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SALINITY EFFECTS IN ESTUARIES

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ABSTRACT

This paper examines the relative importance, in the equation of motion, of the convective acceleration terms and of the extra pressure gradient due to a longitudinal gradient of salinity up an estuary. For estuaries in which the latter is the most important, a criterion is given for the direction of mean water velocity just above the bed.

I. INTRODUCTION

A previous investigation (Abbott, 1960) of an estuary examined the effect of the bed's boundary layer on the tidal velocity; the density gradient, due to varying salinity, was neglected. However, in spite of this and other simplifications which were made to keep the problem tractable, the theory gave results for the Thames that were in reasonable agreement with observation; these results included the effect of the boundary layer on amplitude and phase of the tidal current near the bed, the drift velocity (*i.e.*, the mean velocity at a fixed point) near the bed, and the direction of this drift along the estuary.

Neglect of the density gradient will not give realistic results in every case; in some estuaries this effect may be all important in determining the character of the flow. In § II this paper attempts to assess the effect on the flow of a given density gradient in a given estuary.

Theoretically the basic one-dimensional motion in an estuary is governed by a balance of the time acceleration term $\partial u/\partial t$, the instantaneous surface slope term, and the frictional resistance. To this level of approximation the motion is periodic; however, when convective acceleration terms are taken into account, then the motion is no longer strictly periodic, there being in general a drift velocity at all depths at each point along an estuary in a direction depending on the behaviour of the surface current.

In some estuaries the most important secondary effect may be the salinity gradient rather than the convective effect, and in this case drifts are again likely to occur, since the salinity gradient is constant in sign and does not reverse with the tide; this introduces a preferred direction and a resultant drift at each depth. Even in an estuary in which the salinity gradient is relatively small, there may still be movements of water due to it; this applies particularly to secondary flow in a lateral direction.

Knowledge of the magnitude and direction of the drift in an estuary is of practical importance because of the influence it has on the movement of sediment. The problem of evaluating the drift theoretically is probably intractable when it is due to both density and convective effects and when neither predominate. However, when density differences are more important, a criterion is obtained, in § III, for the direction of drift just above the bed of an estuary. In § IV this is applied to the Mersey, and it is shown that the drift just above the bed reverses to seawards in the vicinity of Eastham.

II. DENSITY EFFECTS IN TIDAL ESTUARIES

First consider a general estuary where the x-axis is taken horizontal in the upstream direction, the y-axis vertically upwards, and the z-axis in a lateral direction. The longitudinal momentum equation may be written

$$\varrho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + \frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} , \tag{1}$$

where (u, v, w) is the water velocity referred to these axes, τ_{xy} etc. the components of stress (turbulent plus viscous), p the pressure, and ϱ the water density. The bed is given by $y = B(x, z)$, the mean tide level up the estuary by $y = s(x)$. The water surface at any time is taken as $y = \eta(x, t)$, where $s = \overline{\eta}$, the mean value of η over a tidal period at a fixed point.

The pressure term in (1) may be evaluated as follows, assuming that the pressure is hydrostatic:

$$p = g \int_y^\eta \varrho \, dy ; \tag{2}$$

therefore

$$\frac{\partial p}{\partial x} = g \varrho \frac{\partial \eta}{\partial x} + g \int_y^\eta \frac{\partial \varrho}{\partial x} \, dy , \tag{3}$$

$$= g\varrho\,\frac{\partial\eta}{\partial x} + g\,(\eta - y)\,\frac{\partial\varrho}{\partial x} \qquad (4)$$

if it is assumed that $\partial\varrho/\partial x$ is independent of y. Pritchard (1954) has observed that this horizontal gradient does not change appreciably with depth in the James River. In general $\partial\varrho/\partial x$ is negative. The first term of (4) is the usual surface slope term; the second term, which is zero on the surface, is the pressure gradient due to the density gradient in an estuary which is related to the known salinity gradient; in fact, neglecting differences of temperature, we may take $\partial\varrho/\partial x$ as proportional to the salinity gradient. The right-hand side of (1) is approximately $\partial\tau_{xy}/\partial y$, since the other two terms are in general small, and they are hereafter neglected. Hence, from (1) and (4) we have

$$\varrho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + g\varrho\,\frac{\partial\eta}{\partial x} + g\,(\eta - y)\,\frac{\partial\varrho}{\partial x} = \frac{\partial\tau_{xy}}{\partial y}\ . \qquad (5)$$

