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A Note on Wave Set-up

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ABSTRACT

Seaward of the breaker zone, the observations of Saville are in good qualitative agreement with the prediction that the mean surface level is increasingly depressed towards the shoreline, proportionally to $F(\eta)$, *i. e.*, to $(\sigma^2 h/g)^{-3/2}$, very nearly. The observed depressions are on the average greater than the theoretical by a factor of about 1.7. Between the breaker zone and the still-water level the surface tends to rise again in the way described by $d\bar{\zeta}/dx = Q(dh/dx)$, with the factor Q equal to 0.15.

1. *Introduction.* It was found experimentally by Fairchild (1958), and confirmed more recently by Saville (1961), that when a steady train of waves is propagated in water of non-uniform depth, the mean level of the water surface may differ appreciably from the still-water level. An effect of this kind was also recently suggested on theoretical grounds (Longuet-Higgins and Stewart, 1962)². This note compares the theory with the experimental results.

2. *Definitions.* Regular, two-dimensional waves, of constant period and amplitude, are supposed advancing into a region of non-uniform depth. Let σ = radian frequency = 2π /wave period; k = wavenumber = 2π /wave length; a = wave amplitude, = $1/2$ wave height; h = local still-water depth; x = horizontal co-ordinate in direction of wave propagation.

It is assumed that the depth varies only gradually, so that $(dh/dx)^2$ is negligible. When the waves originate in deep water ($kh \gg 1$) their amplitude and wavenumber there are denoted by a_∞ and k_∞ respectively. We have simply $k_\infty = \sigma^2/g$.

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² A similar prediction, in less explicit form, is made by Dorrestein (1962). See also a report by Fortak (1962).

3. *Theoretical Predictions.* In the paper by Longuet-Higgins and Stewart (1962; henceforward referred to as [I]), two types of prediction were made:

(1) Assuming that there is no loss of energy in the waves, either by friction, breaking or reflection, then the mean surface level at any point is lowered by an amount

$$-\bar{\xi} = a_{\infty}^2 k_{\infty} F(\eta), \quad (1)$$

where

$$\eta = \sigma^2 h/g, \quad (2)$$

and $F(\eta)$ is a dimensionless function defined as follows:

$$\left. \begin{aligned} F(\eta) &= -\frac{1}{4} \frac{d}{d\eta} \coth \xi, \\ \xi \tanh \xi &= \eta. \end{aligned} \right\} \quad (3)$$

The function $F(\eta)$ is plotted in [I]: fig. 1. For values of η less than 1 (*i. e.*, in moderately deep water), $F(\eta)$ lies close to the asymptote

$$F(\eta) \sim \frac{1}{8 \eta^{3/2}}, \quad \eta \ll 1. \quad (4)$$

For the validity of this result it is necessary that the conditions

$$ak \ll 1, \quad ak \ll (kh)^3 \quad (5)$$

for the small-amplitude theory of water waves be satisfied.

If the waves do not originate in deep water, but in a long channel of depth h_0 , then the generalization of eq. (1) is

$$-\bar{\xi} = a_{\infty}^2 k_{\infty} [F(\eta) - F(\eta_0)], \quad (6)$$

where

$$\eta_0 = \sigma^2 h_0/g, \quad (7)$$

and a_{∞} is a virtual amplitude at infinity, which may be calculated by the condition that the energy flux ($1/2 \rho g a^2 \times$ group velocity) is a constant. A graph giving the ratio of a_0 , the amplitude in water of depth h_0 , to a_{∞} , is given, for example, in Longuet-Higgins (1956).

(2) The second result of [I] applies to the altogether different situation in which the waves are in shallow water ($kh \ll 1$) and are limited in height by breaking. A rough argument suggested that

$$\frac{d\bar{\zeta}}{dx} = Q \frac{dh}{dx}, \quad (8)$$

where Q is a quantity of the order of 0.2.

