at 11 km east of Stellwagen Bank. Each current meter was free to swivel about the mooring-line axis. Since the storage capacity of each instrument was sufficient to allow recording for six semidiurnal tidal periods, each instrument measured a 50-second average of speed and direction each minute. Unfortunately, the time origins of all the velocity data were destroyed by film exposure.

A thermistor, enclosed in a Braincon Navitherm assembly located directly above a 122-cm V-fin, was towed at 16 m below the surface on the western side of Stellwagen Bank on September 19, 1967. The thermistor, which was towed at about 3.25 m/sec, had a time constant of about 0.5 minutes, so that 100 m was the length of the smallest horizontal feature that could be measured. There was a 2-km uncertainty in the exact location of the vessel.

Aerial photographs of a pattern of multiple surface bands were taken on July 26, 1967. The arrival time of the surface bands at St. T was estimated from the average time interval that occurred between the time of maximum flood tidal current recorded in Boston Harbor (U.S. Coast and Geodetic Survey 1966) and the time of onset of successive groups of high-frequency fluctuations measured in 1966 (Fig. 2).

Data Analysis. Fig. 4 contains the vertical distributions of the average temperature and average values of the Brunt-Väisälä frequency for the period July 25–28, 1967. Neglecting the compressibility of water, the average Brunt-Väisälä frequency (radians/second) is given by

$$N^2 = -\frac{g}{\varrho}\frac{d\varrho}{dz}, \tag{1}$$

where g is the acceleration due to gravity, ϱ is the average density, and the vertical coordinate z is positive upward. The average density was computed by using the mean value of the thermistor measurements (Fig. 4) and a single measurement of the vertical distribution of salinity. At depths below 45 m, the temperature, which was measured with a bathythermograph, was nearly isothermal. The average temperatures at the bottom and at the surface were about

3.8°C and about 17.5°C, respectively. Although the local values of the average Brunt-Väisälä frequency show that internal waves of periods as low as 2.5 minutes were possible, the amount of aliasing was probably small since the sampling interval was 0.5 minutes and the thermistor time constants were about 1 minute.

Spectral-density estimates of the high-frequency temperature fluctuations recorded from July 25–27 were computed by a method described by Welch (1967). The time series recorded at each thermistor were divided into five 512-minute (1024 values) sections such that 181 minutes of data occurred prior to each time origin. The time origins were defined as the onset times of the abrupt temperature rises. The Fourier coefficients were calculated for each section using a fast Fourier transform algorithm. To reduce leakage, the time series were first tapered. In Fig. 5, Bartlett-Daniell estimates of 60 degrees of freedom are shown for the interval 146 minutes to 2.16 minutes; in Fig. 5 the depth and frequency are coordinates, and lines are drawn through points of equal spectral density. The maximum values of the spectral-density estimates, which occurred at 10 m below the surface in the interval 6 minutes to 8 minutes, contained two peaks. Spectral-density estimates of 30 degrees of freedom indicated that there was no significant difference between these two peaks at the 95% confidence level. The time-rate of change of the ensemble-averaged estimates, computed from the data obtained at 10 m below the surface, indicated that the frequency associated with the spectral peak remained the same throughout its history. The vertical displacements corresponding to the 7-minute temperature fluctuations, which were significant at the 95% confidence level at 10–25 m below the surface (Fig. 6), were evaluated from the conservation of heat, assuming small-amplitude motion in a fluid of zero mean motion:

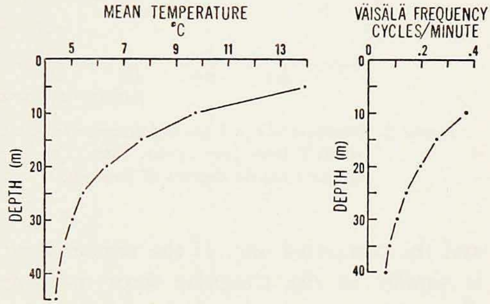

Figure 4. The vertical distributions of average temperature and average Brunt-Väisälä frequency at St. T during the period July 25–28, 1967.

$$\eta = \frac{T'}{<dT/dz>}. \tag{2}$$

These vertical displacements (Table I), which were larger than displacements of the thermistors associated with mooring motion, had a z dependence, with a maximum at 20 m below the surface. A principal source of error occurring in the computation of η is the difference between the real temperature gradient

