Circulation in a Wind-swept and Cooled Ocean[1]

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ABSTRACT

A two-layer model of circulation is developed in an open-ocean basin where a vertically homogeneous layer overlays a thermoclinic region. In the latter, the temperature changes in an exponential manner to a constant abyssal value. The motions are driven by an Ekman suction and cooling (or heating) of the ocean surface. The ocean-surface temperature is not specified but is a dependent variable of the problem. When the cooling of a subtropical basin follows strong heating, north of a fixed latitude, a deep pool of homogeneous water is formed in the northwestern portion of the basin, similar to what is found in the northwestern Atlantic and Pacific oceans.

1. *Introduction.* The models of the steady midoceanic circulation fall broadly into two classes. In the case of multilayered models (for a review, see Jacobs 1968, Welander 1968), it is assumed that the wind is the vorticity source for the oceanic gyre, via the shallow Ekman layers. In the continuously stratified models (see Veronis 1969), the vorticity structure of the basin is a reflection of both the wind-produced Ekman divergence as well as a latitudinal variation in the surface temperature. Estimates show that both effects are of a comparable order of magnitude (Robinson 1965). This paper combines the two classes of circulation models into a two-layer circulation pattern in a deep ocean, where the heat flux on the surface is specified along with the Ekman divergence produced by the wind stress.

Our model consists of a layer of vertically isothermal water of variable thickness above a thermoclinic region. In the thermoclinic region, the temperature changes in an exponential manner to a constant abyssal value. The vertically isothermal layer provides a neutrally stable layer for the exchange of heat with the atmosphere, and the turbulent vertical transfer of heat in the isothermal

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layer is a linear function of depth. As in previous models of flow over a broad deep basin, a net downward heat flux is required to maintain the thermocline at all latitudes. Our model differs fundamentally from its predecessors in that a transfer of heat to the atmosphere is possible via the neutrally stable layer rather than via a negative temperature gradient—a dynamically unstable configuration. When heat is transferred to the atmosphere, a lateral advection of warm water in this layer supplies the heat for both the thermocline and the atmosphere. In the thermoclinic region, heat is advected in all three directions and is diffused vertically, but the motion still conserves potential vorticity. In the vertically isothermal layer, heat is advected horizontally; this advection is balanced by a turbulent transfer of heat in the vertical direction. Throughout the entire water column, the velocity field, the density field, and the pressure field are continuous.

In § 2 the theory is presented, and in § 3, solutions are found for conditions when both the Ekman divergence and the heat input are functions of latitude only. As in all previous work, this midoceanic circulation is not unique and must, in the final analysis, be coupled to the regions of the boundary currents and the global circulation pattern.

2. *The Model.* We divide the vertical thermal structure of the ocean into three regimes. The layer nearest to the surface—the Ekman layer— is the region where momentum is exchanged with the atmosphere; we refer to this layer as the ocean surface, with $z = 0$. Let $\vec{x}$ denote the horizontal coordinate vector, with y north, x east. A second layer, of depth $D_1(\vec{x})$, is characterized by the temperature distribution $T_1(\vec{x})$ and by the homogeneous density in the vertical direction. Below the second layer is the thermocline, with a temperature distribution $T(\vec{x},z)$; within the thermocline the temperature decreases continuously from $T_1(\vec{x})$ to T_0, the temperature at the level of no horizontal motion in the abyss—$z \to -\infty$ (Fig. 1). The density distribution is $\varrho = \varrho_0[1 - \alpha(T - T_0)]$, with α as the coefficient of thermal expansion.

The field equations for the thermoclinic layer are:

$$\varrho_0 f v = p_x, \tag{1}$$

$$-\varrho_0 f u = p_y, \tag{2}$$

$$g\varrho = -p_z, \tag{3}$$

$$u_x + v_y + w_z = 0, \tag{4}$$

$$uT_x + vT_y + wT_z = kT_{zz}. \tag{5}$$

In (1)–(5) and henceforth, u,v,w are the x,y,z components of velocity; $f = f_0 + \beta y$ is the Coriolis parameter, k is the constant vertical Austausch coefficient for heat transfer, and g is the gravitational acceleration.

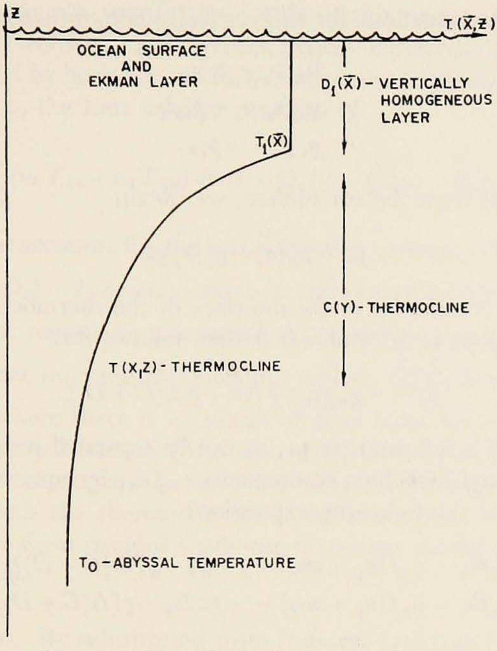

Figure 1. Schematic presentation of the vertical temperature structure in the model.

