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Geostrophy and Direct Measurements in the Straits of Florida[1]

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ABSTRACT

Quantitative agreement between geostrophic computations and direct measurements of current and transport in the Straits of Florida, and elsewhere as well, is found only when observations are made under synoptic conditions. Under other conditions, geostrophic computations often show a multiaxial structure (i.e., bands of current) not found with direct measurements; this apparent multiaxial structure could be caused largely by periodic disturbances related to the tides. The reference level must be determined by direct measurement in order to obtain reasonable agreement between direct and geostrophic methods of transport determination. Another mandatory condition for good agreement between the two methods is that sampling bottles must be properly spaced in the upper layers; otherwise interpolation of the observed parameters may result in a false picture of the density field.

Introduction. The Florida Current, flowing northward through the Straits of Florida between Miami, Florida, and Bimini, Bahamas, has been cited by numerous authors to illustrate the practicality of the geostrophic method. However, the few attempts to compare geostrophically computed currents with direct, or nearly direct, measurements (towed electrodes, electric potential differences on submarine cables, current meters suspended from a ship) have generally produced unconvincing results. Such measurements have rarely been simultaneous, and the direct measurements were probably no more accurate than those that were indirect. This paper describes and analyzes a series of experiments in the Straits of Florida, comparing geostrophic computations with accurate direct measurements taken simultaneously.

Geostrophic Computations vs. Direct Measurements. The magnitudes and spacial distribution of current speed and of transport, as determined by geostrophic computations of typical crossings, differ markedly in three ways from

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the magnitudes and distributions obtained by means of the recent direct measurements employing the Richardson and Schmitz method (1965).

First, direct measurements by Richardson and Schmitz have indicated that the northern component of the surface current reaches a maximum at about one-third the distance across the Strait from Miami and then gradually decreases eastward. The integrated transport values behave similarly, except that the maximum transport is found near the center or somewhat to the east of center. On the other hand, geostrophic computations, in many cases, indicate two or more maxima across the Strait. This phenomenon, referred to hereafter as biaxial or multiaxial, appears in the current as well as in the transport.

Second, the volume transport, computed geostrophically, is consistently lower than that measured directly. This discrepancy is particularly noticeable in the middle of the Strait. As is shown later, this disagreement is not always rectified by merely selecting a deeper reference level for the dynamic computations (see Fig. 3).

Third, there is a large difference in the vertical distribution of the current as determined by the two different methods. At many locations, even after the computed transport has been forced to agree with the measured (direct) transport by addition of a constant of integration, the transport values at mid-depths are different. Also, when the measured surface currents are applied to the computed vertical profiles of the current, the resulting integrated transport is different from the measured transport.

Some analyses of data from the Florida Current and from other segments of the Gulf Stream indicate multiaxial structure. A striking example is the 13-month survey, from May 1962 to May 1963, by the Institute of Marine Sciences, University of Miami, aboard the R/V GERDA. During the middle of each month, for 13 consecutive months, we occupied eight oceanographic stations between Fowey Rocks off Florida and Cat Cay, Bahamas, and seven stations between Palm Beach, Florida, and Settlement Point, Bahamas. The time of the observations provided a random distribution as to time of day and tides, and the station locations were approximately the same during each cruise. The data obtained on this survey are listed in several reports (Broida 1962, 1963, 1964).

Fig. 1 shows computed surface currents for the vertical sections between Miami and Cat Cay, only the north-south component of the current being considered. Remarkably, 11 of the 13 sections have multiaxial or biaxial structure while only two sections have a uniaxial structure. The transects between Palm Beach and Settlement Point (not shown here) show similar biaxial or multiaxial structure. Fig. 1 also shows the computed transport for the Miami-Cat Cay sections. Again, the departure from uniaxial structure is evident in at least 50°/₀ of the sections.

Other examples of double and triple maxima in the surface current of the Gulf Stream (downstream from the Florida Strait) have been cited by Martineau