Finally, we note that, according to the theory of solitary waves (Munk, 1949), the depth h_b at the initial breaking point is related to the wave amplitude in deep water by

$$\frac{h_b}{a_\infty} = 1.14 (a_\infty k_\infty)^{-1/3}. \quad (9)$$

4. *Experimental Data.* For comparison with eq. (6), the most suitable and extensive data are those given by Saville (1961: table 4). These are the observed differences in level $\bar{\zeta}$ when waves approached a ramp of slope 1:10, topped by another ramp of slope 1:6 or 1:3. The wave amplitude a_0 was measured in a depth of water equal to 10 feet (on the model scale), and the mean levels at four different distances from the shoreline. By reconstructing the beach graphically, one finds for the mean depth h at these positions the following:

	Upper slope 1:6				Upper slope 1:3			
Position:	1	2	3	4	1	2	3	4
Distance from shoreline:	18.5	23.6	30.8	51.0	6.5	11.6	18.8	39.0 feet
Depth h :	3.1	3.9	4.6	6.6	2.2	3.9	4.6	6.7

In order to plot all of the experimental points on the same diagram we have calculated a_∞ , k_∞ and η_0 as defined in §§ 2 and 3, and plotted

$$\frac{-\bar{\zeta}}{a_\infty^2 k_\infty} + F(\eta_0) \quad (10)$$

against $F(\eta)$, the above expression being the value of $F(\eta)$ given by eq. (6). The result is shown in Fig. 1. The circles refer to the 1:6 slope (scale model 1:10); the triangles refer to the 1:3 slope (prototype tank data), and the squares to the 1:3 slope (scale model 1:10).

In Fig. 1 a distinction has been made between whether the breaking point had or not had been reached, according to eq. (9). If $h > h_b$, *i. e.*, the waves had not yet broken, the plotted symbols have been filled in solidly. At the largest value of h for which $h \leq h_b$ (at any fixed value of a_∞ and σ), a line has been drawn through the center of the symbol; these presumably correspond to waves on the point of breaking. All other plotted symbols in Fig. 1 are left empty; these represent waves which had almost certainly broken.

