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On the Roles of Vertical Velocity and Eddy Conductivity in Maintaining a Thermocline[1]

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ABSTRACT

Steady-state solutions of the vertical heat-balance equation have been obtained for various assumed vertical profiles of the vertical component of velocity and eddy conductivity. Relationships have been found for the depth and the thickness of the thermocline in terms of parameters that characterize the vertical velocity and eddy conductivity. Two of the solutions agree with observations in regions having respectively divergent and convergent Ekman mass transport.

1. *Introduction.* Existing theoretical models of the thermocline can be placed in two broad classifications, namely, (i) localized studies primarily concerned with the processes that generate and maintain the mixed surface layer, and (ii) large-scale circulation models in which the temperature and velocity fields in the permanent thermocline are nonlinearly coupled.

The vertical models of Munk and Anderson (1948) and Kraus and Rooth (1961), representative of the first category, explain many of the features of the mixed layer and shallow (seasonal) thermocline.

Below the Ekman layer, the thermohaline circulation models of the second type become appropriate. Here the heat and momentum equations have been attacked mainly through the series of similarity solutions obtained by Welander (1959), Robinson and Stommel (1959), Stommel and Webster (1962), Robinson and Welander (1963), Blandford (1965), and Needler (1967). Most of these studies are consistent with Stommel's (1957) hypothesis on thermohaline circulation. According to this hypothesis, there is a slow upwelling of deep water over the major portions of the oceans to compensate for localized high-

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latitude sinking, and the permanent thermocline is primarily maintained by the upward advection of cold water, which balances the downward conduction of heat from surface insolation.

Questions remain on the relationship of both kinds of models to oceanic conditions, and inversely, to what can be determined from observed oceanic conditions about the processes necessary for their maintenance. This paper considers the simplest possible model that can shed some light on these questions.

From previous studies on the thermohaline circulation, it appears that the terms in the heat-balance equation representing the vertical advection and conduction of heat are the most important for determining the basic vertical temperature structure. Munk (1966) has recently shown that this simple vertical model accounts for the distribution of properties below 1000 m in the Pacific Ocean. The basic features of the thermohaline models are retained in the present study. That is, an upward flow is assumed at great depth, and a vertical velocity at the bottom of the Ekman layer is assumed proportional to the curl of the wind stress.

The effects of wind-induced mixing present in models of the mixed layer are introduced into this study by taking the eddy conductivity to be a function of depth. It is realized that the eddy conductivity depends on wind, waves, and stratification and that this dependence necessarily would be included in a complete model. The virtue of this simple model is that it avoids the above complications and yet shows the essential processes controlling the vertical distribution of temperature in certain regions of the ocean.

2. *Formulation of the Problem.* The steady-state equation expressing the conservation of heat is given by

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) - \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z}\right) = 0, \quad (1)$$

where T is the time-mean temperature; x, y are the horizontal coordinates, and z is the vertical coordinate measured positively downward; u, v, w are the velocity components in the x, y, z directions, respectively; and k is the eddy coefficient for the indicated direction. The present model is governed by the following approximate equation, which results from the neglect of horizontal transfer processes:

$$\frac{d}{dz}\left(k_z \frac{dT}{dz}\right) - w \frac{dT}{dz} = 0. \quad (2)$$

Although previous studies confirm the validity of (2) in describing the gross features of the vertical temperature distribution, it is interesting and enlightening to make order-of-magnitude estimates of the three advection terms in

(1). In the subtropical and equatorial North Atlantic, away from meridional boundaries, characteristic values of the temperature gradients and velocity components are:

$$\partial T/dx \sim -0.6\times 10^{-8}\,^{\circ}\mathrm{C\,cm^{-1}}, \quad \partial T/\partial y \sim -3\times 10^{-8}\,^{\circ}\mathrm{C\,cm^{-1}},$$
$$\partial T/dz \sim -4\times 10^{-4}\,^{\circ}\mathrm{C\,cm^{-1}},$$
$$u \sim -10\ \mathrm{cm\,sec^{-1}}, \quad v \sim 1\ \mathrm{cm\,sec^{-1}},$$
$$|w| \sim 5\times 10^{-5}\ \mathrm{cm\,sec^{-1}}.$$

The three advection terms are then:

$$u(\partial T/dx) \sim 6\times 10^{-8}\,^{\circ}\mathrm{C\,sec^{-1}},$$
$$v(\partial T/\partial y) \sim -3\times 10^{-8}\,^{\circ}\mathrm{C\,sec^{-1}}, \quad |w(\partial T/dz)| \sim 2\times 10^{-8}\,^{\circ}\mathrm{C\,sec^{-1}}.$$

Although these figures are crude, it is apparent that, individually, neither $u(\partial T/\partial x)$ nor $v(\partial T/\partial y)$ is negligible compared with $w(\partial T/\partial z)$. Thus the condition for neglecting the horizontal terms is that they approximately cancel each other. Since these terms have been shown to be of the same order of magnitude and of opposite signs, this condition is possible. Ichiye (1958) has shown that, with baroclinic geostrophic flow, these terms exactly cancel when the density distribution is expressible in the form

$$\varrho = F(x,y)\,\varrho_0(z) + \varrho_1(z) + \varrho_2(x,y),$$

where

$$(\partial F/\partial x)\,(\partial \varrho_2/\partial y) - (\partial F/\partial y)\,(\partial \varrho_2/dx) = 0.$$

Classically, this statement is embodied in the "law of parallel fields" (Defant 1961: 477).

In the equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{3}$$

however, the corresponding horizontal terms do not cancel. It is essential in thermocline models that $\partial w/\partial z \neq 0$.

The horizontal diffusion is neglected here, as in most studies, purely on the grounds of the resulting simplicity. Our poor knowledge of the magnitude of the horizontal eddy coefficient and of the difficulty of computing curvatures in the horizontal temperature field make it difficult even to estimate the order of magnitude of these terms. The results of previous studies suggest, however, that the vertical term dominates the diffusion process. Note that, since the horizontal diffusion terms are not coupled with the mean velocity field, as are the advection terms, it is immaterial whether the former are neglected on the basis of being individually small or of canceling each other.

