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Some Measurements of the Directional Wave Spectrum[1]

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ABSTRACT

A sequence of ten records of the directional wave spectrum has been obtained from the motions of a floating buoy located at 56°45′N, 18°57′W. The records were taken to check the results derived from a numerical scheme for forecasting ocean waves in the North Atlantic. The buoy was situated about midway between ocean weather stations India and Juliett so that wave records taken with wave recorders on the weather ships at these stations would provide information on the spatial variation in the wave spectrum, thus providing a further check on the numerical scheme. The wave records, obtained from 22 to 25 June 1967, covered a period of about 2.5 days. During the first half of this period, the observed swell waves were measured when the local wind was very light, but during the last half of the recording period, local winds of about 17 knots prevailed. Under these cirsumstances, the exponential wave growth due to the undulatory turbulent flow has been estimated as $\mu = 9 \times 10^{-5}$ at $c/u_* = 30$ (notation after Phillips 1966). The high-frequency end of the wave elevation and slope spectra follows Phillips' (1958) equilibrium law.

Introduction. At present a numerical scheme for forecasting ocean waves in the North Atlantic ocean is being developed. To check results derived from this scheme, records of the directional wave spectrum and its spatial variation

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Table I. Data concerning wave records.

Record	Date (1967)	Time (GMT)	Wind speed (kts) and direction	Variance of wave elevation (m^2)	Significant wave height (m)	Variance of absolute wave slope ($deg.^2$)
1	22-VI	2140	<5; 050°	0.173	1.7	6.1
2	23-VI	0340	0; –	0.171	1.7	6.3
3	„	1010	5–10; 050°	0.097	1.2	4.0
4	„	1540	0; –	0.101	1.3	3.7
5	„	2140	0; –	0.095	1.2	2.5
6	24-VI	0950	16–18; 020°	0.070	1.1	4.6
7	„	1550	18–20; 020°	0.096	1.2	9.3
8	„	2200	16–20; 030°	0.138	1.5	10.6
9	25-VI	0350	14–18; 030°	0.139	1.5	10.3
10	„	0950	14–18; 035°	0.165	1.6	13.0

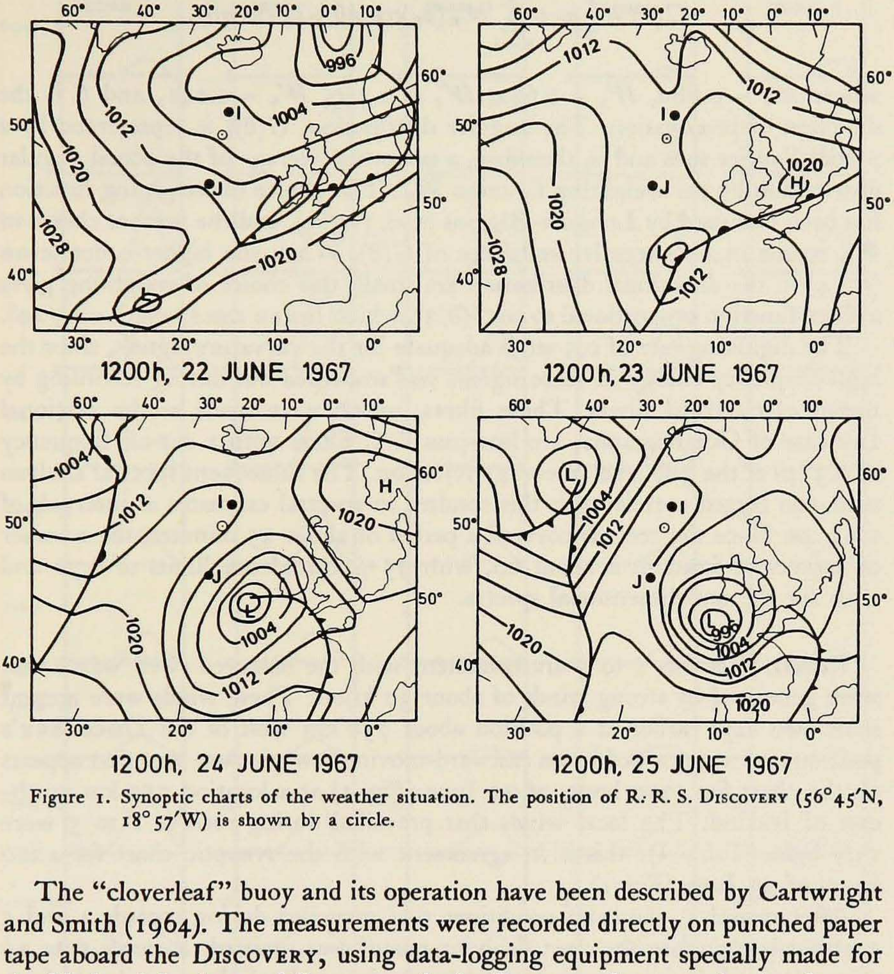
have been obtained. Ten records of the directional wave spectrum have been recorded from the motions of the "cloverleaf" buoy of the National Institute of Oceanography, using the R.R.S. DISCOVERY, which was stationed nearby at 56°45′N, 18°57′W. The buoy was located almost midway between ocean weather stations India and Juliett, which are separated by a distance of 720 km. The weather ships at these stations provided data from the NIO shipborne wave recorder that would give information on the spatial variation of the spectrum. The observations—from 22 to 25 June 1967—covered a period of about 2.5 days.