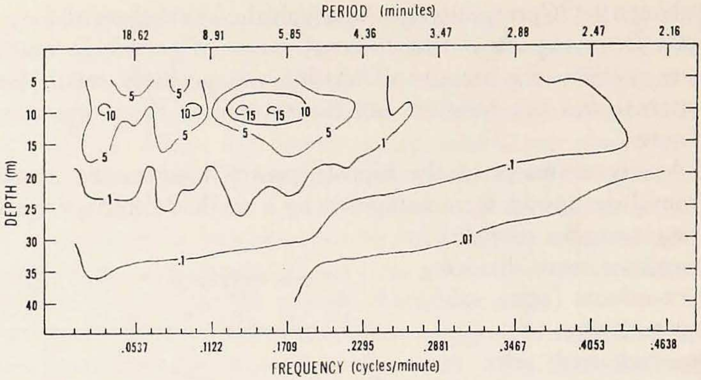

Figure 5. Contours of equal spectral density of the high-frequency temperature fluctuations recorded at St. T from July 25–27, 1967. The spectral units are °C²/cycle/minute. Each spectral estimate has 60 degrees of freedom.

and the computed one. If the temperature decrease in a seasonal thermocline is similar to the irregular steps measured in the main thermocline, then discrete measurements of the vertical temperature field provide insufficient information.

From approximately 8 m to 30 m below the surface, the vertical coherency in the interval 8.8–5.0 minutes was significant at the 95% confidence level (Fig. 7). In this frequency interval, the phase differences $Ø_{32}$, $Ø_{34}$, $Ø_{35}$ and $Ø_{36}$ were the same. The slight bias of the phase toward the negative side of zero may have been caused by unequal thermistor time constants. For example, if two thermistors with time constants equal to 1 minute and 2 minutes measure 7-minute fluctuations, then a 20° phase difference occurs between the two sets of data.

For 1.5 hours on July 26, a pattern of multiple surface bands, which appeared as dark lines on the sea surface, was examined from an altitude of about 230 m. The northern edge of the bands occurred about 2.5 km north of St. T, and the length of the bands, which were parallel to Stellwagen Bank, was about 9.75 km. At their northern and southern ends the bands were wrinkled and compressed along their axes. There was 175 ± 25 m of slightly rippled water or calm water separating the approximately 30-m-wide bands of choppy water. The time of arrival of the lead band at the eastern buoy coincided, within ± 2 minutes, with the time of onset of a burst of high-frequency temperature fluctuations (Fig. 8), suggesting that the surface bands were probably a permanent, or at least a semipermanent, feature of the near-surface temperature oscillations. The westward speed of individual bands was determined by measuring their travel time between the buoys with an accuracy of ± 30 seconds. Beginning at 1345 EST, narrow band 2 moved 200 m westward from the eastern buoy in a period of 5 minutes. Beginning at 1350 EST, band

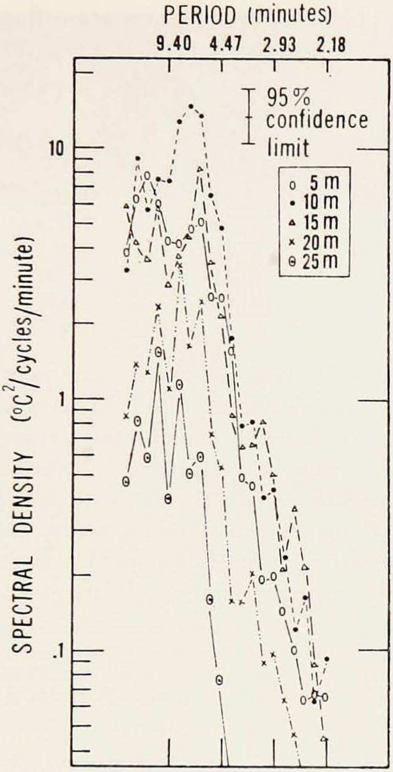

Figure 6. Spectral-density estimates of the temperature fluctuations recorded at St. T from July 25–27, 1967. Each estimate has 120 degrees of freedom.

3 traveled 100 m in 2 minutes. The average speed determined from a total of four such travel-time measurements was 88 ± 20 cm/sec.