There is an alternate approach for obtaining a vertical heat equation having a different form from (2). Although not used in this paper, this alternate formulation merits some comment. If (3) is substituted into (1) and the resulting equation is averaged over a sufficiently large horizontal domain L^2, then, to order $1/L$,

$$\frac{d}{dz}\left(\bar{k}\frac{d\bar{T}}{dz}\right) - \frac{d}{dz}(\overline{wT}) = 0, \tag{4}$$

where $\bar{k} = k_z - [\overline{w'T'}/(d\bar{T}/dz)]$; the overbar denotes the spatial average, and the prime denotes the deviation from this average. Since the vertical velocity is included in the differentiation, (4) can be integrated once to give

$$-\bar{k}\frac{d\bar{T}}{dz} + \overline{wT} = \text{constant.}$$

Here the horizontal terms have been removed, without assumptions, through a proper averaging process. The result, however, is a nondivergent mean vertical heat flux. If the usual conditions are imposed—that the vertical velocity is zero at the surface and that the bottom is level and nonconducting—then the horizontal averaging area must be sufficiently large (oceanic dimensions) so that the mean heat flux at the surface is zero. This result, although physically correct, does not allow a description of local thermoclines. Hence the formulation given by (2), where the local heat flux is horizontally divergent, is used in place of (4).[2]

The boundary conditions to be satisfied by (2) are:

$$T(0) = T_S, \quad T(H) = T_B, \tag{5}$$

where T_S and T_B are, respectively, the temperatures at the upper ($z = 0$) and lower ($z = H$) boundaries of the region of interest.

It is convenient to express (2) and (5) in the normalized dimensionless form

$$\frac{d}{d\eta}\left(k\frac{d\theta}{d\eta}\right) - wH\frac{d\theta}{d\eta} = 0, \tag{6}$$

$$\theta(0) = 1, \quad \theta(1) = 0, \tag{7}$$

where

$$\theta = \frac{T - T_B}{T_S - T_B}, \quad \eta = \frac{z}{H}, \quad k = k_z. \tag{8}$$

2. If the horizontal diffusion terms are neglected in (1), then the vertical heat flux must be horizontally divergent to prevent conduction into the bottom. Hence, $(\partial/\partial x)(uT) + (\partial/\partial y(vT) \neq 0$.

The solution of (6) is

$$\theta = a\int_0^{\eta}\frac{1}{k}\exp\left(\int_0^{\eta'}\frac{wH}{k}\,d\eta''\right)d\eta' + b, \tag{9}$$

where the constants a and b, determined by (7), are understood to be functions of the horizontal position.

3. *The Model.* The forms of temperature profiles maintained by vertical processes fall into two natural classifications, depending upon whether the Ekman transport is (i) divergent or (ii) convergent; i.e., whether the flow through the bottom of the Ekman layer is advecting cold water upward or warm water downward.

Studies by Stommel (1956, 1964: 53–58) and others indicate that, in the open ocean, below the Ekman layer, the vertical component of velocity is generally in the range 10^{-5}–10^{-4} cm sec^{-1}. Wyrtki (1961), using a vertical model somewhat more restrictive than the present one, estimated the ratio w/k to be of the order 10^{-4} cm^{-1} within the Ekman layer and 10^{-5} cm^{-1} in the thermocline region below. Using vertical models for both temperature and carbon-14, Munk (1966) obtained separate estimates for (constant) values of w and k below 1000 m in the Pacific. His values ($w \sim 10^{-5}$ cm sec^{-1}, $k \sim 1$ cm^2 sec^{-1}) are consistent with those of Wyrtki. For our model, magnitudes of w and k have been chosen to include the above range. The direction of the vertical motion at the bottom of the Ekman layer is determined by the curl of the wind stress while that at depth is assumed to be upward.

(i) The Divergent Ekman Transport. In this case, the vertical motion is assumed to be upward throughout the depth range of interest. For simplicity, we assume that w and k are constant with depth.

$$w = -w_0 = \text{constant}, \quad k = k_0 = \text{constant}. \tag{10}$$

These quantities may be considered to represent mean values over the water column.

Substitution of (10) into (9) gives

$$\theta = \frac{e^{-P\eta} - e^{-P}}{1 - e^{-P}}, \tag{11}$$

where $P = w_0H/k_0$ is a turbulent Péclet number representing the ratio of advective to conductive heat transfer.

Fig. 1 shows the effect of variations of the Péclet number upon the temperature profile given by (11). The value $H = 1500$ m is assumed throughout the model. The values of P chosen for Fig. 1 then include the range of w/k that is likely to occur in the ocean.

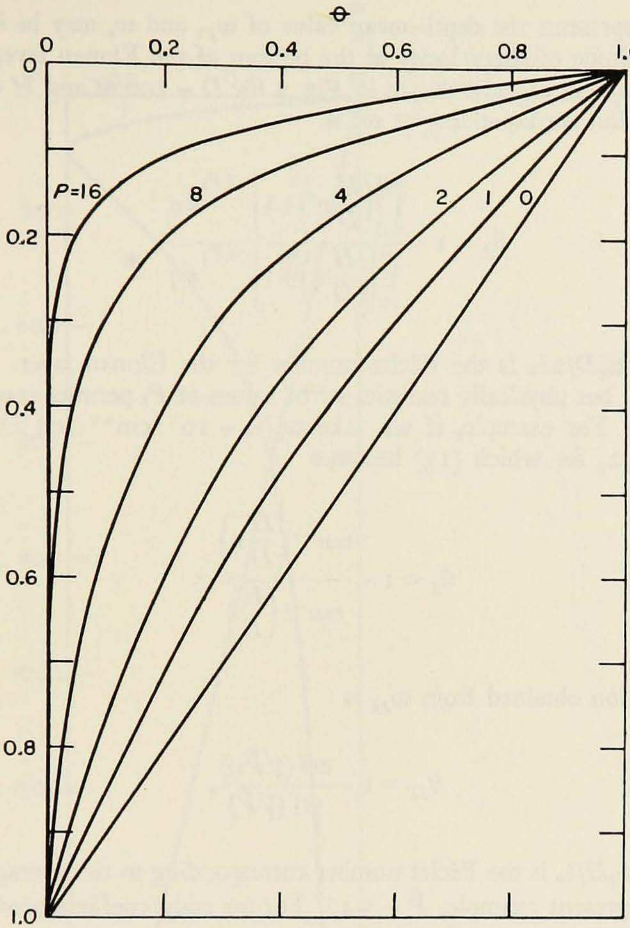

Figure 1. Form of the vertical profile of temperature for various values of the Péclet number P. Divergent Ekman transport.