The buoy can be used to determine the angular distribution of energy in each frequency band of the directional wave spectrum from measurements of the slopes and curvatures of the wave surface together with the vertical acceleration of the buoy.

Since the wave records consist of measurements of swell (records 1 to 5) and then of waves developing under fairly steady wind conditions in a direction opposite to that of the swell (records 6 to 10), it was considered possible to obtain estimates of the wave growth parameters from measurements of the spectrum.

Although the scheme for forecasting ocean waves is still under development, the results derived from the above measurements are considered to be of sufficient interest to be published at this time.

Data and Method of Analysis. Table I gives the dates and times of the observations together with the wind speeds recorded from the DISCOVERY's anemometer (19.5 m above the water surface) as well as the wind direction, which was estimated visually. The ten records were separated in time by about 6 hours, excepting records 5 and 6, which were separated by 12 hours. Fig. 1 shows synoptic charts of the weather situation at noon on each of the four days of observation (22 to 25 June).



1200h, 22 JUNE 1967

1200h, 23 JUNE 1967

1200h, 24 JUNE 1967

1200h, 25 JUNE 1967

Figure 1. Synoptic charts of the weather situation. The position of R. R. S. Discovery (56°45′N, 18°57′W) is shown by a circle.

The "cloverleaf" buoy and its operation have been described by Cartwright and Smith (1964). The measurements were recorded directly on punched paper tape aboard the Discovery, using data-logging equipment specially made for the National Institute of Oceanography by Dynamco Systems; the signals were recorded simultaneously at 0.5-sec intervals. The wave data taken on the weather ships at stations India and Juliett are not included in this paper but will presented later when results from the numerical wave-forecasting scheme are available.

The method of analysis is an extension of that originally applied by Longuet-Higgins et al. (1963) to a buoy that measures vertical acceleration and wave slopes only. The present method differs from that of the above authors in that it includes measurements of the curvatures of the wave surface. By measuring the curvatures as well as the wave slopes (Cartwright and Smith 1964), it is possible to estimate the first nine angular harmonics a_n, b_n of the directional distribution of the form

$$G(\theta) = \frac{1}{2} a_0 + \sum_{n=1}^{4} W_n (a_n \cos n\theta + b_n \sin n\theta),$$

where $W_1 = 0.889$, $W_2 = 0.622$, $W_3 = 0.339$, $W_4 = 0.141$, and θ is the direction of propagation. The angular distribution, $G(\theta)$, is represented by a partial Fourier sum and is, therefore, a smoothed average of the actual angular distribution by the weighting function W_n. The choice of weighting function has been discussed by Longuet-Higgins et al. (1963), and the present choice of W_n results in non-negative estimates of $G(\theta)$. When the higher-order terms ($n > 4$) in the directional distribution are small, this choice of weighting gives a filter function proportional to $\cos^{16}(\theta/2)$, which has an r.m.s. width of ±29°.