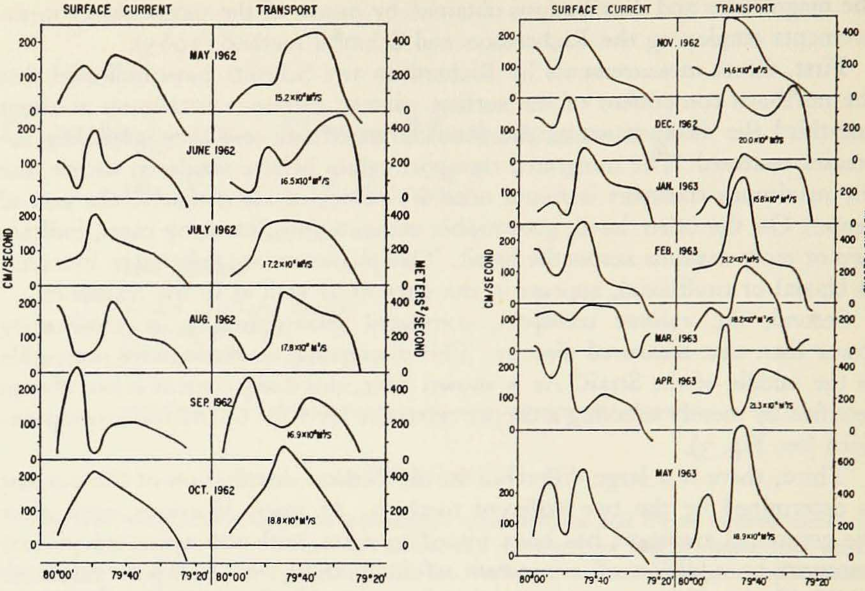

Figure 1. Surface current and transport, Miami-Cat Cay section, May 1962 to May 1963. The numbers under the transport curves are the total integrated transport.

(1957). His computed values agree, in many cases, with direct current measurements by GEK and LORAN-DR. In addition, Parr's (1937) vertical profiles across the Florida Current reveal variations in the slope of isopycnals in support of a multiaxial structure.

In contrast to the above examples, the most convincing evidence for one major axis (uniaxial structure) in the Florida Current is manifested in the direct measurements made by Richardson and Schmitz (1965) of the Institute of Marine Sciences, University of Miami. Also, the measurements by Pillsbury (1890) uphold uniaxial structure. And the remarkable agreement of Wüst's (1924) dynamic computations with Pillsbury's current-meter measurements has contributed largely to the widespread acceptance of the geostrophic method. However, not only were Wüst's temperature data and Pillsbury's current measurements widely separated in time, but many of the density values were derived from average salinities in adjacent waters. Furthermore, Stommel (1958) has cited computed current cross sections of the Gulf Stream that showed "streakiness" (multiaxial structure); he believed that this was caused by internal waves that distorted the density field and that direct measurements (GEK and LORAN) show uniaxial structure.

Procedure. The direct measurements of the surface current and volume transport were made with free-fall instruments and with the Hi-Fix Navigational System of Richardson and Schmitz (1965). Essentially, this method employs weighted instruments that are dropped to a desired depth — usually, one to the bottom and one or two to selected mid-depths — and then released from their weights so that they can return to the surface. A time-versus-depth recorder in the package and the precise recording of the time and position of release and recovery provide the information necessary for measuring not only the integrated volume transport in the water column but the surface-current velocity as well. With a fast boat, a transect of the Florida Strait, consisting of many stations with measurements to several depths at each station, can be completed within a few hours with the above method, whereas a typical crossing time for bottle-cast stations is 20 to 24 hours.

In this paper, an axis is defined as that location, on a line directed normal to the current, where a relative maximum flow occurs. This maximum must be preceded by and followed by a decrease in current of at least 10% before any minimum occurs.

The geostrophic currents and transport were computed according to standard oceanographic procedures, with one exception — the selection of a reference level. Preliminary direct measurements had shown that the current extended almost to the bottom. Therefore, the deepest observation at each pair of stations was used as the reference level for that pair. Where extrapolation of values for the shallower station was necessary, the method suggested by Bennett (1959) was used. Bennett has described this simple method and claims that it is no more uncertain than the older methods of extrapolation near a sloping bottom.

Experiments. A series of five experiments has been designed to compare the transport and current values of nearly simultaneous measurements, employing both methods — direct and computed.

(i) This experiment was performed on 20 October 1964, using two vessels. One vessel occupied 7 oceanographic stations across the Strait; serial samples of both temperature and salinity were obtained. The observarions were taken as deep and as rapidly as possible during a 22-hour period. The other vessel occupied 8 direct-measurement stations; these measurements were obtained with the free-fall instruments and Hi-Fix during a 4-hour period that ended at the same time as the last station made by the other vessel.

Fig. 2 (top) shows the north-south component of the transport plotted against a distance of 80 km from Virginia Key (Miami), Florida. The total transport for the entire section is, by integration, $35 \times 10^6 m^3/sec$ measured directly, and $20 \times 10^6 m^3/sec$ computed geostrophically. The observed value is integrated from the surface to the reference levels selected. The surface currents[2] are shown in Fig. 2 (bottom). The observed values were determined by

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[2] The weather during this experiment was nearly optimal for the methods employed. Mostly light southerly breezes prevailed, rarely exceeding five knots; the sea was nearly calm for the period.