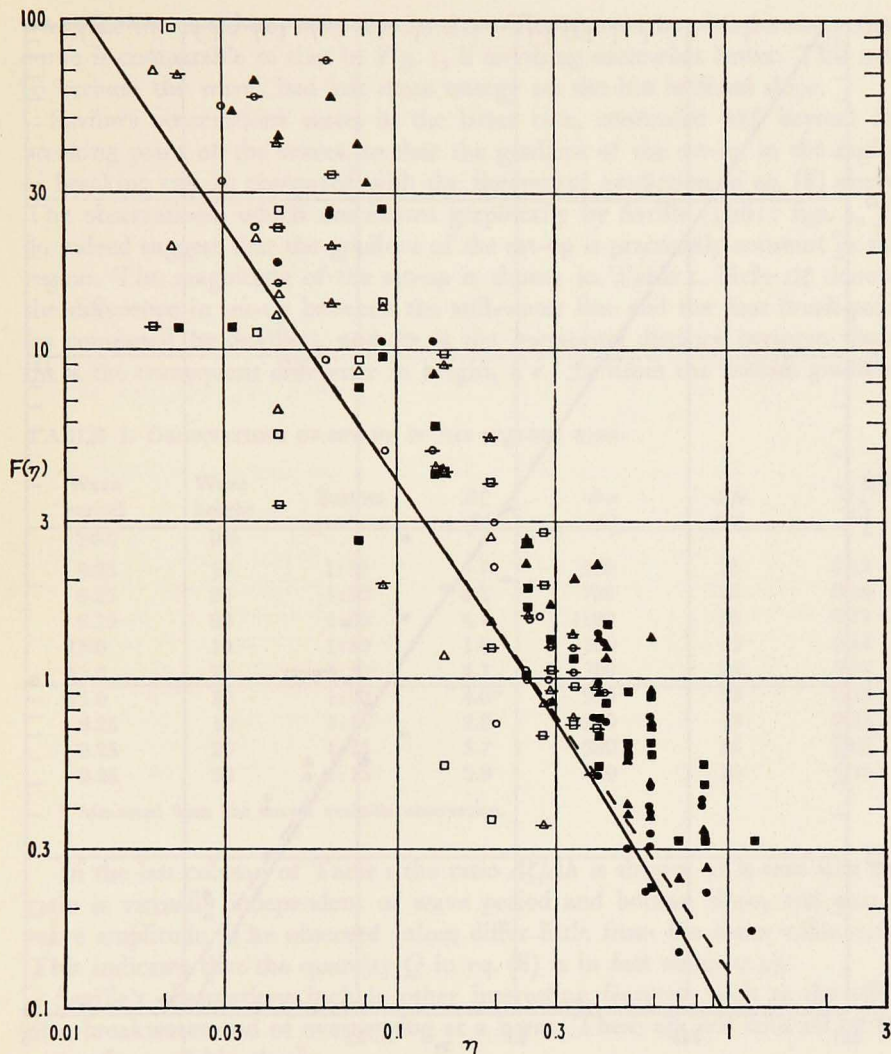


Figure 1. Observations of the surface depression [in the form of eq. (10)] compared with the theoretical function $F(\eta)$ (solid line). The horizontal co-ordinate is proportional to the local mean depth. Data are from Saville (1961): table 4.

The full curve in Fig. 1 represents the theoretical value of $F(\eta)$, and the broken curve is the asymptote, eq. (4).

It is seen that generally the trend of the observations is very similar to that of the theoretical curve, over a range of 1:1000. The plotted points corresponding to waves which had not broken lie generally above the theoretical

