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# *Small-scale, Horizontal Vortices in the Surface Layer of the Sea<sup>1</sup>*

Robert W. Owen, Jr.

*U.S. Bureau of Commercial Fisheries  
Tuna Resources Laboratory  
La Jolla, California*

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## ABSTRACT

A vortex system in the water near the sea surface has been inferred from collection into parallel rows of a positively phototactic zooplankton population. Water velocity in the upper part of convergence planes of the system has been equated with estimated swimming speed of the animals.

A thermohaline driving force was inferred to have produced the circulation system, and the depths to which the vortices extended were derived from heat-flux observations. The computed depth of 61 cm is close to the predicted depth of 64 cm.

*Introduction.* This paper describes a vortex system encountered at sea. Meteorological conditions and the small scale of the circulation suggest that it was a consequence of thermal energy exchange across the sea surface. Vertical velocity in the circulation has been estimated by observing the swimming rate of a zooplankter, and the depth to which the circulation extended has been derived from meteorological and oceanographic observations.

The accumulation of materials in parallel rows at the sea surface often has been attributed to their concentration at convergence lines of horizontal, parallel vortices (e.g., Bainbridge 1957, Sutcliffe et al. 1963, Faller and Woodcock 1964). Two general mechanisms have been proposed for vortical cell generation in the surface layers of the sea: Langmuir (1938) and subsequent authors attributed rows of foam, leaves, and the buoyant alga *Sargassum* to their collection along convergence lines of wind-generated vortices, whereas Montgomery et al. (1947) examined the nature of cells in free convection, driven by thermal energy exchange. To date, no specific mechanism has been proposed to account for vortical cell generation by wind, nor has an observational study been made that distinguishes between the two modes of generation.

Suspension of a particle near the surface in a convergence plane requires that the intrinsic ability of the particle to move upward (by positive buoyancy

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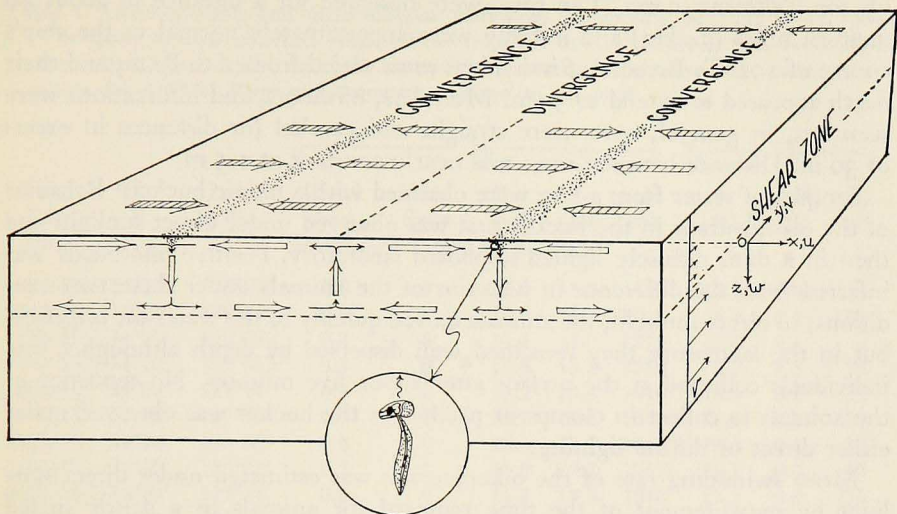


Figure 1. Water motion in the  $x, z$  plane of a vortex system and the resulting distribution of upward-moving particles introduced at the system boundaries. The selection of  $\partial v / \partial x > 0$  is arbitrary.

or upward swimming) be greater than the extrinsic tendency to its downward transport by the drag of the surrounding fluid. All such particles in the region of a vortex system will of necessity collect near the surface along convergence sites (Fig. 1). By contrast, to concentrate particles near the surface along convergences requires that the particles only possess this intrinsic tendency, no matter how small their upward movement relative to the fluid may be. In the case where the intrinsic tendency is small and the particle velocity is close to the fluid velocity, particles are nowhere stationary and are present elsewhere in the vortex as well as at the convergence site.