Fig. 9 shows temperature measurements obtained from the towed thermistor. The sections of the curves beginning at 1020 EST and 1133 EST have features similar to the observations obtained from the moored buoys at St. T; i.e., an abrupt rise in temperature of about 5°C defines the onset of a discrete group of high-frequency fluctuations; the mean water temperature preceding the onset was colder and nearly constant; and, beginning at 1133 EST, a series of six surface bands was observed. During this 73-minute interval, the group of high-frequency fluctuations had a westward speed of about 100 ± 45 cm/sec. The horizontal wavelength of the high-frequency fluctuations was about 200 m.

Fig. 10 contains curves of hourly averages of the eastward (u) and northward (v) components of velocity. Since the real-time origins associated with these data had been destroyed, a time base common to the three sets of observa-

Table I. Statistics of the high-frequency temperature fluctuations.

Depth (m)	Mean temp. (°C)	Mean temp. gradient (°C/m)	Periods between 5.3 and 8.5 min. T_{rms} (°C)	η_{rms} (m)
10	9.84	0.670	0.982	1.47
15	7.64	.349	.625	1.79
20	6.35	.215	.421	1.96
25	5.49	.130	.231	1.77
30	5.05	.073	.131	1.79

tions was defined by making the onsets of discrete groups of short-period large-amplitude speed fluctuations coincide in time. The u curves at 25.8 m and 42.6 m below the surface contained a semidiurnal periodicity that seemed to be in phase. At 10.6 m, the v curve contained a semidiurnal periodicity, yet at 25.8 m and 42.6 m this periodicity was poorly developed. Although tidal currents generally dominated the current measurements, some of the nontidal motion had larger amplitudes than the tides. Speed measurements recorded at St. T contained regularly occurring bursts of large-amplitude fluctuations.

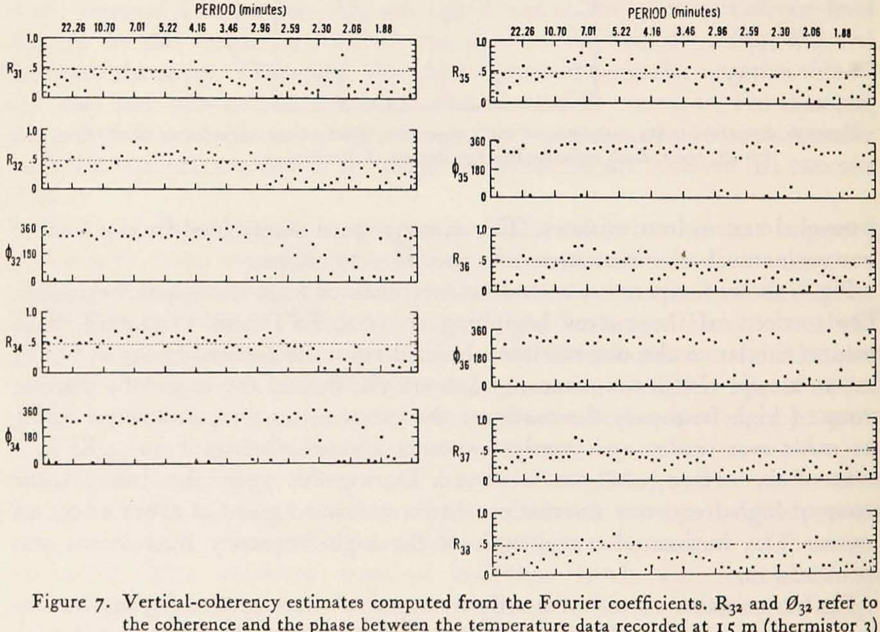

Figure 7. Vertical-coherency estimates computed from the Fourier coefficients. R_{32} and $\emptyset_{32}$ refer to the coherence and the phase between the temperature data recorded at 15 m (thermistor 3) and 10 m (thermistor 2). Each estimate has 25 degrees of freedom. The dashed horizontal line in each coherence diagram is equal to 0.47 and represents the 95% confiderence level.

