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A Harmonic Method for Predicting Shallow-water Tides[1]

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ABSTRACT

The development of an objective technique for identifying significant hidden frequencies in the spectrum makes it possible to accurately predict shallow-water tides by harmonic methods. For Anchorage, Alaska, the 114 constituents used include frequencies in every species (cycles per day) from 0 to 12. The larger set of constituents improved the predictions in times of high and low waters, range of tide, and shape of curve. The stationary characteristics of some of the added constituents have been tested with three years of Philadelphia data.

This study was initially designed for a specific purpose—to improve tidal predictions at Anchorage, Alaska. Anchorage is located at the northern end of Cook Inlet near the western end of the Gulf of Alaska. The mean range of tide at Anchorage is about 25 feet. After oil was found in the area, huge deep-draft tankers required more accurate tidal predictions than were available by using the standard U.S. Coast and Geodetic Survey procedures for tidal analysis and prediction (9).

There was no question as to why the problem existed. Those harmonic constants that were determined in the Coast and Geodetic Survey analysis showed large amplitudes for some shallow-water constituents—clear indications of additional significant compound tides that are not included in the routine analysis. The compound tides are associated with the distortion of the sinusoidal shape of the tidal curve as the tidal wave travels over shallow depths (5).

British authorities have had to cope with shallow-water tides to a much greater extent than the U.S. Coast and Geodetic Survey, and therefore they have traditionally used 60 tidal constituents (3) compared with the 37 used by the Coast and Geodetic Survey. In some extreme cases, the 60 constituents do not adequately describe the shape of the curve, and the British have developed a non harmonic modification of their procedures to cope with these tides (4).

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A logical approach, therefore, was to send the Anchorage data to the Tidal Institute and Observatory, University of Liverpool, for analysis and prediction until such time as the Coast and Geodetic Survey could learn the British techniques. However, this did not solve the problem. We were informed that the Tidal Institute routine analysis obviously would not match the shape of the curve and that a continuous record of hourly heights for one year would be required for the nonharmonic method. Inasmuch as the harbor at Anchorage freezes every winter, the required length of record was not available. Although an effort to get a continuous year of record was initiated by installing a pressure gauge on the bottom, there remained an element of doubt as to whether the tidal characteristics remain unchanged during the winter freeze.

Fortunately, recent technical changes in tidal analysis and prediction made possible another approach. Essentially, the principal change provided greater flexibility in both analysis and prediction in that additional constitutents can now be included. Until now there was a constraint to work only with a fixed set of constituents; no others could be readily analyzed for, nor could they be included in, the prediction.

Traditional analysis included (i) a modified Fourier analysis for particular frequencies, (ii) a modification of the results for the interference effects of nearby frequencies, and (iii) an elimination of sideband contributions of frequencies in the same species—the same number of cycles per day (9,3). A least-square analysis in which any combination of frequencies is inserted as a model is now readily accomplished on a computer. A recent study showed that the harmonic constants for the same set of constituents are slightly more accurate than those obtained by the previous Coast and Geodetic Survey or British (Doodson) methods (10).

The Coast and Geodetic Survey tidal predictions until 1964 were made on a mechanical analog computer having gears designed for a fixed set of frequencies. The predictions now made on an electronic computer do not have the above restriction; in this particular study, 114 constituents have been included.

In shallow water, the nonlinear interaction among large-amplitude constituents generates additional constituents whose frequencies are integral sums or differences of the frequencies of known constituents (8). It is necessary to identify the important additional compound tides, include them in a new analysis, and then predict, using the enlarged set of harmonic constants. The availability of the BOMM (1) programs for time-series analysis makes some of these steps relatively easy.

First, a routine 37-constituent analysis was made of 192 days of Anchorage hourly data for the middle of 1964. Using the derived constants, hourly heights were predicted and subtracted from the observations, and then a spectral analysis was made of the residuals. This identified frequency bands of greatest energy in the residuals, and a Fourier analysis, using maximum resolution, was made for these bands. Wherever large values stood out above the continuum