In this discussion we adopt Needlers (1967) solution of (1)–(5);

$$p = -g\left[\varrho_0 z - C\Delta(\vec{x})\exp\frac{(z+D_1)}{C}\right]; \tag{6}$$

here $C = C_0(f/f_0)$ and $\Delta(\vec{x}) = -\varrho_1(\vec{x}) + \varrho_0 = \varrho_0\alpha(T_1(\vec{x}) - T_0)$; $(\varrho_0 - \Delta)$ is the density at $z = -D_1$. The motion implied by (6) via (1), (2) has a potential vorticity distribution, $f\varrho_z$, that is a function of ϱ only; C is the characteristic depth of the thermocline, and C_0 is the characteristic depth at the latitude where $f = f_0$. Furthermore, a vertical flux of cold water is required to maintain this thermal field, $\tilde{\tilde{w}}(x, z \to -\infty) \to (k/C)$.

We now couple the thermoclinic solution with that of the homogeneous layer to obtain expressions for Δ and D_1. The vertical-boundry conditions on (6) require that the mass flux normal to $z = -D_1(\vec{x})$ as well as the pressure and heat flux be continuous; the solution is already written in a form that allows a continuity in density. In the subsequent analysis we see that the wind stress on the ocean surface and the heat flux into the vertically homogeneous layer determine these quantities to within an arbitrary function of y.

The momentum equations for the vertically homogeneous layer are

$$\varrho_0 f v_1 = p_{1x}, \tag{7}$$
$$\varrho_0 f u_1 = -p_{1y}, \tag{8}$$
$$\varrho_1 g = -p_{1z}. \tag{9}$$

By integrating (9) from the sea surface, we obtain

$$p_1 = p_0(\vec{x}) - \varrho_1(\vec{x}) g z. \tag{10}$$

Pressure is to be continuous at the interface of the thermocline, $z = -D_1(\vec{x})$, whence, from (6), p_0 is determined; it then follows that

$$p_1 = -\varrho_0 g z + g\Delta z + g\Delta(C + D_1). \tag{11}$$

From (7) and (8) it follows that v_1, u_1 can be separated into barotropic components v_p, u_p, and baroclinic components v_c, u_c, by equating like powers in z on either side of the following expression:

$$\varrho_0 f v_1 = \varrho_0 f(v_p + z v_c) = gz\Delta_x + g[\Delta(C+D)]_x, \tag{12}$$
$$\varrho_0 f u_1 = \varrho_0 f(u_p + z u_c) = -gz\Delta_y - g[\Delta(C+D)]_y. \tag{13}$$

The continuity equation, $\vec{\nabla}\cdot\vec{v}_1 = 0$, for this layer is now integrated from $z = -D_1$ to $z = 0$:

$$\int_{-D_1}^{0} \{u_{1x} + v_{1y}\}\, dz = -w_e(0) + w_1(-D_1). \tag{14}$$

Here $w_e(0)$, the flux of water into the Ekman layer, is related to the surface wind stress by $w_e = \hat{z}\cdot\{\vec{\nabla} x (\vec{\tau}/f)\}$ (see Robinson 1965a). Evaluating the left-hand side of (14), we obtain

$$\left.\begin{aligned}&(u_p D_1)_x + (v_p D_1)_y - 1/2(u_c D_1^2)_x - 1/2(v_c D_1^2)_y = \\ &= -w_e(0) + w_1(-D_1) + u_1(-D_1)D_{1x} + v_1(-D_1)D_{1y}.\end{aligned}\right\} \tag{15}$$

The portion on the right-hand side of (15) with the subscript 1 is merely the flux of water normal to the surface, $z = -D_1(\vec{x})$, and this flux must be equated to the flux from the thermoclinic region; i.e.,

$$\left.\begin{aligned}&w_1(-D_1) + u_1(-D_1)D_{1x} + v_1(-D_1)D_{1y} = w(-D_1) + \\ &\quad + u(-D_1)D_{1x} + v(-D_1)D_{1y}.\end{aligned}\right\} \tag{16}$$

To complete the system, we form the heat-balance equation for the D_1 layer. We write $\dot{Q}_0(\vec{x})$ for the rate of heat flow into the basin at $z = 0$. Because details of the process by which momentum is mixed into the Ekman layer are not specified, we do not specify the details of the process by which heat is mixed through the vertically homogeneous layer. However, heat can be transferred in this model by both vertical turbulent processes as well as lateral advection. Since $\varrho_{1z} = 0$, the heat balance equation is

$$\int_{-D_1}^{0} (u_1 T_{1x} + v_1 T_{1y})\, dz = +\dot{Q}_0(\vec{x}) - \dot{Q}(\vec{x}, -D_1). \qquad (17)$$