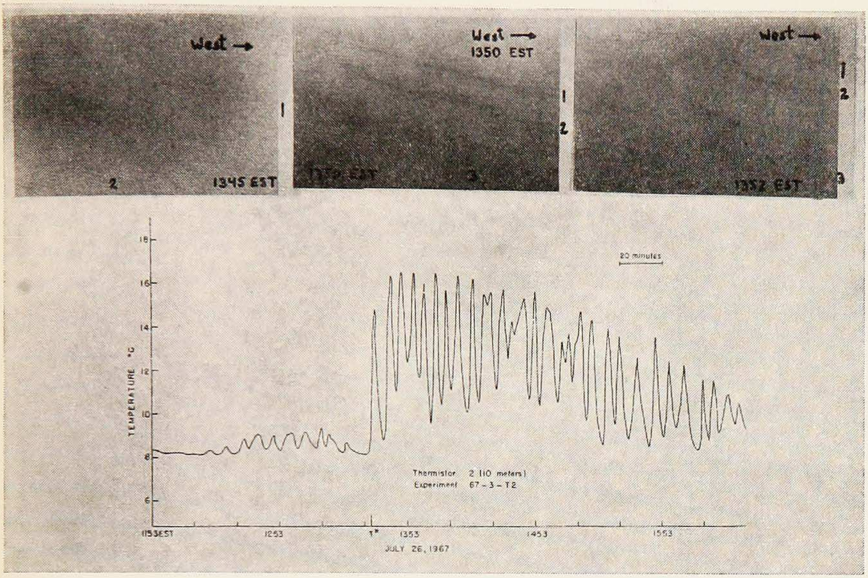

Figure 8. Temperature data recorded at St. T at 10 m below the surface and three photographs of surface bands obtained at 230 m above sea level. Numbers on the photographs identify the narrow bands, e.g., 1 designates the lead band. The two buoys, which appear as small dots in the central portion of the lower half of each photograph, are separated by 100 m. The time is written on each photograph.

However, these discrete groups were absent at Stellwagen Bank and at 11 km east of the Bank. Each group lasted about an hour, and the average time interval between the initiations of successive groups was about 12.4 hours. The measurements in Fig. 11 are assumed to represent a vector summation of a mean flow and of a horizontal motion associated with the vertical oscillations recorded by the thermistors. The mean velocity, defined as the velocity recorded during the 1-hour interval occurring just prior to the onset, was: 29.5 cm/sec, 131° at 10.6 m; 11.3 cm/sec, 209° at 25.8 m; 8.8 cm/sec, 181° at 42.6 m below the surface. This mean flow was subtracted from the velocities measured at the time of the speed maxima, giving velocities of: 50 cm/sec, 206° at 10.6 m; 44 cm/sec, 184° at 25.8 m; 54 cm/sec, 0° at 42.6 m below the surface. This extremely simplified analysis suggests that the instantaneous horizontal velocity, which probably occurred at the same time as the short-period temperature fluctuations, reverses its direction with depth. Since the observations of the long-crested narrow surface bands and the results of the thermistor tow seemed to indicate that the high-frequency temperature fluctuations represented two-dimensional motion propagating in a westward direction, it is perplexing that the dominant velocity component of the resultant velocities was north-south.

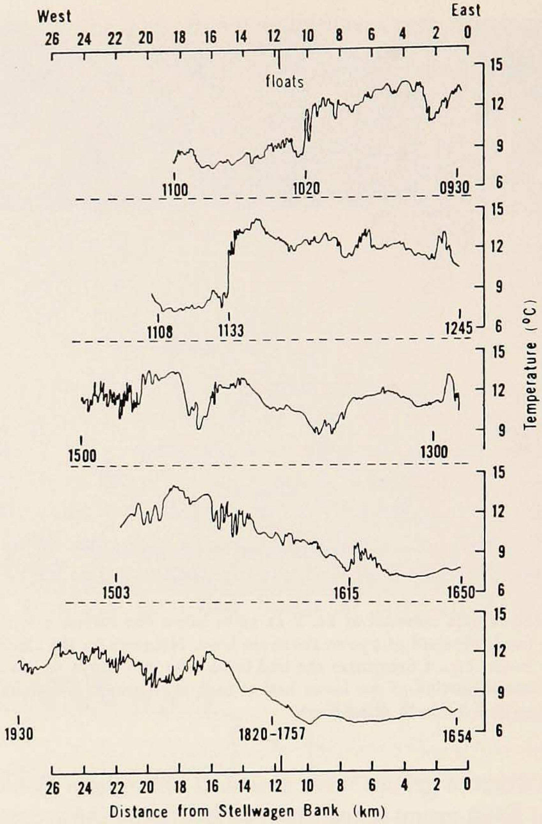

Figure 9. Temperature measurements recorded by a towed thermistor. The time at various locations during the tow is given beneath each curve.