It is convenient to define the thermocline thickness δ as the depth that corresponds to one e-folding of the maximum temperature gradient. From (11) it is seen that this thickness corresponds to the length scale k_0/w_0.

As an indication of the conditions under which the mean-velocity approximation is adequate, comparison is made between the solutions that correspond to two dissimilar velocity profiles having the same mean value. For exemplary purposes, the following distributions are assumed:

$$w_I = -2w_e \frac{\dfrac{z}{D}}{\left(\dfrac{z}{D}\right)^2 + 1}, \quad w_{II} = -2|\overline{w_I}|\frac{z}{H}, \quad k = k_0 = \text{constant}, \qquad (12)$$

where $\overline{w_I}$ represents the depth-mean value of w_I, and w_e may be interpreted as the magnitude of the velocity at the bottom of the Ekman layer ($z = D$). Profiles of w_I and w_{II} are shown in Fig. 2 for $D = 100$ m and $H = 1500$ m.

The solution corresponding to w_I is

$$\theta_I = 1 - \frac{\int_0^\eta \left[\left(\frac{H}{D}\eta'\right)^2 + 1\right]^{-2P_1} d\eta'}{\int_0^1 \left[\left(\frac{H}{D}\eta\right)^2 + 1\right]^{-2P_1} d\eta}, \qquad (13)$$

where $P_1 = w_e D/2k_0$ is the Péclet number for the Ekman layer.

A limited, but physically realistic, set of values of P_1 permits exact integration of (13). For example, if we take $w_e/k_0 = 10^{-4}\text{cm}^{-1}$ and $D = 100$ m, then $P_1 = 1/2$, for which (13) becomes

$$\theta_I = 1 - \frac{\tan^{-1}\left(\frac{H}{D}\eta\right)}{\tan^{-1}\left(\frac{H}{D}\right)}. \qquad (14)$$

The solution obtained from w_{II} is

$$\theta_{II} = 1 - \frac{\text{erf}\,(\sqrt{\bar{P}}\eta)}{\text{erf}\,(\sqrt{\bar{P}})}, \qquad (15)$$

where $\bar{P} = w_I H/k_0$ is the Péclet number corresponding to the average velocity $\overline{w_I}$. In the present example, $\bar{P} = 5.42$. For an eddy coefficient of unity, it follows that $\overline{w_I} = 2.4 \times 10^{-5}\text{cm sec}^{-1}$.

Fig. 3 shows the vertical temperature profiles given by (14) and (15), respectively. Also shown is the profile for (11), where the velocity is assumed constant and equal to the mean value, $\overline{w} = \overline{w_I} = \overline{w_{II}}$. The mean-value approximation correctly represents the form of the temperature profiles for w_I and w_{II} but does not adequately describe the details of either. Obviously, as the scale of the vertical velocity becomes very small, the conductive state is approached, and any differences in velocity distribution can produce only small effects upon the temperature field.

The distributional effects of velocity may be discussed on a semiquantitative basis by means of a heat-flux ratio R, which represents the ratio of the total advective heat transfer above a given depth to the diffusive heat flux at the surface. Under steady-state conditions and with the assumption of a constant eddy coefficient, the flux ratio is given by

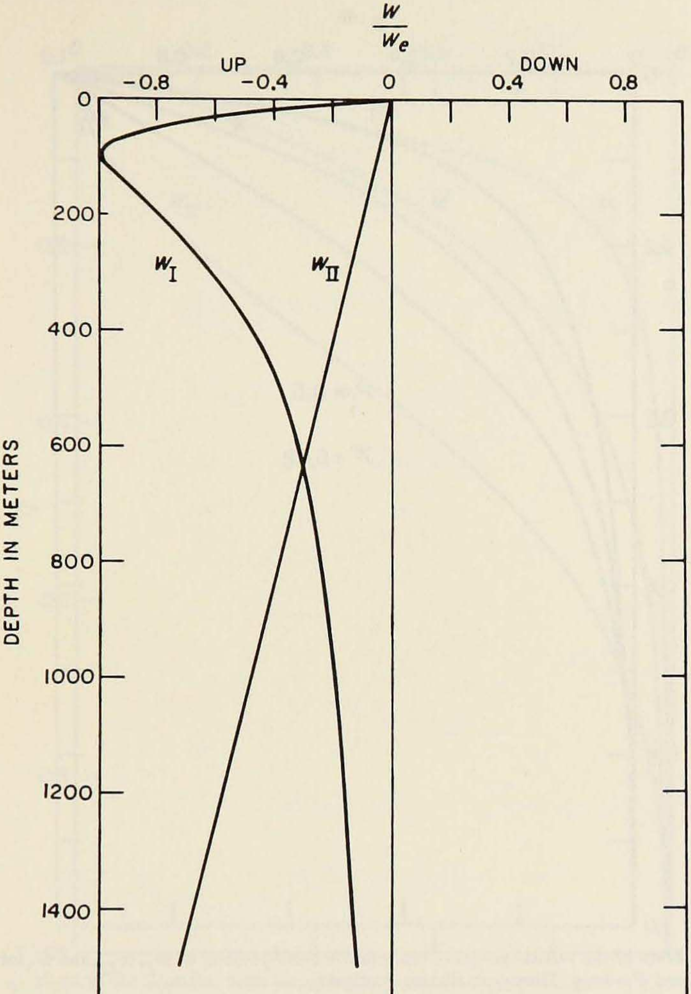

Figure 2. Profiles of vertical velocity, w_I and w_{II}, having the same depth-mean value $\bar{w}$. Divergent Ekman transport.