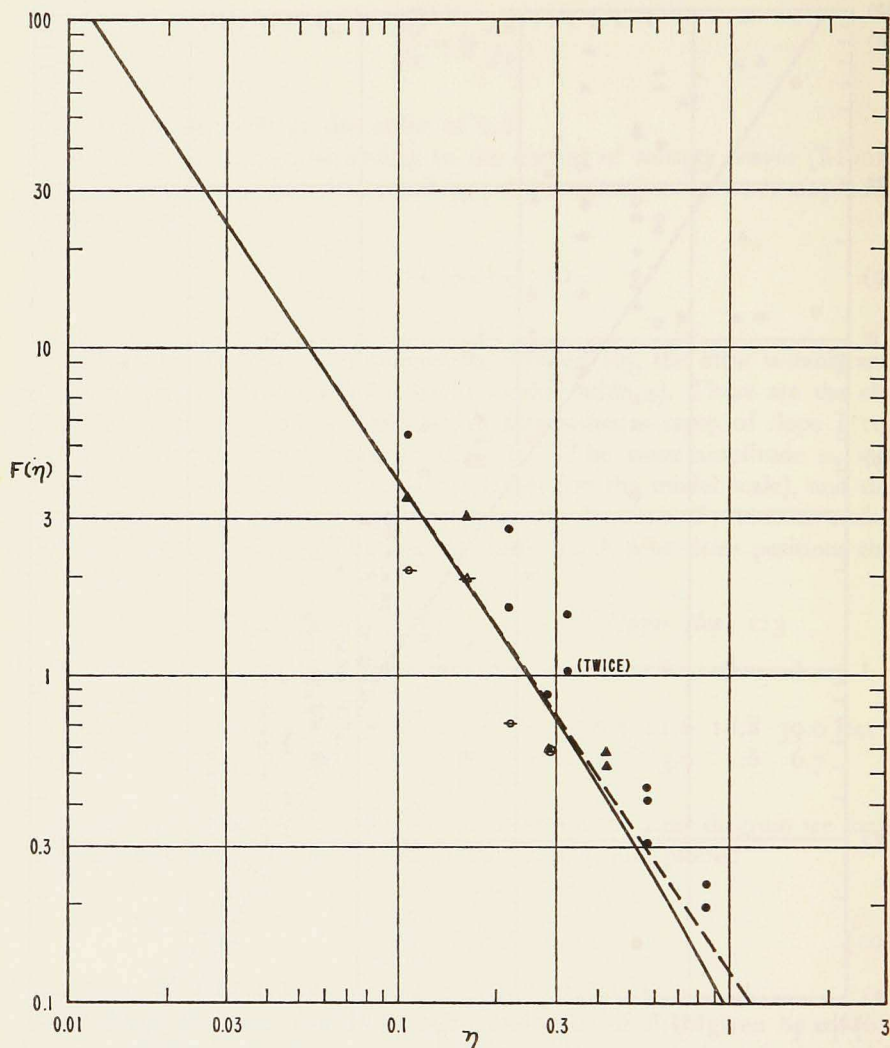


Figure 2. As in Fig. 1. Data also from Saville (1961): table 1.

curve, *i. e.*, the depression of the mean level is greater than predicted, by an average factor of about 1.7.

A second set of data, derived from Saville (1961: table 1), is shown in Fig. 2. These measurements were made on uniform slopes of 1:30 and 1:15 (indicated by circles and triangles respectively). Only those data are shown for which the waves had not yet broken (full plots) or had only just broken (plots with horizontal lines). Also the plots have been confined to those cases

when the waves did not over-top the slope. The agreement with the theoretical curve is comparable to that in Fig. 1, if anything somewhat better. This may be because the waves had lost more energy on the less inclined slope.

Saville's observations were, in the latter case, continued well beyond the breaking point of the waves, so that the gradient of the set-up in the region of breaking can be compared with the theoretical prediction of eq. (8) above. The observations, which are shown graphically by Saville (1961: figs. 4, 5), do indeed suggest that the gradient of the set-up is practically constant in this region. The magnitude of the set-up is shown in Table I. Here $\Delta\zeta$ denotes the difference in set-up between the still-water line and the first break-point (as computed by Saville³), and Δx is the horizontal distance between them. Δh is the consequent difference in height, *i. e.*, Δx times the bottom gradient.

TABLE I. OBSERVATIONS OF SET-UP IN THE BREAKER ZONE.

Wave period (sec)	Wave height (ft)	Bottom gradient	$\Delta\zeta$ (ft)	Δx (ft)	Δh (ft)	$\frac{\Delta\zeta}{\Delta h}$
9.25	10	1:30	1.7	390	13	0.13
9.25	20	1:30	4.2	780	26	0.16
9.25	30	1:30	4.1	1180	39	0.11
15.0	10	1:30	1.8	390	13	0.14
15.0	20	1:30	4.7	780	26	0.18
15.0	30	1:30	4.0*	900	30	0.13
9.25	10	1:15	2.0	200	13	0.15
9.25	20	1:15	5.7	390	26	0.22
9.25	30	1:15	5.9	580	39	0.15

* Measured from the lowest available observation.

In the last column of Table I the ratio $\Delta\zeta/\Delta h$ is shown. It is seen that this ratio is virtually independent of wave period and bottom slope, and also of wave amplitude. The observed values differ little from the mean value 0.15. This indicates that the quantity Q in eq. (8) is in fact about 0.15.

Saville's observations include other interesting features, such as the effect of a breakwater and of overtopping at a berm. These are not covered by the presently available theory.

5. *Conclusions.* Seaward of the breaker zone, the observations of Saville are in good qualitative agreement with the prediction of eq. (6); that is to say, the mean surface level is increasingly depressed towards the shoreline, proportionally to $F(\eta)$, *i. e.*, to $(\sigma^2 h/g)^{-3/2}$ very nearly. The observed depressions are on the average greater than the theoretical by a factor of about 1.7. Between the breaker zone and the still-water level the surface tends to rise again in the way described by eq. (8), with the factor Q equal to 0.15.

³ Saville's predicted break-points do not differ significantly from those given by eq. (9) above.

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