Long-period Waves over California's Continental Borderland[1] *Part II. Tsunamis*

Gaylord R. Miller, Walter H. Munk, and Frank E. Snodgrass

Institute of Geophysics and Planetary Physics
University of California, San Diego

ABSTRACT

During the Chilean tsunami of 22 May 1960, precise readings of sea level were digitally recorded every 15 seconds by the La Jolla low-frequency wave instrument. The tsunami remained above background for a week. The record gave a good opportunity to measure the decay of tsunami energy as a function of frequency. Energy is reduced by $1/e$ about once each half day. The decay is somewhat more rapid at high frequencies and at high energy densities. The total energy of the tsunami is estimated to be of the order of 3×10^{23} ergs.

Introduction. At the time of the Chilean earthquake of 22 May 1960 we were fortunate to have in operation a precision long-period wave recorder at the end of Scripps Pier whose output was recorded in digital form once each 15s for the week of tsunami activity. The installation was part of the San Clemente Island–La Jolla coherence study described in this issue (Snodgrass, *et al.*, 1962; henceforth referred to as 'Part I'). Unfortunately the San Clemente instrument was out of operation at the time, so we attempted to make use of a temporary tide gauge on the island (see fig. 1 of Part I for location), but the quality of this record was so poor and the timing so unreliable that no further attempt has been made to use it in this study; some of the record is shown for comparison in Fig. 1.

A collection of tsunami records and a preliminary description are available (Symons and Zetler, 1961). The epicenter was at 41° S, 73.5° W, and the time of the earthquake 1911 GMT, 22 May 1960. There were no aftershocks of comparable severity. The arrival time in La Jolla was 0850 GMT, 23 May (0050 PST, 23 May), giving a travel time of 13.65 h, or 0.57 days; the epicentral distance is 84.75°, or roughly 8500 km.

[1] This work has been sponsored by the National Science Foundation under grant-in-aid No. G 10186.

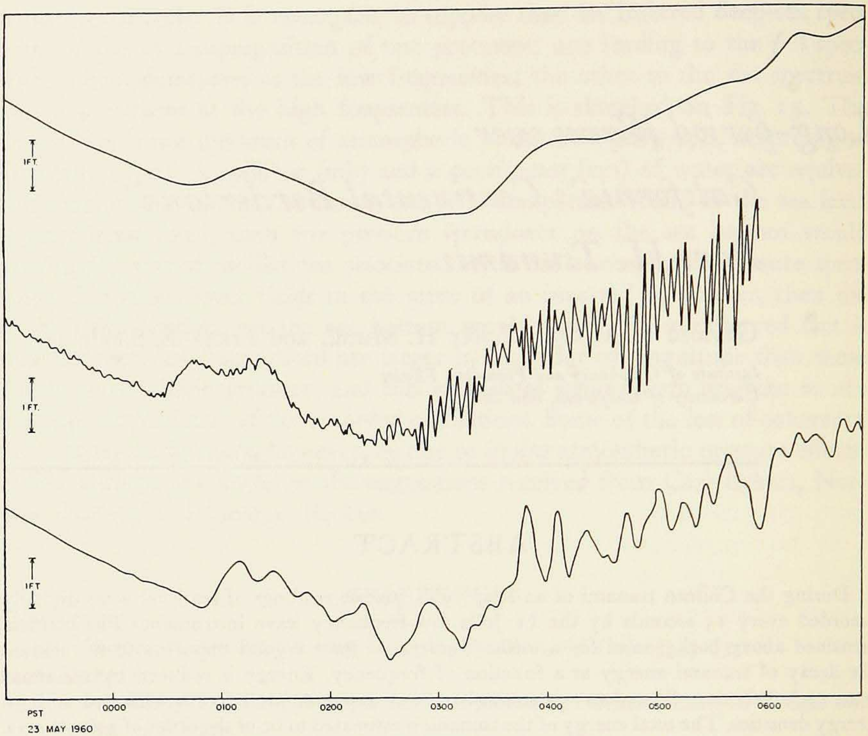

Figure 1. The tsunami as recorded on the La Jolla Vibrotron (Top and Bottom) and on the San Clemente tide gauge. The upper curve has been greatly smoothed.

