A Technique for the Direct Measurement of Transport with Application to the Straits of Florida[1]

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ABSTRACT

A method is described by which the volume transport per unit width of a water column is obtained from measurements of the run time, depth, and horizontal deflection of a freely falling instrument. Measurements of the north-south component of transport in the Straits of Florida using this technique gave $35.5 \pm 1.2 \times 10^6$ m³/sec on August 16 and 17, 1964. The surface currents and east-west components of transport are also given.

I. *Introduction.* This paper describes a technique for the direct measurement of transport and presents data on the flow through the Straits of Florida based on this technique.

Consider an instrument that falls freely to a preselected depth, where it releases its ballast weights and returns to the surface under its own buoyancy. If the speeds of fall and rise are constant (not necessarily equal), and if stationary conditions over the time and space intervals of the instrument's fall and rise are assumed, it is easily shown that, for any distribution of horizontal water velocity, $\vec{v}$, with depth, z,

$$\vec{T} = \int_0^D \vec{v}\,dz = \vec{R}D/t\,, \tag{1}$$

where $\vec{T}$ is the transport per unit width to depth D, $\vec{R}$ is the horizontal separation of the drop and surfacing points, and t is the total time of the run. It is clear that this type of measurement will be neither instantaneous in time nor localized at a point in space. Time and space scales will be discussed in § IV.

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The transport can then be measured with an instrument that attains terminal velocity within a few meters after release, records pressure (depth) internally as a function of time, and casts off ballast at the bottom (or at some prescribed mid-depth) by means of a release mechanism. Of course, the instrument must be used with a navigational system that is capable of measuring the horizontal deflection to the desired accuracy.

In this paper, per-unit-width transport is considered to be the fundamental measurable. Since the vertically averaged velocity, $\vec{v}_{avg}$, is

$$\vec{v}_{avg} = 1/D \int_{0}^{D} \vec{v}\, dz = \vec{R}/t \; ; \tag{2}$$

one could alternatively regard vertically averaged velocity as the fundamental measurable. The choice here is a matter of personal preference. Using the hydrostatic approximation, one can estimate mass transport per unit width, $\vec{T}_m$, by

$$\vec{T}_m = \int_{0}^{D} \varrho \vec{v}\, dz = 1/g \int_{0}^{D} \vec{v}\, dP = \vec{R}P/gt \, , \tag{3}$$

since pressure is recorded directly. However, the accuracy of the pressure measurement used is an order of magnitude too low to resolve variations in (3) because of the actual density variations encountered.

II. *Instrument and Methods.* The instrument consists of an aluminum pipe 15 cm (6 in.) diameter and 150 cm (5 feet) long, fitted with flat end caps that are streamlined by addition of free-flooding fiberglass nose and tail cones (Fig. 1). Within the pressure case is a 16-mm camera that takes time-lapse pictures of a pressure gauge, an electric wrist watch, and a mercury thermometer. The instrument has a net buoyancy of 9 kg and weighs about 23 kg in air. When this instrument is used for transport measurements, the lower end of it carries a 2-m rope attached to a 4.5-kg weight that leads to another 2-m rope, where the release mechanism is attached. This weight provides vertical stability during the fall and ascent and is separated from the instrument case for ease of handling.

Two types of release mechanism are used. The first is a simple lever arrangement that drops two 4.5 kg weights when the weights contact the bottom. The second is a mid-depth release consisting of a piston held at the end of its travel in a cylinder by a brass screw and eyenut. The brass screw is weakened as desired by reducing its diameter at one point so that it will break under tension generated by hydrostatic pressure on the piston. When the screw breaks, the eyenut and the attached 9 kg of weight fall off. The instrument and attached weights for bottom release are shown in Fig. 1.

The navigational system used is the controlling factor in the accuracy of measurements of this type. Clearly, if some percentage precision is specified, the location of the drop and surfacing points must be fixed to about one-half of that percentage of the horizontal deflection. The system employed is known as Hi-Fix (Decca Navigator System, Inc.). This is a radio location system in which the master station, located on the boat, alternately interrogates two slave stations to determine range from them. The system has a range of about 115 km and a precision of about ± 1 m. The readout is in the form of two 5-digit counters showing range to the two stations. The least digit on the counters corresponds to 0.86 m. The slave stations (Virginia Key and Ft. Lauderdale, Florida) are 37 km apart. Along a line from Virginia Key to Bimini, Bahamas, the system provides an accuracy of about ± 1 m on the western side of the Straits of Florida and ± 2 m on the eastern side.

