Effects of Deep-sea Stratification and Current on Edgewaves[1]

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ABSTRACT

The effects of deep-sea density stratification and longshore current on trapped edgewaves traveling over a sloping continental shelf that drops off vertically to deep water of constant depth are investigated. The stratification is idealized with a two-layer model, and the current is assumed to be confined to the deep-sea region and to the upper layer of fluid. It is shown that (i) the influence of the current on the wave modes is small, (ii) each eigenwave on the shelf is coupled to two waves in the deep-sea region, one of which travels parallel to the coast (a barotropic wave), the other toward the coast (a baroclinic wave), and (iii) the waves in each region are damped, the e-folding time being much greater than the wave period.

1. *Introduction.* The purpose of this paper is to report an investigation of the effects of two oceanic phenomena on trapped edgewaves (Munk et al. 1956) that propagate over a gently sloping continental shelf of finite width. The phenomena considered are deep-sea density stratification and a deep-sea current that flows alongside the shelf. This work has been motivated by an earlier study (Mysak 1967), which showed that both of these phenomena significantly affect the phase velocities of continental shelf waves (Robinson 1964).

As in Mysak (1967), a two-layer model for the stratification is considered here, and it is assumed that the deep-sea current is confined to the upper layer of fluid (Fig. 1). For geophysically realistic values of the quantities involved, it is shown in § 3 that the influence of the current on the wave modes is small. In § 4 it is shown that, while density stratification does not significantly affect the phase velocities of the waves, its presence does imply that the waves decay slowly with time. In § 4 it is also shown that each eigenwave on the shelf is weakly coupled to two waves in the deep-sea region, one

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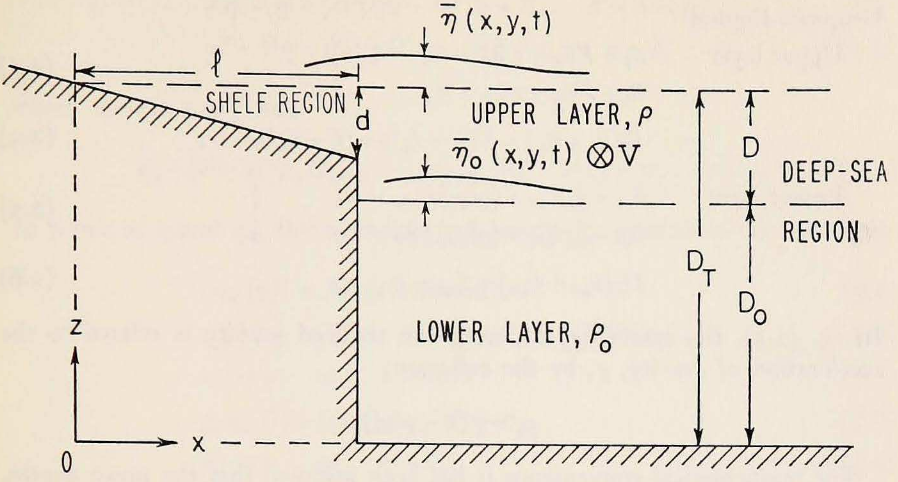

Figure 1. Cross-section diagram of the model used in the present theory. The longshore coordinate, y, increases into the paper, and the basic state uniform flow, V, is in the positive y direction and is confined to the deep-sea region and to the upper layer. Geophysically realistic values of the quantities shown are $V = 200$ cm/sec, $l = 10^7$ cm, $d = 2 \times 10^4$ cm, $D = 3 \times 10^4$ cm, $D_0 = 4.7 \times 10^5$ cm, and $(\varrho_0 - \varrho)/\varrho_0 = 2.5 \times 10^{-3}$.

of which propagates parallel to the coast (a barotropic wave), the other toward the coast (a baroclinic wave).

2. *Equations of Model.* Consider a semi-infinite ocean basin that is bounded on one side by a straight coastline and has a bottom topography consisting of a linearly sloping shelf that drops off vertically to deep water of constant depth (Fig. 1). For the basic state of motion of the fluid in the basin, it is supposed that there exists a uniform flow that is confined to the deep-sea region and to an upper layer of fluid of constant density; below this upper layer lies a motionless layer of fluid of slightly greater density. Upon this basic state is imposed a small-amplitude surface edgewave motion that in turn induces an internal wave motion.

