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Edgewaves on a Gently Sloping Continental Shelf of Finite Width¹

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ABSTRACT

The properties of free unattenuated long waves of small amplitude that travel parallel to the coast over a bottom topography consisting of a gently sloping shelf of finite width that drops off vertically to deep water of constant depth are investigated. For a homogeneous uniformly rotating fluid, it is shown that two classes of waves exist: relatively high-frequency waves with periods of about two hours or less (class I waves) and very low-frequency waves with periods of about one day or more (class II waves). While class I waves can travel in both directions along the coast, class I waves, for a specified hemisphere, can travel in only one direction. For wavelengths less than or equal to the shelf width, the wave modes are independent of the shelf width. In this case the modes of the class I and class II waves correspond, respectively, to those of edgewaves and quasigeostrophic waves that travel along a shelf of semi-infinite width (Reid 1958). For wavelengths considerably greater than the shelf width, however, the wave modes depend very strongly on the shelf width. In particular, for the class I waves (and for this class only), there is a long wavelength cutoff for trapped modes. Also, in the long wavelength limit the class II waves correspond to continental shelf waves (Robinson 1964).

1. *Introduction.* In previous theoretical studies of long unattenuated surface waves of small amplitude that travel parallel to the coast over a gently sloping continental shelf,³ most authors (Munk et al. 1956, Greenspan 1956, Reid 1958) have considered the shelf to be of semi-infinite width. Robinson (1964), however, has developed a theory of long unattenuated waves of small amplitude that travel parallel to the coast over a bottom topography consisting of a gently sloping shelf of finite width that drops off vertically to deep water of constant depth. These waves, which Robinson has termed continental shelf

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3. For the case of a small shelf slope, the hydrostatic pressure approximation is applicable, thereby greatly simplifying the mathematical analysis. The theory of edgewaves traveling over a shelf of arbitrary slope and of semi-infinite width has also been developed (Stokes 1846, Ursell 1952, Johns 1965) but will not be discussed here.

waves, are quite distinct from the familiar edgewaves discussed by Munk et al. and by Greenspan or from edgewaves as modified by the Coriolis force (Reid 1958), for the following reasons: (i) Shelf waves have a very low frequency (considerably less than the Coriolis parameter, which is assumed to be constant in the theory) and a very long wavelength (considerably greater than the shelf width), (ii) they are nondispersive, (iii) in a specified hemisphere they can travel in only one direction along the shelf (for example, southward off the eastern coast of the United States), (iv) they decay in the deep-sea region, and (v) they are characterized by a nearly geostrophic balance of forces in the direction normal to the coast.⁴

Reid (1958) has also established that a low-frequency class of unattenuated waves (quasigeostrophic waves) can travel parallel to the coast over a gently sloping shelf of semi-infinite width. These waves are similar to continental shelf waves in that, for a specified hemisphere, they too can travel in only one direction along the shelf (the same direction as that of the shelf waves). While quasigeostrophic waves are generally dispersive, in the long wavelength limit they are also nondispersive, with wave velocity gs/f ; here s is the shelf slope, f the Coriolis parameter (assumed constant), and g the acceleration of gravity. However, because of the different geometries used by Robinson and by Reid, this wave velocity differs markedly from the shelf-wave velocity, which is simply proportional to fl , where l is the shelf width.

To determine the effect of a cutoff in shelf width on the modes of the high-frequency edgewaves (class I waves) and to gain a deeper understanding of the differences between the low-frequency quasigeostrophic and continental shelf waves (class II waves), this paper reports an investigation of the properties of all free long waves that travel parallel to the coast over the bottom topography considered by Robinson. Numerical computations reveal that, for both the class I and class II waves, the shelf width plays an increasingly important role in the determination of the wave modes as the longshore wavelength increases beyond the shelf width.