The equipment is installed on the Stormy Petrel, a small high-speed boat capable of reaching a speed of 25 knots. Because of its maneuverability, this boat is ideal for this type of work.

Details of the instrument will be made available on request.

III. *Operational Procedure.* The boat is brought to the drop position on a preselected heading and the instrument is dropped from the starboard quarter. At the time of drop, a photograph is taken of the Hi-Fix position counters and a clock. [If several drops to various depths are to be made at the same location, the deepest drop is made first and the shallowest drop last; a picture is taken of each drop position.] The boat is then maneuvered close to the estimated surfacing point of the instrument; when the instrument is sighted (generally at a range of less than 100 m), the boat is brought smartly alongside on the same heading as that used for the drop, with the instrument close to the starboard quarter. A photograph of the fix position is again taken. Twice thereafter the instrument is allowed to drift for 2 or 3 minutes to obtain two additional fixes. The instrument is then recovered.

The computational procedure (carried out mostly on an IBM 1620 computer) is as follows: The record of pressure vs. time in the instrument is read and plotted as depth vs. time (Fig. 2). The curve is extrapolated slightly at the ends to obtain the run depth as well as the surface and drop times. The surface current is computed from the three surface fixes. To obtain the surfacing fix, the position of the instrument is extrapolated back from the first surface fix to the time of surfacing (from the instrument record). From eq. (1), the transport is then computed with knowledge of the drop fix, the corrected surfacing fix, and the depth and duration of the run.

IV. *Errors and Scale.* There are two sources of error in this technique: (i) the system errors themselves, and (ii) possible interpretive errors arising from the assumption of stationary conditions over the time and space scales of a run.

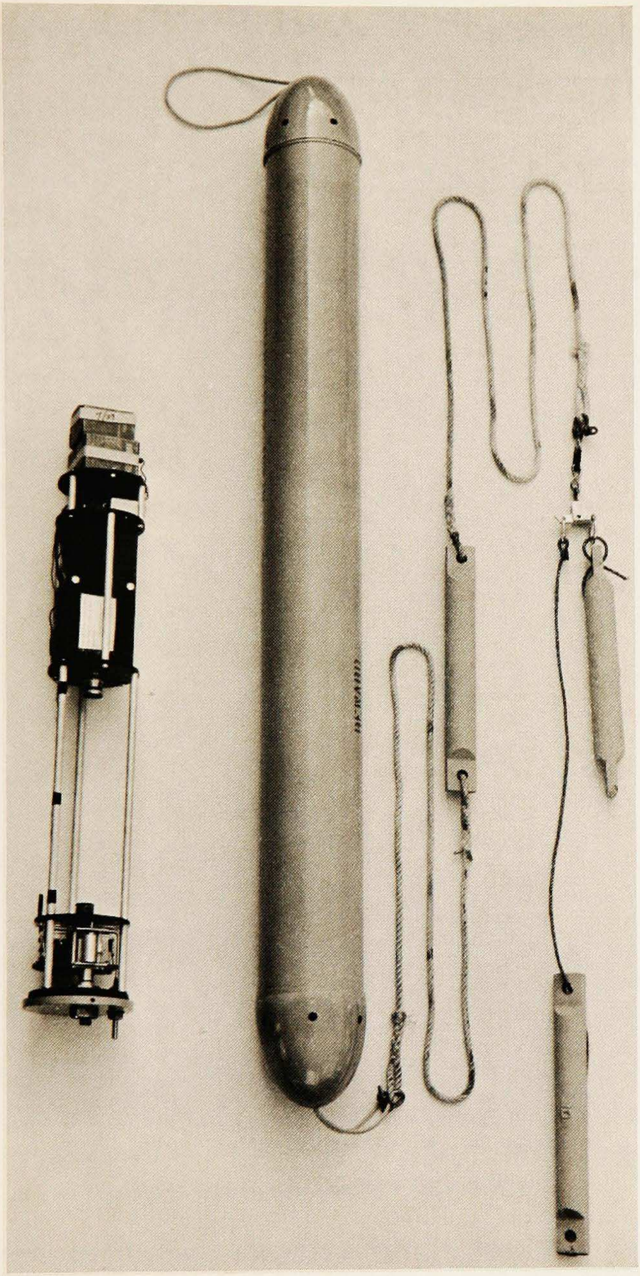

Figure 1. The time-, temperature-, depth-recording instrument with attached bottom release and weights.