Assume that there exists a hydrostatic balance of forces in the vertical direction and that all the unknown quantities are independent of depth. Then, in terms of the notation shown in Fig. 1, the unforced linearized nondissipative equations for the sea level and interfacial distortions, $\bar{\eta}$ and $\bar{\eta}_0$, and the horizontal velocity components of the upper and lower layers, $(\bar{u}, \bar{v})$ and $(\bar{u}_0, \bar{v}_0)$, are given by:

Shelf Region:
$$\left.\begin{aligned} \bar{u}_t + g\bar{\eta}_x &= 0, \\ \bar{v}_t + g\bar{\eta}_y &= 0, \end{aligned}\right\} \tag{2.1}$$

$$[(dx/l)\bar{u}]_x + (dx/l)\bar{v}_y + \bar{\eta}_t = O; \tag{2.2}$$

Deep-sea Region:

Upper layer
$$\left.\begin{aligned} &\bar{u}_t + V\bar{u}_y + g\bar{\eta}_x = 0, \\ &\bar{v}_t + V\bar{v}_y + g\bar{\eta}_y = 0, \end{aligned}\right\} \tag{2.3}$$

$$D(\bar{u}_x + \bar{v}_y) + V(\bar{\eta} - \bar{\eta}_0)_y + (\bar{\eta} - \bar{\eta}_0)_t = 0; \tag{2.4}$$

Lower layer
$$\left.\begin{aligned} &\bar{u}_{0t} + g_0\bar{\eta}_{0x} + (\varrho/\varrho_0)g\bar{\eta}_x = 0, \\ &\bar{v}_{0t} + g_0\bar{\eta}_{0y} + (\varrho/\varrho_0)g\bar{\eta}_y = 0, \end{aligned}\right\} \tag{2.5}$$

$$D_0(\bar{u}_{0x} + \bar{v}_{0y}) + \bar{\eta}_{0t} = 0. \tag{2.6}$$

In eq. (2.5), the symbol g_0 denoting the reduced gravity is related to the acceleration of gravity, g, by the equation:

$$g_0 = g(1 - \varrho/\varrho_0).$$

For mathematical convenience it has been assumed that the mean depths, D and D_0, are constants. An interesting but more difficult problem, which is not discussed here, arises if the mean interface slopes downward in the seaward direction, corresponding to a current that is baroclinic and geostrophic in character. Also, note that the Coriolis force (effect of rotation) has been neglected. For the class of waves and geometry under consideration, the rotational effects are very small. In essence, rotation gives rise to a small splitting of the frequencies of the edgewave modes (Mysak 1968). That is to say, without rotation the dispersion relationship is of the form $\omega \propto \pm k^{1/2}$ whereas with uniform rotation $\omega \propto - k^{1/2} - \beta f F_1(k)$ and $\omega \propto k^{1/2} - \beta f F_2(k)$; here ω is the wave frequency, k is the longshore wave number, f is the Coriolis parameter, β is a small positive dimensionless parameter much less that unity, and $k^{-1/2} f F_i(k) = O(1)$ $(i = 1,2)$.

To uniquely determine the quasisteady trapped-edgewave solution for the system (2.1) – (2.6), the following boundary and continuity conditions are adopted. At the coast $\bar{\eta}$ must be well behaved; far from the coast $\bar{\eta}$ and $\bar{\eta}_0$ must tend to zero. At the edge of the shelf, $\bar{\eta}$ and the normal transport component must be continuous in the upper layer and the normal velocity component must vanish in the lower layer.

3. *Derivation of Dispersion Relationship.* In accordance with the discussion in § 2, we look for solutions of the form

$$(\bar{\eta}, \bar{u}, \bar{v}, \bar{\eta}_0, \bar{u}_0, \bar{v}_0) = (\eta, u, v, \eta_0, u_0, v_0) \exp\left[i(ky + \omega t)\right], \tag{3.1}$$

where k is assumed to be real and positive and η, u, ... are all functions of x alone. In this section an implicit functional relationship between ω and k for the problem posed in § 2 is determined. Eqs. (3.1) and (2.1) – (2.6) imply that η and η_0 satisfy

$$x\eta'' + \eta' + (\omega^2 l/gd - xk^2)\eta = 0, \quad 0 < x < l, \tag{3.2}$$

$$\eta'' - k^2\eta + (\hat{\omega}^2/gD)(\eta - \eta_0) = 0, \quad x > l, \tag{3.3}$$

where $\hat{\omega} = \omega + kV$, and

$$\eta_0'' - (k^2 - \omega^2/g_0 D_0)\eta_0 + (\varrho g/\varrho_0 g_0)(\eta'' - k^2\eta) = 0, \quad x > l. \tag{3.4}$$