2. *Derivation of Dispersion Relationship.* A right-hand Cartesian coordinate system (x, y, z) is introduced; in this system x, y measures, respectively, the distances normal and parallel to a straight infinite coastline orientated in a N-S direction, and z measures the distance vertically upward from the undistorted sea surface. Let $\eta(x, y, t)$ be the sea-level distortion, and let $u(x, y, t)$, $v(x, y, t)$ be the velocity components in the x, y directions, respectively. Since our attention is to be confined to the study of free waves that move parallel to the coast (that is, "trapped" free waves), η, u , and v will have a (y, t) dependence on the form $\exp[i(ky + \omega t)]$, in which ω is real and

4. In this study our attention is restricted to theoretical considerations, but it is noted that, while edgewaves were first observed some time ago (Munk et al. 1956), shelf waves have been observed only recently (Hamon 1966, Mooers and Smith 1968).

$k > 0$; also, it is assumed that, in the x direction, the solutions are of a decaying character. Then the linearized nondissipative equations of shallow-water theory for a homogeneous uniformly rotating fluid imply that the wave amplitude, $\eta(x)$, satisfies the following equation (for a detailed derivation, see Reid 1958):

$$h\eta'' + h'\eta' + [h'fk/\omega + (\omega^2 - f^2)/g - hk^2]\eta = 0. \quad (1)$$

In eq. (1), $z = -h(x)$ is the equation of the sea bottom, g the acceleration of gravity, and $f = 2\Omega \sin \varphi$ the Coriolis parameter; Ω is the earth's angular velocity and φ the latitude (assumed constant). In terms of $\eta(x)$, the velocity amplitudes are given by

$$\left. \begin{aligned} u(x) &= ig(\omega\eta' + kf\eta)/(\omega^2 - f^2), \\ v(x) &= -g(f\eta' + k\omega\eta)/(\omega^2 - f^2). \end{aligned} \right\} \quad (2)$$

Following Robinson (1964), $h(x)$ has the form

$$h(x) = \begin{cases} D, & x > l \text{ (deep-sea region),} \\ dx/l, & 0 < x < l \text{ (shelf region),} \end{cases} \quad (3)$$

where D is the depth in the deep-sea region, d the depth at the edge of the shelf, and l the shelf width.⁵ Eq. (1) is to be solved separately for the deep-sea and shelf regions subject to the boundary conditions $\eta(x) \rightarrow 0$ as $x \rightarrow \infty$ and $|\eta(0)| < M$ (a constant). Then, at $x = l$, $\eta(x)$ and $h(x)u(x)$ (normal transport component) are to be made continuous.

For the bottom topography given in (3), eq. (1) together with the above-stated boundary conditions imply that

$$\eta(x) = \begin{cases} A \exp(-Kx), & x > l, \\ B \exp(-kx) L_\nu(2kx), & 0 < x < l. \end{cases} \quad (4)$$

Here A, B are constants, $K = [k^2 + (f^2 - \omega^2)/gD]^{1/2}$ (assumed to be real and positive), $\nu = -1/2 + [f/\omega + (\omega^2 - f^2)l/gkd]/2$, and $L_\nu(z)$ is the Laguerre function, which has the series representation

$$L_\nu(z) = 1 - \nu z + (-\nu)_2 z^2 / (2!)^2 + \dots, \quad (5)$$

where $(-\nu)_n = -\nu(-\nu+1)(-\nu+2)\dots(-\nu+n-1)$, $(-\nu)_0 = 1$. $L_\nu(z)$ is an entire function and satisfies the differential equation

$$[zd^2/dz^2 + (1-z)d/dz + \nu]L_\nu(z) = 0,$$

5. This choice of $h(x)$ is particularly convenient for studying the effects of deep-sea stratification (and current) on shelf waves and edgewaves (see Mysak 1967, 1968). A study of long small-amplitude waves in a homogeneous uniformly rotating fluid that travel parallel to the coast over a bottom topography having a variable but continuous slope that approaches zero far from the coast, viz., $h(x) = h_0[1 - \exp(-ax)]$, has recently been presented by Ball (1967). Such a choice for $h(x)$, however, is not convenient for a stratified study.