The San Clemente and La Jolla records commence with a positive surge and a relatively steep forward front. The appearance is typical of California stations, and the tendency toward a bore formation becomes increasingly more pronounced as the wave travels northward over the continental borderland. The lower curve in Fig. 1 has been low-passed by numerical convolution methods (see Part I, *Prefiltering*) to remove frequencies above 24 cycles per hour (cph). Actually the spectrum is already poor in these high frequencies, and the appearance of the record is hardly altered by this amount of smoothing. The upper record, however, is severely altered by removing frequencies above 0.7 cph. The smoothed curve is plotted in Fig. 2 for comparison with the normal tide record[2]. All analysis parameters are listed in Table 1.

Example. Spectrum in Fig. 5 was based on 15^s readings low-passed and decimated 4:1 to 1^m intervals, low-passed and decimated again to 4^m intervals,

[2] We are grateful to Mr. Kenneth S. Ulm of the U. S. Coast and Geodetic Survey for making this prediction on the basis of the La Jolla tidal constants.

TABLE 1. ANALYSIS PARAMETERS.

Figure		Filters	N	m	v	Δf cph	lc cm
1, bottom	La Jolla record	15^{s}LP 1^{m}	–	–	–	–	–
1, top	La Jolla record	15^{s}LP 1^{m}LP 4^{m}LP 16^{m}	–	–	–	–	–
2	La Jolla record	15^{s}LP 1^{m}LP 4^{m}LP 16^{m}	–	–	–	–	–
3, 4	*La Jolla tsunami spectrum*						
	For the first two days in 0.5-day intervals*	15^{s}HP 15^{s}	2880	480	12	0.25	0.208
3	The next three days in 1-day intervals.......	15^{s}HP 15^{s}	5760	480	24	0.25	0.208
5	The first 160 hours in three 40-hour intervals	15^{s}LP 1^{m}LP 4^{m}HP 4^{m}	600	480	2.5	.0156	0.208
3	*La Jolla Background*						
	BEFORE 1:1000 19 May to 2230 19 May.. BEFORE 2:2230 19 May to 1100 20 May . AFTER 1: 1000 29 May to 2330 29 May . AFTER 2: 2230 29 May to 1100 30 May .	15^{s}HP 15^{s}	3000	480	12	0.25	0.208

* Zero time is 0854 GCT or 0054 PST.

and then high-passed. The analysis is based on $N = 600$ successive readings at intervals $\Delta t = 4$ minutes $= (1/15)$ hour. The energy was distributed among $m = 480$ frequency bands, each of width $\Delta f = (2m\Delta t)^{-1} = [2 \cdot 480 \cdot (1/15)^{h}]^{-1} = 0.0156$ cph giving $v = 2N/m = 2.5$ degrees of freedom. The least count (last significant digit) in the recorded data is 0.208 cm.

The San Clemente arrivals lead those at La Jolla by about 20 minutes, but the timing of the San Clemente record is too poor to permit a precise estimate of the phase difference.

Immediately following the original arrivals, the record consists of irregular wiggles which last for the better part of a week. The remaining paper is devoted to a spectral description of the oscillations at La Jolla.

Spectral Decay of Tsunami. The four curves in the upper part of Fig. 3 show the tsunami spectra for the first two days in 0.5-day intervals. For the remaining period the decay was slower, and the records were broken into 1-day intervals. These have been plotted separately (for clarity) in the central figure. In all such experiments a comparison with background activity is essential, preferably before and after the experiment. This was the procedure followed here, and the bottom curves show two spectra before and two after

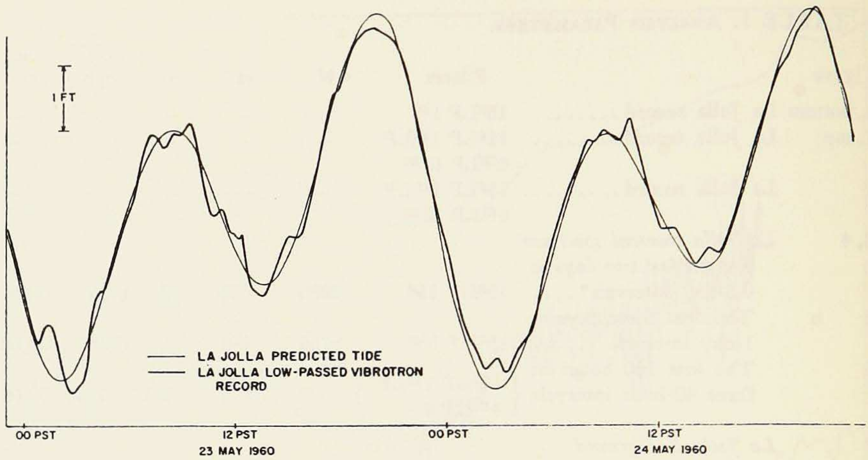

Figure 2. Predicted and recorded (smoothed) sea level at La Jolla during tsunami.

the event. The shaded band represents a subjective estimate of the expected range in background activity, and this band is replotted in the upper displays. It will be noted that above 10 cph the tsunami activity dips into the background after only one day, whereas at 2 to 4 cph it is still above background after 5 days!