$$R = 1 - \frac{k\dfrac{dT}{dz}}{k\dfrac{dT}{dz}\bigg|_{z=0}} = 1 - \frac{\dfrac{d\theta}{d\eta}}{\dfrac{d\theta}{d\eta}\bigg|_{\eta=0}}. \tag{16}$$

Fig. 4 shows the vertical distribution of the flux ratio corresponding to each of the three temperature profiles in Fig. 3. For discussion purposes, we propose that the form of R suggests that the water column can be separated into

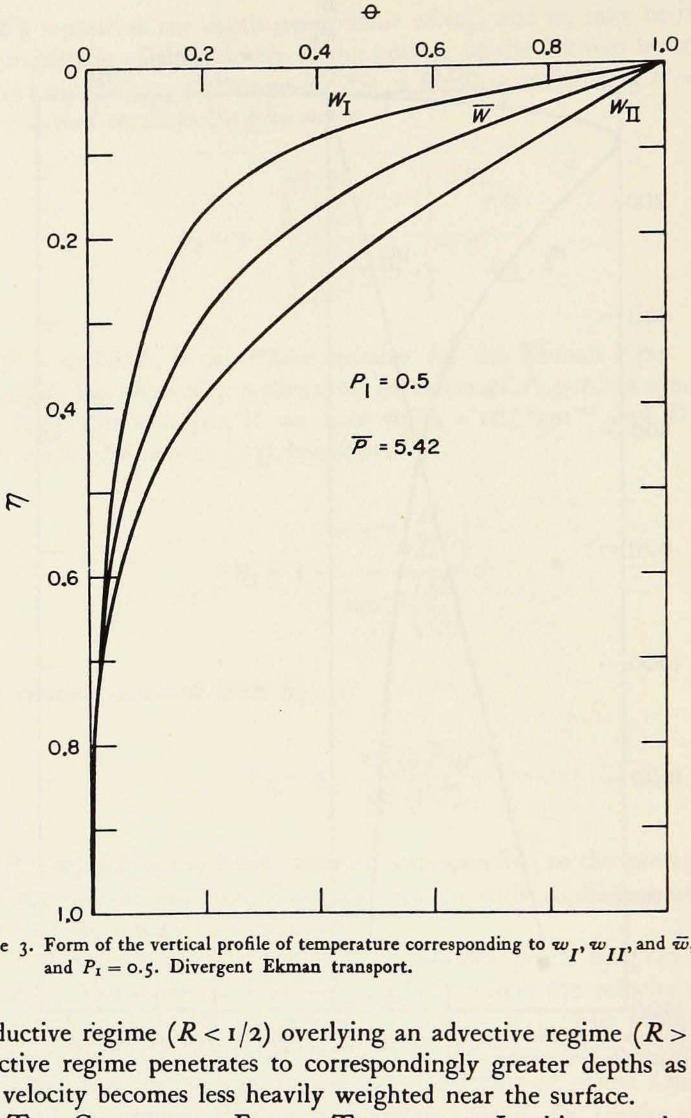

Figure 3. Form of the vertical profile of temperature corresponding to w_I, w_{II}, and $\bar{w}$, for $\bar{P} = 5.42$ and $P_I = 0.5$. Divergent Ekman transport.

a conductive regime ($R < 1/2$) overlying an advective regime ($R > 1/2$). The conductive regime penetrates to correspondingly greater depths as the given mean velocity becomes less heavily weighted near the surface.

(ii) THE CONVERGENT EKMAN TRANSPORT. In this case, the following distributions of velocity and eddy coefficient are assumed:

$$\left.\begin{aligned} w_1 &= w_e \frac{z}{D}, & 0 \le z \le D; \\ w_2 &= w_e \frac{h-z}{h-D}, & D \le z \le h; \end{aligned}\right\} \quad (17)$$

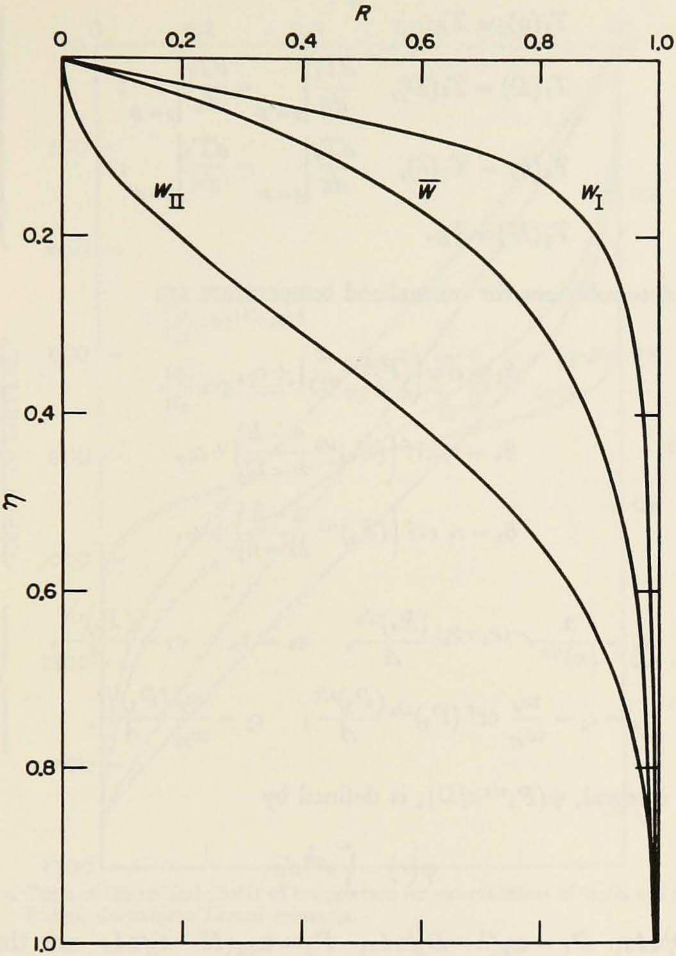

Figure 4. Form of the heat-flux ratio corresponding to w_I, w_{II}, and $\bar{w}$, for $\bar{P} = 5.42$ and $P_I = 0.5$. Divergent Ekman transport.

$$\left.\begin{aligned} w_3 &= -w_e \frac{z-h}{H-h}, \quad h \le z \le H; \\ k &= k_0 = \text{constant}; \end{aligned}\right\} \tag{17}$$

here the numerical subscripts on w refer to the indicated depth ranges. Since the velocity changes signs at $z = h$, it is generally not permissible to use a mean velocity for the present model.