The vertical component of water velocity,  $w$ , is zero at the upper and lower limits of a vortex system. Between these limits, in each of the convergence planes,  $w$  attains a single maximum. If a nonbuoyant plankton population is found in these convergence planes and is absent elsewhere in the system, then its members are presumably swimming up at the same speed as the downward flow at their depth in the plane. Further, if  $w$  may be said to approach its maximum value within a short distance,  $\delta$ , of the sea surface, then the swimming speed of members of the above population that are located at  $z > \delta$  must numerically equal the maximum value of  $w$  in the vortices.

*Observations.* Rows of dense reddish concentrations of zooplankton organisms, consisting mainly of *Oikopleura longicauda* (Vogt), were observed during calm weather from an oceanographic vessel about 180 nautical miles (335 km)

off the California coast. The rows were observed for a distance of about 40 nautical miles (74 km) and initially were approximately normal to the ship's course of  $102^{\circ}\text{T}$ . Breadth of individual rows varied from 2 to 8 cm, and their depth appeared to extend to 5 cm. Meanders, offshoots, and bifurcations were seen but, in general, rows were straight and parallel for distances in excess of 30 m. Distance between rows was nearly constant at 1.5 m.

Samples of water from a row were obtained with a plastic bucket.<sup>1</sup> Behavior of the oikopleurans in the bucket first was observed under direct sunlight and then in a dim, diffusely lighted shipboard laboratory. Positive phototaxis was inferred from the difference in behavior of the animals under these two conditions; in direct sunlight, the animals moved quickly to the water-air interface, but in the laboratory they remained well dispersed by depth although a few individuals collected at the surface after about five minutes. No tendency of the animals to collect in clumps or patches in the bucket was observed under either direct or diffuse lighting.

Mean swimming rate of the oikopleurans was estimated under direct sunlight by measurement of the time required for animals in a gently mixed sample to reach the water-air surface. After most of the water motion stopped, animals still appeared to be well distributed through the 25-cm depth of water in the bucket. Their ascent appeared to proceed at a uniform rate; no animal was seen to pass or lag behind others. In two minutes, 13 to 15 *Oikopleura* were seen just below the 0.5-cm surface layer; the remaining ones, about 85, were within this layer. If we assume (i) initial uniform dispersion of animals throughout the water depth, and (ii) no large-scale vertical water motion in bucket, at least five of the 20 *Oikopleura* present initially in the lowest 5 cm of the bucket arrived in the surface layer within two minutes; hence each of these five animals swam at an average rate of at least  $0.2 \text{ cm sec}^{-1}$ . Furthermore, the uniform swimming speed indicated that the 13 to 15 *Oikopleura* seen just below the 0.5-cm surface layer after two minutes had in fact begun their trip from the lowest 5 cm of the bucket. Therefore, each of these swam at an average rate of less than  $0.2 \text{ cm sec}^{-1}$  but more than  $0.16 \text{ cm sec}^{-1}$ . The best estimate of swimming speed of this isolated population thus appears to be  $0.2 \text{ cm sec}^{-1}$ .

Selected oceanographic and meteorological observations made on June 16, 1964, the day the rows were observed, are summarized in Table I. For comparison, data taken the day before and the day after observation of the rows are included. The ship was underway between oceanographic and meteorological observations, and therefore the data must be regarded in terms of space as well as time. Bathythermograms provided the main source of subsurface data.

Wind speed, which was essentially zero at the ship for 24 hours before rows were sighted at 1230 PST, June 16, increased shortly thereafter. Greater

1. Samples collected at 1540 PST, June 16, 1964,  $33^{\circ}48' \text{N}$ ,  $124^{\circ}45' \text{W}$ , from R/V BLACK DOUGLAS of the U.S. Bureau of Commercial Fisheries.

Table I. Oceanographic and meteorological conditions at the ship on June 15-17, 1964. The quantity  $\Delta_z\theta$  measures extent of the transient thermal structure (see text),  $D_p$  is the mixed-layer depth, and  $p$  is barometric pressure. Air observations made at 6 m above sea surface. Barometric pressure adjusted to sea level.