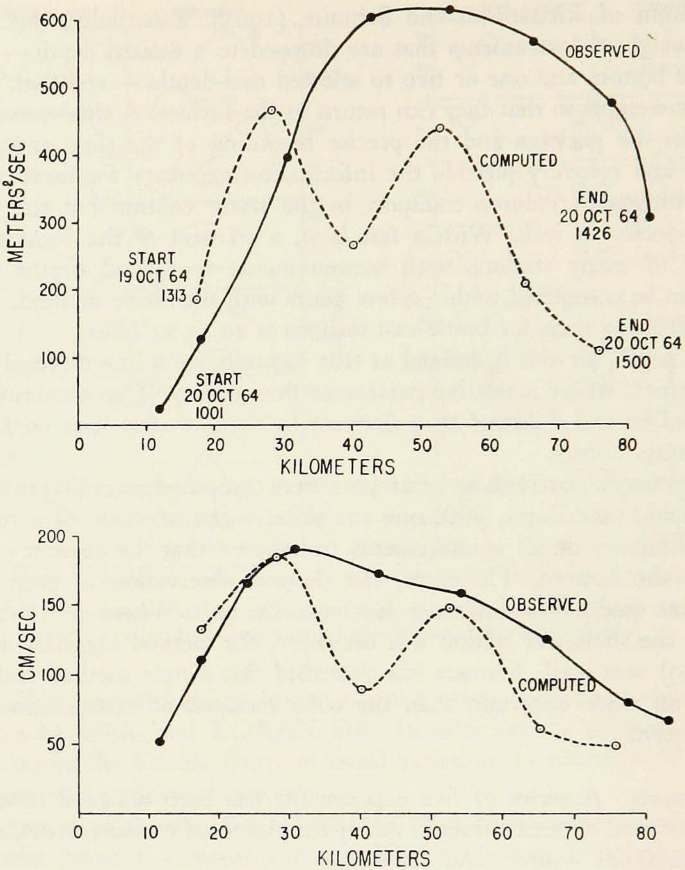

Figure 2. Experiment (i). Top. North-South component of the computed and observed transport extending 80 km from Virginia Key across the Florida Strait. Bottom. Computed and observed surface current.

the drift of the free-fall instruments after they reached the surface; the computed values are relative to the selected reference levels.

It would seem that the very large deficit in the geostrophically computed transport could be rectified by extending the reference level to greater depths. In order to accomplish this, it was decided to employ the observed (direct-measurement) transport for this purpose rather than extrapolate values of temperature and salinity, thereby introducing questionable and subjective values for depths beyond the maximum observations.

Consider Helland-Hansen's equation as an indefinite integral,

$$v - v_0 = \frac{1}{fL} \int (\alpha_B - \alpha_A)\, dp, \tag{1}$$

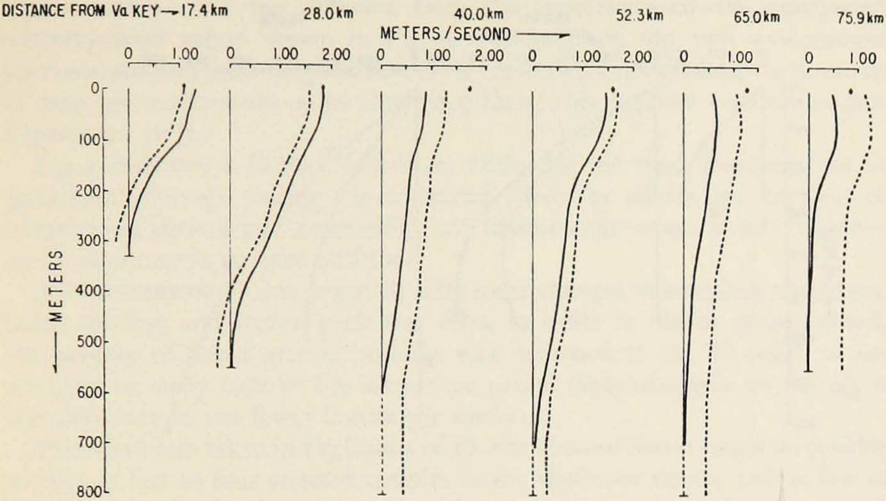

Figure 3. Experiment (i). Vertical distribution. Solid line, unadjusted. Dashed line, adjusted. Arrow at surface, observed current.

where v is the current speed at the isobaric surface being considered, v_0 the current speed at the reference level, f the Coriolis parameter $(2\,\omega \sin \varphi)$, L the distance between Sts. A and B, α the specific-volume anomaly at either station, and p the pressure interval between isobaric surfaces. Then,

$$v = V(z) + C, \tag{2}$$

where $V(z)$ is the standard geostrophic computation and C is a constant of integration, which, when added to the computed current, forced agreement of the computed transport with the observed transport relative to the bottom rather than to some arbitrary reference level. The constant was determined simply as

$$C = \frac{T_0 - T_c}{h}, \tag{3}$$

where T_0 is the observed transport, T_c the computed transport referred to the original reference level, and h the depth to the bottom.