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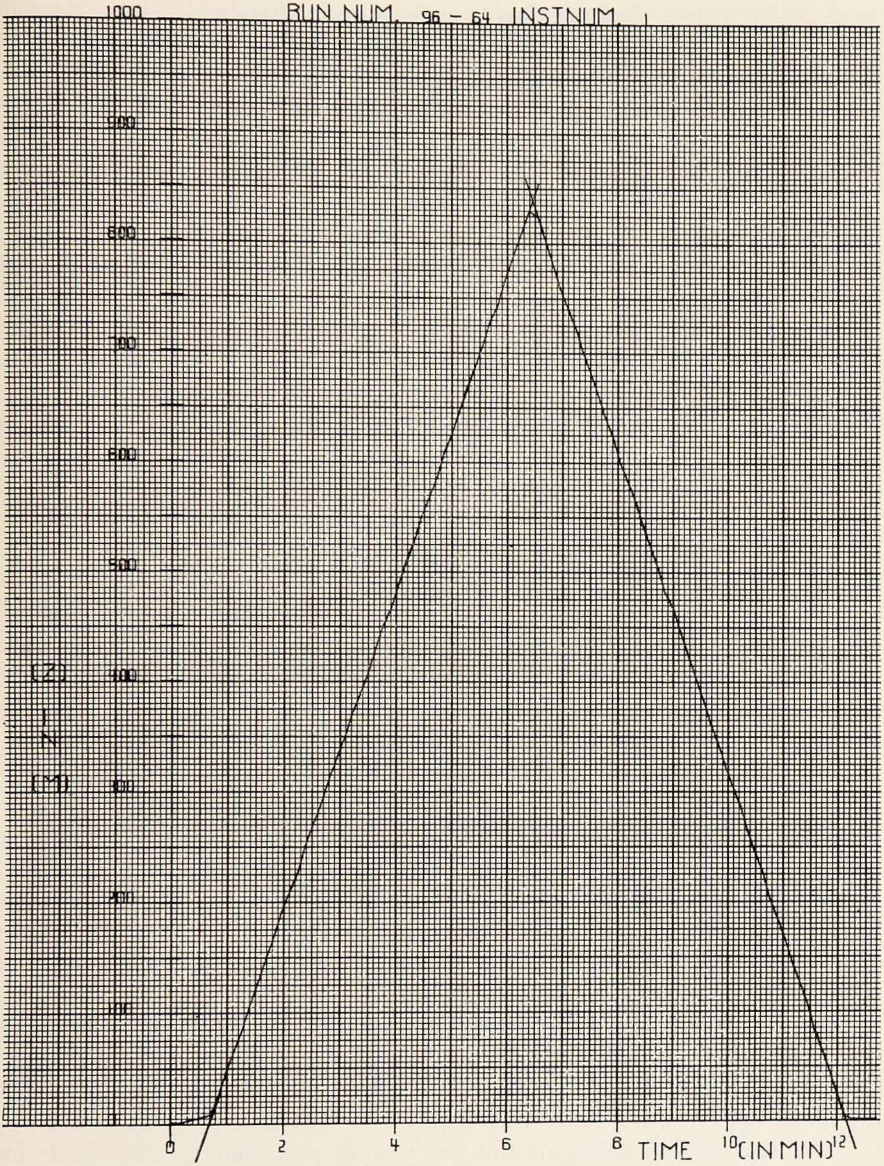

Figure 2. Depth-vs.-time plot from the computer.