Time-Date PST	Lat. N.	Long. W.	Air					Sea		
			Cloud cover fraction	Wind force Beaufort	Dry bulb $\theta$ °C	Wet bulb $\theta$ °C	$p$ mb	$\theta_0$ °C	$\Delta_z\theta$ °C	$D_p$ m
0600-15	34°33'	129°09'	0.8	0	15.6	15.3	1019	15.7	0.0	38
0900-15	34°29'	128°53'	0.8	2	15.5	14.6	1019	16.2	0.4	35
1210-15	34°25'	128°29'	0.8	0	16.4	14.0	1019	16.0	0.1	-
1500-15	34°20'	128°05'	0.3	0	18.3	15.3	1018	18.3	2.3	33
1800-15	34°20'	127°44'	0.7	0	17.5	15.3	1018	19.3	3.2	35
2055-15	34°15'	127°19'	0.7	1	16.7	14.4	1018	17.2	1.0	32
2330-15	34°10'	126°50'	0.8	0	16.6	13.7	1018	16.5	0.4	38
0200-16	34°05'	126°23'	0.7	1	15.8	13.7	1017	16.1	0.0	42
0630-16	34°00'	125°56'	0.8	0	15.3	13.9	1018	15.7	0.0	35
1015-16	33°54'	125°32'	0.6	0	17.8	15.6	1018	16.2	0.3	36
1400-16	33°51'	125°01'	0.2	2	16.1	13.3	1018	16.7	2.2	38
1730-16	33°46'	124°28'	0.3	3	16.0	14.0	1018	15.6	1.2	35
1930-16	33°42'	124°16'	0.2	2	15.3	13.4	1018	15.3	0.5	(30)
2320-16	33°37'	123°38'	0.3	4	15.0	13.4	1018	16.1	0.1	33
0230-17	33°32'	123°18'	0.2	4	15.8	13.9	1017	15.8	0.0	30
0600-17	33°25'	122°40'	0.0	4	15.3	13.9	1017	14.3	0.0	(50)
1000-17	33°21'	122°16'	0.8	5	15.6	14.2	1019	14.3	0.1	(32)
1500-17	33°07'	121°32'	0.7	4	16.7	15.3	1017	14.9	0.2	44
1800-17	33°02'	121°02'	0.7	5	15.4	13.5	1016	14.4	0.1	40

irregularities in individual rows accompanied the increase in wind speed, but rows persisted at least until sunset on the 16th.

During the day, a near-surface transient thermal discontinuity was encountered. The thermal magnitude of the discontinuity was estimated by the difference ( $\Delta_z\theta$ ) between sea-surface temperature ( $\theta_0$ ) and the zero-depth temperature extrapolated from bathythermograms (see insert on Fig. 2). Therefore, the quantity  $\Delta_z\theta$  is the difference between the sea-surface temperature observed and that to be expected if no transient structure had developed. Data in Fig. 2, depicting diurnal development and decay of the transient near-surface thermal structure, suggest that the structure was not some steady-state phenomenon. These data permit development of the transient thermal structure on a nearly calm day to be compared with its development on a windy day.

The depth of the transient thermocline could not be measured from bathythermograms with sufficient accuracy to determine heat exchange across the sea surface; consequently, this exchange was estimated from the meteorological observations. Incoming solar radiation was monitored with a 10-junction Eppley pyranometer; the resulting insolation curve was integrated with respect

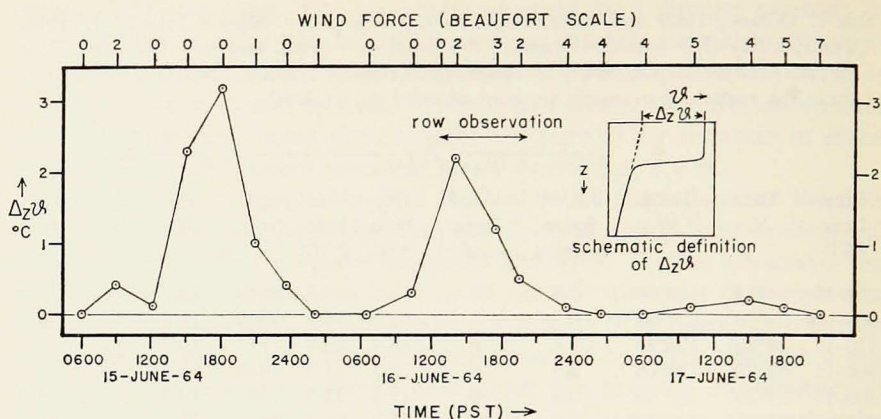


Figure 2. Development and decay of transient thermal structure, June 15-17, 1964. Points represent BT observations.