The vertical distribution of the current between station pairs is illustrated in Fig. 3. The current profiles were referred to the original reference levels (solid lines), and their appearance after adding the constant of integration is the dashed line. The small arrow at the surface is at the observed surface-current value. The transport versus depth for each station pair is shown in Fig. 4. The solid line indicates the transport computed from the surface to the original reference level. The dashed line is the transport profile as computed following

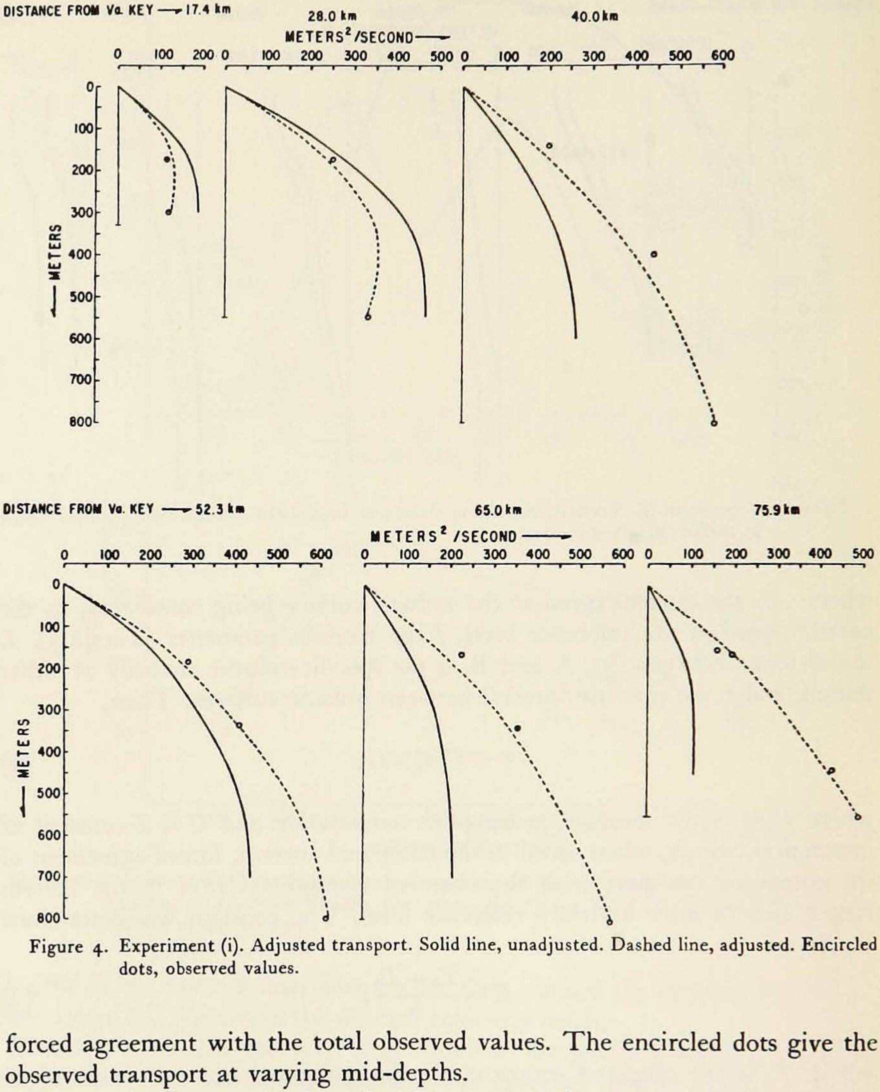

Figure 4. Experiment (i). Adjusted transport. Solid line, unadjusted. Dashed line, adjusted. Encircled dots, observed values.

forced agreement with the total observed values. The encircled dots give the observed transport at varying mid-depths.

Fig. 3 clearly reveals that the disagreement between the observed and computed values of current and transport was not resolved by adjustment of the reference level. Except for the values at 17.4 km and 52.3 km from Virginia Key, the surface current speeds differed markedly from observed speeds. Note also that extension of the computed current profiles to the bottom, with their origin at the observed surface currents, did not give the observed transport. Especially noticeable is the large discrepancy found at 40 km from Virginia Key, which gave the computed current and transport the appearance of a biaxial stream — not too different from the appearance of the unadjusted reference-level values shown in Fig. 2. Furthermore, the two westernmost positions indicate, following addition of the constant of integration, the presence of deep countercurrents up to about 0.5 knot; this requires verification [see Experiment (iv)].