The left-hand side accounts for the total advective process, whence $\dot{Q}(\vec{x}, -D_1) = (-k/\alpha)\,\varrho_z(\vec{x}, -D_1) = k\,(\Delta/\alpha C)$—the heat transferred by irreversible processes to the thermoclinic region. In this model, Δ is always positive; thus $\dot{Q}(\vec{x}, -D_1)$ is an influx of heat into the thermoclinic region; $\dot{Q}(\vec{x})$, however, can be of either sign, and where there is an efflux of heat from the ocean, $(\dot{Q}(\vec{x}) < 0)$, as is the case above 30° latitude (Malkus 1962), there must be a lateral influx of warm water to this region. In the latter case, the homogeneous layer must supply heat for both the thermocline and the atmosphere. Equations (12)–(14) and (17) now form coupled nonlinear first-order partial differential equations for Δ and D_1.

3. *The Solutions.* By substituting from (12) and (13) into (17) and recalling that $C = C(y)$, we obtain equations for Δ, D_1:

$$\left\{\frac{g\Delta}{\varrho_0}\left[\frac{1}{2}\frac{\beta_0}{f^2}D_1^2 + \frac{\beta_0 C_0}{ff_0}D_1 + \frac{C_0^2\beta_0}{f_0^2}\right\}_x = \hat{z}\cdot\vec{\nabla}\times(\vec{\tau}/f) - k/C, \qquad (18)$$

$$D_1\Delta\,\{\Delta_y(D_1 + C)_x - \Delta_x(D_1 + C)_y\} = \frac{\varrho_0\alpha f}{g}\left[\dot{Q}_0 - \frac{k\Delta}{\alpha C}\right]. \qquad (19)$$

Equation (20), the Sverdrup transport equation for the total transport below the Ekman layer, could have been obtained from the sum of the vertical intergrals of (1)–(4), (6) and (7), (9), and (14). In the above derivation we have demonstrated the separation of the velocity structure in this two-layer model of the density distribution in the basin. Note that all three components of velocity in the entire water column, $z[0, -\infty]$, are continuous.

An Ocean of Constant Surface Temperature in the Trade-Wind zone. The summertime equatorial oceans are characterized by a wide latitudinal belt of homogeneous surface water, $\Delta = \Delta_0$—a constant. At the same time, the trade winds are well developed, $\vec{\tau} = [-\tau_0(y), 0]$ (Malkus 1962). Equation (18) can be integrated directly from some longitude, $x = L$, to yield

$$\frac{1}{2}\left(\frac{D_1}{C}\right)^2 + \left(\frac{D_1}{C}\right) + 1 - \left(1 - \frac{x}{L}\right)F(y) - G(y) = 0, \qquad (20)$$

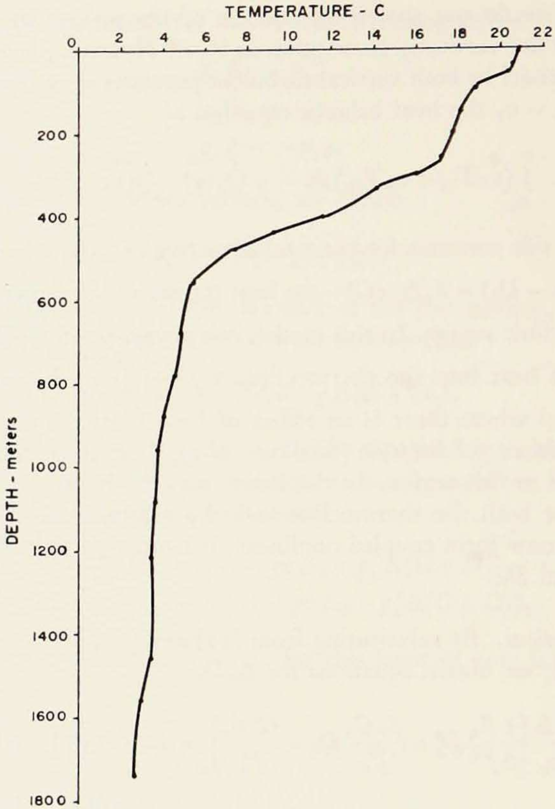

Figure 2. Vertical temperature structure at St. 5440 (50°30′W, 36°35′N) in the North Atlantic. From Fuglister 1960.

where

$$F(y) = \frac{L\varrho_0}{\Delta_0 g}\frac{f_0^2}{C_0^2\beta_0}\left\{\frac{k}{C} + \left(\frac{\tau_0}{f}\right)y\right\} \tag{21}$$

and $G(y)$ is the value of $\left[\frac{1}{2}(D_1/C)^2 + (D_1/C) + 1\right]$ evaluated at the longitude $x = L$.

