

# YALE PEABODY MUSEUM

P.O. BOX 208118 | NEW HAVEN CT 06520-8118 USA | PEABODY.YALE. EDU

## JOURNAL OF MARINE RESEARCH

The *Journal of Marine Research*, one of the oldest journals in American marine science, published important peer-reviewed original research on a broad array of topics in physical, biological, and chemical oceanography vital to the academic oceanographic community in the long and rich tradition of the Sears Foundation for Marine Research at Yale University.

An archive of all issues from 1937 to 2021 (Volume 1–79) are available through EliScholar, a digital platform for scholarly publishing provided by Yale University Library at <https://elischolar.library.yale.edu/>.

Requests for permission to clear rights for use of this content should be directed to the authors, their estates, or other representatives. The *Journal of Marine Research* has no contact information beyond the affiliations listed in the published articles. We ask that you provide attribution to the *Journal of Marine Research*.

Yale University provides access to these materials for educational and research purposes only. Copyright or other proprietary rights to content contained in this document may be held by individuals or entities other than, or in addition to, Yale University. You are solely responsible for determining the ownership of the copyright, and for obtaining permission for your intended use. Yale University makes no warranty that your distribution, reproduction, or other use of these materials will not infringe the rights of third parties.



This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.  
<https://creativecommons.org/licenses/by-nc-sa/4.0/>



# Long Surf

D. V. Ho,<sup>1</sup> R. E. Meyer, and M. C. Shen<sup>2</sup>

*Brown University  
Providence, R. I.*

---

## ABSTRACT

Theoretical investigations by the authors on surf due to long swell are summarized and presented here in a form unencumbered by mathematical technicalities. The conclusions derived thus far are mainly qualitative, but in some respects the results are quite detailed and differ radically from results of earlier investigations; some new observations are also presented. It appears that a simple, nonlinear model is capable of describing the principal features of the entire phenomenon of breaker formation, breaker collapse, run-up, and backwash for a representative type of surf on a shallow beach. The model predicts a marked conversion of potential energy into kinetic energy as the breaker collapses, a notably sudden start of the run-up, a thin sheet of run-up that gets progressively thinner as time elapses, and a curious secondary bore in the backwash.

*Introduction.* A new mathematical approach to an analysis of surf movement on a beach has been given in three recent papers (Ho and Meyer, 1962; Shen and Meyer, 1963a, b). For the logical development of the subject, it was necessary in these papers to employ complicated technical argument in terms familiar only to mathematical specialists. In the following discussion, an effort is made to make the theory understandable to a wider audience and thus stimulate research on the subject by experimental physicists and related groups. The mathematical arguments are omitted.

The analysis possesses a feature that is unusual in theoretical oceanography. All approximations and idealizations of the theory are contained in a set of four assumptions that constitute the 'model' of surf here proposed. This set of assumptions formulates, so to speak, an abstract surf as a system of partial differential equations with boundary conditions. But after these assumptions were stated in the mathematical papers mentioned above, no further idealizations or approximations of any kind were introduced into the mathematical work. Indeed, the authors did not attempt to solve those differential equations; instead, they proved a set of lemmas, theorems, and corollaries that establish rigorously some properties of the exact solutions of the equations. The fol-

<sup>1</sup> Now at the Mathematics Department, Georgia Institute of Technology, Atlanta, Georgia.

<sup>2</sup> Now at the Courant Institute of Mathematical Sciences, New York University, New York City.

lowing account therefore begins with a statement of all the assumptions constituting the mathematical model, and it then proceeds directly to a statement of the conclusions that have been derived to date.

*Formulation.* Consider a two-dimensional motion of water on a beach of uniform slope—e. g. swell approaching the shore from the sea with crest parallel to a straight shore. To avoid additional parameters involved in explaining the interaction of a breaker with the backwash from the preceding wave, consider only very long swell. Or, more precisely, consider only a wave traveling shoreward into water at rest.

(i) Assume that the water motion is governed by the first-order nonlinear long-wave equations (Stoker, 1957):

$$\partial h/\partial t + \partial(hu)/\partial x = 0, \quad (1)$$

$$\partial u/\partial t + u\partial u/\partial x + g\partial(h - h_0)/\partial x = 0, \quad (2)$$

where  $x$  denotes the horizontal distance (positive toward the beach),  $t$  the time,  $h(x, t)$  the total local water depth (Fig. 1),  $h_0(x)$  the equilibrium water depth,  $g$  the gravitational acceleration, and  $u$  the  $x$ -component of the water velocity. If  $u$  is interpreted as the vertical mean of the horizontal velocity component, then (1) is an exact kinematic statement of mass conservation on a nonporous beach. If viscosity and vorticity are neglected, moreover, (1) and (2) can be shown (Meyer and Taylor, 1963) to represent the first approximation to the exact Eulerian equations of water motion, not too far from the shore, on a beach of small slope.