The systematic errors arise in the measurement of depth, time, and position due to possible instrumental and computational errors. By selecting an instrument with a pressure gauge of appropriate range, the depth error can be kept to about 0.5 °/o; if a high-range instrument is used on a shallow drop, this may be degraded to 1 %. Both the timing and the navigational errors are essentially independent of the length of the run; thus associated percentage errors can be reduced as desired by increasing the run time (by changing the ballast in the case of transport measurements or by allowing longer drift periods in the case of surface-current determination). In the measurements described below, fall and rise speeds (2.2 and 2.0 m/sec, respectively) and the 2-minute drift period were selected to keep the navigational errors at about the 1 % level. In the case of transport measurement, an additional error of about 1 % arises in the back extrapolation of the surface drift to obtain the surfacing position. As an example, consider a typical run on the western side of the Straits of Florida where the depth is 250 m, the transport 150 m^2/sec, and the surface current 1 m/sec. With navigational errors of $\pm$ 1 m and timing errors $\pm$ 1 second, the estimated errors are: surface speed $\pm$ 3 %, surface-current direction $\pm 2^\circ$, transport magnitude $\pm$ 5 %, transport direction $\pm 3^\circ$. The above error estimates are near the expected maximum and in practice should be considerably less. It is clear that percentage errors will be smaller for deeper, longer runs and in regions of higher transport. As a measure of precision, three runs taken in rapid succession at a point where the transport was somewhat lower than in the above example gave 129 ± 2 m^2/sec.

The assumption of stationary conditions over the space and time scales of a run is inappropriate if the variation of the transport over these scales is an appreciable fraction of the observed transport. Typical run times used in the Straits of Florida are from 1 to 10 minutes, and horizontal separations of drop and recovery are typically 10 to 500 m. One would not expect overly energetic fluctuations over these scales in a system such as the Florida Current, particularly in the downstream direction, and it is our empirical experience that the measurements are stable (reproducible) over these scales. One can view these measurements as implicit averages over the time and space intervals of the run, the instrumental averaging of high frequency fluctuations being desirable for most oceanographic purposes. The averaging properties of the instrument become more explicit over the full scale of the rise and fall intervals as the fall and rise speeds become more nearly equal.

Although it is accepted practice to estimate velocities by Lagrangian measurements over space and time scales much larger than those used by us in the Florida Current, one must always be concerned whether the parcel of water being tracked is representative. It is recommended that prospective users of this technique assess the stability of this type of measurement empirically for a given region; it is not a difficult task.

V. *A Measurement of the Total Transport of the Straits of Florida.* On August 16 and 17, 1964, a series of measurements was made across the Straits of Florida on a line east from Virginia Key (25°43.5′N). The wind was very light from the southeast on both days. On the passage to Bimini, measurements were spaced about 10 km apart, starting near the color change at the western boundary of the Florida Current. On the return passage, in the hope of canceling the predominately semidiurnal tide, observations were made approximately midway between and close to six hours out of phase with those of the previous day. The series of runs in each direction took approximately five hours. In addition to the 18 drops to the bottom, 14 were made to mid-depths between 151 and 205 m, and seven to mid-depths between 432 and 499 m.

The results are shown in Figs. 3 through 7 and Table I; the small arrow on the abcissa of Figs. 3 and 5 indicates the position of the 10-m depth contour. Fig. 3 shows the north component of transport. Observations indicated by triangles and accompanied by a depth in meters are mid-depth drops that are not used in drawing the transport curves. The area under the August 16 curve is 34.3×10^6 m³/sec; under the August 17 curve it is 36.8×10^6 m³/sec; the average is $35.5 \pm 1.2 \times 10^6$ m³/sec. Mid-depth measurements to an average