Internal Wave Model. The Eulerian equation describing the vertical velocity $w(x,z,t)$ of small-amplitude two-dimensional motion in a nonrotating incompressible stably stratified Boussinesq fluid, which has a mean velocity distribution $\bar{U}(z)$, has been given as (Phillips 1966)

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)+\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\frac{\partial w}{\partial x}\frac{d^2\bar{U}}{dz^2}+N^2\frac{\partial^2 w}{\partial x^2}=0. \quad (3)$$

The direction of the mean velocity is parallel to the positive x axis. The vertical axis, z, is positive upward. A possible solution is a sum of progressive harmonic waves propagating in the direction of increasing x. Let one such constituent of frequency $\sigma = 2\pi/\text{period}$ and of horizontal wavenumber $k = 2\pi/\text{wavelength}$ be represented by the real part of

$$w(x,z,t) = W(z)\exp(jkx-j\sigma t), \quad (4)$$

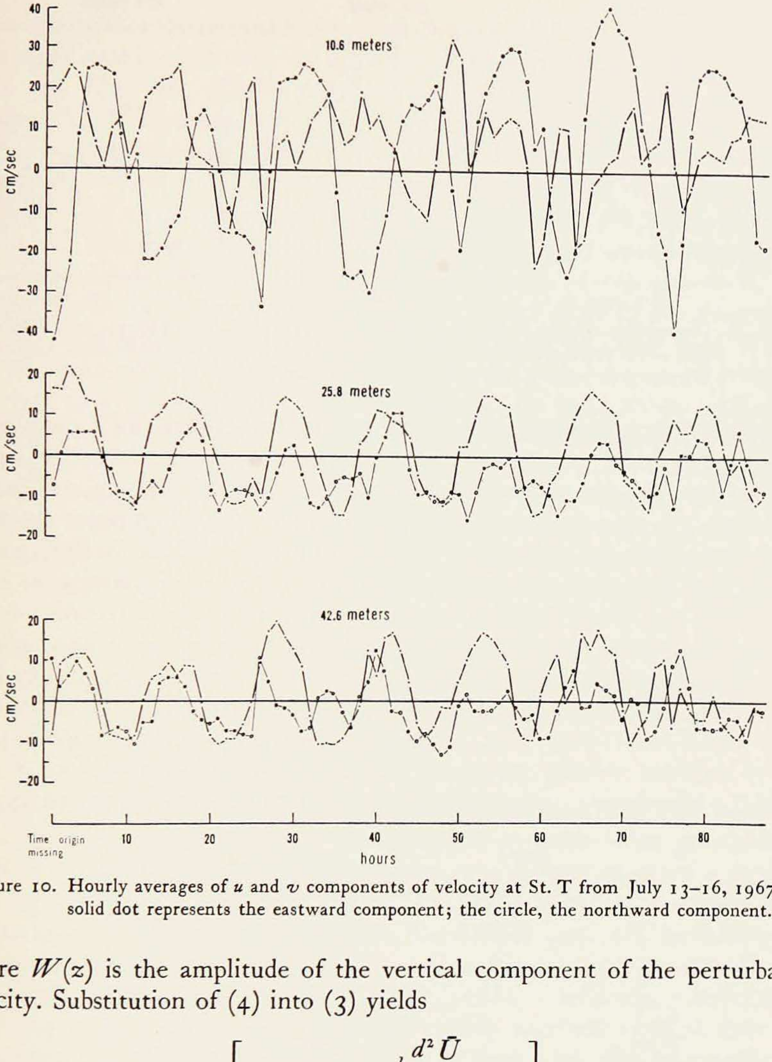

Figure 10. Hourly averages of u and v components of velocity at St. T from July 13–16, 1967. The solid dot represents the eastward component; the circle, the northward component.

where $W(z)$ is the amplitude of the vertical component of the perturbation velocity. Substitution of (4) into (3) yields

$$\frac{d^2W}{dz^2} + \left[\frac{N^2k^2}{(\sigma - k\bar{U})^2} + \frac{k\dfrac{d^2\bar{U}}{dz^2}}{(\sigma - k\bar{U})} - k^2\right]W = 0. \tag{5}$$

When the factor multiplying $W(z)$ is positive, $W(z)$ has an oscillatory behavior and $w(x,z,t)$ describes the vertical velocity of internal-gravity waves. Several features of internal-gravity waves of the first mode (such as a single maximum vertical displacement, a uniform vertical phase, and a reversal in the direction of the horizontal particle velocity) are common to the discrete bursts of 7-minute oscillations measured at St. T.