which is a special case of the confluent hypergeometric equation. In the terminology of generalized hypergeometric functions, $L_\nu(z) = {}_1F_1(-\nu; 1; z)$ (Erdelyi 1953: 248). An extensive discussion of the properties of the Laguerre function, which reduces to a Laguerre polynomial of degree n when $\nu = n = 0, 1, 2, \dots$, has appeared elsewhere (Pinney 1946) and will not be given. Upon application of the continuity conditions to (4), the following implicit dispersion relationship is obtained:

$$L_\nu(2\lambda) \{ [1 + \delta\Delta(1 - \sigma^2)/\lambda^2]^{1/2} - 1/\sigma - \Delta(1 - 1/\sigma) \} + 2\Delta L'_\nu(2\lambda) = 0, \quad (6)$$

where $\sigma = \omega/f$, $\lambda = kl$, $\delta = f^2 l^2/gd$, and $\Delta = d/D$; the quantity ν is related to σ , λ , and δ by

$$\sigma^3 - [1 + (2\nu + 1)\lambda/\delta]\sigma + \lambda/\delta = 0. \quad (7)$$

Eq. (7) is a cubic in σ ; in § 3 it is seen that two of the roots correspond to the modes of class I (high-frequency) waves, and the third root, to the modes of class II (low-frequency) waves.

For the case of the inertial oscillation, $\omega = f$, the velocity amplitudes in (2) are indeterminate so that (6) does not apply. [In fact, for $\omega = f$ ($\sigma = 1$), (6) is identically satisfied.] In this case an analysis of the original conservation equations together with the appropriate boundary and continuity equations yields the dispersion relationship

$$(f^2/gk - s/2) + \exp(-kl)[s(kl + 1/2) - kD] = 0, \quad (8)$$

where $s = d/l$ and $k > 0$. For fixed s and k , (8) implies that $k = 2f^2/gs$ in the limit $l \rightarrow \infty$, which is in agreement with Reid (1958). Hence, in this limiting case, the wave velocity in the positive y direction, $-\omega/k$, is given by $-gs/2f$. This corresponds to a wave moving in the negative (positive) y direction if $f > 0$ ($f < 0$). For finite l such that $s \ll 1$, $f^2 l \ll g$, and $d \ll D$, the only solution to (8) is that for which $kl \ll 1$. With this approximation, it follows that the wave velocity in the positive y direction is $-(gD)^{1/2}$ or $(gD)^{1/2}$ according to whether $f > 0$ or $f < 0$.⁶ Thus, for a finite-width shelf, the inertial edgewave behaves like a long gravity wave traveling in water of depth D . Also, it is analogous to a Kelvin boundary wave (Proudman 1952: 253-255) in that, for a specified hemisphere, both can propagate in only one direction along the coast (for example, southward off the eastern coast of the United States).

Prior to discussion of the solution of (6) for an explicit relationship, $\sigma = \sigma(\lambda)$, the dispersion relationship obtained by Reid (1958) for waves traveling along a gently sloping shelf of semi-infinite width is presented.

6. For the case $\omega = -f$, the wave velocities corresponding to infinite and finite l are identical to those given above, so that all inertial waves are included in the above discussion.

In this case, $h = sx$, $x > 0$, where s is the shelf slope, so that the equation for $\eta(x)$ is identical to that for the case of a finite-width shelf, provided d/l is replaced in the latter with s . However, in order to ensure that $\eta \rightarrow 0$ as $x \rightarrow \infty$ on the shelf, it is required that ν be a non-negative integer, n , since for ν not a non-negative integer,

$$L_\nu(z) \sim -(1/\pi) \sin(\pi\nu) \Gamma(\nu+1) \exp(z)/z^{\nu+1}, \quad z \gg 1,$$

whereas for $\nu = n = 0, 1, 2, \dots$,

$$L_n(z) \sim (-1)^n z^n/n!, \quad z \gg 1$$

(Pinney 1946). Hence, for the shelf of semi-infinite width, the implicit dispersion relationship takes the simple form

$$\omega^3 - [f^2 + (2n+1)gks] \omega + gksf = 0, \quad k > 0, \quad n = 0, 1, 2, \dots \quad (9)$$

Note that (9) is equivalent to (7) if, in the former, n is replaced by ν , and s by d/l .