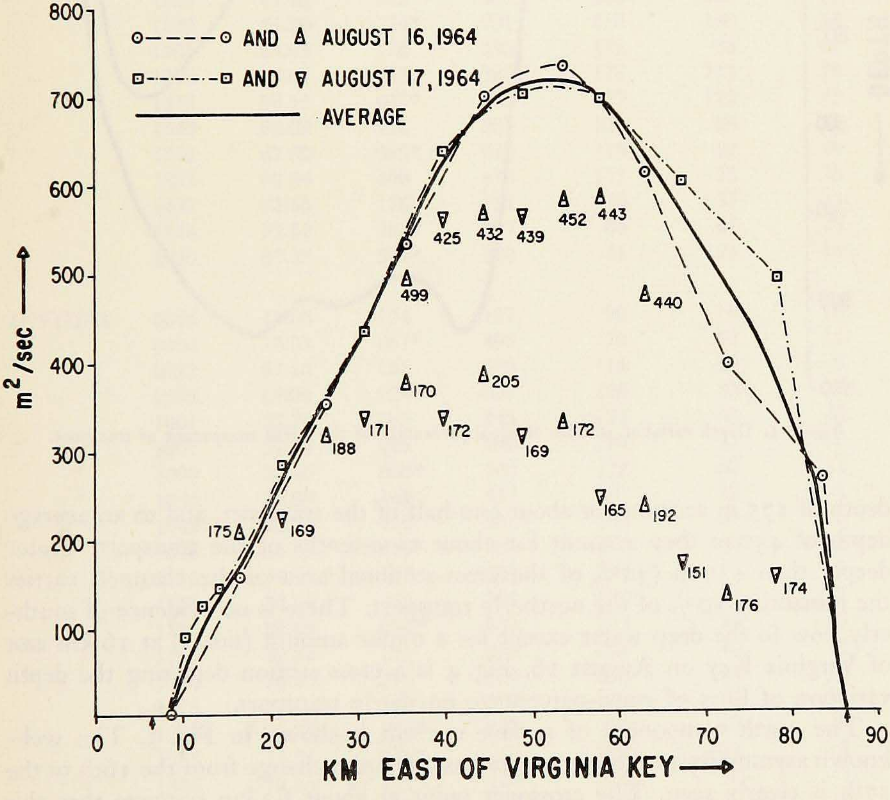

Figure 3. The north component of transport vs. distance across the Straits of Florida. Measurements denoted by circles and squares are to the bottom. Measurements denoted by triangles are to mid-depths, the depth in meters being shown.

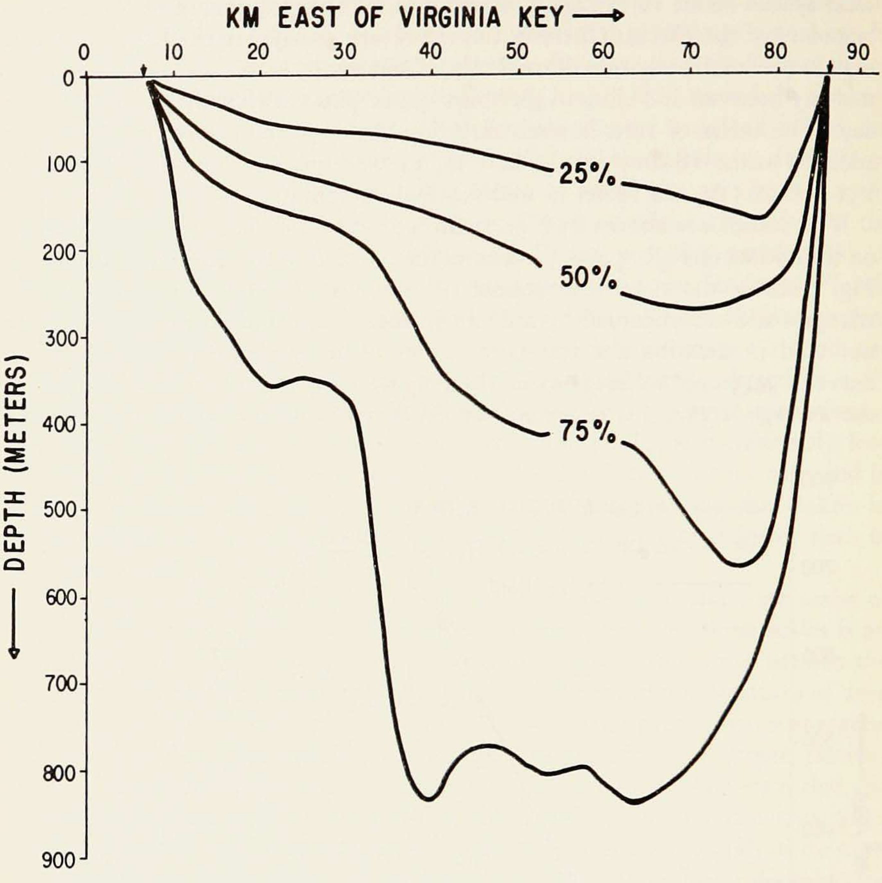

Figure 4. Depth variation of lines of equal percentage of the north component of transport.

depth of 175 m account for about one-half of the transport, and to an average depth of 450 m they account for about nine-tenths of the transport. Water deeper than 450 m (30% of the cross-sectional area of the channel) carries the remaining 10% of the northerly transport. There is no evidence of southerly flow in the deep water except for a minor amount (tidal?) at 16 km east of Virginia Key on August 16. Fig. 4 is a cross section depicting the depth variation of lines of equal-percentage northerly transport.