In terms of η and η_0, the boundary and continuity conditions take the form

$$|\eta_<(0)| < M \text{ (a constant)}, \tag{3.5}$$

$$\eta_<(l) = \eta_>(l), \quad (d/\omega)\eta_<'(l) = (D/\hat{\omega})\eta_>'(l), \tag{3.6}$$

$$\eta_> \to 0 \quad \text{as} \quad x \to \infty, \tag{3.7}$$

$$g_0\eta_0'(l) + (\varrho/\varrho_0)g\eta_>'(l) = 0, \tag{3.8}$$

$$\eta_0 \to 0 \quad \text{as} \quad x \to \infty. \tag{3.9}$$

In (3.5) – (3.8), the subscript symbols $<$ and $>$ denote the solutions in the regions $0 < x < l$ and $x > l$, respectively.

Before the solution to (3.2) – (3.9) is determined, the relative importance of the terms arising from the deep-sea current must be examined. For $\omega = o(10^{-3}\,\text{sec}^{-1})$, $k = o(10^{-7}\,\text{cm}^{-1})$ [typical orders of magnitude of the frequency and wave number associated with trapped edgewaves (Mysak 1968)] and for $V = 2 \times 10^2\,\text{cm sec}^{-1}$ (a realistic upper bound for the surface velocity of the most intense deep-sea boundary currents), note that $\hat{\omega} = \omega$ to within $o(10^{-2})$. That is to say, the influence of the deep-sea current is, for all practical purposes, negligible. Henceforth, $\hat{\omega}$ in eqs. (3.3) and (3.6) will be approximated by ω.

The solution to (3.2) and (3.5) is given by

$$\eta = A_s \exp(-kx)\, L_\nu(2kx), \quad 0 < x < l, \tag{3.10}$$

where A_s is a constant, L_ν is the Laguerre function (Pinney 1946), and $2\nu + 1 = \omega^2 l/gd$. In (3.3) (with $\hat{\omega}$ replaced by ω) and (3.4), let $\chi = kx$; then, after some rearrangement, these become

$$\begin{aligned} &d^2\eta/d\chi^2 + (\theta^2 - 1)\eta - \theta^2\eta_0 = 0, \quad \chi > kl, \\ &(1 - \varepsilon^2)(d^2\eta/d\chi^2 - \eta) + \varepsilon^2 d^2\eta_0/d\chi^2 + (\theta^2\delta - \varepsilon^2)\eta_0 = 0, \quad \chi > kl, \end{aligned} \tag{3.11}$$

where $\theta^2 = \omega^2/k^2 gD$, $\delta = D/D_0$, and $\varepsilon^2 = 1 - \varrho/\varrho_0$. The solutions to (3.11), (3.7), and (3.9) can be written in the form

$$\begin{Bmatrix} \eta_j \\ \eta_{0j} \end{Bmatrix} = \begin{Bmatrix} A_j \\ A_{0j} \end{Bmatrix} \exp[-r_j(\chi - kl)] \quad (j = 1,2), \tag{3.12}$$

where $\mathrm{Re}(r_j) > 0$ and A_j, A_{0j} are constants. From (3.11) and (3.12) it follows that nontrivial solutions exist provided

$$r_j^2 \equiv r_\pm^2 = \{-\theta^2(1+\delta) + 2\varepsilon^2 \pm \theta^2(1+\delta)[1 - 4\varepsilon^2\delta/(1+\delta)^2]^{1/2}\}/2\varepsilon. \quad (3.13)$$

Henceforth, the notation $r_1^2 = r_+^2$ and $r_2^2 = r_-^2$ is adopted. Thus, the solutions for η and η_0 take the form

$$\eta = \begin{cases} A_s \exp(-kx) L_\nu(2kx), & 0 < x < l, \\ A_1 \exp[-r_1 k(x-l)] + A_2 \exp[-r_2 k(x-l)], & x > l, \end{cases} \quad (3.14)$$

$$\begin{aligned} \eta_0 = {} & [(r_1^2 + \theta^2 - 1)/\theta^2] A_1 \exp[-r_1 k(x-l)] \\ & + [(r_2^2 + \theta^2 - 1)/\theta^2] A_2 \exp[-r_2 k(x-l)], \quad x > l, \end{aligned} \quad (3.15)$$