(ii) Assume further that the front of the incoming wave consists of a bore. As used in the following, this concept requires clarification. It is known that progressive waves described by (1) and (2) form steep fronts, similar to those observed in swell approaching a beach. As such waves develop further, their mathematical description exhibits a singularity of water acceleration and surface slope that represents a breakdown in the applicability of (1) and (2) (Stoker, 1957). On the other hand, observation and analysis indicate that the region in which (1) and (2) fail is of relatively short horizontal extent. Thus it is of considerable interest to consider a model approach to the problem, in which the study of the details of the water motion in that narrow region is avoided by assuming only over-all conservation of mass and momentum for the narrow region. This idea permits the development of a consistent mathematical model (Stoker, 1957) in which the narrow region is represented by a discontinuity (the "bore"); and within the framework of the model, analysis of the water motion outside the narrow region then proceeds, regardless of whether (or how) a real breaker is formed within that region.

Comparison with the work of Carrier and Greenspan (1958) shows that the assumption of bore formation has a crucial influence on the results of the



investigation discussed here. Carrier and Greenspan studied certain particular bore-free solutions of (1) and (2) that show a near-shore behavior which is radically different from the behavior predicted in the following pages. Note that the distinction depends on whether or not the waves steepen locally to a greater extent than is consistent with (1) and (2)—not on whether they

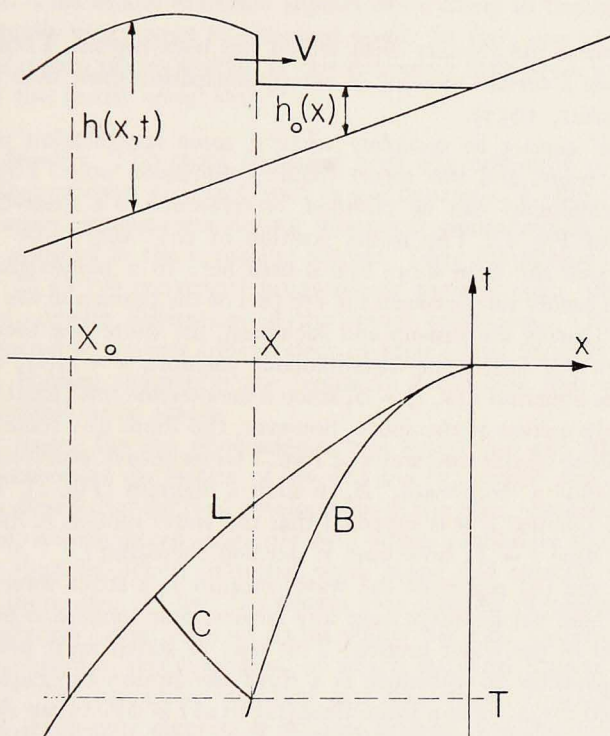


Figure 1.

do or do not form actual breakers. For this study, the U. S. Weather Bureau made available unpublished computational results which indicate that even swell of very small amplitude, if it is governed by (1) and (2), is very likely to develop the singularity associated with bore formation as the wave approaches the shore. The same is indicated by an analytical investigation (Greenspan, 1958).

By analogy with studies in gas dynamics, the initial formation and early development of a bore are fairly well understood (Stoker, 1957; Meyer, 1960), and this part of the problem was therefore not studied during the present investigation. The analysis discussed here concerns the later development of the water motion, after it has reached a stage where the bore is fairly well developed and has moved to the very front of the wave (Fig. 1). For a bore traveling

shoreward into water at rest, the conservation conditions require (Stoker, 1957) that the water level rise from  $h_0$  on the landward side to  $h_b$  on the seaward side of the bore, and that the water velocity rise similarly from zero to  $u_b$ . The quantities  $h_b$  and  $u_b$  are related by

$$u_b/V = 1 - h_0/h_b, \quad 2V^2 = gh_b(1 + h_b/h_0), \quad (3)$$

where  $V$  denotes the velocity with which the bore travels. These conditions also imply that a certain amount of energy dissipation must take place within the bore (Stoker, 1957).