The north component of surface current is shown in Fig. 5. The well-known asymmetry to the west as well as a definite change from the 16th to the 17th is clearly seen. The crossover point at about 63 km suggests that this change is tidal, although it is curious that it is not reflected in the transport (see, for instance, Wertheim 1954). An indication of the precision of the sur-

TABLE I. SUMMARY OF TRANSPORT AND SURFACE-CURRENT MEASUREMENTS.

Date	Time EST	Km. East of Virginia Key	Depth (m)	N/S Comp. Transport (m^2/s)	N/S Comp. Surface Current (cm/s)	E/W Comp. Transport (m^2/s)	E/W Comp. Surface Current (cm/s)
16/VIII/64	0950	8.55	54*	1	13	10	–7
	1017	16.35	289*	157	215	31	23
	1022	16.36	175	208	215	30	20
	1058	26.32	188	318	252	68	31
	1103	26.28	349*	352	253	53	34
	1136	35.53	170	376	249	53	48
	1141	35.47	688*	533	250	73	47
	1144	35.55	499	496	254	107	48
	1223	44.30	205	383	213	78	59
	1229	44.28	432	568	215	106	55
	1230	44.26	774*	701	218	100	58
	1305	53.63	178	330	172	63	82
	1312	53.62	452	583	176	117	76
	1312	53.55	807*	735	177	125	72
	1348	62.89	192	263	120	86	38
	1352	62.90	839*	615	119	92	40
	1353	62.94	440	476	112	75	38
	1437	72.55	176	131	78	31	23
	1444	72.51	763*	397	80	61	19
	1520	83.32	382*	269	31	24	13
17/VIII/64	0844	78.06	174	127	90	18	12
	0850	78.05	661*	496	79	32	15
	0922	67.10	151	170	114	18	–6
	0933	67.09	824*	607	120	33	–8
	1001	57.70	165	245	135	30	10
	1007	57.70	443	589	128	52	16
	1008	57.65	800*	700	132	62	11
	1038	48.69	169	317	167	32	–1
	1045	48.72	439	563	169	87	–6
	1046	48.71	775*	705	174	84	–2
	1116	39.78	172	339	204	55	5
	1122	39.76	425	564	209	110	12
	1124	39.69	838*	639	215	94	5
	1150	30.83	171	338	226	40	12
	1154	30.83	375*	434	233	52	12
	1225	21.34	169	222	242	22	14
	1229	21.28	356*	282	209	15	9
	1255	14.31	270*	146	200	–24	–4
	1306	12.25	241*	124	186	16	–1
	1319	10.39	141*	90	189	18	–3

* Denotes bottom drop.

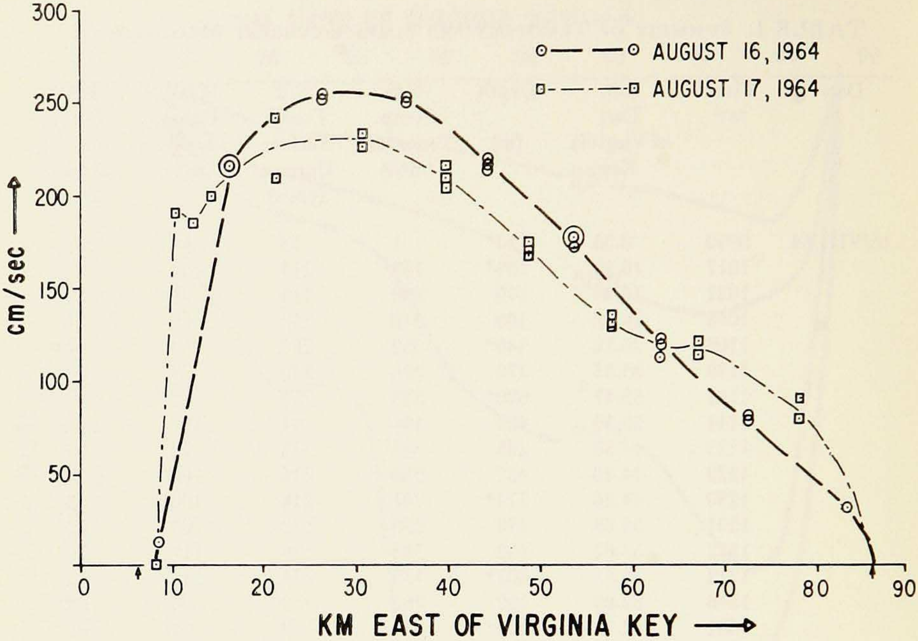

Figure 5. The north component of surface current vs. distance across the Straits of Florida.

face-current measurements is given by the multiple values shown at several of the stations where two or three instrument drops were made.