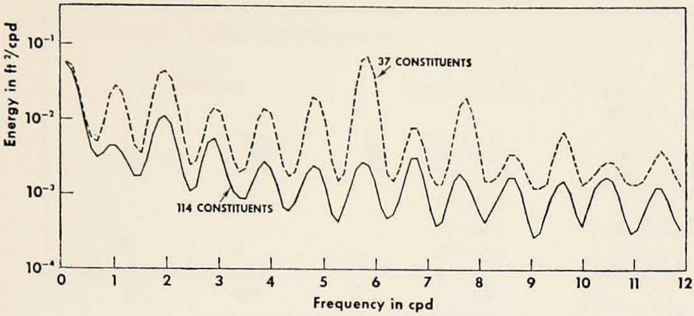

Figure 1. Spectral analysis of Anchorage residuals.

in a plot of the Fourier amplitudes, an effort was made to identify an integral combination of frequencies of constituents that were known to be important and that closely matched the frequencies of these peaks. A new least-square analysis was performed, adding these new frequencies to the original 37. As a check, a new total prediction was prepared, new residuals were determined, and Fourier- and power-spectrum analyses were conducted with these data.

Fig. 1 shows the comparative power spectra with residuals from 37 and 114 constituents, respectively. The solid line below the dotted line shows that a significant improvement has been made in every species.

Fig. 2 shows the results of Fourier analysis near two cycles per day for the

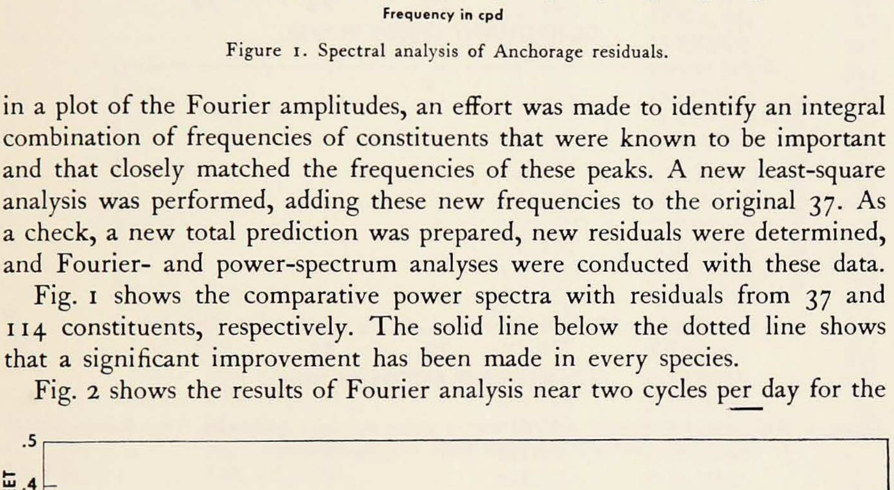

Figure 2. High-resolution Fourier analysis near 2 cycles per day; Anchorage. Top: Residuals from 37 constituents. Bottom: Residuals from 114 constituents.

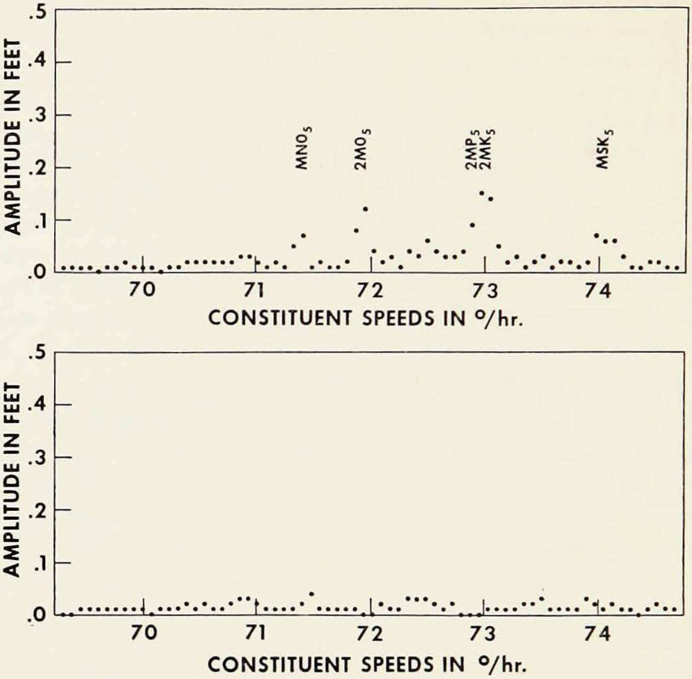

Figure 3. High-resolution Fourier analysis near 5 cycles per day: Anchorage. Top: Residuals from 37 constituents. Bottom: Residuals from 114 constituents.

two sets of residuals. Although a major portion of the energy has been removed, it is obvious that some remains. In a recent paper on tidal cusps (7), it was shown that the continuum rises in cusps near the large tidal lines. There are indications of a systematic residual midway between the groups (identified by the first two digits of the Doodson number), including speeds of about 28.2°, 28.7°, 29.2°, and 29.8° per hour. Combinations of constituents significantly more contrived than those used would be required to approximate these speeds, and this has not been done because the cusps are now believed to include both line-line and line-noise interactions (W. H. Munk, personal communication).