The instruments were less sensitive than in the coherence study (Part I), and the instrumental noise level is estimated at 10^{-3} cm²/cpks (*cf.* 10^{-5} cm²/cpks in Part I). Over most of the frequency range the tsunami is very comfortably above this instrumental noise level.

The sharp cut-off below 1 cph is the result of high-pass filtering. In addition, the high frequencies are somewhat depressed by hydraulic filters (Part I). We have deliberately not corrected for these effects, for a plot of the raw spectrum gives a better impression of the expected contamination from side bands, and besides only the *relative* spectra are needed for estimates of relative decay. Fig. 4 shows the spectrum (corrected for all prefiltering) for the first interval (the initial 12 hours) over the interesting range of frequencies.

Fig. 5 shows the corrected tsunami spectra at very low frequencies. Here the record has been divided into coarser intervals, 40 hours each; even so the statistics are very poor. But the indication is that the decay is very slow and that some activity above background at 0.5 to 1.0 cph might be expected for fully 10 days.

Three features are evident: (1) The *shape* of the spectrum remains remarkably unchanged during the decay, but there is an indication that (2) high frequencies decay somewhat more rapidly than low frequencies, and (3) the logarithmic rate of decay does not vary markedly during the decay.

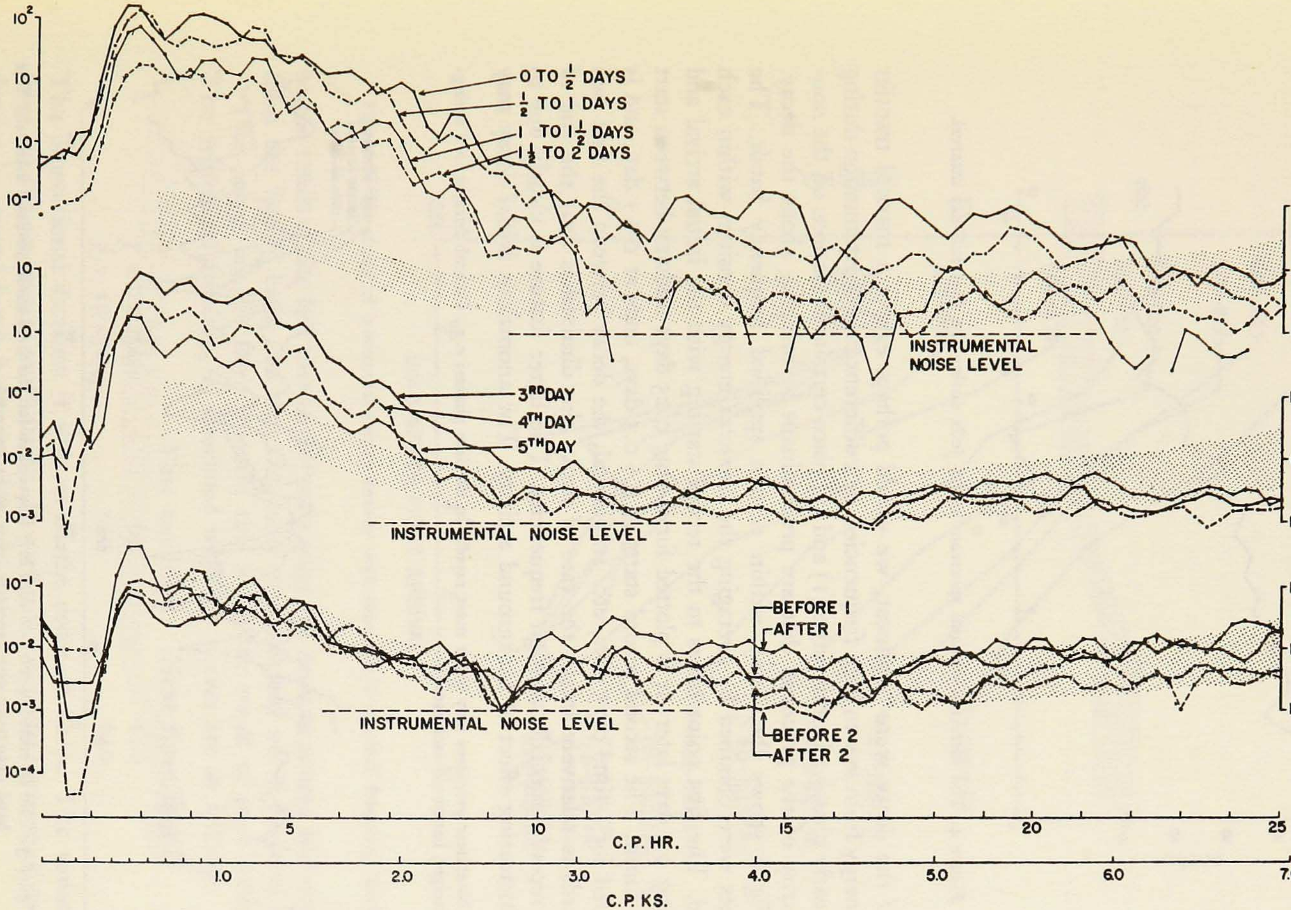