Equations (2) and (17) yield solutions for each of the three layers subject to the boundary and matching conditions:

$$\left.\begin{aligned} &T_1(0) = T_S; \\ &T_1(D) = T_2(D), \quad \left.\frac{dT_1}{dz}\right|_{z=D} = \left.\frac{dT_2}{dz}\right|_{z=D}; \\ &T_2(h) = T_3(h), \quad \left.\frac{dT_2}{dz}\right|_{z=h} = \left.\frac{dT_3}{dz}\right|_{z=h}; \\ &T_3(H) = T_B. \end{aligned}\right\} \tag{18}$$

The complete solutions for normalized temperature are

$$\left.\begin{aligned} \theta_1 &= c_1\varphi\left((P_1)^{1/2}\frac{z}{D}\right) + c_2, \\ \theta_2 &= c_3 \operatorname{erf}\left((P_2)^{1/2}\frac{z-h}{h-D}\right) + c_4, \\ \theta_3 &= c_5 \operatorname{erf}\left((P_3)^{1/2}\frac{z-h}{H-h}\right) + c_6, \end{aligned}\right\} \tag{19}$$

where

$$\left.\begin{aligned} &c_1 = \frac{2}{(\pi)^{1/2}} e^{-(P_1+P_2)}\frac{(P_1)^{1/2}}{\Delta}, \quad c_2 = 1, \quad c_3 = \frac{(P_2)^{1/2}}{\Delta}, \\ &c_4 = c_6 = \frac{w_e}{w_H}\operatorname{erf}(P_3)^{1/2}\frac{(P_3)^{1/2}}{\Delta}, \quad c_5 = \frac{w_e}{w_H}\frac{(P_3)^{1/2}}{\Delta}. \end{aligned}\right\} \tag{20}$$

Dawson's integral, $\varphi(P_1{}^{1/2}z/D)$, is defined by

$$\varphi(r) = \int_0^r e^{\sigma^2}d\sigma.$$

$P_1 = w_e D/2k_0$, $P_2 = w_e(h-D)/2k_0$, $P_3 = w_H(H-h)/2k_0$ are the Péclet numbers for the three layers and Δ is given by

$$\Delta = (P_2)^{1/2}\operatorname{erf}(P_2)^{1/2} + \frac{w_e}{w_H}(P_3)^{1/2}\operatorname{erf}(P_3)^{1/2} + \frac{2}{(\pi)^{1/2}}e^{-(P_1+P_2)}(P_1)^{1/2}\varphi(P_1)^{1/2}.$$

For a given total depth and Ekman-layer thickness, the temperature distribution given by (19) is characterized by the three Péclet numbers P_1, P_2, and P_3. An equivalent but more convenient characterization is given by the quantities h, $(w_e/k_0)^{-1}$, and $(w_H/k_0)^{-1}$; i.e., the depth at which the vertical velocity passes through zero and the relative magnitudes of the length scales above and below this depth.

Fig. 5 shows temperature profiles for $h = 800$ m and various choices of w_e/k_0 and w_H/k_0. The value $D = 100$ m has been chosen for this and for

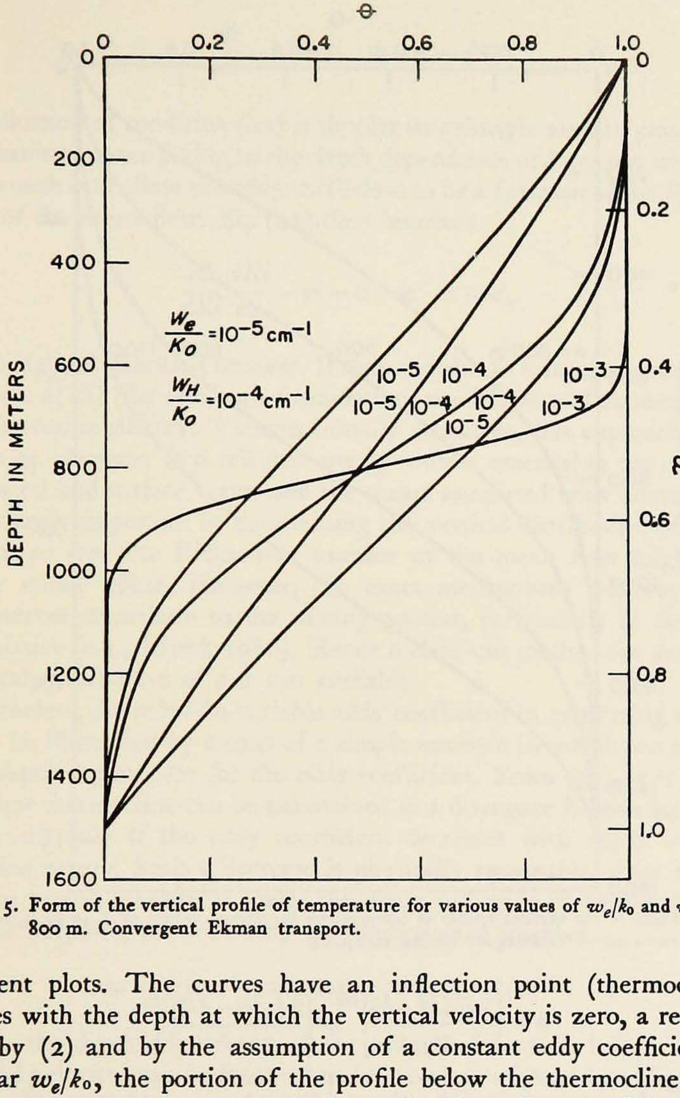

Figure 5. Form of the vertical profile of temperature for various values of w_e/k_0 and w_H/k_0, at $h =$ 800 m. Convergent Ekman transport.

subsequent plots. The curves have an inflection point (thermocline) that coincides with the depth at which the vertical velocity is zero, a result necessitated by (2) and by the assumption of a constant eddy coefficient. For a particular w_e/k_0, the portion of the profile below the thermocline exhibits a dependence upon w_H/k_0 that is similar to that for a divergent Ekman layer—stronger upward flow producing a colder bottom layer and thinner thermocline. For a given w_H/k_0, an increasing w_e/k_0 results in a warmer and more uniform upper layer and a thinner thermocline.