The digitizing rate of 0.5 sec is adequate for the curvature signals, since the high-frequency energy in these signals was smoothed out during recording by means of electrical filters. These filters, which were made at the National Institute of Oceanography, are low-pass R-C filters with a cut-off frequency of 0.3 cps at the 3db level of energy rejection. The subsequent spectral analysis used 100 lagged correlations; this resulted in spectral estimates at intervals of 0.01 cps. Since the records covered a period of about 25 minutes, the number of degrees of freedom is about 60, with 95 % confidence limits of 0.72 and 1.49 for the one-dimensional spectra.

Results. Records 1 to 5 are consistent with the observed swell waves that were generated by strong winds of about 30 knots. These winds were present some two days earlier at a position about 750 km west of the DISCOVERY's position and were caused by an eastward-moving low-pressure area that appears on the chart for 1200 hours of 22 June (Fig. 1) at a location 750 km northeast of Iceland. The local winds that prevailed during records 1 to 5 were very light (Table I); this is in agreement with the synoptic chart for 1200 hours of 23 June (Fig. 1).

After record 5, the calm conditions that prevailed during records 4 and 5 continued; therefore the next six-hour record was omitted. Records 6 to 10 were obtained during a period of fairly steady winds of about 17 knots from 030°; these winds originated from a low-pressure area about 370 km southwest of Ireland.

Table I gives the variance of the wave elevation together with the significant wave height, defined as $4 \times$ (variance of wave elevation)$^{1/2}$. Also shown is the variance of the absolute wave slope, defined as the sum of the pitch and roll variances of the buoy.

Figs. 2 and 3 show the main set of results for the smoothed estimates of the directional wave spectrum, $E(f,\theta)$, as a function of the wave frequency, f(cps), and the direction, θ (referred to north). Fig. 2 shows measurements of the swell waves that were present in records 1 to 5, when the local wind was very light. The predominant wave direction is nearly the same for all frequency

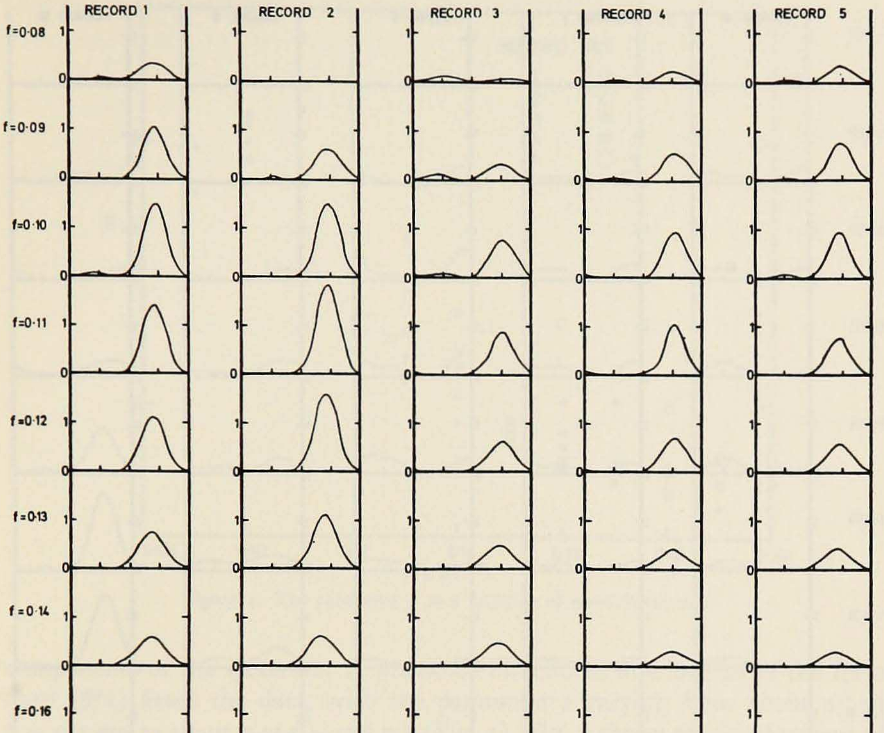


Figure 2. Smoothed estimates of the directional wave spectrum for records 1 to 5. The vertical scale is in units of m² sec. There is a change in scale to 0.4 m² sec, for the maximum vertical-scale value for frequencies greater than 0.16 cps.