The model cannot be complete without some specification of the initial shape of the wave, and this aspect requires discussion now. The description of bore development<sup>3</sup> can be clarified by reference to a space-time diagram (lower part of Fig. 1). The initial position of the "shore line" is taken as  $x = 0$ ; note that the term shore line is used here in a nongeographical sense to denote the border line between the dry part of the beach and the part covered with water. During the run-up and backwash, the shore line therefore moves up and down the beach; its instantaneous position,  $x = x_s(t)$ , is given implicitly by the equation  $h(x, t) = 0$ , since  $h$  denotes the total local water depth. Until the bore arrives at the shore, however, the shore line remains at  $x = 0$ , because the bore travels into water at rest. The successive positions of the bore,  $x = x_b(t)$ , define a 'bore path,'  $B$ , in such a diagram (Fig. 1). The problem at hand is as follows: if it is supposed that the water motion is fully known at some initial time  $t = T$ , how does it develop thereafter?

Since (1) and (2) represent the water motion as a strict wave propagation process, the bore development over any chosen time interval depends only on a limited part of the water motion. This may be stated more precisely as follows. It is plausible to introduce as a third assumption the requirement that the bore reach the shore at a finite time (taken as  $t = 0$ )<sup>4</sup>. It can then be shown that the wave on the seaward side of the bore must possess a "limiting ray of propagation." This is a curve ( $L$  in Fig. 1) in the space-time diagram defined by two requirements. First, its local tangent is given by  $dx/dt = u(x, t) + c(x, t)$ , where  $c(x, t)$  is the positive square root of  $gh(x, t)$ ; and second, the curve meets the bore path at the time when the bore reaches the shore line. The part of the wave extending at time  $T$  from the limit ray position,  $X_0$ , to the bore position,  $X$  (Fig. 1), has the following role. Specification of the initial wave shape,  $h(x, T)$ , and the initial velocity distribution,  $u(x, T)$ , from  $x = X_0$  to  $x = X$  is necessary and sufficient for determining the bore development from  $t = T$  to  $t = 0$  by means of (1) to (3). The wave shape seaward of the limit ray,  $L$ , is irrelevant to the bore development.

<sup>3</sup> This term is used to denote the variation with time of  $h_b$ ,  $u_b$ ,  $V$ , and the bore position,  $x_b$ ; no other information concerning the bore can be obtained from the model here adopted.

<sup>4</sup> With this normalization, the value  $T < 0$  of the initial time is one of the unknowns of the problem.



The second major distinction between this work and that of Carrier and Greenspan (1958) is in the types of waves specified. They consider only a few particular types of wave shapes; most of them are characterized by the peculiarity that a time exists at which the water is simultaneously (a) everywhere at rest and (b) in dis-equilibrium. By contrast, the fourth assumption of the present model, to be discussed now, admits a rather general class of wave profiles. This model therefore appears more likely to include those profiles that actually correspond to observed swell. In fact, one of the striking mathematical results of this analysis is that extremely little needs to be specified in regard to the initial wave shape.

*Selective Memory.* The work discussed here was originally undertaken to elucidate a phenomenon of importance in gas dynamics. Keller *et al.* (1960) noticed the same phenomenon during numerical computations pertaining to the problem outlined in the preceding section. They found that three sample solutions of (1) to (3) converged progressively towards each other with increasing time, despite differences in the initial conditions. Thus the three solutions appeared to become independent of their initial wave shapes as time elapsed, in a manner suggesting the figurative description that the solutions forget their initial wave shapes.

The basic reason for this phenomenon was found (Ho and Meyer, 1962) in the degeneration of the system of eqs. (1) and (2), which may occur when  $h = 0$ . It was shown that the final development of the bore, just before it reaches the shore line, is determined primarily by a single *qualitative* property of the swell, rather than by details of initial wave shape and velocity distribution.

Precisely, then, eqs. (1) and (2) possess two families of propagation rays, viz. the "advancing" rays (such as  $L$ ), of which the local tangent is given by  $dx/dt = u + c$ , and the 'receding' rays, of which the local tangent is given by  $dx/dt = u - c$ . The particular receding ray that passes through the point  $(X, T)$  of the space-time diagram is of some interest, and the segment of this ray, which extends from the bore-path,  $B$ , to the limiting ray,  $L$ , will be denoted by  $C$  (Fig. 1). Instead of setting the initial data from  $x = X_0$  to  $x = X$ , as discussed above, it is more convenient and mathematically equivalent to set the quantity

$$u(x, t) - 2c(x, t) - gth_0(x)/x = \alpha$$

as function of time on the ray segment  $C$ . However, rather than specify this function  $\alpha(t)$  in detail, Ho and Meyer (1962) have taken as their fourth assumption only the requirement that  $\alpha$  increase<sup>5</sup> with time on  $C$ . A satisfac-

<sup>5</sup> More precisely,  $\alpha(t_2) > \alpha(t_1)$  whenever  $t_2 > t_1$ . In practice, this amounts to  $d\alpha/dt > 0$ , which can be shown to imply the following inequality for the water acceleration,  $\partial u/\partial t + u\partial u/\partial x$ , and the velocity gradient,  $\partial u/\partial x$ , on  $C$ :

$$\partial u/\partial t + u\partial u/\partial x - c\partial u/\partial x > gh_0(x)/x.$$

This inequality has been studied (Ho and Meyer, 1962, 1963), but the results are of no direct oceanographical interest.

tory physical interpretation of this "monotoneity assumption" has not yet been given. Moreover, while this assumption has been proven to be a sufficient condition, it is not known to be also a necessary condition for the results that follow; hence, it is not certain that a simple physical interpretation exists.