The solution to (19) is merely

$$\dot{Q}_0(y) = \frac{k\Delta_0}{\alpha C(y)}, \tag{22}$$

and the heat input varies inversely with latitude. In this solution the heat flowing into the surface goes directly through the homogeneous layer to supply the heat flow required to maintain the thermocline at each latitude. Conversely, the above solution could have been obtained from (18) and (19) for the special

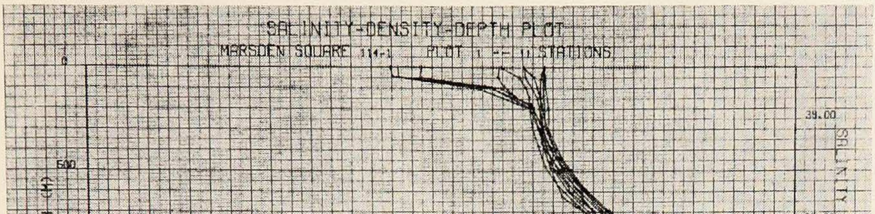

Figure 3. Vertical density structure and salinity structure of Marsden Square 114–1.

case in which (i) the heat input varies inversely with latitude, (ii) the wind-stress curl is a function of latitude only, and (iii) the boundary conditions on Δ are given on the curve $y = y_0$—a constant. In general, Δ must be specified on some curve, $\Gamma(x,y) = 0$, and a free function of y, $G(y)$, has to be specified as the limit of integration of (18). Both conditions are directly related to (i) the boundary conditions along the longitudinal boundaries of the basins and (ii) the nature and extent of the coupling of this oceanic region across the equator to adjacent oceanic basins, the dynamics of which have not been considered here. The functions $G, \Delta(\Gamma)$ are determined from the global circulation pattern.

Note in (21) that F is a dimensionless variable and that F, through its dependence on x, reflects the longitudinal variation in the open-ocean circulation. In the North Atlantic, $[\tau_y^{(x)}/\varrho_0] \sim 1/15 \times 10^{-7}$ cm sec^{-2}, $f_0 \sim 1 \times 10^{-4}$ sec^{-1}, $L \sim 4.5 \times 10^8$ cm, $\Delta g/\varrho_0 \sim 2$ cm sec^{-2}, $C_0 \sim 1 \times 10^5$ cm, $\beta_0 \sim 1.5 \times 10^{-13}$ sec^{-1}, whence $F \sim 10^{-1}$—a quantity that is much less than unity. The change in the depth of the homogeneous layer along the latitudinal belt in this model is much less than the change in the thermocline depth, C, in the latitudinal direction. This implies that the pressure gradients maintained by oceanic continental boundaries, or variations in $G, \Delta(\Gamma)$, may drive a midoceanic circulation of