Fig. 3. shows a comparable Fourier analysis of the residuals near five cycles per day. No other study has been found in which this portion of the spectrum has been examined for tidal lines. There is significantly less evidence of a cusp in the final residuals than in Fig. 2; this could have been anticipated from the spectral analysis in Fig. 1. Furthermore, cusps are found adjacent to large tidal lines; these do not occur at five cycles per day.

Table I identifies all of the 114 constituents and shows their amplitudes. The subscript at the right of the name indicates the number of cycles per day.

Table I. Anchorage Tidal Constituents.

Name	Source	Doodson No.	Speed (°/hr)	Amplitude (feet)
Sa		056.555	.0410686	.519
Ssa		057.555	.0821373	.182
Mm		065.455	.5443747	.139
MSf		073.555	1.0158958	.347
Mf		075.555	1.0980331	.101
$2Q_1$		125.755	12.8542862	.041
σ_1		127.555	12.9271398	.129
Q_1		135.655	13.3986609	.210
ϱ_1		137.455	13.4715145	.030
O_1		145.555	13.9430356	1.197
MP_1	$M_2 - P_1$	147.555	14.0251729	.236
M_1		155.655	14.4966939	.169
χ_1		157.455	14.5695476	.026
P_1		163.555	14.9589314	.588
S_1		164.555*	15.0000000*	.119
K_1		165.555	15.0410686	2.238
J_1		175.455	15.5854433	.088
$2PO_1$	$2P_1 - O_1$	181.555	15.9748272	.060
SO_1	$S_2 - O_1$	183.555	16.0569644	.225
OO_1		185.555	16.1391017	.084
$2NS_2$	$2N_2 - S_2$	217.755	26.8794590	.072
$2NK2S_2$	$2N_2 + K_2 - 2S_2$	219.755	26.9615963	.136
MNS_2	$M_2 + N_2 - S_2$	227.655	27.4238337	.162
$MNK2S_2$	$M_2 + N_2 + K_2 - 2S_2$	229.655	27.5059710	.107
$2MS2K_2$	$2M_2 + S_2 - 2K_2$	233.555	27.8039338	.066
$2N_2$		235.755	27.8953548	.368
μ_2		237.555	27.9682084	.683
N_2		245.655	28.4397295	1.830
υ_2		247.455	28.5125831	.493
$2KN2S_2$	$2K_2 + N_2 - 2S_2$	249.655	28.6040041	.072
OP_2	$O_1 + P_1$	253.555	28.9019669	.145
M_2		255.555	28.9841042	11.039
MKS_2	$M_2 + K_2 - S_2$	257.555	29.0662415	.108
$M2(KS)_2$	$M_2 + 2K_2 - 2S_2$	259.555	29.1483788	.087
$2SN(MK)_2$	$2S_2 + N_2 - M_2 - K_2$	261.655	29.3734880	.095
λ_2		263.655	29.4556253	.274
L_2		265.455	29.5284789	.573
T_2		272.556	29.9589333	.128
S_2		273.555	30.0000000	2.937
R_2		274.554	30.0410667	.200
K_2		275.555	30.0821373	.922
MSN_2	$M_2 + S_2 - N_2$	283.455	30.5443747	.208
$2KM(SN)_2$	$2K_2 + M_2 - S_2 - N_2$	287.455	30.7086493	.054
$2SM_2$	$2S_2 - M_2$	291.555	31.0158958	.181
SKM_2	$S_2 + K_2 - M_2$	293.555	31.0980331	.073
NO_3	$N_2 + O_1$	335.655	42.3827651	.101

* Doodson uses 164.556 with a speed of 15.0000020.

Table I. Anchorage Tidal Constituents (continued).