Consider the various terms of (5). For the Thames their rough maximum orders of magnitude are given in lb-ft-sec units by:

$$\varrho\,\frac{\partial u}{\partial t} \sim \varrho\,\omega\,U_0 \doteqdot 4 \times 10^{-2},$$

$$\varrho u\,\frac{\partial u}{\partial x} \sim \frac{\varrho\,\omega\,U_0^2}{2\,c} \doteqdot 3 \times 10^{-3},$$

$$\varrho v\,\frac{\partial u}{\partial y} \sim \frac{\varrho\omega\,U_0 H}{h} \doteqdot 10^{-2},$$

$$\varrho w\,\frac{\partial u}{\partial z}\ \text{is neglected},$$

$$g\varrho\,\frac{\partial\eta}{\partial x} \sim \frac{g\varrho\,\omega\,H}{c} \doteqdot 8 \times 10^{-2},$$

$$g\,(\eta - y)\,\frac{\partial\varrho}{\partial x} \sim c^2\left|\frac{\partial\varrho}{\partial x}\right| \doteqdot 5 \times 10^{-3},$$

$$\frac{\partial\tau_{xy}}{\partial y} \sim \frac{\tau_{bed}}{h} \sim \frac{g\varrho\,U_0^2}{h\,C^2} \doteqdot 9 \times 10^{-2}.$$

Here U_0 is the current amplitude, H the tidal amplitude, $2\pi/\omega$ the tidal period, $c = \sqrt{gh}$, h the mean depth, and C Chezy's constant. In

deriving the above it is assumed that the tide can be represented by a progressive wave

$$u = U_0 \cos\left\{\omega\left(t - \frac{x}{c}\right)\right\}, \quad \eta = H \sin\left\{\omega\left(t - \frac{x}{c}\right)\right\}; \qquad (6)$$

this is not strictly accurate but will suffice for the present purpose. In estimating $\varrho v \partial u / \partial y$ it is assumed that v varies linearly from zero at the bed to $\partial \eta / \partial t$ at the surface and that $\partial u / \partial y$ is roughly u/y in the boundary layer region where $\varrho v \partial u / \partial y$ has its maximum value. The values for the Thames which are used are: $U_0 = 4$ ft/sec, $H = 8$ ft, $h = 26$ ft, $C = 120$ ft$^{\frac{1}{2}}$/sec, $\partial \varrho / \partial x = -6 \times 10^{-6}$ lb/ft^4.

The dominant terms are therefore the time acceleration term, the surface slope term and the shear term—the three terms which are retained in the usual approximate theory of motion in an estuary. Next in order of magnitude are the sum of the convective acceleration terms (irrespective of any phase difference between them) and the density gradient term, which is the smallest of those considered.

Thus, for tidal motion in the Thames, a second approximation which takes account of the convective acceleration but which neglects variations of salinity is valid in general, though the reservations given in § I apply with regard to lateral effects and secondary flow.

A rough measure of the relative importance of density (salinity) effects in a general estuary may be obtained from the dimensionless number

$$D = \frac{c^3 \left|\dfrac{\partial \varrho}{\partial x}\right|}{\omega\, U_0^2 \varrho}.$$

D is proportional to the ratio of the density gradient term to the sum of the convective acceleration terms in the longitudinal equation of motion. In summing the convective terms it is assumed that $\dfrac{U_0}{c} \propto \dfrac{H}{h}$; for a progressive wave in a frictionless uniform channel there is the exact relation $\dfrac{U_0}{c} = \dfrac{H}{h}$.

Small values of D indicate that the most important second order effects are the nonlinear convective terms in the equation of motion, these giving rise to sizable drift currents in an estuary as shown by Abbott (1960). On the other hand, large values of D indicate that

any drifts are due primarily to salinity differences. Examples are: Thames, $D = 0.8$; Mersey, $D = 7$; James, $D = 12$. Thus in the latter two estuaries, salinity effects predominate; this is confirmed numerically by Pritchard (1956), who has shown from observations in the James River (a tributary estuary of Chesapeake Bay) that the convective terms of mean motion are insignificant compared to the terms dependent on density gradient, surface slope and shear force.