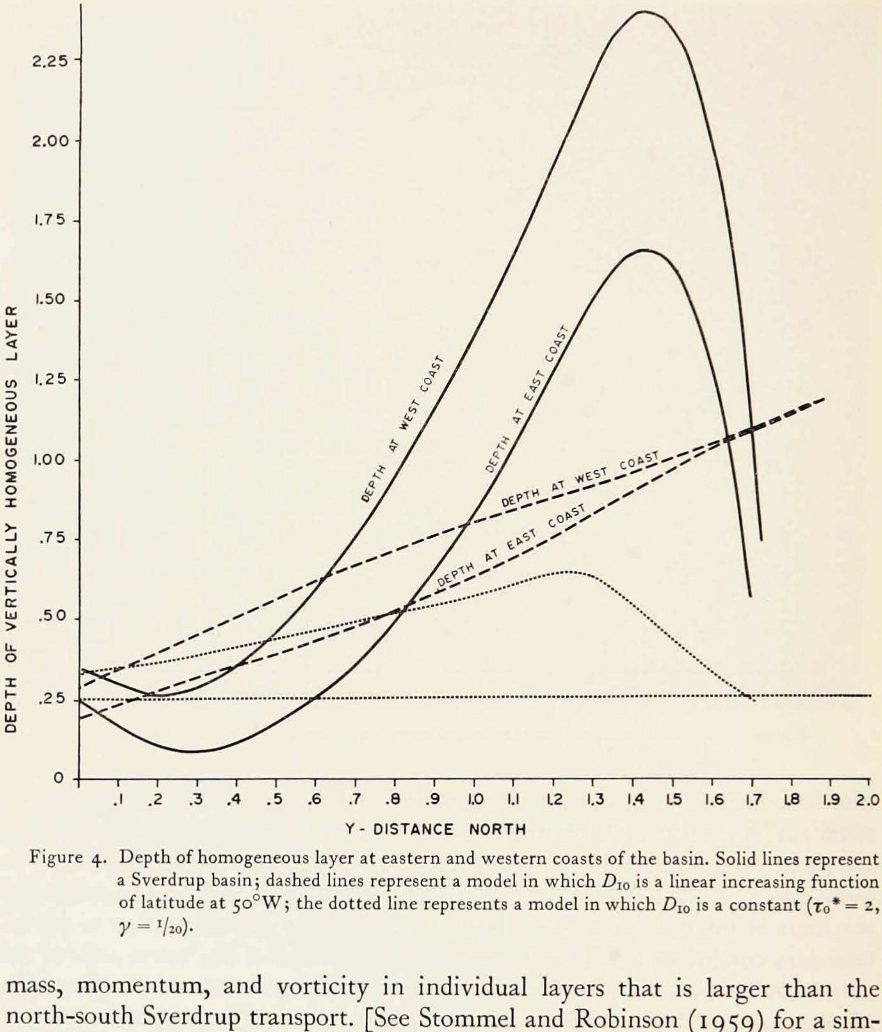

Figure 4. Depth of homogeneous layer at eastern and western coasts of the basin. Solid lines represent a Sverdrup basin; dashed lines represent a model in which D_{10} is a linear increasing function of latitude at 50°W; the dotted line represents a model in which D_{10} is a constant ($\tau_0{}^* = 2$, $\gamma = 1/20$).

mass, momentum, and vorticity in individual layers that is larger than the north-south Sverdrup transport. [See Stommel and Robinson (1959) for a similar computation of the scale of F.]

To formalize the above notions, we present a nondimensional analysis of (18) and (19) and solve the system for a more general $\dot{Q}(y)$.

The Ocean of Variable Surface Temperature. Now we introduce the following scales into (18) and (19): (i) Δ_0, the vertical density difference through the thermocline the scale of Δ; (ii) C_0, the scale of C and D_1 and a measure of the depth of the thermocline at a latitude $f = f_0$; (iii) L, the scale of x and the longitudinal extent of the basin; and (iv) l, the scale of y, defined as $l = f_0/\beta_0$.

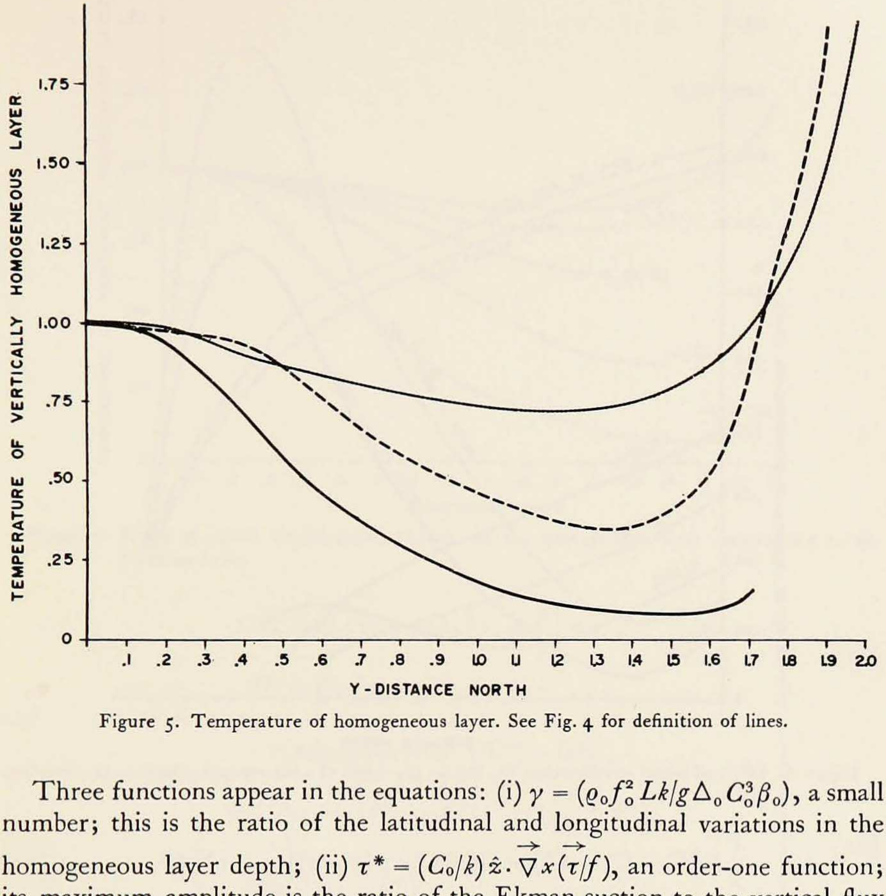

Figure 5. Temperature of homogeneous layer. See Fig. 4 for definition of lines.