3. *Approximate Solution of Dispersion Relationship.* For specified values of the parameters δ and Δ , the determination from (6) of an explicit dispersion relationship in the form $\sigma = \sigma(\lambda)$ for a large domain of λ is a formidable task in view of the involved relationship between ν , σ , and λ [see (7)] and the lack of detailed tables of $L_\nu(z)$ for a wide range of ν and z . However, for $\Delta \ll 1$, a first approximation to $\sigma = \sigma(\lambda)$ can be determined simply from

$$L_\nu(2\lambda) = 0. \quad (10)$$

In connection with many real ocean-bottom topographies, this approximation introduces an error of only a few percent, since typically $\Delta = 4 \times 10^{-2}$ (corresponding to $d = 200$ m and $D = 5000$ m). Thus, the solution of (10) itself for $\sigma = \sigma(\lambda)$ is of considerable interest. Physically, the condition $\Delta \ll 1$ means that the waves on the shelf are weakly coupled to those in the deep-sea region and that, to within $o(\Delta)$, the waves on the shelf have a node at the edge of the shelf.

Before $\sigma = \sigma(\lambda)$ is determined from (10), mention is made here that, for the case of continental shelf waves discussed by Robinson (1964), which are characterized by the conditions $\lambda \ll 1$ and $\sigma \ll 1$, the dispersion relationship $\sigma = \sigma(\lambda)$ is determined from

$$\mathcal{F}_0[2(\lambda/\sigma)^{1/2}] = 0, \quad (11)$$

again with the condition $\Delta \ll 1$. Since $\nu \sim 1/2\sigma$ for $\lambda \ll 1$, $\sigma \ll 1$, and $\delta \ll 1$ (also one of the conditions assumed in Robinson's theory), and since

$$\lim_{\varepsilon \rightarrow 0} L_{\mu/\varepsilon}(\varepsilon) = \mathcal{F}_0[2(\mu)], \quad \text{Re}(\mu) > 0$$

(Pinney 1946), (10) behaves like (11) for this low-frequency long-wave-length limit.

To my knowledge, the only available tables of the zeros of the Laguerre function are of the following form (for example, see Slater 1960: 112): For a specified value of $\nu > 0$, the corresponding positive values of z that satisfy the equation $L_\nu(z) = 0$ are listed. (There are no positive zeros for $\nu < 0$, since $L_\nu(z)$ is a monotonically increasing function of z with $L_\nu(0) = 1$). However, to determine a first approximation to $\sigma = \sigma(\lambda)$, the values of ν corresponding to a specified value of $\lambda > 0$ that satisfy the equation $L_\nu(2\lambda) = 0$ must first be found. These ν 's shall be denoted by $\nu_n (n = 0, 1, 2, \dots)$. Then, for each $\lambda > 0$ and corresponding ν_n , eq. (7) (with δ specified) yields three real values of σ . These dimensionless frequency functions shall be denoted by $\sigma(j, \nu_n; \lambda)$, where $j = 1, 2, 3$ denotes the three different *modes* corresponding to each *order* ν_n . Further, for each $\lambda > 0$ and order ν_n , the product of the three roots is equal to $-\lambda/\delta < 0$; hence, two of the frequency functions are positive and one is negative. $\sigma(1, \nu_n; \lambda)$ shall be adopted as the negative root, which corresponds to waves moving in the positive (negative) y direction according to whether $f > 0$ ($f < 0$). With this notation, the roots of (7) can be written in the form

$$\sigma(j, \nu_n; \lambda) = -2(3)^{-1/2} [1 + (2\nu_n + 1)\lambda/\delta]^{1/2} \cos [\beta + (j-1)2\pi/3], \quad (12)$$

where

$$\cos 3\beta = 3(3)^{1/2} [1 + (2\nu_n + 1)\lambda/\delta]^{-3/2} \lambda/2\delta.$$