Figure 3. The tsunami spectrum (uncorrected) at La Jolla for the first two days in half-day intervals (Top) and the next three days in one-day intervals (Center). The dotted band indicates the expected range in background (again uncorrected). The estimated background is based in part on the four background spectra (Bottom), two of these from records preceeding the tsunami and two from records following the tsunami. The dashed line at 10-3 cm²/cpks gives the instrumental noise level. All energy densities are in cm²/cpks.

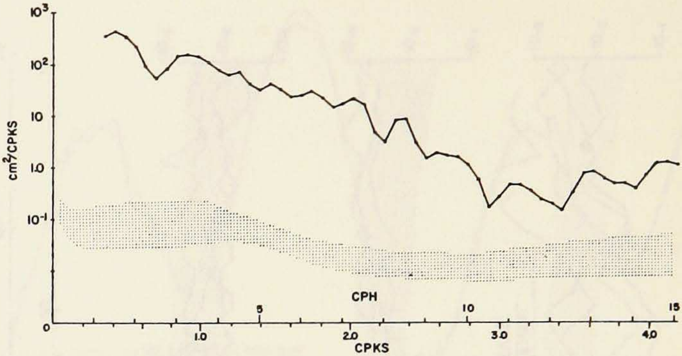

Figure 4. The corrected tsunami spectrum at La Jolla during the first half-day interval.

If the decay were nonlinear, we would perhaps expect a marked transfer of energy from low to high frequencies, or a different rate of attenuation during the early phases.[3] In this sense (1) and (3) place certain limitations on the nonlinearity of the processes that are predominant in bringing about the decay.

Fig. 6 shows the decay within certain specified frequency bands. The values were obtained by averaging the *corrected* energy densities within each band. The first point refers to the record starting with the initial arrival and ending 0.5 days later; it is plotted for a time 0.25 days midway between start and finish. The second record starts at 0 + 0.5 days, ends at 0 + 1 day, and is plotted for a time 0.75 days, etc. In general, the decay between the first two records is relatively small; the rate of decay also diminishes near the end of the record, especially at high frequencies. The latter feature is clearly due to an increasing effect of background activity. The anomalous *initial* decay may

3 Nonlinear processes may occur most rapidly in the late phases; *e.g.*, Colomb friction in a spring-and-weight harmonic oscillator.