In calculating values of D above, the steepest salinity gradient in an estuary is used; though the values given are only rough, they give an indication of the relative importance of salinity effects in the three estuaries.

III. MEAN MOTION

In this section we examine the drifts in an estuary which result when the motion is meaned over a tidal cycle. The components of water velocity in an estuary may be written

$$u = \overline{u} + U_0 \cos (\omega t - \varepsilon) + u', \quad v = \overline{v} + v', \quad w = w', \qquad (7)$$

as in Pritchard (1956), where $(\overline{u}, \overline{v}, o)$ is the steady drift velocity at a point, $U_0 \cos (\omega t - \varepsilon)$ the tidal motion, and (u', v', w') the turbulent fluctuations. Both U_0 and ε are functions of x and y.

Following Pritchard, eqs. (7) are inserted into (1) and the mean is taken over a tidal cycle. First we observe from (4) that

$$\overline{\frac{\partial p}{\partial x}} = g\varrho \frac{dS}{dx} + g (S - y) \frac{\partial \varrho}{\partial x}, \qquad (8)$$

where $S(x)$ is the mean tide level of the water surface in an estuary. It is assumed in deriving (8) that ϱ and $\partial \varrho / \partial x$ vary little over a tidal cycle compared to the variation of $\partial \eta / \partial x$ and η. This is reasonable since a relatively large change in salinity produces only a small change in density. In (8), $\partial \varrho / \partial x$ can be taken, for example, as a mean of values occurring at high slack water and low slack water; for convenience a bar is omitted.

From (7), (1) and (8) we obtain

$$\overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} + U_0 \frac{\partial U_0}{\partial x} + g \frac{dS}{dx} + \frac{g(S - y)}{\varrho} \frac{\partial \varrho}{\partial x} = \frac{1}{\varrho} \frac{\partial \overline{\tau}_{xy}}{\partial y}. \qquad (9)$$

When taking the mean of (1) it is assumed that the time scale of the turbulent fluctuations in (7) is much smaller than the time scale of

the tidal motion. An equivalent equation to (9) occurs in Pritchard (1956).

A criterion for the direction of drift just above the bed will be derived for estuaries which have a high value of D. In this case the convective terms of mean motion can be neglected in (9) in comparison with the density gradient term; this has been confirmed by comprehensive observations in the James River (Pritchard, 1956). Pritchard and Kent (1956) calculated from theory and observation the drift profile over the whole depth of a section; however, for practical considerations of sediment movement we are chiefly interested in the direction of drift just above the bed, and a simple and easily applied theoretical criterion can be obtained for this.

The term $U_0 \partial U_0 / \partial x$ in (9) will, as a first approximation, be neglected, since in many cases the amplitude of the tidal current is nearly constant along an estuary. However, the result will later be quoted when this term is retained but assumed independent of y.

With these assumptions (9) becomes

$$\frac{1}{\varrho} \frac{\partial \overline{\tau}_{xy}}{\partial y} = g \frac{dS}{dx} + g \frac{(S-y)}{\varrho} \frac{\partial \varrho}{\partial x} ; \tag{10}$$

integrating this with respect to y gives

$$\overline{\tau}_{xy} = g \varrho y \frac{dS}{dx} + g \left(Sy - \tfrac{1}{2} y^2\right) \frac{\partial \varrho}{\partial x} + f(x), \tag{11}$$

since the variation of ϱ with y is small. On the surface $\overline{\tau}_{xy} = 0$, hence

$$f(x) = - g \varrho S \frac{dS}{dx} - \tfrac{1}{2} g S^2 \frac{\partial \varrho}{\partial x}. \tag{12}$$

From (11) and (12) we have

$$\overline{\tau}_{xy} = - g \varrho (S-y) \frac{dS}{dx} - \tfrac{1}{2} g (S-y)^2 \frac{\partial \varrho}{\partial x} ; \tag{13}$$

on the bed, therefore,

$$\overline{\tau}_{xy} = g h \left\{ \tfrac{1}{2} h \left(- \frac{\partial \varrho}{\partial x} \right) - \varrho \frac{dS}{dx} \right\}, \tag{14}$$

where $h(x, z) = S(x) - B(x, z)$ is the local depth at mean tide level.