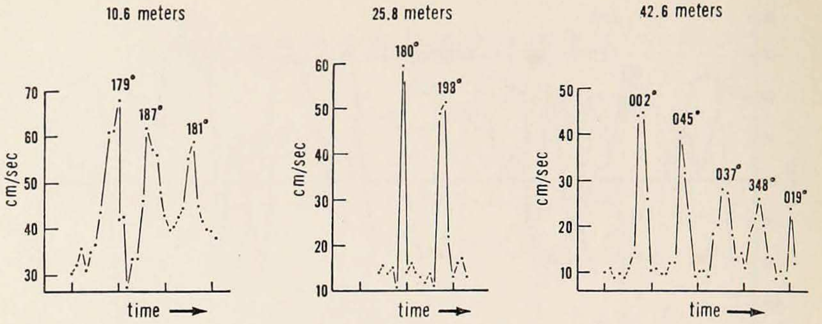

Figure 11. One-minute averages of speed and direction recorded at St. T on July 13, 1967. The direction of the speed measured at the time of a crest is written above the crest. The interval between tick marks on the abscissa is 10 minutes.

Equation (5), with boundary conditions at the free surface and at the bottom, is solved numerically by a matrix formulation (Press and Harkrider 1962). Two extreme cases were considered because the mean velocity occurring at the same time as a discrete group of oscillations contained a significant southerly component. First, the dispersion relation was determined for internal gravity waves of the first mode propagating parallel to a vertical distribution of mean speeds (Fig. 12), which were estimated from the average speeds recorded during the 1-hour interval occurring just prior to the onset of the large-amplitude speed fluctuations. Since the speed profile did not contain a singularity level where $(\sigma/k - \bar{U}) = 0$, the iterative approach did not require any modification (Hines and Reddy 1967). The horizontal wavelength and the horizontal component of phase velocity computed for internal waves of mode one and of period 7.5 minutes are, respectively, 344 m and 77.0 cm/sec. From 10 to 30m

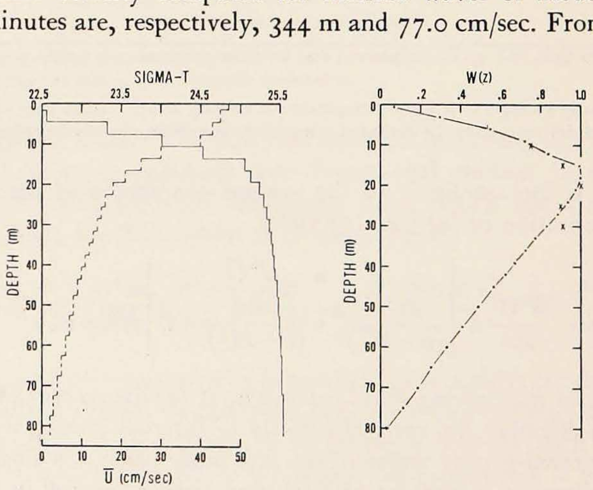

Figure 12. LEFT: Vertical profiles of speed (dashed curve) and sigma-t. RIGHT: Distribution of the amplitude of the vertical velocity for internal gravity waves of mode one. The crosses represent the ratios of the vertical displacements to the maximum vertical displacement.

below the surface the normalized vertical displacements of the 7-minute oscillations agree within 10% with the eigenfunction computed for internal waves of mode one (Fig. 12), providing further indication that the periodic discrete groups of high-frequency temperature fluctuations represent internal waves of mode one. The computed value of the propagation speed is similar to the observed speed of the individual surface bands; however, the computed value of the horizontal wavelength is about twice the spacing between individual surface bands. We may speculate that two surface bands are associated with each wavelength of the internal waves measured at St. T.

The second case describes internal gravity waves of mode one propagating in a direction perpendicular to a mean parallel flow, i.e., $\bar{U}(z) = 0$. For this case the horizontal wavelength and the phase velocity computed for internal waves of mode one and of period 7.5 minutes are, respectively, 158 m and 35.3 cm/sec. The vertical distribution of $W(z)$ was slightly different than that shown in Fig. 12 because the maximum value occurred at 12 m below the surface. In this case the ratios of the rms vertical displacements compared less favorably with the eigenfunction. The calculated value of the horizontal wavelength agrees with the spacing measured between individual surface bands, but the computed phase speed does not agree with the observed speed of individual bands.