Fig. 6 shows temperature profiles corresponding to given values of w_e/k_0 and w_H/k_0 ($w_e/k_0 = w_H/k_0 = 10^{-4}\text{cm}^{-1}$) for various arbitrary choices of h.

Defining the thermocline thickness δ as before, it is apparent in the present case that the thickness must be measured in both directions from $z = h$. It follows from (19) that

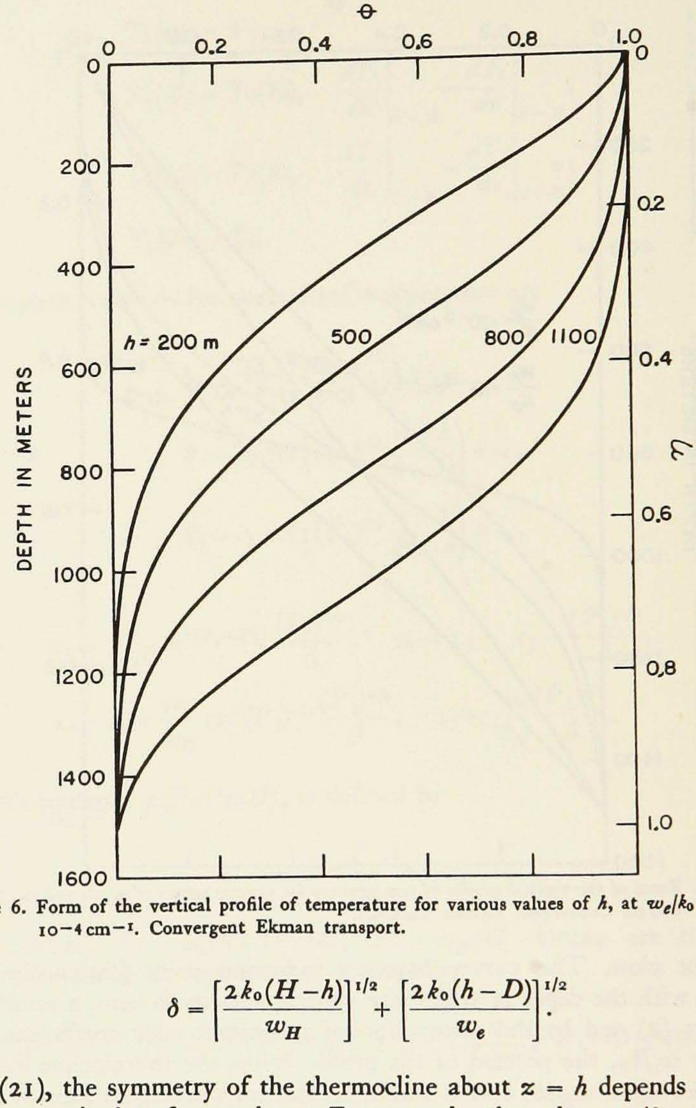

Figure 6. Form of the vertical profile of temperature for various values of h, at $w_e/k_0 = w_H/k_0 = 10^{-4}\,\mathrm{cm}^{-1}$. Convergent Ekman transport.

$$\delta = \left[\frac{2k_0(H-h)}{w_H}\right]^{1/2} + \left[\frac{2k_0(h-D)}{w_e}\right]^{1/2}. \tag{21}$$

From (21), the symmetry of the thermocline about $z = h$ depends upon the relative magnitudes of w_e and w_H. For example, the values $w_e/k_0 = w_H/k_0 = 10^{-4}\mathrm{cm}^{-1}$ and $h = 800$ m result in a symmetric thermocline whose depth is 800 m and whose thickness is 750 m.

(iii) The Variable Eddy Coefficient of Conductivity. In the two preceding sections it is seen that, for a constant eddy coefficient, the thermocline occurs wherever the vertical velocity is zero. If the eddy coefficient is allowed to be depth dependent, it is apparent from (2) that a more general criterion for the thermocline depth d is given by the relationship

$$\frac{dk}{dz} - w = 0 \quad \text{at} \quad z = d. \tag{22}$$

The application of condition (22) is simpler in principle than in practice, since the mechanisms contributing to the depth dependence of k are not well known. One approach is to allow the eddy coefficient to be a function of the Richardson number of the mean flow. Eq. (22) then becomes

$$\frac{dk}{dRi}\frac{dRi}{dz} - w = 0 \quad \text{at} \quad z = d,$$

where Ri is the Richardson number. It then remains to formulate the functional dependence $k(Ri)$ (for one such formulation, see Munk and Anderson 1948) and to determine dRi/dz. Though initially attractive, this approach does not appear to be adequate. It is felt that energy sources external to the mean flow, such as wind and surface waves and the shears associated with internal waves, are sufficiently important in determining the vertical distribution of the eddy coefficient so that the Richardson number of the mean flow might have a relatively minor effect. However, the exact mechanisms whereby external energy sources contribute to the mixing process, particularly in deep water, are speculative (e.g., Munk 1966). Hence a clear-cut method for determining the vertical distribution of k is not available.