Fig. 4 indicates a further difference. Although the total transports are in agreement (through forcing the computed values by adding the constant of integration), there is still some difference between the values at mid-depths — up to about 20% at some positions.

(ii) Experiment (i) was repeated, with some changes in sampling procedure, bottle spacing, and station positions. Also, in order to obtain greater detail, the number of oceanographic stations was increased to 12. In order to accomplish so many casts in the same time period (approximately 22 hours), it was necessary to use fewer bottles per station.

Historical data taken in the Straits of Florida showed that it might be possible to take as few as four selected samples in the shallower depths and as few as eight samples in the deeper waters without sacrificing the major features of the density field. This hypothesis was tested by selecting observed data from Experiment (i) and recomputing. Approximately the same values of current and transport were obtained after the data from certain depths had been eliminated.

On 13 and 14 December 1964, both vessels again obtained a nearly simultaneously observed series of data consisting of 12 oceanographic stations and seven direct-measurement stations. This experiment was performed during weather conditions that were similar to those that prevailed during Experiment (i).

Dynamic heights, computed from the observed-depth data, were plotted versus depth and joined with smooth curves. A third-degree polynomial was fitted, and dynamic heights at standard depths were computed from the polynomial.

These computed values were compared with the dynamic heights computed from graphically interpolated temperatures and salinities. For Experiment (ii), the mean difference between the two methods of interpolation for standard-depth dynamic heights was 1.4 dyn cm, or an average standard deviation of ± 1.2 dyn cm. For Experiment (i), employment of the same methods resulted in a mean difference of 1.0 dyn cm, with an average standard deviation of ± 0.6 dyn cm.

Which method of interpolation is more accurate has not yet been resolved. But if the rather poor agreement between the results achieved with the interpolation methods applied to the Experiment (ii) data is compared with the difference in standard deviation of less than 50% obtained with the two methods applied to the Experiment (i) data, it is suggested that the temperature and salinity interpolations in Experiment (ii) do not reflect the density field as accurately as is desired.[3] The results would have been better with more bottles per cast or with more efficacious spacing of the few bottles used. Proper spacing of the bottles reduces the errors inherent in the numerical integration methods, because the remainder term in the trapezoidal rule is ignored in standard geostrophic computations; yet this term can be large where rapid and irregular changes in density take place (Fomin 1964). Closely-spaced bottles in the upper layers reduce the term's magnitude, thus rendering a closer approximation to the integral of the specific-volume anomaly with respect to depth (dynamic height).

Little confidence can be placed in geostrophic computations between closely-spaced oceanographic stations, because the relative error, dv/v, can be as high as 100°/₀ in many cases (Fomin 1964). It was expedient, therefore, to reduce the probable error by regrouping the station data in this experiment when computing current and transport. By simply computing between all odd-numbered stations and then between all even-numbered stations, the station separation was increased to more than 10 km for each pair.

The computed and observed transports are shown in Fig. 5 (top). Major discrepancies are apparent at about 35 km and 60 km east of Virginia Key and near the eastern boundary as well. Note that a sharp maximum in this experiment occurs at about the same position as the low minimum in Experiment (i).

In Fig. 5 (bottom), the values for the computed and observed surface currents show rather good agreement except for those values near 28 km to 36 km east of Virginia Key and near the eastern edge. Note that the relative minimum at about 60 km appears in both the observed and computed speeds.

The computed transport and the vertical distribution of current were adjusted by a constant of integration in the same manner as that described for Experiment (i). Again, many discrepancies in shear, surface current, and mid-depth transports are clearly evident.

(iii) This experiment, conducted on 6 April 1965, during fine weather, was designed to compare computed and observed transport between a pair of oceanographic stations separated by 15 km; the stations were positioned by Hi-Fix. The direct measurements of transport were made to several depths midway between the two stations.

The deeper part of the Florida Strait (about 65 km east of Virginia Key) was selected for the experiment. A current meter was positioned about 10 m above the bottom at this location. Both oceanographic casts were completed within four hours; the eight direct measurements were made to the bottom and to various mid-depths.

The geostrophic computations were made relative to 650 m, with no extrapolations. The computations were then modified by the constant of integration,

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[3] A difference of 1 dyn cm in geopotential anomaly at an isobaric surface at thisl atitude, between stations 10 km apart, could make a difference of about 16 cm/sec in computed current.