Although the dispersion relation for internal gravity waves of mode one cannot be completely verified from the data, there is considerable evidence that the temperature fluctuations are internal waves of 7-minute period propagating in the same direction as a westerly flowing current.

Discussion. There are many unanswered questions that arise from a study of the data. For example, it would be of interest to know what generates the 7-minute internal waves. It is tempting to suppose that they are a result of shearing instability in the tidal flow. At St. T the Richardson number drops almost to 0.25 for very short intervals, but further upstream the Richardson number appears to be somewhat higher, so that this speculation is inconclusive. According to Miles (1961) and Howard (1961), a sufficient condition for small-amplitude stable motion in parallel stably stratified inviscid flow is a Richardson number that is everywhere greater than 0.25. Fig. 13 contains 10-minute averages of a Richardson number evaluated at about 17.5 m below the surface at St. T during the period July 14–16, using: velocity data recorded at 10.6 m and 25.8 m below the surface; temperature measurements obtained at 15 and 20 m; and a single salinity measurement taken at 15 m and at 20 m below the surface. The Richardson number, though always greater than 0.25, was less than unity and was as low as 0.3 prior to the initiation of the short-period internal waves. Since variations in the Brunt-Väisälä frequency were smaller than variations in the shear, the formation of the discrete groups of short-period internal waves may be dependent upon the velocity shear.

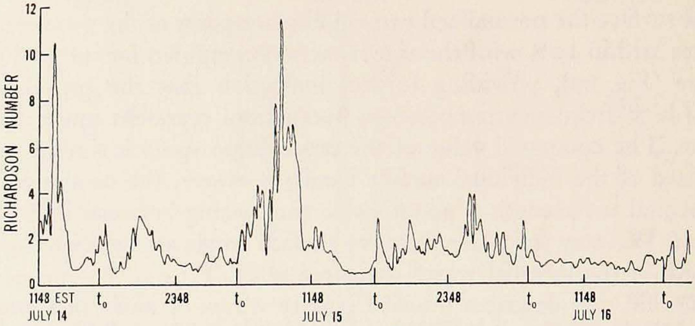

Figure 13. Ten-minute averages of a Richardson number. t_0 represents the time of onset of a burst of high-frequency temperature fluctuations.

Several features of the data indicate that the abrupt rise in temperature or thermal front accompanying a discrete group of short-period internal waves is probably generated by the density and velocity stratified tidal-flood current flowing over the crest of Stellwagen Bank. Although Long's (1954) investigation of density-stratified flow over a barrier suggests that a stationary internal hydraulic jump could originate on the downstream side of Stellwagen Bank during periods of flood tide, the observations imply that the internal waves were propagating in the same direction as the current. Also, it is unlikely that the phenomenon, if stationary, would reach St. T, since the magnitude of the tidal-current excursion is less than 7.5 km. Experimental studies by Gargett (1968) indicate that a long wave passing over a submarine obstacle can degenerate by bore formation into a number of waves of much shorter wavelength. Since the vertical oscillations are confined to the upper region of the water column, it is doubtful that the asymmetry of the temperature curve was caused by internal tidal waves shoaling in coastal water, as was observed by Cairns (1967). The ultimate reason for flooding the surface from the east with a layer of deeper thermal structure is unknown.

Another feature of the data that is not entirely explicable is the actual mechanism of the multiple surface bands. Internal waves are cellular and contain alternating zones of convergence and divergence (Eckart 1961). Bands of calm water in a rippled sea (LaFond 1966) and bands of ruffled water in a smooth sea (Pickard 1961) have been associated with convergence zones of short-period internal waves occurring near the surface. To my knowledge there are no reports in which two narrow surface bands are associated with each wavelength of short-period internal waves. Generation of the bands by a system of longitudinal roll vorticies occurring in the lower atmosphere (Woodcock and Wyman 1946) appears unlikely, because the thermal instability required to initiate Benard convection did not occur in Massachusetts Bay. It is doubtful that the bands are ocean swell, since there were no storms at sea within a few days before the measurements were made.

The temperature fluctuations of semidiurnal frequency contained a large proportion of the total spectral energy of the temperature measurements. The question of whether these baroclinic waves represent a significant energy sink for the barotropic tide remains to be discussed in a later paper.

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