Name	Source	Doodson No.	Speed (°/hr)	Amplitude (feet)
$2MK_3$†	$2M_2-K_1$	345.555	42.9271398	.311
M_3		355.555	43.4761563	.084
SO_3	S_2+O_1	363.555	43.9430356	.157
MK_3	M_2+K_1	365.555	44.0251729	.218
SK_3	S_2+K_1	383.555	45.0410686	.141
N_4	$2N_2$	435.755	56.8794590	.093
$3MS_4$	$3M_2-S_2$	437.555	56.9523127	.117
MN_4	M_2+N_2	445.655	57.4238337	.280
$MNKS_4$	$M_2+N_2+K_2-S_2$	447.655	57.5059710	.085
M_4		455.555	57.9682084	.839
SN_4	S_2+N_2	463.655	58.4397295	.079
KN_4	K_2+N_2	465.655	58.5218668	.134
MS_4	M_2+S_2	473.555	58.9841042	.538
MK_4	M_2+K_2	475.555	59.0662415	.123
SL_4	S_2+L_2	483.455	59.5284789	.056
S_4		491.555	60.0000000	.061
MNO_5	$M_2+N_2+O_1$	535.655	71.3668693	.077
$2MO_5$	$2M_2+O_1$	545.555	71.9112440	.141
$3MP_5$	$3M_2-P_1$	547.555	71.9933813	.064
MNK_5	$M_2+N_2+K_1$	555.655	72.4649023	.061
$2MP_5$	$2M_2+P_1$	563.555	72.9271398	.134
$2MK_5$	$2M_2+K_1$	565.555	73.0092770	.198
MSK_5	$M_2+S_2+K_1$	583.555	74.0251728	.097
$3KM_5$	$K_2+K_1+M_2$	585.555	74.1073100	.042
$3NKS_6$	$3N_2+K_2-S_2$	627.855	85.4013258	.081
$2NM_6$	$2N_2+M_2$	635.755	85.8635632	.090
$2NMKS_6$	$2N_2+M_2+K_2-S_2$	637.755	85.9457005	.102
$2MN_6$	$2M_2+N_2$	645.655	86.4079380	.281
$2MNKS_6$	$2M_2+N_2+K_2-S_2$	647.655	86.4900752	.098
M_6		655.555	86.9523127	.507
MSN_6	$M_2+S_2+N_2$	663.655	87.4238337	.093
MKN_6	$M_2+K_2+N_2$	665.655	87.5059710	.127
$2MS_6$	$2M_2+S_2$	673.555	87.9682084	.483
$2MK_6$	$2M_2+K_2$	675.555	88.0503457	.146
NSK_6	$N_2+S_2+K_2$	683.655	88.5218668	.104
$2SM_6$	$2S_2+M_2$	691.555	88.9841042	.090
MSK_6	$M_2+S_2+K_2$	693.555	89.0662415	.057
S_6		$6E\bar{1}.555$**	90.0000000	.006
$2MNO_7$	$2M_2+N_2+O_1$	735.655	100.3509735	.059
$2NMK_7$	$2N_2+M_2+K_1$	745.755	100.9046318	.079
$2MSO_7$	$2M_2+S_2+O_1$	763.555	101.9112440	.095
$MSKO_7$	$M_2+S_2+K_2+O_1$	783.555	103.0092771	.067
$2(MN)_8$	$2M_2+2N_2$	835.755	114.8476674	.053
$3MN_8$	$3M_2+N_2$	845.655	115.3920422	.125
$3MNKS_8$	$3M_2+N_2+K_2-S_2$	847.655	115.4741795	.062
M_8		855.555	115.9364169	.160

† $2MK_3$ is named MO_3 in Admiralty Manual of Tides, p. 68.

** According to Doodson code, $E = +6$ and $\bar{1} = -6$ above and below base 5.

Table I. Anchorage Tidal Constituents (continued).