Three functions appear in the equations: (i) $\gamma = (\varrho_0 f_0^2 Lk/g\Delta_0 C_0^3 \beta_0)$, a small number; this is the ratio of the latitudinal and longitudinal variations in the homogeneous layer depth; (ii) $\tau^* = (C_0/k)\,\hat{z}\cdot\vec{\nabla} x\,(\overrightarrow{\tau/f})$, an order-one function; its maximum amplitude is the ratio of the Ekman suction to the vertical flux of cold water from the abyss; (iii) $\dot{Q}^* = (\dot{Q}_0 \alpha C_0/\Delta_0 k)$, an order-one function; this is the ratio of the surface heating and the heat required to maintain the thermocline.

The dimensionless equations are:

$$\left\{\Delta\left(\frac{1}{2}\left(\frac{D_1}{C}\right)^2 + \left(\frac{D_1}{C}\right) + 1\right)\right\}_x = -\gamma\left(\frac{1}{C} - \tau^*\right), \tag{23}$$

$$D_1 \Delta\{\Delta_y(D_1 + C)_x - \Delta_x(D_1 + C)_y\} = \gamma(CQ^* - \Delta). \tag{24}$$

We are interested in the solution of (23) and (24) with small γ, with the assumptions that Q^*, τ, are functions of y only.

We integrate (23) to obtain [see (20)]

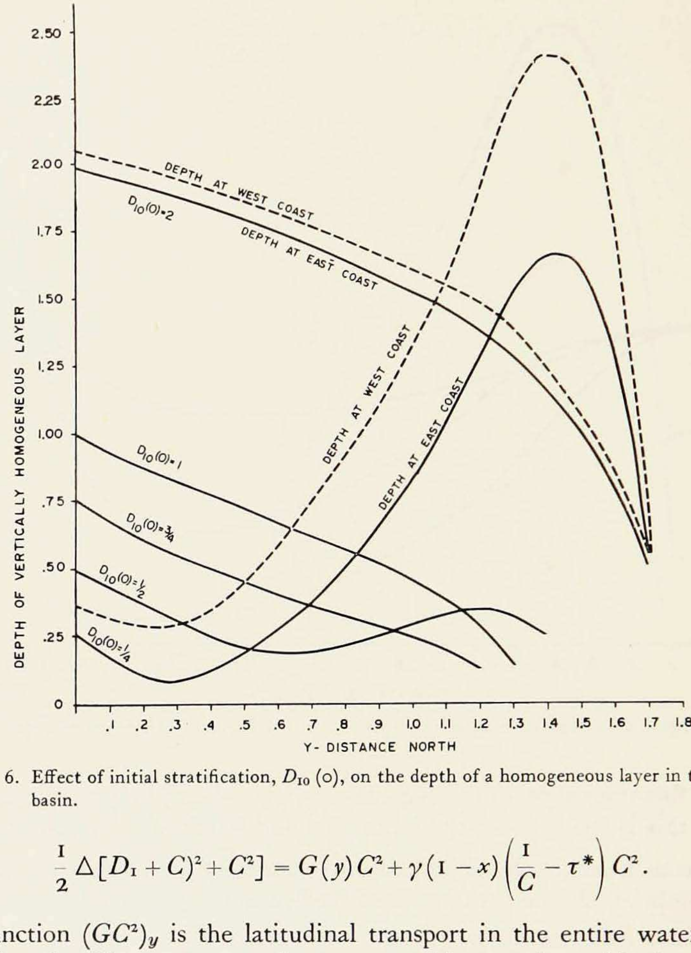

Figure 6. Effect of initial stratification, D_{10} (o), on the depth of a homogeneous layer in the Sverdrup basin.

$$\frac{1}{2}\Delta[D_1 + C)^2 + C^2] = G(y)C^2 + \gamma(1 - x)\left(\frac{1}{C} - \tau^*\right)C^2. \qquad (25)$$

The function $(GC^2)_y$ is the latitudinal transport in the entire water column (excluding the Ekman transport) at $x = 1$; this, together with the curve Γ, along which Δ is specified, serves to determine the solution for the temperature and depth in the homogeneous layer. Here we make a number of assumptions about these functions in solving (24) and (25) for small γ.

By substituting from (23) into (24) we eliminate the derivatives in D_1,

$$\left\{\frac{D_1}{D_1 + C}\right\}\left\{\gamma C^2(1 - \tau^*)\Delta_y + \left[-\Delta CC_y + \gamma(1 - x)\left(C^2\left(\frac{1}{C} - \tau^*\right)\right)_y + \right.\right.$$
$$\left.\left. + (GC^2)_y\right]\Delta_x\right\} = \gamma[\Delta - C\dot{Q}^*]. \qquad (26)$$

Consider the case in which Δ is specified as a constant along a latitudinal circle; thus the asymptotic expansion of (25) and (26) is