Apart from a mathematical regularity assumption that has little physical significance, the monotoneity assumption completes the mathematical model. We therefore turn to the predictions—first of all to those pertaining to the phenomenon of "forgetfulness."

The bore height,  $h_b - h_0$ , falls to zero as the bore approaches the initial shore position,  $x = 0$ . Both the bore velocity,  $V$ , and the value,  $u_b$ , of the water velocity immediately seaward of the bore tend to a finite limit,  $u_0$ ; this much follows already from the first three assumptions of the model (Keller *et al.*, 1960). The full model implies that  $u_0 > 0$ , and that  $V$  and  $u_b$  increase during the final stage of the bore's approach to the shore. This agrees with the commonly observed behavior of breakers on a beach. This final stage of the bore's approach is thus a process in which potential energy is converted to kinetic energy.

As  $t \rightarrow 0$ , the water acceleration immediately seaward of the bore develops a singularity characterized by a parameter,  $a_0$ , which is related to the acceleration distribution of the wave at the initial time,  $T$ . This parameter shares with  $u_0$  the property that the model determines only its sign, not its magnitude. But that is sufficient for a quite detailed prediction of the final stage of bore development. It is an "asymptotic" description, i. e. it establishes the limiting values of various quantities as  $t \rightarrow 0$  (Fig. 1). Such a description distinguishes different "orders" in the sense that increasing the order means establishing the limiting values of additional and more intricate quantities, thereby obtaining a more precise description of the nature of the final stage in bore development.

The model determines such a description to at least the tenth order; the seventh order was derived explicitly (Ho and Meyer, 1962), because it reveals the precise sense in which surf may be said to have a "selective memory." For definiteness, consider the relation between the nondimensional bore velocity,  $V/u_0$ , and the nondimensional bore position,  $\gamma x_b/u_0^2$ , where  $\gamma = -gdh_0/dx$  is the beach slope in acceleration units. To the sixth asymptotic order, the relation between  $V/u_0$  and  $\gamma x_b/u_0^2$  is INDEPENDENT OF THE INITIAL WAVE SHAPE. But the same statement does not apply to the seventh order description, which is plotted in Fig. 2.

The relations between other properties of the bore are derivable from that between  $V/u_0$  and  $\gamma x_b/u_0^2$ , and they show an analogous behavior. Altogether, then, only two properties of the wave forming the bore and propelling it towards shore have an important influence on the final stage of bore development. They are (i) the monotoneity property defined by the fourth assumption,



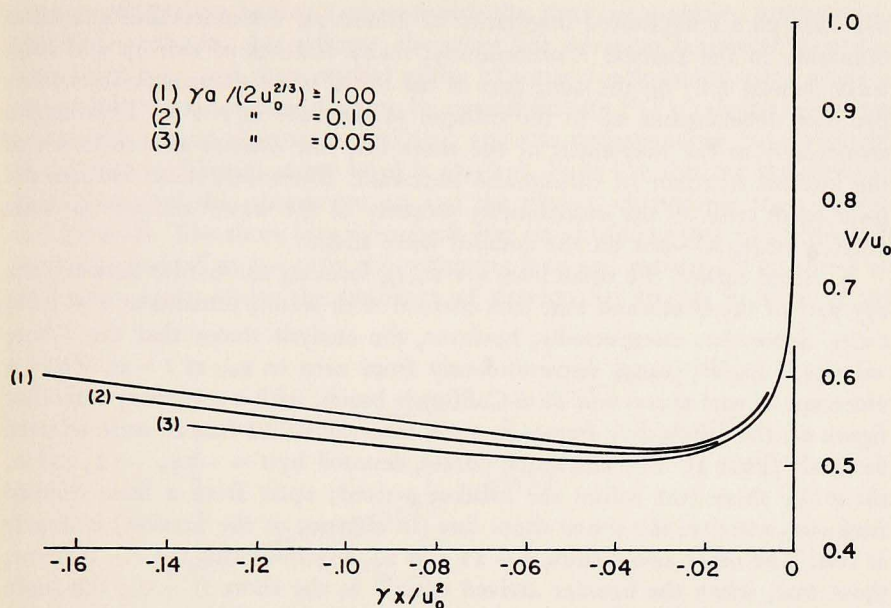


Figure 2.

and (ii) the velocity,  $u_0$ . The latter is, presumably, a measure of the energy in that part of the wave extending from  $x = X_0$  to  $x = X$  at the initial time,  $T$ . All other properties of the initial wave shape have a practically negligible influence on the final stage of bore development, during which the main conversion of potential to kinetic energy takes place.