In general the flow in an estuary is "rough turbulent", so that the shear on the bed is given by

$$\tau_{xy} = k\varrho u|u|, \tag{15}$$

where k is a positive quantity. We now show that the sign of the mean shear is the same as that of the drift so that we can deduce the direction of the latter from (14).

Neglecting turbulent fluctuations, we can write the velocity just above the bed as

$$u = \overline{u} + U_0 \cos \varphi, \tag{16}$$

where $\overline{u} < U_0$ and φ varies from 0 to 2π over a tidal cycle. Let the first zero of u occur at $\varphi = \alpha \ (0 < \alpha < \pi)$; then

$$\overline{u} + U_0 \cos \alpha = 0$$

and

$$\tau_{xy} = k\varrho u^2 \ \text{in} \ 0 < \varphi < \alpha \ \text{and} \ 2\pi - \alpha < \varphi < 2\pi,$$
$$= -k\varrho u^2 \ \text{in} \ \alpha < \varphi < 2\pi - \alpha.$$

Taking the mean of (15) over a tidal cycle gives

$$\overline{\tau}_{xy} = \frac{k\varrho}{2\pi} \left\{ \int_0^\alpha u^2 d\varphi - \int_\alpha^{2\pi-\alpha} u^2 d\varphi + \int_{2\pi-\alpha}^{2\pi} u^2 d\varphi \right\}$$
$$= \frac{k\varrho}{\pi} \left\{ \int_0^\alpha u^2 d\varphi - \int_\alpha^\pi u^2 d\varphi \right\}.$$

This reduces to

$$\overline{\tau}_{xy} = \frac{k\varrho}{\pi} \left\{ 2\overline{u}^2 \alpha + 4\overline{u}\, U_0 \sin \alpha + U_0{}^2 \sin \alpha \cos \alpha \right.$$
$$\left. + U_0{}^2 \alpha - \pi \overline{u}^2 - \tfrac{1}{2}\pi U_0{}^2 \right\}, \tag{17}$$

after substituting (16) and evaluating the integrals. Now,

$$\cos \alpha = -\frac{\overline{u}}{U_0}, \quad \sin \alpha = \frac{\sqrt{U_0{}^2 - \overline{u}^2}}{U_0} \quad \text{as} \quad 0 < \alpha < \pi;$$

therefore, from (17)

$$\overline{\tau}_{xy} = \frac{k\varrho}{\pi} \left\{ 3\overline{u}\sqrt{U_0{}^2 - \overline{u}^2} + \left(U_0{}^2 + 2\,\overline{u}^2\right)\left(\cos^{-1}\left(-\frac{\overline{u}}{U_0}\right) - \frac{\pi}{2}\right) \right\}. \tag{18}$$

It is easily seen that

$$\cos^{-1}\left(-\frac{\overline{u}}{U_0}\right) - \frac{\pi}{2} = \overline{u}P, \tag{19}$$

where P is always positive. From (18) and (19), therefore, we obtain

$$\overline{\tau}_{xy} = A\overline{u}, \tag{20}$$

where A is always positive. Thus the drift and the mean shear have the same sign.

Therefore, from (14), the drift just above the bed will be landwards or seawards according as

$$\tfrac{1}{2} h \left(-\frac{\partial \varrho}{\partial x}\right) \gtrless \varrho \frac{dS}{dx}. \tag{21}$$

This expresses the balance between density gradient and slope of the mean tide level. It can be seen from (21) that, if the density gradient is independent of lateral position in an estuary, any landward drift will reverse near the sides of the estuary first, where $h\,(x, z)$ will be smaller than the value in the centre of the estuary. However, (21) does not apply right up to the banks, since stresses other than τ_{xy} will be important there.

Eq. (21) also applies when flow near the bed is "smooth turbulent"; there is now a viscous layer, and at the bed we have

$$\overline{\tau}_{xy} = \mu \frac{\partial \overline{u}}{\partial y}, \tag{22}$$

where μ is positive, and on the bed $\overline{u} = 0$. Therefore the sign of $\overline{u}$ just above the bed is the same as that of $\partial \overline{u}/\partial y$ at the bed, which by (22) has the same sign as $\overline{\tau}_{xy}$. Again we have shown that the drift and the mean shear have the same sign.