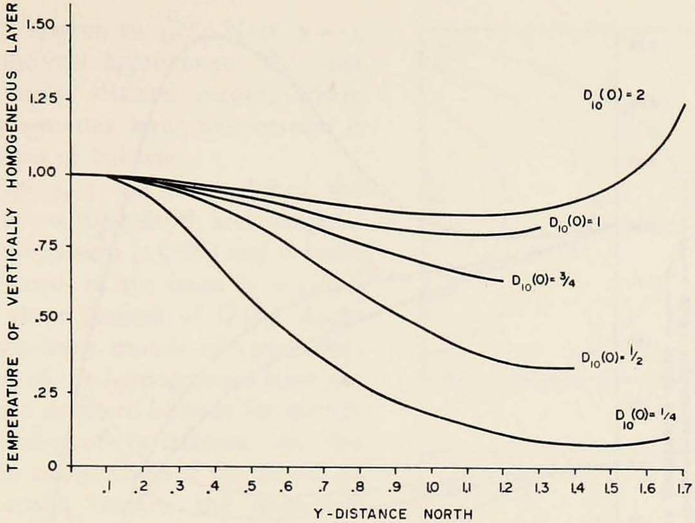

Figure 7. Effect of initial stratification, $D_{10}(0)$, on the homogeneous-layer temperature in the Sverdrup basin.

$$\left.\begin{aligned} \Delta &= \Delta_0(y) + \gamma^2 \Delta_1\left(x, y; \frac{y}{\gamma}\right) + \ldots, \\ D_1 &= D_{10}(y) + \gamma D_{11}(x, y) + \ldots, \end{aligned}\right\} \quad (27)$$

and

$$\left.\begin{aligned} 1/2\, \Delta_0[(D_{10} + C)^2 + C^2] &= GC^2, \\ \Delta_0[D_{10} + C] D_{11} &= C^2(1 - x)\left(\frac{1}{C} - \tau^*\right), \\ \frac{D_{10} C^2\left(\frac{1}{C} - \tau^*\right)}{D_{10} + C} \Delta_{0y} - \Delta_0 &= -CQ^*. \end{aligned}\right\} \quad (28)$$

Equation (28) is now integrated numerically for three cases: (i) the Sverdrup basin, where the easterly mass flux in the entire column vanishes at $x = 15°$W; (ii) a basin where D_{10} is a linear increasing function of y at $x = 50°$W; and a basin where D_{10} is a constant at $x = 15°$W. We identify the vertically homogeneous layer with the depth of the 15° isotherm in the Sargasso Sea (Figs. 2, 3), which deepens at 50°W up to the cold wall of the Gulf Stream; at 15°W the homogeneous layer can no longer be readily identified [Fuglister 1960]. For the second and third set of solutions, $G(y)\,C^2$ is determined a posteriori; for the first set of salutions, $G(y)\,C^2 = G_0$ (a constant) is chosen to satisfy the boundary condition of vanishing total transport normal to the eastern boundary. Recall that if the mass flux in the entire water column is forced to vanish in a baroclinic system, then the velocity is finite in all but one depth. Different

Figure 8. Effect of wind stress on the temperature and depth of the homogeneous layer in a Sverdrup basin $[D_{10}(0) = 2]$.

choices of $G(y)$ are associated with different forms of non-Sverdrup flow in the interior of the basin.

Here we consider examples of basins that span the 10°–40° latitudinal belt, with the β plane centered at 25° latitude. The Coriolis parameter changes by a factor of 3 across this belt, and, in nondimensional units, $f = 1/2\,(1+y) = C$; here $y = 0$ represents the 10° latitude and $y = 2$ the 40° latitude. The wind-stress distribution is a zonal wind,

$$\tau^* = \tau_0{}^* \left[\frac{\cos\left(\frac{\pi}{2} y\right)}{1/2\,(1+y)} \right]_y,$$

and heating is specified as $\dot{Q}^* = \dot{Q}^* \cos(\pi/3y)$. Heat is extracted from the basin above 32° latitude. The parameters of the numerical problem are $\tau_0{}^* = (C_0 \tau_0 / kfL)$ and $D_{10}(0)$—the depth of the homogeneous layer at $y = 0$. With $5 < k < 20$, these parameters vary for the oceanic values between 1/4 and 2. Here we set $\dot{Q}^* = (\dot{Q}_0 C_0 \alpha / \Delta_0 k) \equiv 1$, and we scale G and

Δ_0 with respect to $\dot{Q}^*$. Near $y = 0$, these solutions approximate the case of the basin, derived earlier, where the homogeneous layer temperature is independent of latitude.