This result also contains a suggestion on how experimental data might be plotted profitably. If all velocities are divided by  $u_0$ , all water heights by  $u_0^2/g$ , all horizontal distances by  $u_0^2/\gamma$ , and all times by  $u_0/\gamma$ , then the model predicts that the behavior of the bore, sufficiently close to shore, is the same for all waves and all shallow beaches of uniform slope.

*Run-up and Backwash on the Beach.* Because of the importance of improved understanding of wave run-up in relation to tsunamis, the analysis has been extended beyond the collapse of the bore on the beach. In view of the radiative character of (1) and (2), it is logical that a prediction of the water motion for  $t > 0$  should require a knowledge of the initial wave shape and velocity distribution for  $x < X_0$  (Fig. 1). This is borne out, for example, by the work of Carrier and Greenspan (1958), where extension of the determination of the motion over any given time-interval requires use of initial data over a comparable, additional  $x$ -interval. However, this does not hold when a bore is present. Again, this is due to the degeneration of (1) and (2) when  $h = 0$ ;



this leads to a complicated singularity of almost all nonobservable quantities occurring in the analysis. Consequently, many features of run-up and back wash depend only on the same part of the initial wave shape that determines the bore development up to the collapse of the bore at  $t = 0$ . This applies particularly to the movement of the shore line but extends also to much of the internal structure of run-up and backwash. Moreover, these features depend again only on the monotonicity property of the wave<sup>6</sup> and on the basic velocity scale,  $u_0$ —not on the detailed wave shape.

As noted earlier, the shore line,  $x = x_s(t)$ , forming the border between the dry part of the beach and that part covered with water, remains at  $x = 0$  for  $t < 0$ . Somewhat unexpectedly, however, the analysis shows that the "shore velocity,"  $dx_s/dt$ , jumps discontinuously from zero to  $u_0$ , at  $t = 0$ . From a videotape of surf movement on a California beach, with a stationary reference figure on the beach, five frames at equal intervals of 12 frames were selected for study (Plate 1). The first three frames, denoted by  $t = -24, -12, \text{ and } 0$ , show the shore just before the breaker arrived; apart from a little residual backwash activity, the actual shore line (in contrast to the breaker) is largely at rest. The other two frames,  $t = 12$  and  $24$ , together with the  $t = 0$  frame, show that, when the breaker arrived exactly at the shore ( $t = 0$ ), the shore line rather suddenly assumed a considerable velocity. Of course, the model discussed here cannot decide whether a process is truly discontinuous. Since the bore is treated as a discontinuity, the prediction of another process as discontinuous can be taken only to imply that it involves a change of a suddenness comparable to that with which the water level changes in a bore.

The shore acceleration,  $d^2x_s/dt^2$ , jumps from zero to  $gdh_0/dx = -\gamma < 0$ , at  $t = 0$ , and it retains the same value during the whole run-up and during part of the backwash. This comes about as follows. The water may be considered as being divided into fluid elements, each occupying the space between the sand, the free surface, and two vertical planes at a distance  $dx$  apart (Fig. 3). These fictitious vertical partition surfaces may be regarded as moving with the local water velocity (uniform over each such surface, according to the model discussed here). Then the vertical surfaces do not interfere with the natural water motion, and each element contains the same mass of water at all times. The motion of such an element is controlled by two influences, viz. that of gravity and that of the pressures exerted at the vertical surfaces by the adjacent elements. The theorem of Shen and Meyer for the model discussed here is that, the closer the element is to the shore line, the less do these pressures exert influence on the motion of a water element. The limiting element, at the very tip of the sheet of run-up, moves purely under the in-

<sup>6</sup> Shen and Meyer (1963b) proceed without approximation from the four assumptions stated above, except that they extend the monotonicity assumption on  $C$  over an arbitrary time interval  $\epsilon$  beyond  $L$  (Fig. 1). But then they permit  $\epsilon \rightarrow 0$ , and since the time at which  $C$  and  $L$  intersect in Fig. 1 is an unknown of the problem, no physical extension of the four assumptions is involved.