In the above, the terminology and conventions used by Reid (1958) have been essentially adopted in order to facilitate a comparison of the frequency functions for each model. For the case of the shelf of semi-infinite width, the frequency functions shall be denoted by $\sigma(j, n; \lambda)$ ($n = 0, 1, 2, \dots$; $j = 1, 2, 3$), with the understanding that the "l" in λ is the same as that used in the definition of the shelf slope, viz., $s = d/l$. For $n = 0$, the three modes are given by

$$\sigma(1, 0; \lambda) = -(\lambda/\delta + 1/4)^{1/2} - 1/2,$$

$$\sigma(2, 0; \lambda) = (\lambda/\delta + 1/4)^{1/2} - 1/2,$$

$$\sigma(3, 0; \lambda) = 1.$$

The modes corresponding to $j = 1, 2$ are the familiar lowest-order edgewave modes as modified by the Coriolis parameter; the third mode, the inertial oscillation discussed in § 2, coincides with the special case $\sigma(2, 0; 2\delta)$. For the higher orders ($n \geq 1$), the modes are given by (12), with ν_n replaced by n . The modes corresponding to $j = 1, 2$ characterize the higher-order edgewaves, and the mode corresponding to $j = 3$ characterizes the higher-order quasigeostrophic waves.

Table I. The first three zeros, ν_0, ν_1, ν_2 , of the equation $L_\nu(2\lambda) = 0$ as a function of λ .

λ	ν_0	ν_1	ν_2	λ	ν_0	ν_1	ν_2
0.05	14.155	76.063	186.098	2.25	0.041	1.550	4.033
0.10	6.655	37.313	92.799	2.50	0.027	1.420	3.658
0.20	3.155	18.563	46.362	2.75	0.018	1.322	3.361
0.30	1.967	12.313	30.737	3.00	0.012	1.245	3.118
0.40	1.342	9.063	22.924	3.25	0.008	1.187	2.922
0.50	0.992	7.188	18.237	3.50	0.005	1.142	2.760
0.75	0.542	4.688	12.143	3.75	0.003	1.107	2.626
1.00	0.330	3.469	9.018	4.00	0.002	1.081	2.515
1.25	0.211	2.750	7.205	4.25	0.001	1.060	2.422
1.50	0.137	2.278	5.986	4.50	0.000	1.044	2.344
1.75	0.092	1.950	5.142	4.75	0.000	1.033	2.280
2.00	0.061	1.722	4.509	5.00	0.000	1.024	2.226

In Table I are given the first three orders, ν_n , as a function of λ over the range $0.05 \leq \lambda \leq 5$. In essence, the program used to determine the ν_n 's corresponding to each λ involved the following: The Laguerre function was evaluated by using the first 25 terms in the series (5) for a sequence of ν 's in the neighborhood of an approximate value of ν_n given by $\lambda \approx \pi^2(n + 3/4)^2 / (4 + 8\nu_n)$ (Slater 1960: 107). The sequence of ν 's were chosen in a manner such that the value of L_ν converged to zero, with the computation stopping at that value of ν for which $|L_\nu| < 0.001$.

From Table I it is seen that, as λ increases, each order $\nu_n \rightarrow n$ (the order of the waves on a shelf of semi-infinite width), with the convergence being more rapid as the order decreases. Physically, this means that, for relatively short waves (longshore wavelengths of the order of the shelf width), the wave modes corresponding to each order are insensitive to the finiteness of the shelf width. As the wavelength increases, however, the shelf width plays an increasingly important role in determining the wave modes. In view of the exponential factor, $\exp(-kx)$, in the wave amplitude on the shelf [see (4)], the above result is not surprising: For relatively large k (short waves), the wave amplitude will be essentially zero quite close to the shoreline; for small k (long waves), however, the wave amplitude will be finite at the edge of the shelf.