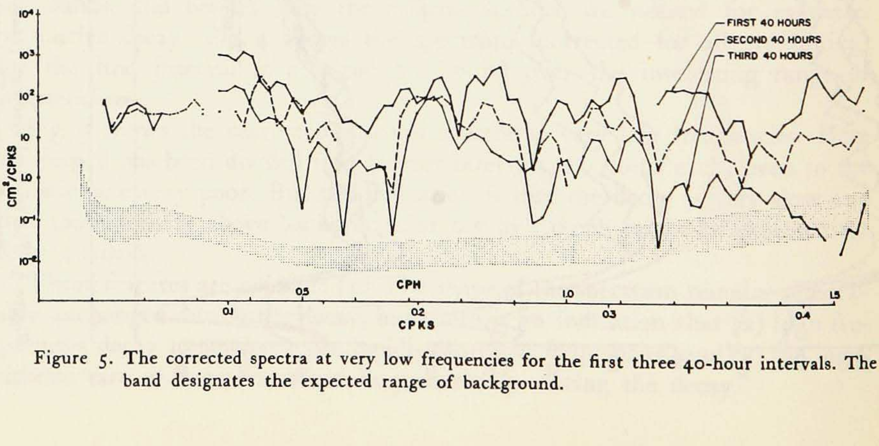

Figure 5. The corrected spectra at very low frequencies for the first three 40-hour intervals. The band designates the expected range of background.

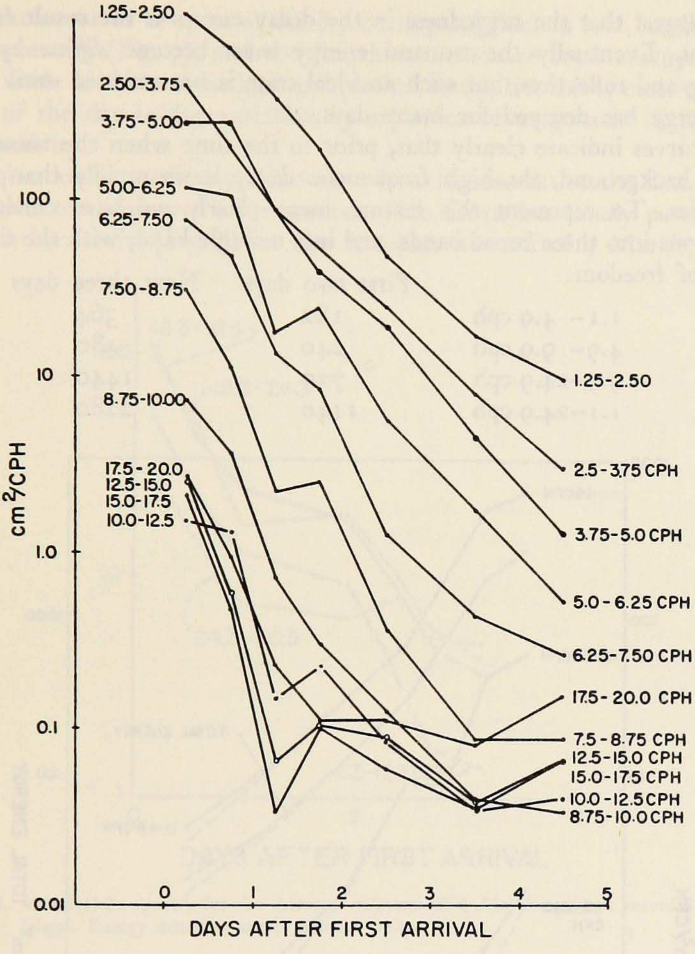

Figure 6. The decay of (corrected) energy density with time within specified frequency bands.

be the result of the fact that at this early stage the tsunami energy had not yet filled the Pacific basin and that the decay processes had not fully developed.

The curves are far from smooth; this is *not* the result of poor statistics. The degrees of freedom, ν, associated with the curves are as follows:

	First two days	Next three days
$f < 10$ cph	60	120
$f > 10$ cph	120	240

The proportional variance is $\nu^{-\frac{1}{2}}$, or of the order of $10\,^0/_0$. The variations from curve to curve are much larger. In particular, note the increase of energy density in several bands between 1.25 and 1.75 days.

We suggest that the raggedness in the decay curves is the result of coastal reflections. Eventually the tsunami energy must become *diffuse* by coastal scattering and reflection, but such an ideal state is not attained until the tsunami energy has decayed for many days.