The east-west components of surface current and of average currents derived from transport measurements on August 16 and 17 are shown in Figs. 6 and 7, respectively. The subsurface values indicated on the figures are average current components over depth intervals marked by X's. There is a pronounced difference (tidal?) between the surface currents for the two days. Although the magnitudes of the subsurface values are somewhat variable, the directional picture is relatively consistent. Easterly flow occurs throughout the upper layers in an area whose lower boundary deepens eastward and approaches the bottom in the eastern 30 km of the channel. Flow is westward in the bottom layers on the western side of the channel. Neglecting the first measurement on the 16th, which was just inshore of the western boundary of the current, the average total current directions were 008 and 005 degrees on the 16th and 17th, respectively. Thus, the stream was moving offshore of the western boundary on both crossings, and although the east-west components are cross-channel components to within ± 2 degrees, they are not cross-stream components.

The asymmetry observed in the north-south surface-current component compares favorably with Pillsbury's (1890) data. Both sets of measurements

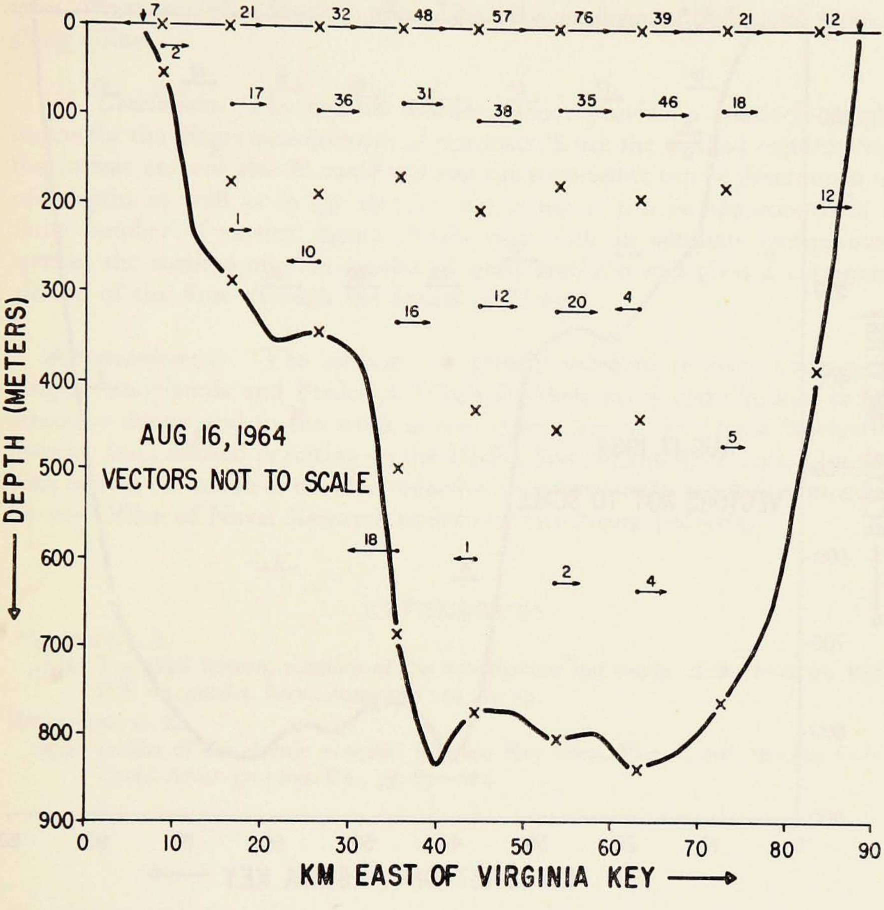

Figure 6. East-west components of average current (cm/sec.) over the depth intervals (indicated by crosses) on August 16, 1964.