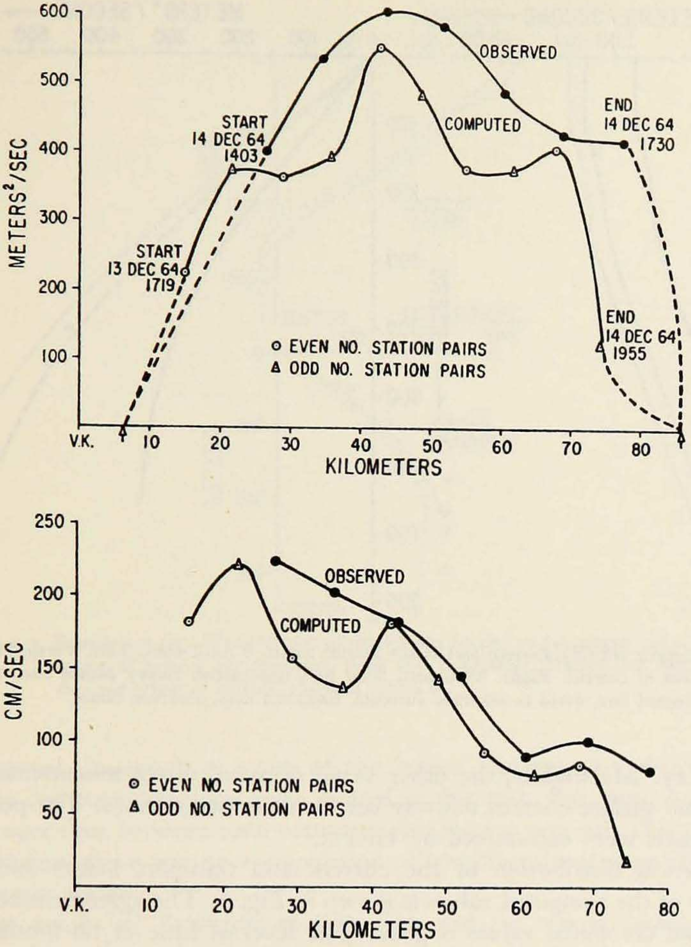

Figure 5. Experiment (ii). Top. Computed and observed transport. Bottom. Computed and observed surface currents.

as in the previous experiments. Also, the computations were modified to fit the observed surface and "bottom" currents. The results are shown in Fig. 6.

The rather good agreement between the observed and computed transport after adjustment is not surprising in this instance. This agreement was expected, because the experiment was brief enough to preclude large fluctuations in the current and in the vertical oscillations of the isopycnals; also, the bottom topography is without extreme irregularities. Experiments (iv) and (v) were planned for the western and middle parts of the Florida Strait, respectively.

(iv) On 30 September 1965, during a three-hour period, one of the two vessels occupied two oceanographic stations, at 20 km and 30 km east of

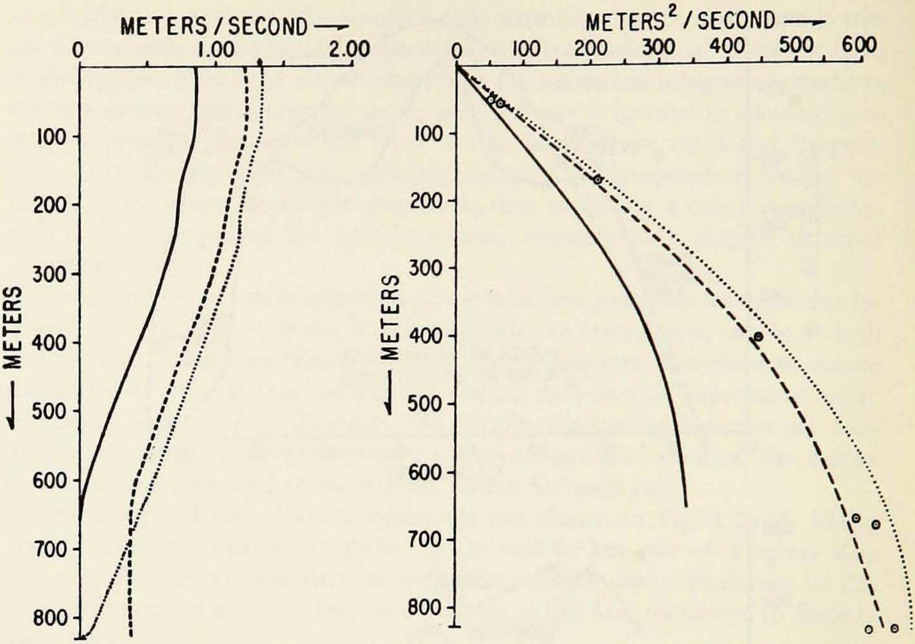

Figure 6. Experiment (iii). Eastern part of the Florida Strait, 6 April 1965. Left. Vertical distribution of current. Right. Transport. Solid line, unadjusted. Heavy dashed line, adjusted. Dotted line, fitted to observed currents. Encircled dots, observed values.