fluence of gravity; hence, it moves with the constant, negative acceleration,  $d^2x_s/dt^2 = gh_0/dx$ . Accordingly, the shore line advances landward up to the time  $t = 2u_0/\gamma$ , and then recedes again; its successive positions,  $x_s(t)$ , are represented in the space-time diagram by a parabolic path  $P$  (Fig. 4); the maximum horizontal run-up distance is  $u_0^2/(2\gamma)$ , and the corresponding run-up height above the equilibrium water level is  $u_0^2/(2g)$ . Note the contrast between the sudden start of the run-up process and the gradual changeover from run-up to backwash. The shore line movement may be likened to that of a pendulum that is kicked off at  $t = 0$  by the collapsing bore and is then left to swing up and down again under the influence of gravity. Of course, in view of the

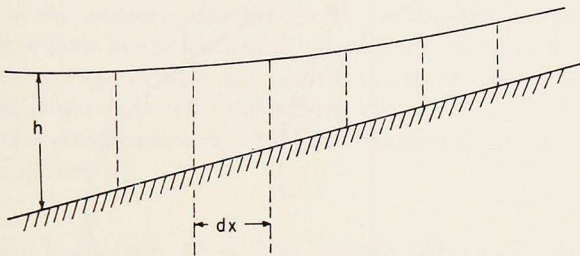


Figure 3.

neglect of friction and various other effects, these quantitative predictions furnish only upper bounds for the real run-up distance, height, and time. Such bounds are valuable, however, since the precise laws of friction applicable to surf are still unknown. Moreover, no method is presently available for estimating the value of  $u_0$  from the properties of swell far from the shore.

The water profile close to the shore line, i. e. the net water height,  $h(x, t)$ , at any fixed time  $t > 0$ , is described to the first order by

$$[x_s(t) - x]^{-2} h(x, t) \rightarrow (3t)^{-2},$$

as  $[x_s(t) - x] \rightarrow 0$ . Again, the model cannot apply to the very tip of the run-up and backwash, where friction and surface tension should be important. But it does indicate a much thinner sheet of run-up than might have been expected from the mathematical model; and in particular, it predicts a very marked progressive thinning of the run-up and backwash sheet with time. Thus it is not necessary to depend entirely on seepage for an explanation of this observed effect, for much of it is already explained in the theory of ideal fluid motion on an impermeable beach.

Although all of these predictions are in marked contrast to those obtained from Carrier and Greenspan's (1958) particular solutions and from the linearized theories (Stoker, 1947), they correspond to features that are visible to the casual observer on the beach. However, we were surprised to find that



the four assumptions of the model imply the presence, in the interior of the backwash, of a singularity of water acceleration of the type generally associated with bore formation. The curve  $D$  in Fig. 4, which marks the successive positions of such a singularity in the space-time diagram, is called "limit line" (Stoker, 1957). Although the genesis and precise course of the limit line,  $D$ ,

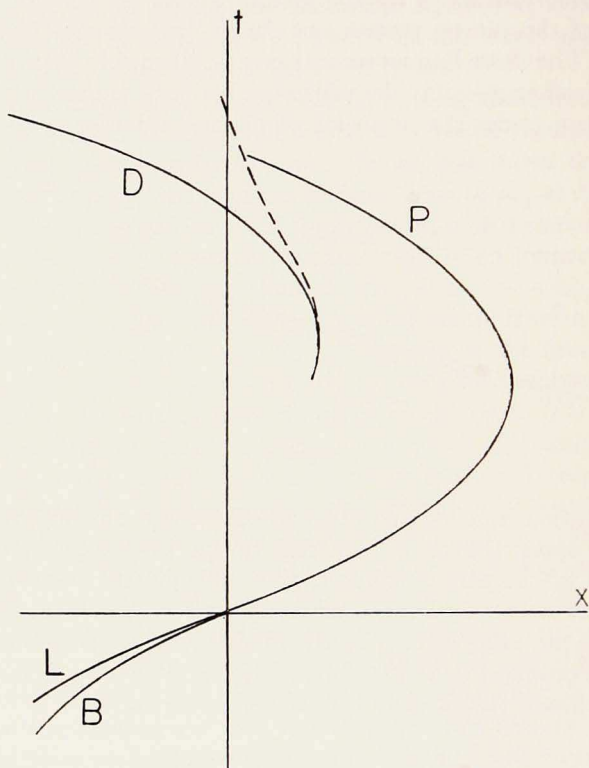


Figure 4.

are not determined by the four assumptions of the model, those assumptions do imply that it must ultimately run towards  $x = -\infty$ , with the parabola  $P$  as asymptote; hence its general course must be as indicated in Fig. 4.