place the surface current axis approximately 20 km offshore of the 10-m curve on the western side of the channel. However, Pillsbury's measurements place the axis of volume transport approximately 10 km to the west of the channel center whereas our measurements place this axis approximately 10 km to the east of the channel center. Pillsbury's observations extend to only 230 m and his linear extrapolation with depth (see Pillsbury 1890: fig. 51) gave zero speed at 200 to 300 m above the bottom for the first two stations to the east of his axis of volume transport (Pillsbury's Sts. 3 and 4). Since these two sets of measurements were made on very different time scales (Pillsbury's fig. 51 depicts averages over 2 to 3 years), one would not necessarily expect close agreement in magnitude of current or transport. Pillsbury's estimate of trans-

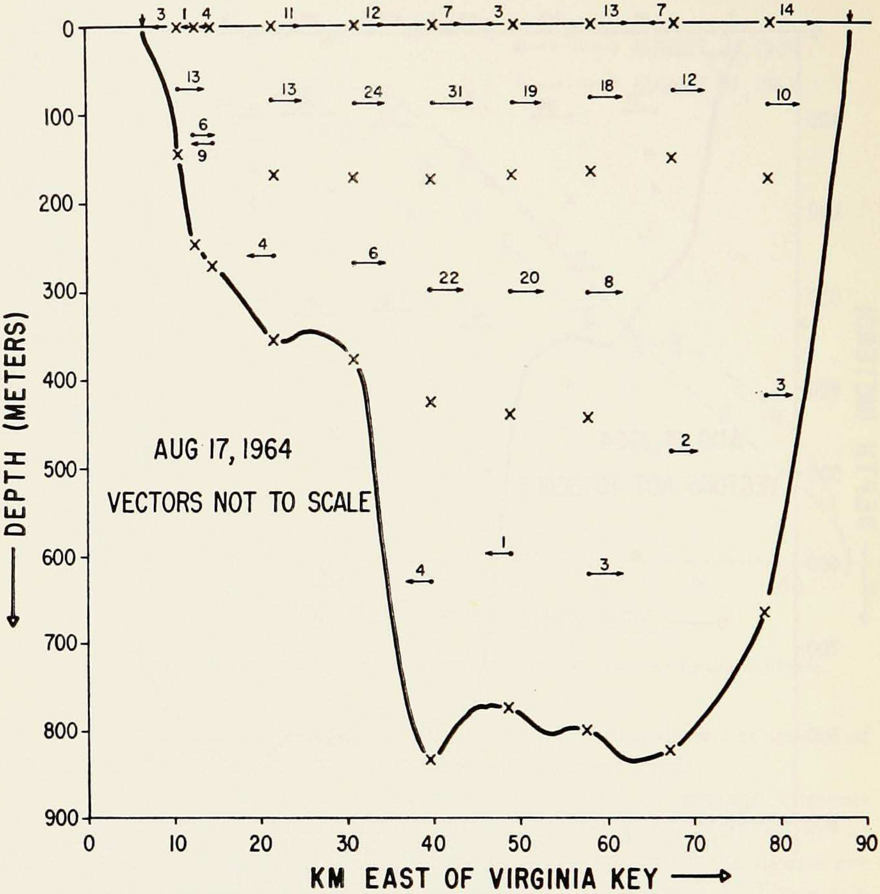

Figure 7. East-west components of average current (cm/sec.) over the depth intervals (indicated by crosses) on August 17, 1964.

port is approximately 70% of our result. One would not expect disagreement on the placement of the axis of volume transport by 25% of the channel width. The probable reason for this disagreement lies in the method of depth extrapolation used by Pillsbury; his curves were brought to zero more rapidly with depth than seems appropriate from our observations. The east-west current components in Figs. 6 and 7 are roughly what would be expected for a steepening density field in a converging channel. The total current was moving offshore of the western boundary, and the total cross-channel flow decayed to zero at the eastern boundary. By continuity, the total northerly flow component was decelerating on the shallow western side of the channel and accelerating on the deeper eastern side of the channel. One cannot unambiguously infer the existance of vertical motion by measurements of horizontal motion along a line.

VI. *Conclusions.* The method described above provides a relatively simple means for the direct measurement of transport. Since the method requires that the surface current also be measured and since transport can be determined to mid-depths as well as to the bottom, the transport can be apportioned to a finite number of vertical layers. When used with an adequate navigational system, the method appears capable of good precision and gives a consistent picture of the flow through the Straits of Florida.

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