to time, and a reflectance correction was applied to express  $Q_r$ , net short-wave radiation penetrating the sea surface. Corrections for reflectance were obtained from Sverdrup et al. (1942: table 26) and were applied in the form

$$Q_r = Q_t (1 - \bar{r}),$$

where  $Q_t$  is the recorded direct and diffuse insolation rate and  $\bar{r}$  is the average reflectance over the integration period.

Effective back-radiation rate, the difference between rates of long-wave radiation from the sea surface and long-wave radiation from the atmosphere, was computed from the formula reported by Budyko (1956: 46):

$$Q_b = s\sigma T_a^4 (0.39 - 0.058 [e_s]^{1/2}) (1 - kC^2) + 4s\sigma T_a^3 (T_s - T_a),$$

where the following definitions obtain:

- $s$ , ratio of emittance of sea surface to that of a black-body = 0.97;
- $\sigma$ , Stefan-Boltzmann constant =  $7.888 \times 10^{-11}$  cal  $\text{cm}^{-2} \text{ } ^\circ\text{K}^{-4} \text{min}^{-1}$ ;
- $T_s, T_a$ , absolute temperature of sea, air,  $^\circ\text{K}$ ;
- $e_s$ , saturation vapor pressure at sea surface corrected for salinity effect, mb;
- $k$ , function of vertical development and base height of clouds, here = 0.64;
- $C$ , fraction of sky covered by clouds.

Heat-exchange rate by evaporation,  $Q_e$ , was computed from an expression adapted from Montgomery (1940):

$$Q_e = L_\theta E = L_\theta \rho_a \kappa_o^2 \Gamma_E^2 (0.623 \frac{\Delta e}{p}) \bar{U}_a,$$

where the following definitions obtain:

- $L_\theta$ , latent heat of evaporation =  $596 - 0.52 \vartheta$  cal gm<sup>-1</sup>°C<sup>-1</sup>;  
 $E$ , evaporation, gm cm<sup>-2</sup>min<sup>-1</sup>;  
 $\rho_a$ , air density =  $1.2 \times 10^{-3}$  gm cm<sup>-3</sup>;  
 $\kappa_o$ , Karman constant = 0.4;  
 $\Gamma_E$ , coefficient for expression of humidity gradient evaluated from Deacon and Webb (1962: fig. 17); at low wind-speed values = 0.1, and at higher wind speeds encountered ( $> 2$  Beaufort)  $\div 0.085$ ;  
 $\Delta e = e_s - e_a$ , vapor pressure at sea surface and height  $a$  (ca. 6 m), respectively [ $0.623 (\Delta e/p) = \Delta q$ , specific humidity gradient];  
 $p$ , barometric pressure at sea surface, mb;  
 $\bar{U}_a$ , wind speed at height  $a$  (ca. 6 m), cm sec<sup>-1</sup>.

Conductive heat-exchange rate,  $Q_c$ , was computed from the evaporative exchange rate by the Bowen ratio,  $R$  (Bowen 1926):

$$Q_c = RQ_e,$$

where  $R = 0.66 \times 10^{-3} p (\Delta \vartheta / \Delta e)$ , ( $\Delta \vartheta = \vartheta_o - \vartheta_a$ , sea temperature less air temperature, °C).

The net heat-exchange rate across the sea surface due to the above processes is then

$$Q_\Sigma = Q_r + Q_b + Q_e + Q_c, \quad (1)$$

wherein the positive sense denotes heat gained by the sea. Computed values for each term above are presented in Table II.