where the first equation in (3.11) has been used to determine the relationship between A_j and A_{0j}. Upon applying the conditions (3.6) and (3.8) to (3.14) and (3.15), three homogeneous equations for A_s, A_1, and A_2 are obtained. If the determinant of coefficients is set equal to zero, the (implicit) dispersion relationship for the problem is obtained:

$$\begin{aligned} & (\varepsilon^2/\theta^2)(r_2^2 - r_1^2) r_1 r_2 L_\nu(2\lambda) + \Delta[2L_\nu'(2\lambda) - L_\nu(2\lambda)] \\ & \{r_2[1 + \varepsilon^2(r_2^2 - 1)/\theta^2] - r_1[1 + \varepsilon^2(r_1^2 - 1)/\theta^2]\} = 0, \end{aligned} \quad (3.17)$$

where $\lambda = kl$ and $\Delta = d/D$.

4. *Discussion of Solution.* In order to exhibit the details of the formal solution derived in § 3, we now exploit the fact that $\theta^2 = O(1)$, $\Delta = O(1)$, $0 < \delta \ll 1$, $0 < \varepsilon \ll 1$, and $\delta/\varepsilon = O(1)$ for $\omega = O(10^{-3}\,\mathrm{sec}^{-1})$, $k = O(10^{-7}\,\mathrm{cm}^{-1})$, $d = 2 \times 10^4$ cm, $D = 3 \times 10^4$ cm, $D_0 = 4.7 \times 10^5$ cm, $g = 9.8 \times 10^2\,\mathrm{cm\,sec}^{-2}$, and $(\varrho_0 - \varrho)/\varrho_0 = 2.5 \times 10^{-3}$. Under these conditions, (3.13) implies that

$$\left.\begin{aligned} r_1^2 &= 1 - \theta^2\delta + O(\delta^2), \\ r_2^2 &= -(\theta^2/\varepsilon^2)[1 + \delta + O(\varepsilon^2)]. \end{aligned}\right\} \quad (4.1)$$

Application of the condition $\mathrm{Re}(r_j) > 0$ to (4.1) yields

$$r_1 = 1 - \theta^2\delta/2 + O(\delta^2), \quad (4.2)$$

$$r_2 = -(i\theta/\varepsilon)[1 + \delta/2 + O(\varepsilon^2)], \quad (4.3)$$

where, in the parameter θ, the frequency has the form $\omega = \omega_1 + i\omega_2$, with $\omega_2 > 0$ so that the waves decay with increasing time. If it is assumed a priori that ω is real and the Sommerfeld radition condition is subsequently applied as the boundary condition far from the coast, a complex dispersion relationship is obtained. Hence, for the frequency and wave-number domain under consideration, damping (or attenuation) must occur. Since diffusive processes have not been included in the model, this result is surprising; an explanation of where the energy of the waves goes is presented in the next paragraph.

Note that the root (4.2) corresponds to a barotropic gravity wave that travels parallel to the coast and has an e-folding distance (in the direction normal to the coast) of $o(k^{-1}\,\text{cm})$. The root (4.3), on the other hand, corresponds to a baroclinic gravity wave. This wave has an e-folding distance of $o[(gD)^{1/2}\varepsilon/\omega_2]$, an interfacial amplitude of $o(1/\varepsilon^2)$ of that of the barotropic wave, and it travels toward the coast with its crests really parallel to the coast [since $k \ll \text{Im}\,(kr_2)$]. This latter result implies that the following transfers of energy occur: (i) from the upper layer of fluid in the deep-sea region to the fluid in the shelf region, and (ii) from the lower layer of fluid in the deep-sea region to the vertical boundary of the deep-sea region. The second implication, (ii), is particularly important in that it appears to explain where the energy of the waves goes.

To determine (i) the strength of the coupling between the waves on the shelf and in the deep-sea region, (ii) the relative change in the phase velocities of the waves, and (iii) the relative magnitude of the e-folding time (decay-time constant), attention is now focused on the dispersion relationship (3.17). By substituting (4.2) and (4.3) into (3.17), utilizing the identity $\theta^2 = (2\nu+1)\Delta/\lambda$, and setting $\varepsilon = \alpha\delta$, where α is a positive constant of order unity, the following relationship is obtained:

$$L_\nu(2\lambda)\{1+\delta[3/2-\Delta(1+(2\nu+1)/2\lambda+i\alpha(\lambda/\Delta(2\nu+1))^{1/2})]+ \\ +o(\delta^2)\}+\delta 2\Delta\, L'_\nu(2\lambda)\{1+i\alpha[\lambda/\Delta(2\nu+1)]^{1/2}+o(\delta)\}=0. \qquad (4.4)$$