The curves indicate clearly that, prior to the time when the tsunami dips into the background, the high frequencies decay more rapidly than the low frequencies. To represent this feature more clearly we have collected the observations into three broad bands, and into a single band, with the following degrees of freedom:

	First two days	Next three days
1.1– 4.9 cph	182	364
4.9– 9.9 cph	240	480
9.9–24.9 cph	720	1440
1.1–24.9 cph	1140	2280

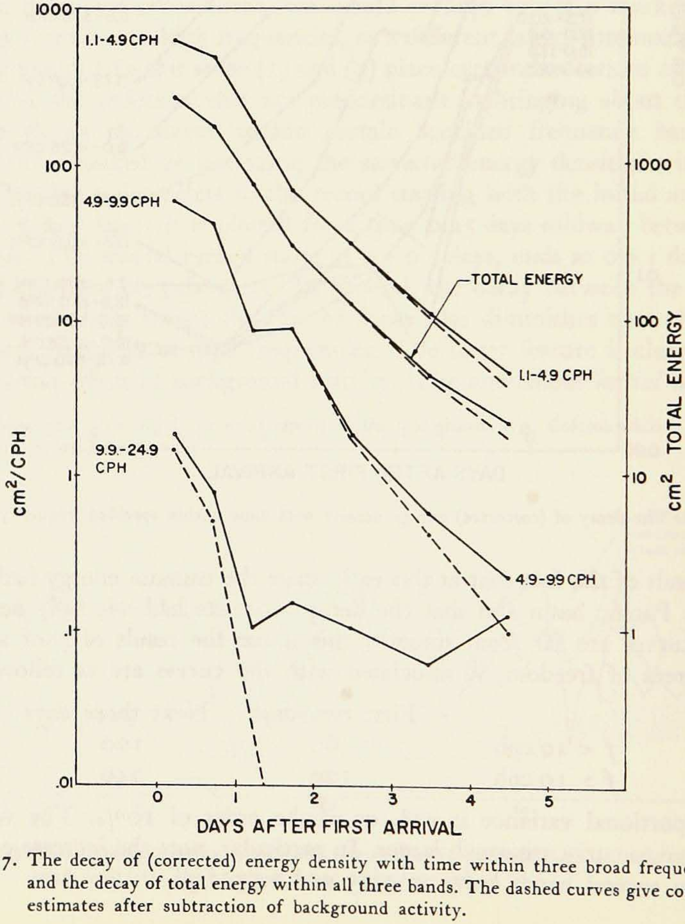

Figure 7. The decay of (corrected) energy density with time within three broad frequency bands, and the decay of total energy within all three bands. The dashed curves give corresponding estimates after subtraction of background activity.

The three curves of spectral density correspond then to low, medium, and high frequencies (Fig. 7), and the fourth curve to the total energy. An attempt is made to subtract background (dashed lines), but this is most uncertain because of the day-to-day variation by an order of magnitude in the background activity.

The question immediately arises as to how representative the decay curves are of other tsunamis and other stations. Van Dorn has kindly permitted us to present his results (Fig. 8) for the Aleutian tsunami of 9 March 1957 as

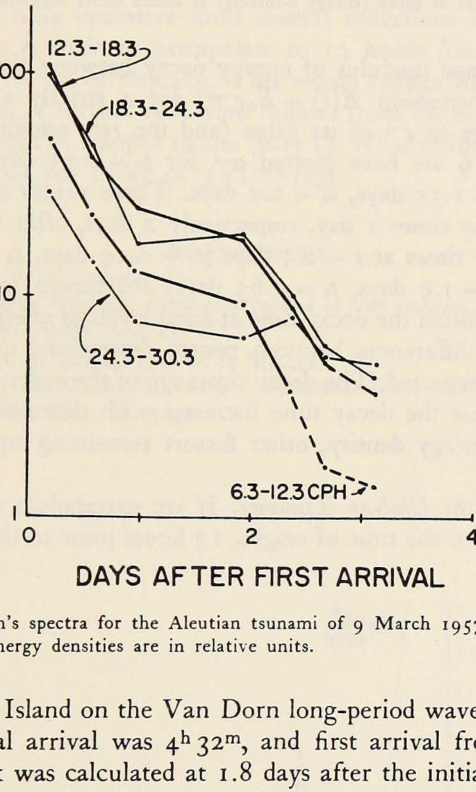

Figure 8. Van Dorn's spectra for the Aleutian tsunami of 9 March 1957 as recorded at Wake Island. Energy densities are in relative units.

recorded at Wake Island on the Van Dorn long-period wave recorder. Travel time for the initial arrival was $4^h 32^m$, and first arrival from a particularly violent after-shock was calculated at 1.8 days after the initial arrival. Contribution from background activity is small except where the lines are dashed. No corrections for background were made. The decay during the first day and after the second day is comparable to our estimates. Between the first and second day the decay is virtually absent, indicating the arrival of strong coastal reflections.