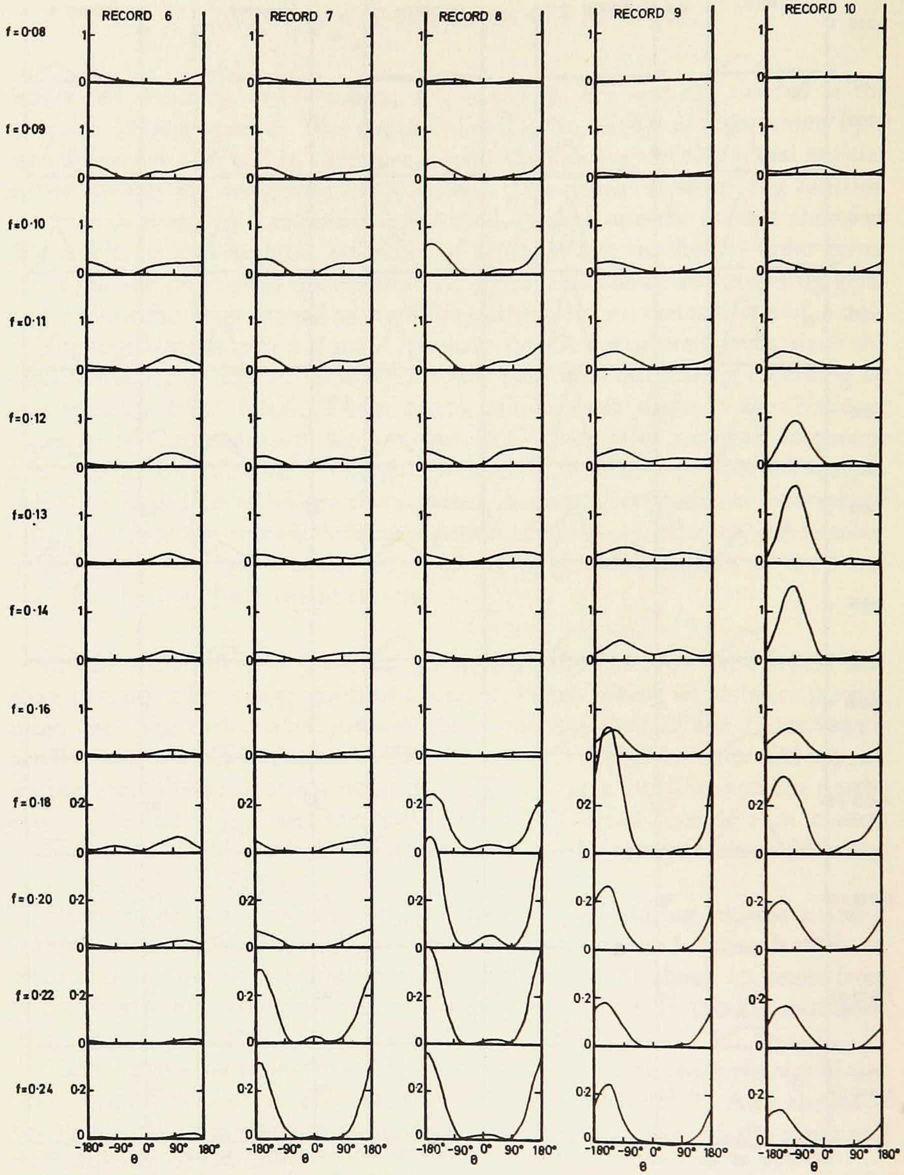


Figure 3. Smoothed estimates of the directional wave spectrum for records 6 to 10. The vertical scale is in units of m² sec. There is a change in scale to 0.4 m² sec for the maximum vertical-scale value for frequencies greater than 0.16 cps.