In Fig. 1 are illustrated the frequency functions for the finite-width and semi-infinite-width shelf corresponding to the first three orders. In computing these functions, a value of $\delta = 0.027$ has been used [$|f| = 0.73 \times 10^{-4} \text{ sec}^{-1}$ ($\varphi = \pm 30^\circ$), $l = 100 \text{ km}$, and $d = 200 \text{ m}$ (or $s = 2 \times 10^{-3}$)]. Note in Fig. 1 that both continental shelf waves⁷ and quasigeostrophic waves, though nondispersive for $\omega \ll f$ and $kl \ll 1$, are dispersive when considered over a larger frequency and wave-number domain. Second, observe that, for the

7. Here, continental shelf waves are regarded as low-frequency waves that travel along a sloping shelf of finite width and that correspond to Robinson's continental shelf waves when $\omega \ll f$ and $kl \ll 1$.

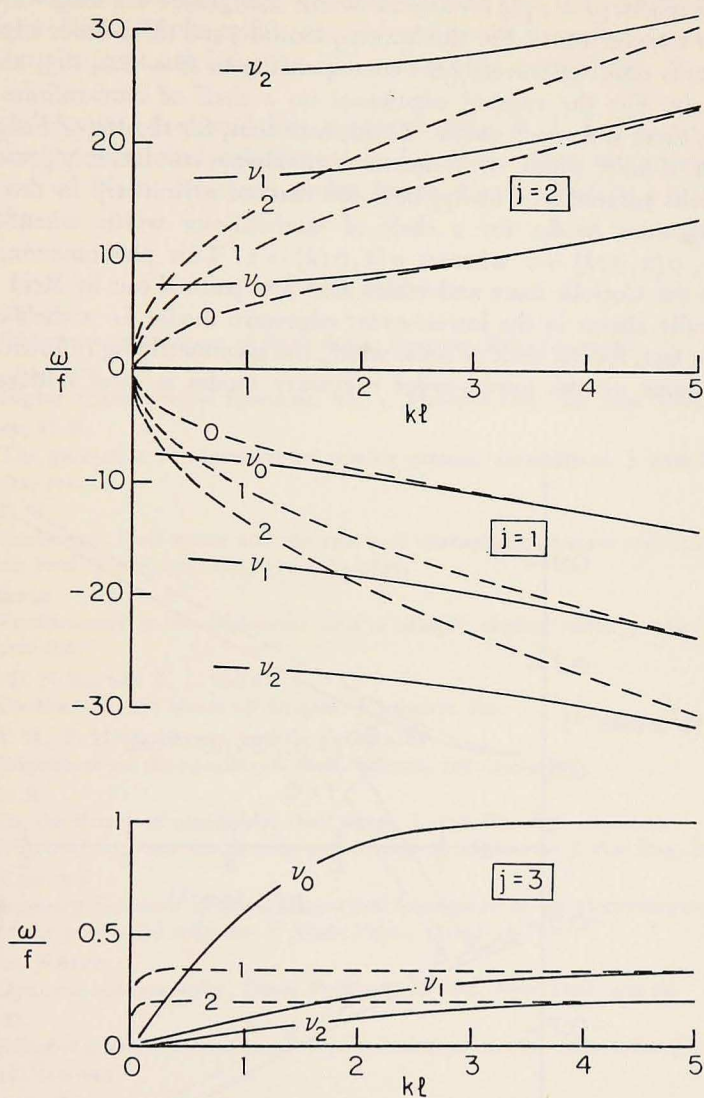


Figure 1. Plots of the frequency functions versus wave number for a shelf of finite width (solid lines) and semi-infinite width (dashed lines) with $\delta = 0.027$ [$|f| = 0.73 \times 10^{-4} \text{ sec}^{-1}$, $d = 200 \text{ m}$, $l = 100 \text{ km}$ (or $s = 2 \times 10^{-3}$)]. The different orders ν_n or n are indicated on the curves. The modes $j = 1, 2$ correspond to edgewaves and the mode $j = 3$ corresponds to continental shelf or quasigeostrophic waves. There is no lowest-order quasigeostrophic frequency function shown since it coincides with the point $\sigma(2, 0; 2\delta)$. Also, the edgewave frequency functions for a shelf of finite width have been plotted for only those values of kl for which $K > 0$, the condition for trapped waves.