Let E_1 designate the mean energy density in a record centered at a time t_1; similarly for E_2; and let

$$\alpha = \frac{\ln E_1 - \ln E_2}{t_2 - t_1}$$

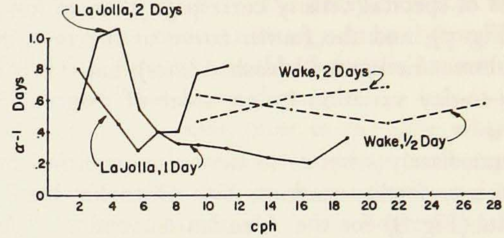

Figure 9. The rate of decay (energy e-folding) at stated times following initial arrival.

designate the average modulus of energy decay between times t_1 and t_2 consistent with the expression $E(t) = E_0 e^{-\alpha t}$ for a strictly exponential decay. This energy decays to e^{-1} of its value (and the *rms* amplitude to $e^{-\frac{1}{2}}$) in a time α^{-1}. In Fig. 9 we have plotted α^{-1} for $t_1 = 0.75$ days, $t_2 = 1.75$ days, and again for $t_1 = 1.75$ days, $t_2 = 2.5$ days. These values of α^{-1} may be regarded as the decay times 1 day, respectively 2 days, after the initial arrival. Van Dorn's decay times at $t = 0.5$ days ($t_1 = 0.19$ days, $t_2 = 0.95$ days) and at $t = 2$ days ($t_1 = 1.9$ days, $t_2 = 2.65$ days) are shown for comparison. In Fig. 10 we have plotted the decay time at fixed levels of energy density. Figs. 9 and 10 represent differences between poorly determined quantities, and the scatter is large, as expected. The decay times are of the order of 0.5 day. There is an indication that the decay time increases with decreasing frequency and with decreasing energy density, other factors remaining equal.

The Energy of the Chilean Tsunami. If we extrapolate the energy of the Chilean Tsunami to the time of origin, 13 hours prior to the initial arrival at

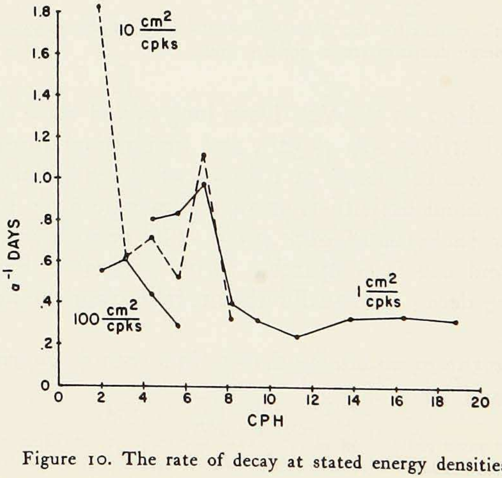

Figure 10. The rate of decay at stated energy densities.

La Jolla, we infer an energy density of roughly $5000\,\varrho\,g$ ergs cm^{-2}. This is the value at the recorder (depth 100 feet). In the open sea the energy density is believed to be reduced in the ratio of the square-root of the depths, and this leads to $400\,\varrho\,g$ ergs cm^{-2}. Coastal resonances appear not to be of much importance in the frequency range of the tsunami (fig. 2 in Part I). For the Pacific the total energy is the $400\,\varrho\,g$ ergs cm$^{-2} \times 1.8 \times 10^{18}$ cm^{2} $= 7.5 \times 10^{23}$ ergs.

There are many uncertainties in this estimate. We have extrapolated backwards as if the signal had been diffuse at the time of origin. Actually the decay processes are not fully operative until several reflections have occurred, and it might be more realistic to extrapolate to 12 hours (one typical reflection time) subsequent to the earthquake. This would reduce the estimated energy by a factor of two. A further reduction follows from the fact that some of the energy is in the form of trapped modes (part I). We conclude that the tsunami energy might be of the order of 3×10^{23} ergs.

REFERENCES

SYMONS, J. M. and B. D. ZETLER
1961. The tsunami of May 22, 1960, as recorded at tide stations. U.S. Coast and Geodetic Survey, Washington 25, D.C.

SNODGRASS, F. E., W. H. MUNK, and G. R. MILLER
1962. Long-period waves over California's continental borderland. Part 1. Background spectra. J. Mar. Res., *20*: 3–30.