Name	Source	Doodson No.	Speed (°/hr)	Amplitude (feet)
$2MSN_8$	$2M_2+S_2+N_2$	863.655	116.4079380	.088
$2MNK_8$	$2M_2+N_2+K_2$	865.655	116.4900752	.055
$3MS_8$	$3M_2+S_2$	873.555	116.9523127	.230
$3MK_8$	$3M_2+K_2$	875.555	117.0344500	.056
$MSNK_8$	$M_2+S_2+N_2+K_2$	883.655	117.5059710	.083
$2(MS)_8$	$2M_2+2S_2$	891.555	117.9682084	.087
$2MSK_8$	$2M_2+S_2+K_2$	893.555	118.0503457	.046
$2M\,2NK_9$	$2M_2+2N_2+K_1$	945.755	129.8887360	.036
$3MNK_9$	$3M_2+N_2+K_1$	955.655	130.4331108	.015
$4MK_9$	$4M_2+K_1$	965.555	130.9774855	.023
$3MSK_9$	$3M_2+S_2+K_1$	983.555	131.9933813	.039
$4MN_{10}$	$4M_2+N_2$	$\overline{10}45.655$	144.3761464	.051
M_{10}	$5M_2$	$\overline{10}55.555$	144.9205211	.055
$3MNS_{10}$	$3M_2+N_2+S_2$	$\overline{10}63.655$	145.3920422	.065
$4MS_{10}$	$4M_2+S_2$	$\overline{10}73.555$	145.9364169	.103
$2MNSK_{10}$	$2M_2+N_2+S_2+K_2$	$\overline{10}83.655$	146.4900752	.052
$3M\,2S_{10}$	$3M_2+2S_2$	$\overline{10}91.555$	146.9523127	.060
$4MSK_{11}$	$4M_2+S_2+K_1$	$\overline{11}83.555$	160.9774855	.033
$4MNS_{12}$	$4M_2+N_2+S_2$	$\overline{12}63.655$	174.3761464	.051
$5MS_{12}$	$5M_2+S_2$	$\overline{12}73.555$	174.9205211	.056
$3MNKS_{12}$	$3M_2+N_2+K_2+S_2$	$\overline{12}83.655$	175.4741795	.042
$4M\,2S_{12}$	$4M_2+2S_2$	$\overline{12}91.555$	175.9364169	.045

In considering the amplitudes, it is important to remember that, although a tidal prediction is a summation of cosine curves, the high and low waters occur when the sum of the first derivatives is equal to zero. In the first derivative, each constituent is weighted according to its speed. Therefore, $2MS_6$, with an amplitude of about one-half foot, contributes as much to the times of high and low waters as a diurnal constituent having an amplitude of three feet.

There are two problems concerning these results that need to be explored. First, as the number of interactions to obtain a particular frequency increases, the number of combinations that add up to this frequency also increases. For example, the Coast and Geodetic Survey uses $2\ MK_3$ for the frequency that is named MO_3 by the British. Although the speeds are the same, the node factors and phase corrections are not. In seeking to identify a peak in the high resolution Fourier plot, the tendency is to accept the first combination of constituents that satisfies the data; there may be a more logical one that has not been discovered and, in any case, the multiplicity of satisfactory combinations is bound to introduce some error due to the different corrections for the longitude of the moon's node. Inasmuch as equilibrium relationships are not necessarily valid, there does not seem to be any way to resolve the problem except, possibly, by analyzing a large number of consecutive years of data and empirically determining node corrections.

Furthermore, are these constituents part of a stationary process? That is, if the harmonic constants are used for future predictions, will they fit the phenomena at that time? This is the characteristic that made reliable tidal predictions possible many years ago. The easiest way to check this point is to compare the harmonic constants derived from different series of data. Data were not available to do this for Anchorage, but the same procedure was used with three years of Philadelphia data (1946, 1952, and 1957) for a smaller set of constituents (Table II). The set of constituents was determined from only the 1957 data.

Using subjective criteria, 16 of the original 37 constituents were unsatisfactory as compared with 9 of the added 24 constituents (roughly a similar ratio). Traditionally, the Coast and Geodetic Survey omits from its predictions analyzed constituents that have amplitudes of less than .03 foot because the phase tends to be unreliable for such small constituents. Of the original 16 poor values, six are larger than .03 foot. However, five of these are long-period constituents (Sa, Ssa, Mm, Mf, and MSf), and it has been shown (6, 7) that the continuum rises sharply in the low frequencies, making the .03-foot limit too low in this portion of the frequency spectrum. This left one unsatisfactory routine constituent, $2N_2$, and two unsatisfactory new constituents, KN_4 and MKN_6, that were greater than .03 foot. A study of all unsatisfactory new constituents showed that the sum of K_2 and N_2 appeared in six of the nine. Furthermore, there appeared to be for these six constituents a consistent pattern in the phase relationships that indicated that a very small change in

Table II. Tidal Constants; Philadelphia.