There are only two known interpretations for a limit line (Meyer, 1960), viz. either the problem is physically unrealistic, or a bore develops. If the latter interpretation is accepted, then again the precise genesis and development of this bore are not determined by the four assumptions of the model; however, on general grounds (Meyer, 1960), (i) the bore position must be shoreward of the limit-line position, and (ii) these two positions must be close together at first. The general course of the bore path in the space-time diagram must therefore be as it is indicated by the broken line in Fig. 4.

Now, there exist two different types of limit lines, and they are associated with different types of bores. The bore types are most easily distinguished by the direction in which the bore "faces." For instance, the original bore that forms the front of the incoming wave is said to face landward (Fig. 1) because the water level rises across it in a landward-seaward direction. If such a bore is reflected from a sea wall, then the reflected bore is of the type facing seaward, i. e. the water level rises across it in a seaward-landward direction. Unexpectedly, the limit line,  $D$  (Fig. 4), is of the type associated with a *landward facing* bore, like the original bore, and therefore the bore that develops in the backwash cannot be interpreted as a reflection of the original bore. On the other hand, the broken line in Fig. 4 shows that the new backward bore *travels in the seaward direction* much in contrast to the original bore (Fig. 1). Such a bore in the backwash appears difficult to observe on Rhode Island beaches, perhaps because the swell is too short. However, three frames taken at equal intervals of 16 frames from the serial film referred to above, show clearly the development of a backwash bore with the features predicted by the model (Plate II).

*Beaches with Nonuniform Slope.* The analysis of (1) to (3) for beaches with nonuniform slope has not yet progressed to the same level of mathematical precision as the analysis discussed above. But functions  $u(x, t)$  and  $h(x, t)$  have been constructed (Shen and Meyer, 1963a); they satisfy (1) to (3) approximately and possess the same type of acceleration singularity on the seaward side of the main bore as the solutions discussed above. It is a plausible conjecture that the new functions once more furnish the asymptotic description of the final stage of bore development. This asymptotic description is again independent of the initial wave shape up to the sixth order; in addition, it is found to be similarly independent of the variations in beach slope.

Since the mathematical shore singularity is the same, it would be expected that the shore line moves again with the constant vertical acceleration,  $-g$ , during the run-up. If so, then the same upper bound,  $u_0^2/(2g)$ , for the maximum run-up height would be obtained, provided the beach rises monotonically landward.

*Acknowledgments.* This work was supported by the U. S. Office of Naval Research under Contracts NONR 562(34) (Geophysics Branch) and NONR 562(07) (Mechanics Branch), with Brown University.

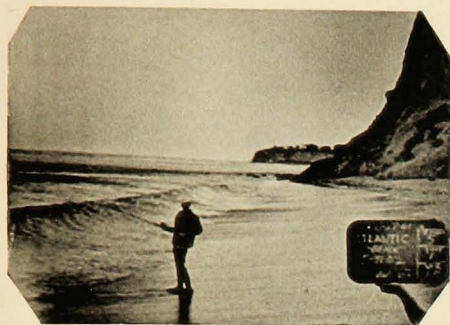
Plates I and II, from a videotape taken under the direction of Mr. B. Pennington for the Atlantic Refining Co., were made available for publication here through the good offices of N. W. Ayer and Sons (New York), KTTV (Hollywood), and WPRO-TV (Providence). Fig. 2 is from Ho and Meyer (1962).



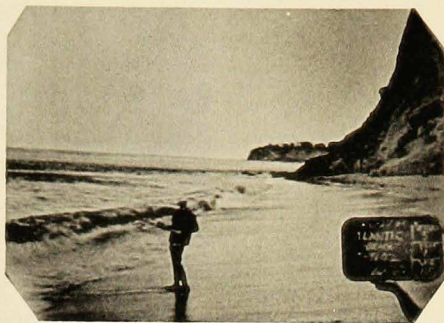
The authors are indebted to the U. S. Weather Bureau (Mr. D. Lee Harris) for making unpublished research material available during the work reported, and to Dr. W. G. Van Dorn (Scripps Institution of Oceanography) for suggesting studies of run-up in connection with the problem of tsunami damage.