*Probable Causes of Row Formation.* Two mechanisms other than convergent flow between cells may produce parallel rows of particles at the sea surface. Roughly isometric patches of plankton, formed initially at the sea surface by behavioral mechanisms, may be drawn out into rows in the presence of current shear. This possibility is discounted in the situation described here, since observed row spacing was nearly constant—not an expected result of this mechanism. Bary (1953) suggested that some organisms may be aggregated at the surface in slicks formed by internal waves. Narrow rows of organisms were observed, however, whereas broad rows would be expected in this circumstance. Furthermore, characteristic wave lengths of 1.5 m are too short for standing internal waves.

Since wind speed was zero when the rows were sighted, the phenomenon was not the wind-driven Langmuir circulation. Furthermore, it is not likely

Table II. Heat budget in  $\text{cal cm}^{-2} \text{min}^{-1}$  on June 15-17, 1964 in the vicinity of  $33^{\circ}45' \text{N}$ ,  $124^{\circ}30' \text{W}$ . Positive quantities denote heat gained by sea, negative quantities denote heat lost by sea. Symbols explained in text.

Time-Date PST	$Q_r^*$	$Q_b$	$Q_e$	$Q_c$	$Q_{\Sigma}$
0600-15	0.030	-0.049	0	0	-0.019
0900-15	0.518	-0.053	-0.007	0.003	0.461
1210-15	1.048	-0.046	-0.005	-0.002	0.995
1500-15	0.960	-0.071	-0.007	0	0.882
1800-15	0.115	-0.062	-0.022	0.008	0.039
2055-15	0	-0.057	-0.012	0.002	-0.067
2330-15	0	-0.048	-0.010	-0.004	-0.062
0200-16	0	-0.054	-0.009	0.001	-0.062
0630-16	0.066	-0.051	-0.016	-0.002	-0.003
1015-16	1.067	-0.049	-0.008	0.007	1.017
1400-16	1.183	-0.081	-0.033	-0.003	1.066
1730-16	0.324	-0.074	-0.017	0.002	0.235
1930-16	0	-0.080	-0.024	-0.001	-0.105
2320-16	0	-0.085	-0.017	-0.003	-0.105
0230-17	0	-0.080	-0.020	0	-0.100
0600-17	0.141	-0.072	0.009	-0.014	0.064
1000-17	0.660	-0.042	0.019	-0.024	0.613
1500-17	0.687	-0.047	0.029	-0.025	0.644
1800-17	0.174	-0.053	0.004	-0.018	0.107

\* By integration of insolation curve over one-hour period around time stated.

that the observed cells represented the remnant of a motion deprived of its driving force; circulation in a vortex may be shown to damp exponentially by viscous effects upon removal of the driving force (see Appendix, p. 66). For the circulation to have been wind-driven, therefore, a lull in the wind would have had to precede the moving ship by a short and constant distance. This event was improbable; flat surface conditions did not support such a contention. Also, the observed row spacing (1.5 m) was an order of magnitude less than the shortest spacing (20 m) reported by Faller and Woodcock (1964) for Langmuir circulations. These authors further observed that rows do not form with winds of Beaufort 2 or less (p. 27). Apparently, therefore, a driving force other than wind stress was acting on the system at the time of observation.

An alternative mechanism is the so-called thermohaline effect. By its effect on local density, transfer of heat and of water across the sea surface apparently was sufficient to produce the observed vortex system, or at least to sustain it after relaxation of wind stress. Formation of denser surface water is possible in spite of the heating effect of insolation. This is because short-wave solar radiation is absorbed throughout a few meters of water whereas the direct effects of long-wave back radiation and evaporation are confined to a surface layer about 1 mm thick. The form of cells in free convection, described by Bénard (1900),



is generally hexagonal, with the sources (divergences) at the cell centers and the sinks (convergences) along the six-sided edges. To produce the elongated form of the cells observed, shear in the direction of the long axes is required. Since negligible shear at the air-sea interface was to be expected, the energy required to produce the observed strip-cell configuration must have been supplied at the lower boundary of the circulation. This is possible if, for example, the surface layer previously had been set in motion by wind stress. The expected tendency for cell axes to veer with inertial period (0.5 pendulum day) was not noted, possibly because the rows were observed for less than 1.5 hours under windless conditions. The direction of the wind that arose by 1400 PST (Fig. 2) prescribed thereafter the orientation of the rows. The extent to which the system of vortices was influenced by shear cannot be specified, and since the depth of the transient thermal structure could be established only once, no statement may be made as to whether the two sets of forces were in equilibrium.