In Figs. 4, 5 we have plotted the homogeneous layer depth and temperature at the eastern (15°W) and western (70°W) coasts of the basin ($\gamma = 1/20$), for the three choices of $G(\gamma)$. As in other two-layer models of circulation, the depth of the homogeneous layer can vanish at a northern latitude for specific combinations of parameters. At that point the assumptions of the model no longer apply, because the latitudinal velocities become very large. The value of $\tau_0^* = 2$, considered above, is for a windy ocean having a small amount of heat diffusion, such that the Ekman suction is much larger than the abyssal vertical flux of cold water that balances the heat diffused down through the thermocline. The effect on the thermal structure of the Sverdrup basin of varying $D_{10}(0)$ and τ_0^* is shown in Figs. 6–8. A small value of $D_{10}(0)$ corresponds to a weakly stratified basin in that C is large. The less windy and strongly diffusive basin corresponds to $\tau_0^* = 1/4$. The two-dimensional plot of the transport in the Sverdrup basin is plotted in Fig. 9A, 9B for $\tau_0^* = 2$ and $\tau_0^* = 1/4$. Recall that the total transport in this model of the Sverdrup basin is independent of both heating and $D_{10}(0)$ [eg. (25)] for all values of γ. In Fig. 9C we have plotted the transport distribution for the ocean $D_{10} = 1/4$; the homogeneous layer is of constant depth at the eastern portion of the basin.

In a number of solutions [solutions with small τ_0^* and large $D_{10}(0)$, or large

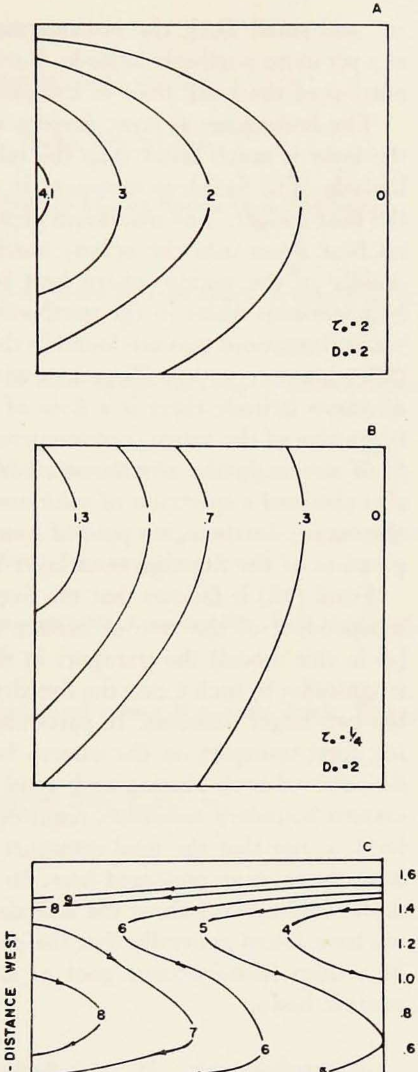

Figure 9. (A) Sverdrup transport in a windy basin [$\tau_0^* = 2$, $D_{10}(0) = 1/4$]. (B) Svedrup transport in a strongly diffusive basin [$\tau_0^* = 1/4$, $D_{10}(0) = 2$]. (C) Horizontal transport in a basin with $D_{10}(y) \equiv 1/4$ at the eastern boundary.

τ_0* and small D_{10}], the homogeneous layer deepens north before it shallows at a yet more northerly latitude. Furthermore, the layer is deeper in the western portion of the basin than in the eastern portion.

The homogeneous layer deepens with latitude because the rate of heating in the layer is much faster than the rate of obsorption in the thermocline at that latitude. The Sverdrup transport, in advecting cold water to the south, balances the heat budget. The maximum depth of the layer occurs at that latitude where no heat flows into the ocean; north of this the homogeneous layer shallows rapidly in the region where heat is extracted from the basin. This pool of homogeneous water in the northwestern portion of the basin is much like the water formations that are found in the North Atlantic and North Pacific oceans (Worthington 1959). Together with the rapid shallowing of the layer at a northern latitude there is a flow of warmer water to the southwest, which is suggestive of the warm countercurrent often found to the south and east of the Gulf stream in the northwestern North Atlantic (Stommel 1965). We have also obtained a spectrum of solutions in which no heat flow was allowed from the ocean. In these, no pool of homogeneous water was found, and the temperature of the homogeneous layer decreased uniformly with latitude.

From (28) it follows that the Sverdrup transport in the basin is $o(\gamma)$ and is independent of the rate of surface heating. However, in a baroclinic system (as in this model) the transport in the individual layers can be of much larger magnitude; in such a case the Sverdrup transport is the small difference between the two larger numbers. In particular, when the Sverdrup condition of vanishing total transport on the eastern boundary is not satisfied, the transport is a function of both heating and wind stress. In a baroclinic system, the proper eastern boundary condition requires that the easterly velocity vanish at every level, z, not that the total transport vanish. This condition cannot be satisfied with the solution presented here. In a limited manner we have investigated different assumptions about the boundary condition on the easterly transport, and we have found generally that the cooling of the ocean results in the formation of a deep homogeneous pool of water in the northwestern portion of the oceanic basin.

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