Virginia Key. Meanwhile, the other vessel obtained direct measurements of transport and surface current midway between the two stations. The positions of both vessels were determined by Hi-Fix.

The vertical distribution of the current and transport before and after adjustment of the computed values is shown in Fig. 7. The agreement between observed and computed values is good. The level of little or no motion was located at about 250 m, as was the deepest bottle — a good reason for the excellent agreement. These results confirm the existence of a southerly flow below this level at the time of measurement. Note that the execution of this experiment coincided with the high spring tide at Miami Beach, and this probably accounts for the southerly flow of about 10 cm/sec (the estimated maximum tidal current).

(v) Experiment (iv) was repeated in the central part of the Strait on 10 November 1965; the two oceanographic stations were located at 38.0 km and 54.6 km east of Virginia Key. Again, the second vessel made direct measurements midway between these two oceanographic stations. The vessels were positioned by Hi-Fix.

The current profile and transport are shown in Fig. 8. Again, the agreement between the observed and computed values is good; this is somewhat

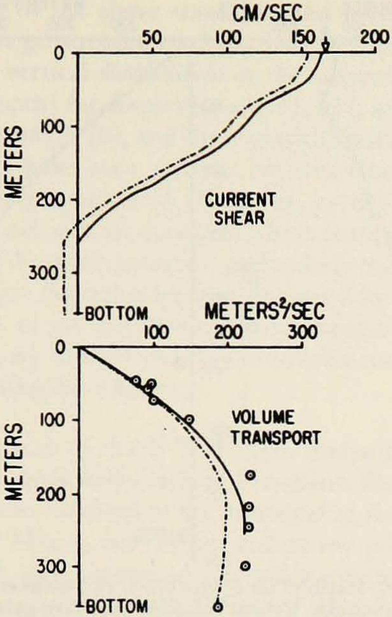

Figure 7. Experiment (iv). Western part of the Florida Strait, 30 September 1965. Vertical distribution of current and transport. Solid line, unadjusted. Dashed line, adjusted. Encircled dots and arrow at surface, observed values.

unexpected, inasmuch as many of the largest discrepancies observed in previously computed transects were located in this part. But, note again that the messenger time between each oceanographic station was only about two hours. In addition, there was no extrapolation of specific-volume anomaly values for the deeper waters.

The results of Experiments (iii), (iv), and (v) clearly indicate that the geostrophic computations are in good agreement with the direct measurements. Conversely, the results of Experiments (i) and (ii) show the unusual biaxial or multiaxial structure (computed geostrophically) that was found in most of the 13-month-survey sections (Fig. 1).

Discussion and Conclusions. Inasmuch as the direct measurements made with free-fall instruments have revealed only a single axis in the Florida Current, it would appear that some forces distort the density field, producing apparent multiaxial or banded structure in the geostrophic current; or, the timing of the temperature and salinity serial observations was such that the disturbed density field produced the illusion of multiaxial structure. This concept is not new in oceanography; for example, it has been discussed by Seiwell (1937) and Defant (1950), and much of the distorting influence has

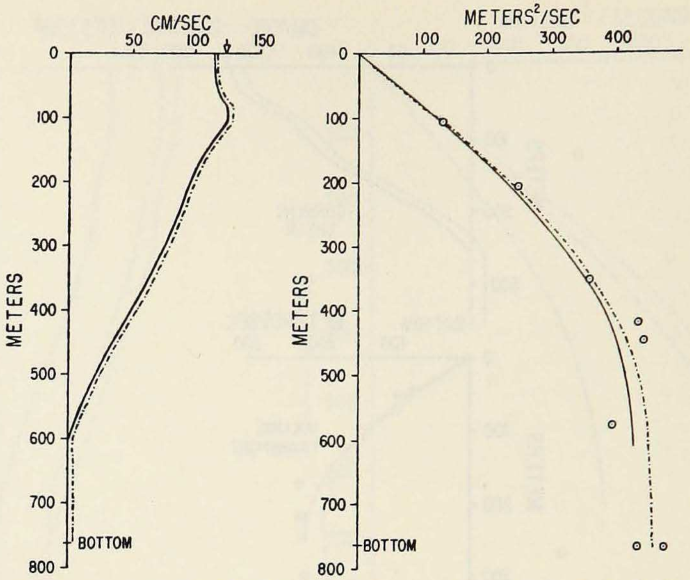

Figure 8. Experiment (v). Middle of the Florida Strait, 10 November 1965. Left. Vertical distribution of current. Right. Volume transport. Solid line, unadjusted. Dashed line, adjusted. Encircled dots and arrow at surface, observed values.

been attributed to periodic changes related to the tides. Therefore, the possibility of changes in the slopes of isopycnals or the variations in dynamic heights, which could be caused by tidal forces, should be examined.