Nevertheless, the role of a variable eddy coefficient in generating a thermocline can be illustrated by means of a simple example in which we assume an *a priori* depth dependence for the eddy coefficient. From (22), it is seen that a subsurface thermocline can be maintained in a divergent Ekman layer ($w < 0$ for all $z > 0$) only if the eddy coefficient decreases with depth within the thermocline region. Such a decrease is physically reasonable, since the upper part of the region involved is under the direct influence of wind mixing. For illustrative purposes, the following distributions are assumed:

$$w = -w_0 = \text{constant}, \qquad k = k_0 + k_1 e^{-z/s}. \tag{23}$$

Since only the depth dependence of the eddy coefficient is being considered, the vertical velocity may be assumed to be constant without loss of generality.

Substitution of (23) into (9) and use of the original depth variable give

$$\theta = \frac{\left[\dfrac{k(0)}{k(z)}\right]^{w_0 s/k_0} e^{-w_0 z/k_0} - \left[\dfrac{k(0)}{k(H)}\right]^{w_0 s/k_0} e^{-w_0 H/k_0}}{1 - \left[\dfrac{k(0)}{k(H)}\right]^{w_0 s/k_0} e^{-w_0 H/k_0}}. \tag{24}$$

Fig. 7 shows temperature profiles corresponding to various values of the reciprocal attenuation coefficient s. The values of the remaining parameters

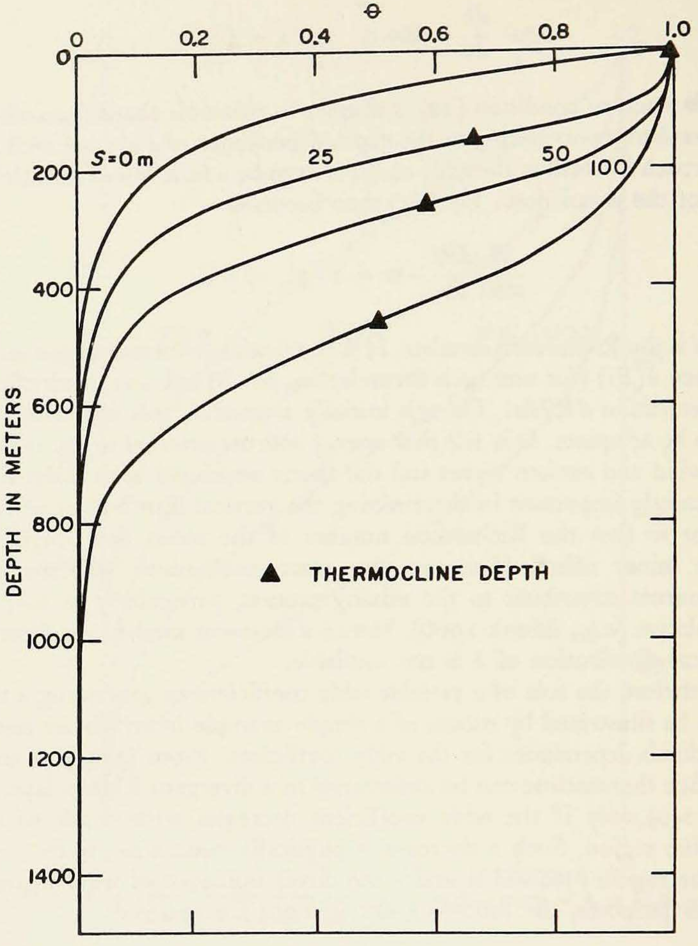

Figure 7. Form of the vertical profile of temperature for various values of the attenuation parameter s, for characteristic values of eddy conductivity $k_0 = 1$ cm² sec⁻¹ and $k_1 = 100$ cm² sec⁻¹, and vertical velocity $w_0 = 10^{-4}$ cm sec⁻¹. Divergent Ekman transport.

have been arbitrarily chosen to be $k_0 = 1\ \mathrm{cm^2\ sec^{-1}}$, $k_1 = 100\ \mathrm{cm^2\ sec^{-1}}$, and $w_0 = 10^{-4}\ \mathrm{cm\ sec^{-1}}$. The profile corresponding to $s = 0$ represents the case of a constant eddy coefficient and is similar to those in Fig. 1. For $s > 0$, the curves are characterized by the presence of a mixed layer that thickens as s is increased. The thermocline depth, determined by (22) and (24), is indicated on each of the profiles in Fig. 7.

4. *Comparison with Observations.* To demonstrate the applicability of the model to the real ocean, comparison is made with data given by Austin (1957)

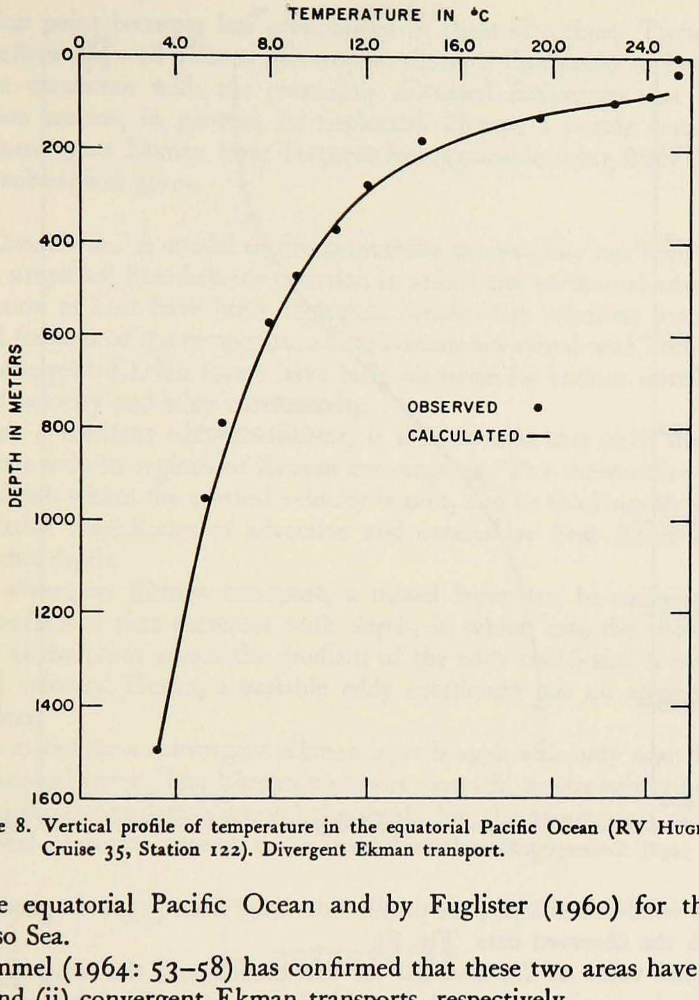

Figure 8. Vertical profile of temperature in the equatorial Pacific Ocean (RV HUGH M. SMITH, Cruise 35, Station 122). Divergent Ekman transport.

for the equatorial Pacific Ocean and by Fuglister (1960) for the central Sargasso Sea.