*Characteristics of the Vortices.* The swimming velocity of oikopleurans found in the vortex system has been estimated to be  $-0.2 \text{ cm sec}^{-1}$ . An additional remark, that none of these highly visible animals was seen between convergence lines, permits the inference that the sampled population consisted only of oikopleurans that were successfully resisting downward transport by the water. Since some oikopleurans were seen at 5 cm below the sea surface, vertical velocity of water,  $w$ , at this depth did not exceed  $0.2 \text{ cm sec}^{-1}$ . Further, if the distance within which  $w$  approached its single maximum value in the convergence plane was 5 cm or less, then  $w_{\text{max}} \approx 0.2 \text{ cm sec}^{-1}$  in this plane.

It is likely that the depth of the transient thermocline marked the lower limit of convective circulation on June 15 and 16. Therefore, if shear was applied at the lower boundary of the convecting cells, then this thermocline was the shear zone. Its nocturnal disappearance apparently was due to heat loss through the sea surface from the whole of the convecting layer. Diminution of the thermal magnitude,  $\Delta_2\theta$ , of the transient structure was obvious on the 17th and may be attributed to a surface-heating rate that barely exceeded the rate of downward transfer by wind mixing.

Components of the heat-budget equation may be grouped according to whether their instantaneous effect is confined to the upper centimeter or so of the water column. Under this convention, equation (1) becomes

$$Q_{\Sigma} = Q_r + Q_s,$$

where  $Q_s = Q_b + Q_e + Q_c$ , all essentially surface phenomena. In the absence of short-wave radiative transfer, i.e., during hours of darkness,  $Q_r \approx 0$  and  $Q_{\Sigma} = Q_s$ .

On June 16, 1964, between 1930 and 2320 (PST), the average rate of net heat transfer in this area was

$$\bar{Q}_\Sigma = \bar{Q}_s = -0.105 \text{ cal cm}^{-2} \text{ min}^{-1}.$$

Over the same period, the temperature of water overlying the shallow transient thermocline decreased by  $0.4^\circ\text{C}$ .<sup>2</sup>

Under the assumptions that temperature change in the convecting layer resulted from heat flux across only the air-sea boundary and that the convecting layer was isothermal from the sea surface to some depth,  $z_*$  (thermocline depth), this depth to which the change in thermal magnitude,  $\Delta_t(\Delta_z\vartheta)$ , took place had to be

$$z_* = -\frac{tQ_\Sigma}{c_p\rho\Delta_t(\Delta_z\vartheta)} = 61.6 \text{ cm}, \quad (Q_r = 0)$$

where  $c_p = 0.956 \text{ cal gm}^{-1}^\circ\text{C}^{-1}$ ,  $\rho = 1.0246 \text{ gm cm}^{-3}$  and  $t = 230 \text{ min}$ . Bathythermograms substantiate that this layer of water was isothermal. The depth  $z_*$  represents the lower limit of the vortical convective circulation during this period. The value of  $z_*$  calculated above is close to the depth derived from the theoretical ratio of cell width to depth reported by Stommel (1947: table II) for strip cells having one free and one rigid boundary. For the observed cell width  $l = 150 \text{ cm}$ , cell depth,  $z_*$ , is predicted from Stommel's ratio to be 64.1 cm. These cell dimensions thus appear to be specified adequately by the theoretical method. Alternatively, the close agreement probably means that these semiempirical heat-budget calculations adequately describe net heat exchange at the sea surface.

The meteorological conditions recorded where the cells were observed are not unusual for this area in summer. Therefore, not considering the small heat loss from evaporation, heat fluxes similar to that on June 16 (Table II) are not unusual. It is, therefore, possible that vortex systems of the type observed occur widely and with sufficient frequency that their biological and physical consequences should not be ignored when considering near-surface oceanographic problems on small time scales. Masking and modification of the circulation by wind effects and absence of visible particles that can mark convergence lines may account for previous lack of comment on the widespread nature of the phenomenon.