The principal conclusion to be drawn from Experiments (iii), (iv), and (v) is that the geostrophic approximation, when applied to the Florida Current in the Florida Strait, is quantitatively valid only under certain exacting conditions.

The most important condition is that the stations be nearly synoptic, preferably simultaneous, in order to minimize the effect of the extremely rapid changes that take place with time in the density field.

A second condition is that the reference level be determined by some accompanying direct measurement. For example, the assumed reference level used by some investigators in the deeper part of the Strait appears to be valid only when the opposing tidal current (flooding) exceeds the northerly flow of deep water; this tidal-current opposition may reveal a southerly flow of deep water [see Experiments (i), (iv)]. At other times, the northerly current appears to extend almost to the bottom.

A third condition is that the spacing of the sample bottles for serial observations be very close in the upper 250 m in order to ascertain accurately the vertical density gradients and minimize the errors in interpolation and numerical-integration techniques.

Application of some of the above conditions did much toward eliminating the differences between geostrophic computations and direct measurements in surface current, in the vertical distribution of the current, and in the transport (total as well as mid-depth) for Experiments (iii), (iv), and (v).

Regarding Experiments (i), (ii), and the 13-month-survey sections, the three conditions noted should have been applied; but this was not possible nor was their necessity completely appreciated. However, even if the three conditions had been applied, there can be little doubt that these multistation transects would have exhibited some of the discrepancies—particularly the multiaxial structure.

It should be noted that discrepancies may be caused by aperiodic oscillations, by periodic oscillations as yet undetermined, by changes introduced by water masses from upstream, by diurnal changes in salinity and temperature at the naviface, or by meteorological effects.

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REFERENCES

BENNET, E. B.
1959. Some oceanographic features of the Northeast Pacific Ocean during August, 1955. J. Fish. Res. Board Canada, *16* (5): 565–633.

BROIDA, SAUL
1962. A report of data obtained in Florida Straits and off the west coast of Florida. January–June, 1962 (ONR Geophysics Branch). Inst. Mar. Sci., Univ. Miami, No. 62S11. 138 pp. Unpublished.
1963. A report of data obtained in Florida Straits and off the west coast of Florida. July–December, 1962 (ONR Geophysics Branch). Inst. Mar. Sci., Univ. Miami, No. 63-3. 196 pp. Unpublished.
1964. A report of data obtained in Florida Straits and off the west coast of Florida. January–June, 1963 (ONR Geophysics Branch). Inst. Mar. Sci., Univ. Miami, No. 64–1. 179 pp. Unpublished.

DEFANT, ALBERT
1950. Reality and illusion in oceanographic surveys. J. Mar. Res., *9* (2): 120–138.

FOMIN, L. M.
1964. The dynamic method in oceanography. Elsevier Publishing Co., New York. 212 pp.

MARTINEAU, D. P.
1957. The Gulf Stream as an equivalent-barotropic system. Doctoral dissertation, New York Univ. 84 pp.

PARR, A. E.

1937. Report on hydrographic observations at a series of anchor stations across the Straits of Florida. Bull. Bingham oceanogr. Coll., *6* (3): 1–62.

PILLSBURY, J. E.

1890. The Gulf Stream, methods of the investigations and results of the research. U.S. Cst. Geodet. Survey Publ. Rep. 1890. Append. No. 10: 461–620.

RICHARDSON, W. S., and W. J. SCHMITZ, JR.

1965. A technique for the direct measurement of transport with applications to the Straits of Florida. J. Mar. Res., *23* (2): 172–185.

SEIWELL, H. R.

1937. Short period vertical oscillations in the western basin of the North Atlantic. Pap. phys. Oceanogr. Meteorol., *5* (2): 1–44.

STOMMEL, HENRY

1958. The Gulf Stream, a physical and dynamical description. Univ. of California Press, Berkeley. 202 pp.

WÜST, GEORG

1924. Florida- und Antillenstrom, eine hydrodynamische Untersuchung. Veroff. Inst. Meereskunde, N.F. *12*: 1–48.