	Amplitude			Phase lag*		
	1946 (feet)	1952 (feet)	1957 (feet)	1946	1952	1957
Mm	0.187	0.067	0.048	16.3	72.2	335.0
MSf	0.166	0.056	0.146	19.8	67.9	348.8
Mf	0.045	0.123	0.108	80.8	292.6	304.6
Ssa	0.129	0.123	0.153	44.6	115.3	1.0
Sa	0.370	0.153	0.267	104.5	55.6	91.2
MP_1†	0.027	0.042	0.039	308.2	308.2	294.0
χ_1†	0.022	0.020	0.036	119.5	146.3	155.4
KP_1†	0.024	0.011	0.037	123.1	355.2	76.8
$2Q_1$	0.013	0.003	0.031	343.0	100.0	140.1
Q_1	0.030	0.051	0.031	230.3	225.2	209.3
ϱ_1	0.018	0.013	0.030	225.8	81.3	199.8
O_1	0.264	0.289	0.266	199.0	198.6	202.0
M_1	0.019	0.008	0.018	342.0	15.2	252.3
P_1	0.067	0.061	0.096	222.2	207.7	195.6
K_1	0.316	0.331	0.342	212.7	209.2	211.1
S_1	0.089	0.065	0.081	170.4	175.5	151.5
J_1	0.030	0.006	0.010	257.5	130.7	108.2

Table II. Tidal Constants, Philadelphia (continued).

	Amplitude			Phase lag*		
	1946 (feet)	1952 (feet)	1957 (feet)	1946	1952	1957
OO_1	0.010	0.010	0.003	210.2	195.9	205.2
MSN_2†	0.020	0.020	0.038	281.9	267.4	282.9
$2N_2$	0.084	0.080	0.041	332.4	59.3	48.5
μ_2	0.158	0.168	0.138	175.5	164.6	174.5
N_2	0.403	0.425	0.417	29.8	31.9	31.5
ν_2	0.143	0.145	0.120	33.9	21.8	22.4
M_2	2.583	2.676	2.506	47.1	44.8	45.3
λ_2	0.090	0.095	0.061	52.8	58.2	55.7
L_2	0.221	0.238	0.283	52.2	70.0	56.4
T_2	0.032	0.026	0.015	68.7	74.9	227.2
S_2	0.322	0.346	0.328	83.9	79.8	78.2
R_2	0.019	0.005	0.016	29.2	191.4	237.4
K_2	0.108	0.094	0.078	73.9	75.4	60.1
$2SM_2$	0.014	0.019	0.022	316.6	326.7	266.1
$2NP_3$†	0.015	0.015	0.016	143.4	337.5	8.2
NO_3†	0.022	0.019	0.024	77.0	100.8	77.8
SO_3†	0.026	0.026	0.030	145.4	125.9	131.0
M_3	0.025	0.029	0.014	194.7	170.3	37.9
MK_3	0.081	0.083	0.068	124.7	120.9	125.6
$2MK_3$	0.080	0.088	0.068	97.1	95.8	95.7
ML_4†	0.046	0.061	0.056	343.5	348.4	337.3
SL_4†	0.014	0.019	0.028	26.6	51.8	26.2
MK_4†	0.032	0.034	0.026	17.5	16.4	6.2
MN_4	0.121	0.122	0.114	336.0	334.3	330.9
MS_4	0.098	0.115	0.095	33.6	26.3	25.3
M_4	0.345	0.378	0.311	350.3	346.1	346.1
S_4	0.007	0.006	0.010	359.4	328.8	73.5
MNO_5†	0.022	0.022	0.017	349.0	342.0	336.7
$2MO_5$†	0.044	0.053	0.039	355.3	342.0	346.7
$2MP_5$†	0.028	0.031	0.021	28.1	26.6	7.9
$2MK_5$†	0.051	0.055	0.045	7.8	0.5	358.1
MNK_5†	0.018	0.023	0.020	49.3	313.9	326.0
$2MN_6$†	0.076	0.071	0.075	195.5	188.4	179.2
$2ML_6$†	0.046	0.060	0.061	205.1	209.0	189.0
$2MS_6$†	0.063	0.068	0.059	243.4	239.0	229.5
MSL_6†	0.018	0.024	0.033	235.7	272.2	245.7
S_6	0.004	0.000	0.005	324.7	297.0	33.6
M_6	0.148	0.159	0.138	206.4	205.0	196.4
$2(MN)_8$†	0.018	0.022	0.015	108.4	124.2	118.1
$3MN_8$†	0.048	0.045	0.041	131.7	129.0	120.4
$3ML_8$†	0.024	0.033	0.025	153.5	130.2	117.1
$3MS_8$†	0.038	0.041	0.033	187.1	174.3	171.9
$2MSL_8$†	0.012	0.020	0.021	183.7	195.9	178.4
M_8	0.059	0.067	0.052	146.3	146.8	140.9

* Referred to 75°W (g).