*Summary.* Concentrations of the zooplankter, *Oikopleura longicauda* (Vogt), were observed along lines of convergent surface-water flow. Positive phototaxis observed in these animals aboard ship confirmed the possibility suggested by Bainbridge (1957) that this combination of physical and behavioral processes results in plankton concentration in parallel rows.

2. Measured by  $\Delta_z\vartheta_{1930} - \Delta_z\vartheta_{2320} = \Delta_t(\Delta_z\vartheta) = 0.4^\circ\text{C}$ .

Estimated swimming speed of the animals was equated with downward water velocity in a vortex pattern. This velocity was  $0.2 \text{ cm sec}^{-1}$ .

Absence of wind suggests that the Langmuir circulation is unlikely; the thermohaline driving force is indicated as an alternate mechanism to produce the circulation.

Depth of the vortices was computed by equating observed temperature changes in the convecting layer with heat exchange across the sea surface. The result (61 cm) is close to the depth predicted from theory (64 cm).

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## APPENDIX

Viscous damping of motion in a vortex can be shown by a development from the Navier-Stokes equation. Assumptions of incompressibility and constant viscosity of the fluid permit this equation to be written

$$\frac{\partial \vec{v}}{\partial t} - \vec{v} \times \vec{\zeta} = -\nabla \left( \frac{p}{\rho} + \frac{v^2}{2} - \Omega \right) + \nu \nabla^2 \vec{v}.$$

Taking the curl,

$$\frac{D\vec{\zeta}}{Dt} = (\vec{\zeta} \nabla) \vec{v} + \nu \nabla \times (\nabla^2 \vec{v}).$$

Noting that  $\nabla \vec{\zeta} = 0$  and hence that  $\nabla \times (\nabla^2 \vec{v}) = \nabla^2 \vec{\zeta}$ , this becomes

$$\frac{D\vec{\zeta}}{Dt} = (\vec{\zeta} \nabla) \vec{v} + \nu \nabla^2 \vec{\zeta}. \quad (1)$$

Under the approximations that the motion is circular about the vortex axis, and that velocity is a function of distance,  $r$ , from the axis, we obtain from (1)

$$\frac{\partial \zeta}{\partial t} = \nu \left( \frac{\partial^2 \zeta}{\partial r^2} + \frac{1}{r} \frac{\partial \zeta}{\partial r} \right),$$

which is satisfied by the solution

$$\zeta = \frac{\nu r}{4 \nu t} \exp \left( -\frac{r^2}{4 \nu t} \right).$$

Circulation after time  $t$  is then

$$C_t = \int_0^r \zeta 2\pi r dr = \pi r v_0 \left[ 1 - \exp \left( -\frac{r^2}{4 \nu t} \right) \right].$$

This equation exposes the exponential nature of damping of the circulation from its initial value,  $C_0 = \pi r v_0$ . Rearranging terms to evaluate the time factor, we get

$$t = -\frac{r^2}{4 \nu} \frac{1}{\ln [1 - (C_t/C_0)]}. \quad (2)$$

A reasonable value for initial velocity in the convergence plane of a Langmuir circulation is  $1 \text{ cm sec}^{-1}$  (e.g., Langmuir 1938) so that, in a radius of  $50 \text{ cm}$ ,  $C_0 = 157 \text{ cm}^2 \text{ sec}^{-1}$ . After time  $t$ , this velocity is  $0.2 \text{ cm sec}^{-1}$  so that  $C = 31.4 \text{ cm}^2 \text{ sec}^{-1}$ . Taking  $1 \text{ cm}^2 \text{ sec}^{-1}$  for the eddy value of kinematic viscosity,  $\nu_e$ , we find that the time required for the circulation to damp to its observed state is 0.8 hour. Although other values of eddy viscosity are available for use in (2), it is seen that, in any case other than laminar, the circulation damps to some part (here, 20%) of its initial value in a rather short time.