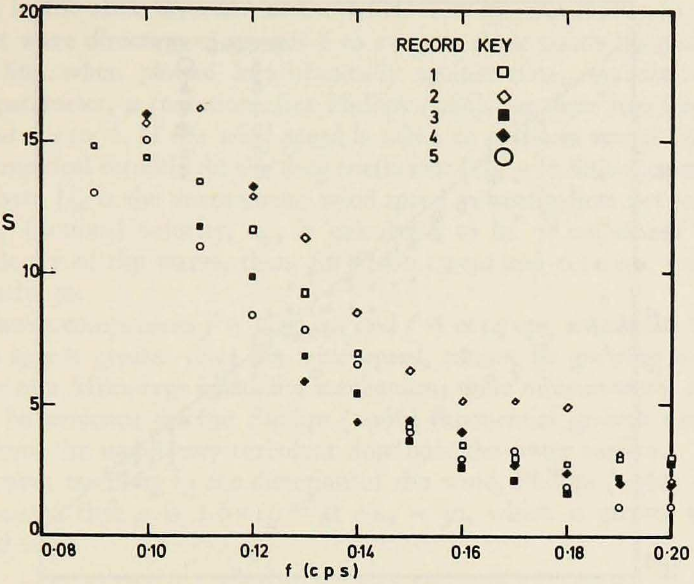


Figure 4. The parameter S as a function of wave frequency.

components of the spectrum. A unimodal directional distribution of the form $\cos^{2s}(\theta/2)$ fitted the data, with the parameter s varying from about 15 at $f = 0.1$ cps to about 2 at $f = 0.2$ cps (Fig. 4). Fig. 3 shows the development of the directional wave spectrum caused by winds of about 17 knots from a direction of 030°. The higher-frequency wave components ($f > 0.16$ cps) grew very rapidly during the interval between records 6 to 8; the subsequent slow decrease is attributed to a decrease in wind speed and a small change in wind direction during records 9 and 10.

Fig. 5 shows, in a double logarithmic plot for records 5 to 10, the spectra of wave elevation, $E(f)$, for frequencies greater than 0.2 cps. The approach to a state of equilibrium, defined by a $(\text{frequency})^{-5}$ power law (Phillips 1958), is well illustrated by the data of records 5, 6, and 7. The straight line corresponding to the relation $E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \ldots \text{m}^2\,\text{sec}$, with $\alpha = 8.1 \times 10^{-3}$ (Pierson and Moskowitz 1964), fits the data fairly well at high frequencies.

Fig. 6 shows the spectra of absolute wave slope, $C_{22} + C_{33}$ (in the notation of Longuet-Higgins et al. 1963); the straight line corresponds to the relation $10^{-2} \times f^{-1} \ldots \text{rad}^2\,\text{sec}$. The spectra appear to follow a $(\text{frequency})^{-1}$ law at high frequencies in accordance with Phillips' equilibrium spectrum.

Discussion. It was possible to derive fairly convincing estimates for exponential growth of the wave spectrum at only two frequencies—$f = 0.13$ cps and 0.14 cps—near the peak of the wave spectrum and when the wave direc-

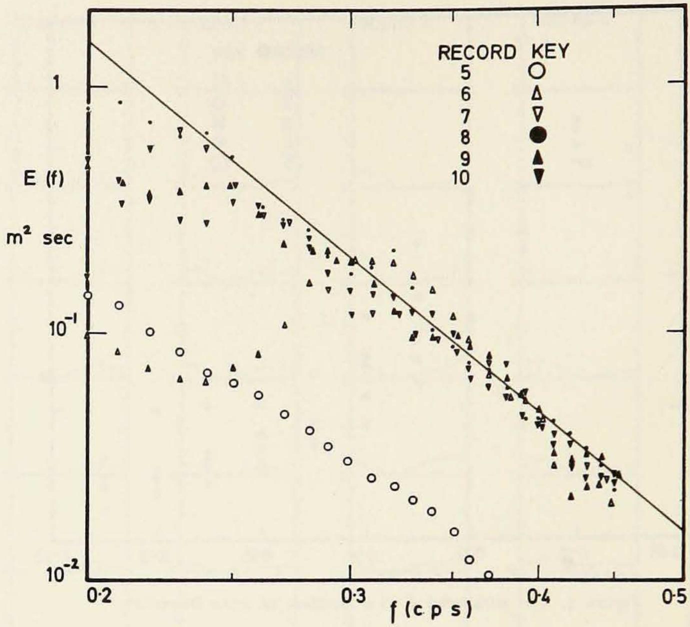


Figure 5. Frequency spectra of wave elevation.