Note that, for fixed values of the parameters α, Δ, and δ, (4.4) can be regarded as a functional relationship between an independent variable, λ, and a dependent variable, ν. Hence, a solution of the form $\nu = \nu(\lambda;\alpha,\Delta,\delta)$ can be obtained from (4.4). By combining this with the relationship

$$\omega^2 = (2\nu+1)gd\lambda/l^2, \qquad (4.5)$$

a dispersion relationship of the form $\omega = \omega(k)$ can readily be determined. Before this procedure is carried out, it is noted that, to first order in small quantities, (4.4) reduces to the (implicit) dispersion relationship for edgewaves in a homogeneous ocean in the limit $\alpha \to 0$ (corresponding to $\varrho_0-\varrho\to 0$), $D\to 0$, and $D_0 \to D_T$ (the mean deep-sea total depth), viz.,

$$L_\nu(2kl)[1-\delta_T(1+\omega^2/2k^2gd)+o(\delta_T^2)] \\ +2\delta_T L'_\nu(2kl)=0, \qquad (4.6)$$

where $\delta_T = d/D_T \ll 1$ and k, ν, and ω are all real [Mysak 1968: eq. (6) with $f=0$].

The presence of the small parameter δ in (4.4) suggets that $\nu = \nu(\lambda)$ be determined by means of an ordinary perturbation expansion. Hence, we let

$$\left.\begin{aligned} \nu &= \nu^0 + \delta\nu^1 + \dots, \\ L_\nu &= L_{\nu^0} + \delta\nu^1 [\partial L_\nu/\partial\nu]_{\nu=\nu^0} + \dots, \end{aligned}\right\} \tag{4.7}$$

with a similar representation for L'_ν. Since (4.4) is a complex function, it is assumed that $\nu^j (j = 0, 1, 2, \dots)$ is of the form

$$\nu^j = \nu^j_1 + i\nu^j_2, \tag{4.8}$$

where ν^j_1 and ν^j_2 are real. Substitution of (4.7) into (4.4) yields, to lowest order,

$$L_{\nu^0}(2\lambda) = 0, \tag{4.9}$$

where $\nu^0 = \nu^0_1 + i\nu^0_2$. For $\lambda > 0$, eq. (4.9) implies that $\nu^0_2 = 0$. Note that the solution of (4.6) for $\nu = \nu(\lambda)$ by means of an ordinary perturbation expansion in δ_T also yields $L_{\nu^0}(2\lambda) = 0$(ν^0 real) to lowest order. Hence, for the two cases, the dispersion relationships are identical to lowest order in small quantities: it is also evident that even with stratification present, each eigenwave on the shelf is weakly coupled[3] to the waves in the deep-sea region. A table of the real zeros, $\nu^0 \equiv \nu^0_1$, of eq. (4.9) as a function of λ has been given elsewhere (Mysak 1968); here, only a brief summary of the properties of the zeros is given. For each $\lambda > 0$ there exists a sequence of ν^0's that are denoted by ν^{0n} $(n = 0, 1, 2, \dots)$, with the property that $0 < \nu^{00} < \nu^{01} < \dots$. Further, for $\lambda^2 \gg 1$ (relatively short waves), $\nu^{0n} \approx n$, whereas for $\lambda^2 \ll 1$ (relatively long waves), $\nu^{0n} \gg 1$. [These results can also be readily deduced respectively from the asymptotic and series representations of L_ν (see Pinney 1946).] Hence, we observe that, for each $\lambda > 0$ and for a corresponding ν^{0n}, (4.5) yields two real zeroth-order values of ω, which we denote by $\pm\omega^{0n}$. According to the terminology used by Mysak (1968), there are two edgewave *modes* corresponding to each *order* ν^{0n}. For a graphical representation of the relationships $\omega^{0n} = \omega^{0n}(\lambda; \nu^{0n})$ $(n = 0, 1, 2)$, see Mysak (1968).

To the next order, (4.4), (4.7), and (4.8) imply that

$$\begin{aligned} \nu^{1n}_1 &= -\Delta a^n, \\ \nu^{1n}_2 &= -\alpha a^n [\lambda\Delta/(2\nu^{0n} + 1)]^{1/2}, \end{aligned} \tag{4.10}$$

where

$$a^n = 2[L'_\nu(\partial L_\nu/\partial\nu)^{-1}]_{\nu=\nu^{0n}}.$$

In the homogeneous case, it is readily verified that

$$\nu^{1n} = -a^n. \tag{4.11}$$

3. This means that, to within order δ (or δ_T in the case of a homogeneous ocean), the edgewaves have a node at the edge of the shelf.