† Not included in original 37 constituents.

speed could make the harmonic constants acceptable. The sum of M_2 and L_2 varies from the sum of K_2 and N_2 by .009°/hr (one cycle in about 4.5 years). These speeds are too close together to be separated with only one year of data. When the change in speed was made on the six constituents and the data were reanalyzed, the constants for all became satisfactory, leaving only 3 of the added 24 constituents unsatisfactory; all three are less than .03 foot. Furthermore, a new spectral analysis of the residuals showed a lower level of energy in species 4, 6, and 8—the species containing the six modified constituents—thereby indicating an improvement in fitting the data by virtue of the changed speeds.

In retrospect, it is now obvious why the combination of M_2 and L_2 should have been given preference over N_2 and K_2. In equilibrium theory, the four largest semidiurnal constituents are M_2, S_2, N_2, and K_2, in that order. L_2 is only 22°/₀ of K_2. Hence, the choice of species-2 constituents was limited to the first four in trying various combinations to match the frequencies where high resolution Fourier analysis showed large peaks. However, the analysis for Philadelphia shows L_2 to be roughly three times larger than K_2. Therefore, it is obvious that larger interactions should be expected from the sum of M_2 and L_2 than from N_2 and K_2.

Essentially then, 3 of 24 new constituents are unsatisfactory compared with 16 of the original 37—an amazing improvement considering that the 37 include the major constituents that are not subject to question. These results raise questions as to whether some corrections in the speeds of the smaller standard constituents may be indicated. At one time the manpower requirements for successive analyses of a large number of years of data made such a study virtually impossible. The present availability of both computer programs for analysis and data in a format compatible with computers makes such a study now only a modest effort; plans for future tidal research in ESSA will include this study.

Tables III and IV show comparisons of observations and predictions for Anchorage and Philadelphia, respectively. If the time differences were smaller and if the ratio of observed range to predicted range were closer to 1.00, the fit would be better. Tide level minus sea level is a measure of

Table III. Comparisons of observations and predictions for May, July, and October 1964; Anchorage.

No. of constituents in predictions	37	114
Time differences (pred.-obs.)		
High water (hours)	−0.13	−0.03
Low water (hours)	−0.35	−0.19
Ratio, obs./pred.		
Mean range	1.05	1.01
Tide level minus sea level		
Observed (feet)	−1.09	−1.09
Predicted (feet)	− .70	−1.00
Residual variance (192 days, 1964) in feet²	.9767	.2954
°/₀ of original variance	1.33	0.40

Table IV. Comparisons of observations and predictions for January, March, July, and October 1957; Philadelphia.

No. of constituents in predictions	37	61
Time differences (pred.-obs.)		
High water (hours)	−0.18	−0.14
Low water (hours)	−0.33	−0.28
Ratio, obs./pred.		
Mean range	1.06	1.04
Tide level minus sea level		
Observed (feet)	−0.23	−0.23
Predicted (feet)	−0.12	−0.14
Residual variance (355 days, 1957) in feet²	.3467	.3299
% of original variance	8.49	8.08

the distortion of the curve from a pure cosine curve, primarily due to the contributions of compound constituents. An optimum fit would show identical values for observed and predicted values. The residual variance, which should be as small as possible, is significant only as a comparative number. The absolute numbers depend primarily on the range of tide and the energy in the meteorological continuum. The improvement in practical predictions by using more constituents is clearly shown in the results presented in Tables III and IV.

The use of "harmonic" in the title warrants some explanation. Even though annual or seasonal node corrections (amplitude factors and phase corrections) are applied to lunar constituents by all organizations preparing tidal tables, the predictions are ordinarily regarded as harmonic if subsequent corrections are not added to the derived times, the heights of high and low waters, or both. It is used in the title of this paper in this sense to differentiate the procedure from Doodson's shallow-water technique (4). Doodson (2) developed a purely harmonic set of constituents (calling the usual method, "quasi-harmonic"), but he never used the method for practical tidal predictions.

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