Stommel (1964: 53–58) has confirmed that these two areas have (i) divergent and (ii) convergent Ekman transports, respectively.

(i) DIVERGENT EKMAN TRANSPORT. Comparison of the general shape of Austin's (1957) observed temperature profile with the profiles in Fig. 2 suggests that the vertical velocity decreases with depth below the Ekman layer. In the light of this observation, the velocity profile w_I of (12) has been chosen for the model. The eddy coefficient is assumed to be constant. For $w_e/k_0 = 1.25 \times 10^{-4}\,\text{cm}^{-1}$ and $D = 40$ m, $P_1 = 0.25$; the solution given by (13) becomes

$$\theta_I = 1 - \frac{ln\left(\frac{H}{D}\eta + \left[\left(\frac{H}{D}\eta\right)^2 + 1\right]^{1/2}\right)}{ln\left(\frac{H}{D} + \left[\left(\frac{H}{D}\right)^2 + 1\right]^{1/2}\right)}. \tag{25}$$

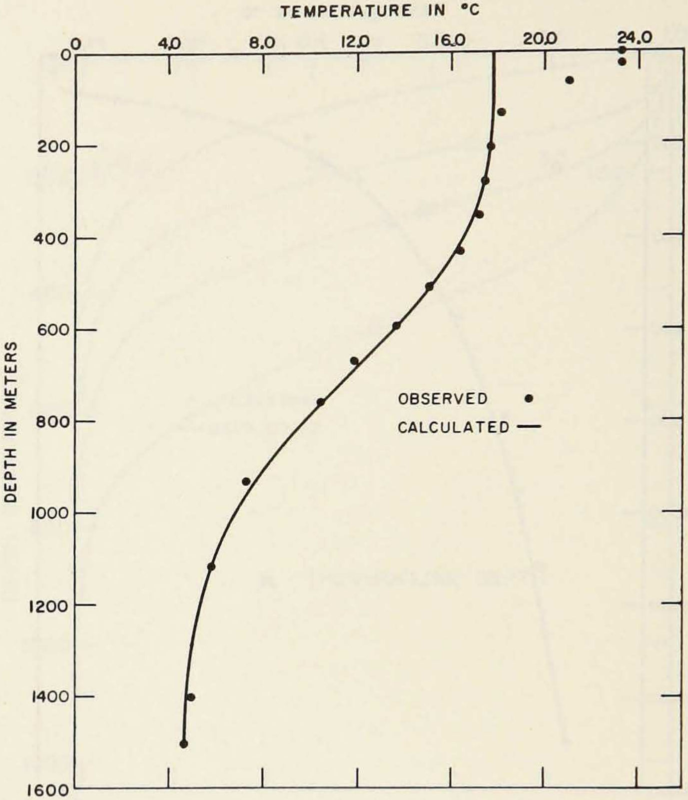

Figure 9. Vertical profile of temperature in the Sargasso Sea (RV Atlantis, Cruise 299, Station 5448). Convergent Ekman transport.

Below the wind-mixed layer, the profile calculated from (25) is in good agreement with the observed data (Fig. 8).

The small value of D, assumed for the model, and the relatively rapid decrease in the vertical velocity below this depth, dictated by the form of w_I in (12), are consistent with the suggestion of Cromwell (1953, 1958) that the upwelling in the equatorial Pacific is a shallow process.

(ii) Convergent Ekman Transport. In Fuglister's (1960) data, the position of the inflection point and the slight asymmetry of the thermocline thickness about this point suggest that $h = 650$ m and that $(w_e/k_0) > (w_H/k_0)$. Fig. 9 shows that the profile calculated from (19) for $w_e/k_0 = 1.25 \times 10^{-4}\text{cm}^{-1}$, $w_H/k_0 = 0.8 \times 10^{-4}\text{cm}^{-1}$, and $h = 650$ m agrees well with the data in the range 200–1500 m.

Although available data indicate a strong convergence of the Ekman transport southward and eastward of the center of the Sargasso Sea, observed temperature distributions show that the thermocline depth decreases while the inflection point becomes less pronounced in these directions. These observations reflect the well-defined geostrophic currents that occur in these regions and are consistent with the previously discussed limitations that horizontal advection cannot, in general, be neglected. Hence, a purely vertical model for a convergent Ekman layer becomes less applicable away from the centers of the subtropical gyres.

5. *Conclusions.* A model of the permanent thermocline has been developed from a simplified heat-balance equation in which the horizontal advection and conduction of heat have been neglected. Steady-state solutions describing the general features of the temperature distributions associated with both divergent and convergent Ekman layers have been obtained for various distributions of vertical velocity and eddy conductivity.

Given a constant eddy coefficient, it is concluded that deep thermoclines can occur only in regions of Ekman convergence. The thermocline is located at the depth where the vertical velocity is zero, and its thickness depends upon the relative magnitudes of advective and conductive heat fluxes above and below this depth.

For divergent Ekman transport, a mixed layer can be modeled with an eddy coefficient that decreases with depth, in which case the thermocline is located at the point where the gradient of the eddy coefficient is equal to the vertical velocity. Hence, a variable eddy coefficient has an apparent advective effect.

The model for a convergent Ekman layer is applicable only near the centers of subtropical gyres. The Ekman transport may still be strongly convergent in the peripheral circulation around the gyres, but the assumption of negligible horizontal advection appears to be invalid in these regions.

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