From (4.5), (4.10), (4.11), and the series expansions for ν, it follows that, to first order in small quantities,

$$\left(\frac{C_s^n}{C_H^n}\right)^2 = \frac{2\,[\nu^{0n} - (d/D_0)\,a^n] + 1}{2\,[\nu^{0n} - (d/D_T)\,a^n] + 1}, \qquad (4.12)$$

where C_s^n and C_H^n are respectively the n^{th}-order phase velocities of the edgewaves in a stratified and homogeneous ocean. Since $\nu^{0n} > 0$ and $D_0 < D_T$, (4.12) implies that $C_s^n \lesseqgtr C_H^n$ according to whether $a^n \gtreqless 0$. However, since $\nu^{0n}/a^n = O(1)$ and since typical values of the ratios d/D_0 and d/D_T are 4.3×10^{-2} and 4.0×10^{-3}, respectively, the ratio C_s^n/C_H^n is nearly equal to unity. Therefore, the latter ratio as a function of λ is not computed here. For the special cases $\lambda^2 \ll 1$ and $\lambda^2 \gg 1$, it is fairly easy to show analytically that $a^0 > 0$, which implies that stratification produces a small decrease in the lowest-order phase velocity.

Finally, by writing the n^{th}-order complex frequency in the form

$$\omega^n \equiv \omega_1^n + i\omega_2^n = \omega^{0n} + \delta(\omega_1^{1n} + i\omega_2^{1n}) + \ldots,$$

it is readily established that

$$|\omega_2^n/\omega_1^n| = \delta\,|\nu_2^{1n}/(2\,\nu^{0n} + 1)| + o(\delta). \qquad (4.13)$$

Hence, the decay time of the waves is much greater than the wave period. For $\omega_1^n = O(10^{-3}\,\text{sec}^{-1})$, (4.13) implies that $\omega_2^n = O(10^{-5}\,\text{sec}^{-1})$; hence, the e-folding distance of the baroclinic wave in the deep-sea region is of $O(10^7\,\text{cm})$, which is the same order as that of the barotropic wave (see 2, § 4).

5. *Concluding Remarks.* In the following paragraphs the main results obtained in the present paper are compared with those obtained by Mysak (1967) in a study of the effects of deep-sea current and stratification on continental shelf waves, which are characterized by a frequency of $O(10^{-5}\,\text{sec}^{-1})$ and wave number of $O(3 \times 10^{-8}\,\text{cm}^{-1})$.

Since $kV \ll \omega$ for edgewaves whereas $kV \lesssim \omega$ for shelf waves, only the latter waves are significantly affected by an essentially surface deep-sea longshore current having a characteristic speed of $V = 200\,\text{cm sec}^{-1}$. In particular, if c^0 denotes the lowest-order phase velocity of a shelf wave when deep-sea stratification is present, then c^0 changes by about 20% if a current with speed $V = 200\,\text{cm sec}^{-1}$ is introduced.

In the stratified theory of edgewaves, each eigenwave on the shelf is weakly coupled to a longshore-traveling barotropic wave and an onshore-traveling baroclinic wave. In the stratified theory of shelf waves, however, each eigenwave on the shelf is strongly coupled to a barotropic and a baroclinic wave, both of which travel parallel to the coast.[4] Further, in the stratified theory of edgewaves, the e-folding distances of the deep-sea waves away from the coast are of the same order of magnitude. In the stratified theory of shelf waves, on the other hand, the e-folding distance of the baroclinic wave is much less than that of the barotropic wave.

Only in the case of shelf waves does stratification produce a significant change in the phase velocities. For the lowest-order shelf wave in particular, the phase velocity is increased by nearly a factor of two.

Edgewaves are slightly damped when deep-sea stratification is present. When deep-sea stratification is incorporated into the theory of shelf waves, the waves still propagate without any energy loss.

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4. In an earlier paper (Mysak 1967) it was shown that, when first-order solutions to the characteristic equation (5.12) are used in the analysis, each shelf eigenwave is coupled to a baroclinic wave only. However, if second-order solutions to (5.12) are used, we find that each shelf eigenwave is in fact coupled to both a baroclinic and a barotropic wave. These higher-order approximations, however, do not give rise to any new terms of o(1) in the eigenvalue equation (5.15).