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A Necessary Condition for the Existence of an Inertial Boundary Layer in a Baroclinic Ocean [1]

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ABSTRACT

A criterion for the existence of an inertial boundary current in a baroclinic ocean is derived. It is shown that the effect of topographical variations can play a decisive role in determining whether an inertial boundary current can exist, even if the velocity field is highly baroclinic.

Introduction. The question of the existence of inertial boundary layers plays a central role (Charney 1955) in most theories of the steady general oceanic circulation. Greenspan (1962) showed that a necessary condition for the existence of an inertial boundary layer in a homogeneous ocean could be found in the form

$$(\vec{U_I} \cdot \vec{n}) \frac{\partial}{\partial s} (f/h) < 0. \tag{1}$$

Here $\vec{U_I}$ is the fluid velocity of the interior flow at the edge of the boundary layer, f is the Coriolis parameter, h is the ocean depth, and $\vec{n}$ is a unit normal vector directed toward land at the ocean boundary. The scalar s measures arc length (counterclockwise) along the perimeter of the basin. The criterion (1) suggests that bottom topography could play a decisive role in determining whether an inertial boundary current can exist.

One important weakness of this theorem exists in the assumption that the ocean is essentially vertically homogeneous in both density and velocity. If, on the other hand, the velocities at great depths are considerably smaller than the surface currents, as in a baroclinic ocean, one might expect the influence

of topography to be correspondingly reduced. To investigate this question further it is necessary to consider the condition for the existence of an inertial boundary current in a baroclinic ocean.

The Baroclinic Criterion. For the sake of convenience only, consider that the relevant boundary runs south to north, along which the y axis is aligned; let x measure positive-eastward into the ocean, and z measure positive-upward from the mean ocean bottom. A scaling analysis can show that for geophysically relevant phenomena in middle latitudes the appropriate equation governing the boundary-layer motion in the asymptotic region near the edge of the boundary layer is the quasigeostrophic potential vorticity equation. The density is assumed to be a conservative property for any fluid element. In addition, the scale height of the density field is assumed large compared to the depth of the fluid. In the asymptotic region, the potential vorticity equation can be linearized about the interior motion, and one obtains as the relevant asymptotic equation governing the boundary-layer motion:

$$U_I(y,z)\left[(\gamma v_z)_z + v_{xx}\right] + v\left[\frac{df}{dy} - (\gamma U_{Iz})_z\right] = 0. \tag{2a}$$

In (2a), $U_I(y,z)$ is the interior eastward flow velocity at the edge of the boundary layer and is a function of latitude and depth, v is the northward velocity of the boundary current, and γ is the positive ratio, $-f^2/g(\partial ln\varrho_I/\partial z)$. The quantity ϱ_I is the density field of the interior flow at the edge of the boundary layer. The appropriate boundary conditions for (2a) express the vanishing of the vertical velocity at the top of the ocean, $z = H$, while at the ocean's bottom a possible south-north variation of bottom topography, η_y, will produce a vertical velocity, $v\eta_y$. The east-west variation of topography is ignored for purposes of simplicity. Unless the topographical variation is very large in the x direction, this may be done without loss of generality (see Greenspan 1962). If the equation expressing the conservation of density is utilized to evaluate the vertical velocity, it can be shown that the boundary conditions may be expressed as follows:

$$U_I v_z - v U_{Iz} = 0 \qquad\qquad \text{at } z = H, \tag{2b}$$

$$\gamma\left(U_I v_z - v U_{Iz}\right) = -vf\eta_y \qquad\qquad \text{at } z = \eta. \tag{2c}$$

Two assumptions are now made. First, it is assumed that the tangent of the angle made by the horizontal-velocity vector to the shore is always finite, i.e. there is always some component of the boundary-layer flow parallel to the coast. This may be expressed in the asymptotic region as

$$v/U_I \neq 0. \tag{3}$$

Second, it is assumed that, in the asymptotic region,

$$\frac{v_{xx}}{v} > 0 . \tag{4}$$

This insures a boundary-layer character for the flow in the asymptotic region. The equation for v/U_I can be shown to be

$$(\gamma U_I^2 (v/U_I)_z)_z + (v/U_I) \left(U_I \frac{df}{dy} \right) + \left(\frac{v}{U_I} \right)_{xx} U_I^2 = 0 . \tag{5}$$

Multiply (5) by the nonsingular quantity $(v/U_I)^{-1}$ and then integrate from $z = \eta$ to $z = H$ to obtain

$$\int_\eta^H U_I^2 \frac{(v/U_I)_z^2}{(v/U_I)^2} \gamma \, dz + \int_\eta^H U_I^2 \frac{v_{xx}}{v} \, dz + \int_\eta^H U_I \frac{df}{dy} \, dz + U_I(\eta) f \eta_y = 0 . \tag{6}$$

Since the first two terms in (6) are positive, one must have, as a necessary condition for the existence of a baroclinic inertial boundary current,

$$\frac{df}{dy} \int_\eta^H U_I \, dz + f U_I(\eta) \eta_y < 0 , \tag{7}$$

or

$$(H - \eta)^2 U_I(\eta) \frac{d}{dy} \left(\frac{f}{H - \eta} \right) + \frac{df}{dy} \int_\eta^H [U_I(z) - U_I(\eta)] \, dz < 0 . \tag{8}$$

If U_I is independent of z, i.e. if the ocean is barotropic, (8) reduces to (1)— the criterion derived by Greenspan for the homogeneous ocean.

Define the vertically averaged flux out of the boundary layer, M, as

$$M = \int_\eta^H U_I \, dz . \tag{9}$$

Then the criterion (8) can be written

$$\frac{df}{dy} M + f U_I(\eta) \frac{d\eta}{dy} < 0 . \tag{10}$$

Thus, if the bottom of the ocean is flat, (10) states that the flux, M, must be negative. The vertically integrated interior velocity must be directed toward the coast. On the other hand, if $d\eta/dy$ is not zero and if the *bottom* velocity, $U_I(\eta)$, is not zero, the effect of topography may completely alter this criterion

on the transport. If $d\eta/dy$ is $o(10^{-2})$ and H is $o(10^5 \text{ cm})$, which are reasonable Gulf Stream values, then, if $U_I(\eta)H/M > 10^{-2}$, the effect of topography will be at least as important as the variation of the Coriolis parameter. Assume that the interior flow, U_I, takes on its maximum value at the surface, $z = H$; then, if the ratio of the bottom velocity to the surface velocity is as large as one to one hundred, the topographical effect will play a decisive role, even in such a highly baroclinic ocean, in determining whether an inertial boundary current may exist. Thus, such a phenomenon as the separation of the Gulf Stream at a point where (10) is violated, may, in fact, be completely determined by topographical effects, as Greenspan originally argued on the basis of his treatment of the homogeneous-ocean problem.

Finally, I feel that assumptions (3) and (4) are probably not independent and that only one need be assumed. However, such a speculation has not as yet been proven.

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