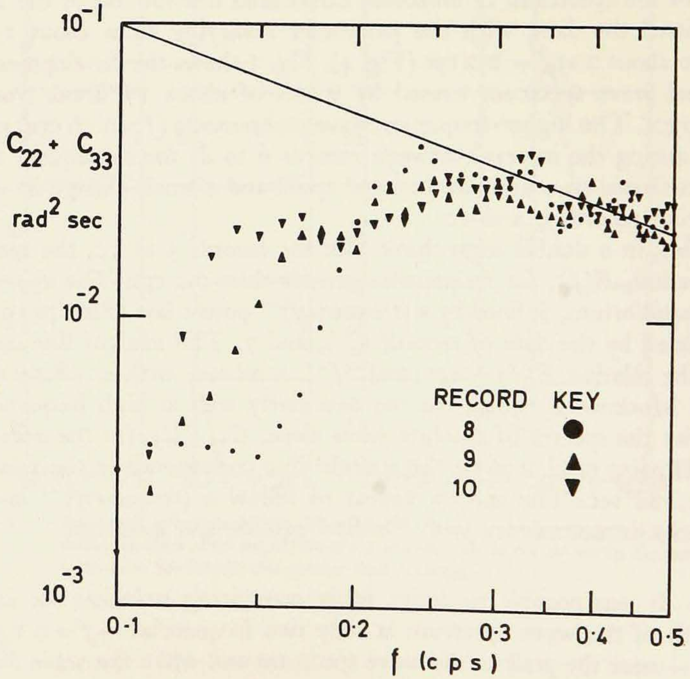


Figure 6. Frequency spectra of absolute wave slope.

tion was in the same direction as the wind. The spectral energy in the predominant wave direction of records 8 to 10 gave three points lying close to a straight line when plotted logarithmically against time; the dimensionless growth parameter, μ (notation after Phillips 1966), for these two frequencies was about 9×10^{-5}. If the wind speed is taken as 17 knots and if Sheppard's (1958) empirical formula for the drag coefficient [$C_D = (0.80 + 0.00114\, U_a) \times 10^{-3}$, where U_a is the anemometer wind speed in centimeters per second] is used, the frictional velocity, u_*, is calculated to be 37 cm/sec. If c is the phase velocity of the waves, then, for $f = 0.13$ cps and 0.14 cps, c/u_* is approximately 30.

The wave components $f = 0.13$ cps and $f = 0.14$ cps, which are traveling at phase speeds greater than the wind speed, cannot be growing under the influence of a Miles-type instability mechanism; these measurements therefore seem to be evidence for the Phillips (1966) exponential growth mechanism arising from the undulatory turbulent flow near the water surface.

For waves traveling in the direction of the wind, Phillips (1966: fig. 4.6) has calculated that μ is 1.6×10^{-4} at $c/u_* = 30$, which is greater than the measured value.

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REFERENCES

CARTWRIGHT, D. E., and N. D. SMITH
1964. Buoy techniques for obtaining directional wave spectra. Buoy Technology Symposium, Washington, D.C., 112–121. Published by Marine Technology Society; 504 pp.

LONGUET-HIGGINS, M. S., D. E. CARTWRIGHT, and N. D. SMITH
1963. Observations of the directional spectrum of sea waves using the motions of a floating buoy; *In* Ocean wave spectra, pp. 111–132. Prentice Hall, Inc., New Jersey. 557 pp.

PHILLIPS, O. M.
1958. The equilibrium range in the spectrum of wind generated waves. J. fluid Mech., *4*: 426–434.
1966. Dynamics of the upper ocean. Cambridge University Press; 261 pp.

PIERSON, W. J., and L. I. MOSKOWITZ
1964. A proposed spectral form for fully-developed wind seas based on the similarity theory of S. A. Kitaigorodskii. J. geophys. Res., *69*: 5181–5190.

SHEPPARD, P. A.
1958. Transfer across the earths' surface and through the air above. Quart. J. R. met. Soc., *84*: 205–224.