## REFERENCES

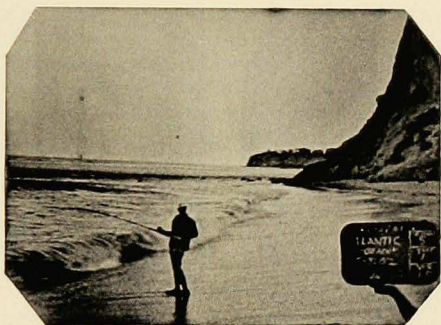
- CARRIER, G. F., AND H. P. GREENSPAN  
1958. Water waves of finite amplitude on a sloping beach. *J. Fluid Mech.*, 4: 97-109.
- GREENSPAN, H. P.  
1958. On the breaking of water waves of finite amplitude on a sloping beach. *J. Fluid Mech.*, 4: 330-334.
- HO, D. V., AND R. E. MEYER  
1962. Climb of a bore on a beach. Part I. Uniform beach slope. *J. Fluid Mech.*, 14: 305-318.  
1963. Notes on nonuniform shock propagation. *J. Acoust. Soc. Amer.*, 35: 1126-1132.
- KELLER, H. B., D. A. LEVINE, AND G. B. WHITHAM  
1960. Motion of a bore over a sloping beach. *J. Fluid Mech.*, 7: 302-316.
- MEYER, R. E.  
1960. Theory of characteristics of inviscid gas dynamics. *Encyclopedia of Physics*, 9. Springer-Verlag, Heidelberg.
- MEYER, R. E., AND A. D. TAYLOR  
1963. On the equations of waves on beaches. *Brown University Tech. Rep. NONR 562 (34)/5*.
- SHEN, M. C., AND R. E. MEYER  
1963a. Climb of a bore on a beach. Part II. Non-uniform beach slope. *J. Fluid Mech.*, 16: 108-112.  
1963b. Climb of a bore on a beach. Part III. Run-up. *J. Fluid Mech.*, 16: 113-125.
- STOKER, J. J.  
1947. Surface waves in water of variable depth. *Quart. Appl. Math.*, 5: 1-54.  
1957. *Water waves*. Interscience Publishers, New York. 595 pp.



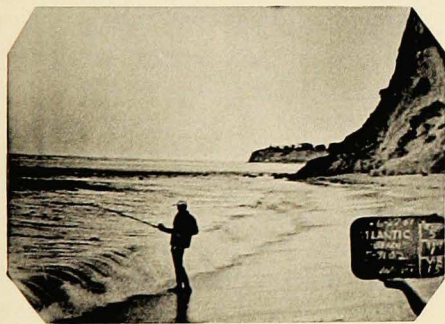
1



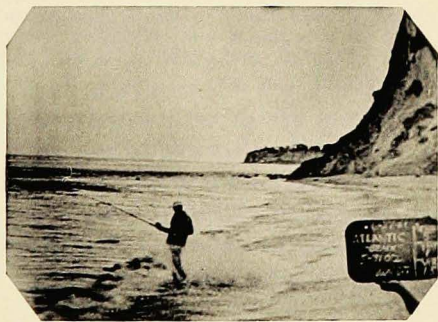
2



3



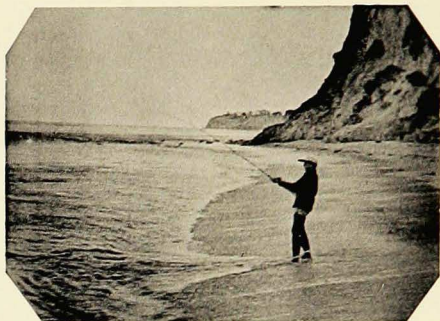
4



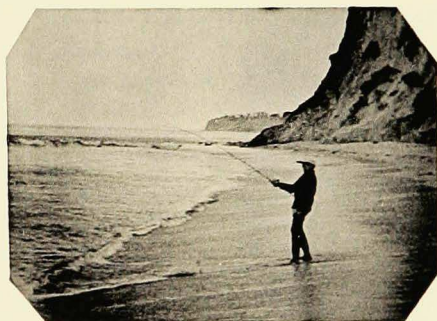
5

PLATE I. Frames from a videotape showing surf movement on a California beach (see p. 226).  
1,  $t = -24$ ; 2,  $t = -12$ ; 3,  $t = 0$ ; 4,  $t = 12$ ; and 5,  $t = 24$ .

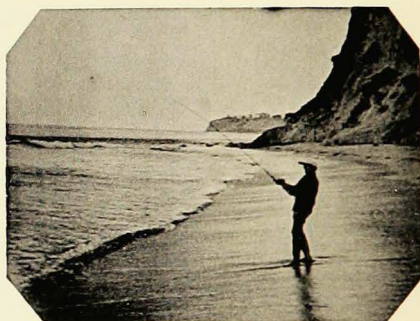




1



2



3

PLATE II. Frames from a videotape showing surf movement on a California beach (see p. 229).  
1,  $t = 500$ ; 2,  $t = 516$ ; and 3,  $t = 532$ .