A Note on the Western Intensification of the Oceanic Circulation[1]

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The purpose of this note is to provide a simple physical explanation for the westward intensification of the oceanic circulation found in the several dynamically different existing theoretical models (e.g. Stommel 1948, Carrier and Robinson 1962). It has the advantage of showing why the *western* oceanic boundary is singled out as the boundary-layer region that closes the interior Sverdrup solution.

Consider the fundamental characteristics of Rossby waves. If β is the northward derivative of the Coriolis parameter, k the component of the wave-number vector in the eastward direction, l the wave-number component in the northward direction, then the frequency, ω, of a Rossby wave in a resting fluid on the β plane is (Rossby et al. 1939)

$$\omega = -\frac{\beta k}{k^2 + l^2}, \tag{1}$$

while the group velocity in the eastward direction, C_{gx}, is

$$C_{gx} = \frac{\partial \omega}{\partial k} = \beta \frac{(k^2 - l^2)}{(k^2 + l^2)^2}. \tag{2}$$

Thus energy at small scales ($k^2 > l^2$) will be transmitted to the east while energy at large scales ($k^2 < l^2$) will move to the west. For slow, large-scale geophysical systems, such as the ocean, the group velocity of Rossby waves is the significant signal velocity.

Now, suppose energy of varying scales is put into the ocean, perhaps by the action of a wind stress. The short-scale components will move toward

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the eastern boundary of the ocean, where they will be reflected as components of a large east-west scale. Meanwhile, the large-scale components will move toward the western boundary, where they will be reflected as small-scale motions. Thus it is the *western boundary* that acts as a *source* of small-scale motions. This is the fundamental reason for the preference for the westward intensification.

Let us consider some examples in detail. In Stommel's original theory, the scale of the western boundary current is

$$L = \frac{R}{\beta D}, \tag{3}$$

where R/D is the time for the decay of the motion due to bottom friction. We can derive this length scale on the basis of the simple argument presented above.

In the dissipative time $[R/D]^{-1}$, a short ($k >> l$) wave component generated at the western boundary can travel a distance $L = C_{gx}[R/D]^{-1}$ before it is dissipated. Now, $C_{gx} \sim \beta/k^2$ (if $k^2 >> l^2$), so that $C_{gx} \sim \beta L^2$. Therefore,

$$L = \beta L^2 \left[\frac{R}{D}\right]^{-1}, \tag{4}$$

or

$$L \sim \frac{R}{\beta D}, \tag{5}$$

which is the same as (3). Scales of motion larger than L will be essentially unaffected by bottom friction and will be propagated into the interior. Stommel's model contains a *western* boundary current because all small-scale motions generated at the western boundary are dissipated before they can appear in any other region of the ocean.

In the inertial theories, on the other hand, the scale of the western-boundary current is (Carrier and Robinson 1962).

$$L = \sqrt{\frac{-U}{\beta}}, \tag{6}$$

where U is the zonal velocity induced at the western boundary by the direct action of the wind stress; it is measured positive for an eastward flow. The theory predicts (in the absence of bottom topographical effects) a western boundary layer only if $U < 0$, i.e. only if the flow is toward the west. Let us consider how this is contained in the simple group-velocity argument. The short-wave components generated at the western boundary will have their intrinsic signal velocity, C_{gx}, augmented by the drift velocity, U, so that the total group velocity eastward, $\bar{C}_{gx}$, in the presence of the Sverdrup advection, is given by

$$\bar{C}_{gx} = C_{gx} + U \tag{7}$$

or

$$\bar{C}_{gx} \sim \beta L^2 + U. \tag{8}$$

All scales of motion for which $\bar{C}_{gx} \leqq 0$ will be trapped at the western boundary, i.e. scales for which

$$L \leqq \sqrt{\frac{-U}{\beta}}. \tag{9}$$

The maximum scale trapped in the western region (which fixes the size of the region of intensification) is given by the equality and is identical to (6). This explanation also reveals why the, by now, classical difficulty in the inertial theory arises where $U > 0$. For in such cases the short-wave components generated at the western boundary, rather than being trapped there, are actually encouraged to leave the western boundary of the ocean and are radiated into the oceanic interior as Rossby waves.

In fact, one way to view the various theories of the Gulf Stream is to classify them according to the mechanism (viscous or inertial) used to prevent motions of a boundary-layer scale, which are most naturally generated at the western boundary, from "leaking" into the oceanic interior.

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