edgewave modes ($j = 1, 2$) for a finite-width shelf, there is a long wavelength cutoff for trapped waves. For the lowest-, second-, and third-order edgewaves, these cutoffs occur at wavelengths corresponding to $kl \approx 0.2, 0.5,$ and $0.75,$ respectively. For the trapped edgewaves on a shelf of semi-infinite width, however, there is no such cutoff. Third, note that, for the trapped edgewaves on a shelf of finite width, the frequencies are always considerably greater than the Coriolis parameter. Finally, note the marked asymmetry in the lowest-order edgewave modes for a shelf of semi-infinite width when $kl \ll 1$: as $\lambda \rightarrow 0, \sigma(2, 0; \lambda) \rightarrow 0$ whereas $\sigma(1, 0; \lambda) \rightarrow 1$. This phenomenon, which is due to the Coriolis force and which was first pointed out by Reid (1958), is practically absent in the lowest-order edgewave modes for a shelf of finite width. In fact, for the shelf of finite width, the asymmetrizing influence of the Coriolis force on the lowest-order edgewave modes is most noticeable for

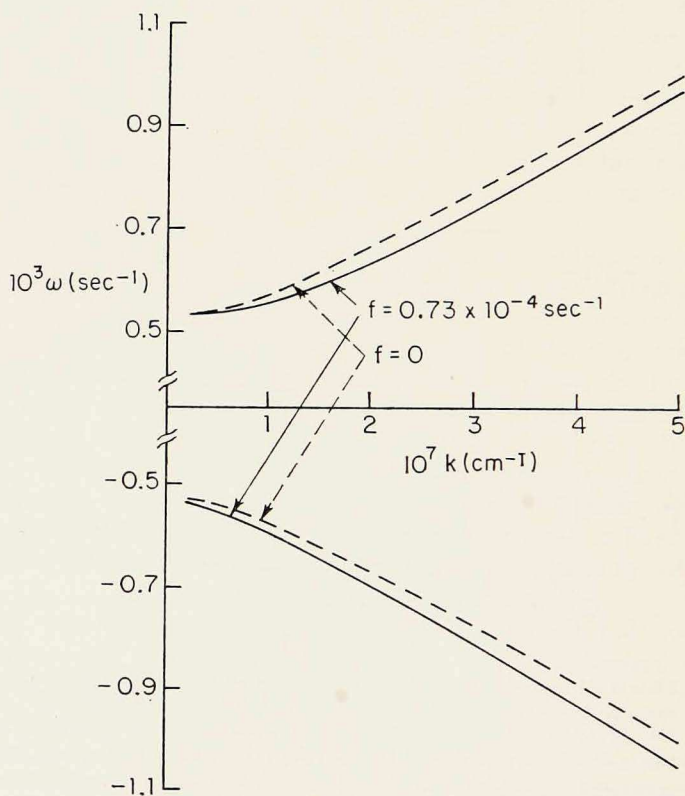


Figure 2. Lowest-order edgewave frequency functions for a shelf of width 100 km and maximum depth 200 m, depicting the asymmetrizing influence of the Coriolis force. With the Coriolis force neglected (dashed lines), the frequency functions are symmetrical about the wave-number axis.

relatively short waves (Fig. 2). For the second- and third-order edgewave modes for a shelf of finite width, the changes in the frequency functions due to the Coriolis force are less than 1% and 0.5%, respectively.

Acknowledgment. I should like to thank Miss P. A. Jaskun for her assistance in the numerical calculations in § 3. This work was supported by the Office of Naval Research under NONR Grant 1866(51) to Harvard University.

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