If account is taken of the $U_0 \partial U_0/\partial x$ term occurring in (9), we obtain in place of (21)

$$\tfrac{1}{2} h \left(-\frac{\partial \varrho}{\partial x}\right) \gtrless \varrho \left(\frac{dS}{dx} + \frac{U_0}{g}\frac{\partial U_0}{\partial x}\right) \tag{23}$$

for landward or seaward drift respectively. It is assumed in deriving (23) that U_0 is independent of y.

In the case of the Thames it can be shown from (21) that the density gradient alone (as determined from the measured values of salinity), without the convective effect, cannot sustain a landward drift near the bed. Table I gives results of these calculations.

TABLE I

x	$\frac{1}{2} h \left(-\frac{\partial \varrho}{\partial x} \right)$	$\varrho \frac{dS}{dx}$
Miles from London Bridge	lb/ft^3 × 10^4	lb/ft^3 × 10^4
− 40	0.3	1.3
− 35	0.6	1.3
− 30	0.9	1.3
− 25	1.4	3.0
− 20	1.2	3.0
− 15	1.0	3.0
− 10	1.2	3.0
− 5	0.8	3.0

The values of h, $\partial \varrho/\partial x$ and dS/dx are from Inglis and Allen (1957). Thus, throughout the range of x considered, $\frac{1}{2} h \left(-\partial \varrho/\partial x \right) < \varrho(dS/dx)$, indicating a *seaward* drift near the bed; but both observation and theory (Abbott, 1960) indicate a *landward* drift up to $x = -15$.

IV. APPLICATION TO THE MERSEY ESTUARY

It has been shown in § II that the effects of salinity differences are important in the Mersey Estuary. The direction of drift just above the bed will now be investigated from (21). The derivation of this criterion assumed that the convective acceleration terms may be neglected in comparison with the contribution of the longitudinal salinity gradient to the equation of motion and that the term $U_0 \partial U_0/\partial x$ is negligible. With these assumptions we can proceed as follows.

The longitudinal salinity up this estuary has been obtained from a survey carried out at both low and high water on 14 December 1957; the mean of the low water and high water salinity gradient was then taken. The mean tide level used corresponds to the tide on the above day. Finally the channel depth has been taken from a recent survey of this estuary. Results of these calculations are given in Table II.

TABLE II

Place	Distance from Gladstone Dock (*miles*)	$\frac{1}{2}h\left(-\dfrac{\partial\varrho}{\partial x}\right)$ (lb/ft³ × 10⁵)	$\varrho\dfrac{dS}{dx}$ (lb/ft³ × 10⁵)
Gladstone Dock	0	7.6	0
Prince's Pier	3	8.8	0
Eastham	9	71	77
Hale Head	16	4.4	920

The slope of the mean tide level at Hale Head is only approximate since the tide gauge dries out before low water is reached. However, the main conclusion which can be drawn is independent of this approximation.

Table II shows that, according to the present theory, the drift just above the bed reverses from landward to seaward in the vicinity of Eastham. The tidal curves up to this point are still fairly sinusoidal in form. Though the ebb is rather longer than the flood, there is still symmetry about the mean tide level. Also, the variation of $\partial\varrho/\partial x$ over a tidal cycle is not yet excessive, as it is beyond c 11 or 12 miles from Gladstone Dock. For these reasons the use of criterion (21) is probably justified in the case of the Mersey. The theory is extremely sensitive to the mean tide level, and when (21) is applied the tidal information used should correspond to the tide occurring during the salinity sampling.

A model of the Mersey Estuary is in operation at the Hydraulics Research Station, Wallingford, and, when salinity is correctly represented, the position of zero drift just above the bed occurs slightly seaward of Eastham, which is in satisfactory agreement with the theoretical prediction. Further, across the section from Cammell Laird to Brunswick Dock, the model indicates a mean landward drift velocity just above the bed of about 0.4 ft/sec in the prototype. A rough theoretical value for comparison can be obtained as follows. From (14) and (18) we have approximately

$$\frac{k\varrho}{\pi}\left\{3U_0\bar{u}+U_0^2\left(\cos^{-1}\left(-\frac{\bar{u}}{U_0}\right)-\frac{\pi}{2}\right)\right\}=gh\left\{\frac{1}{2}h\left(-\frac{\partial\varrho}{\partial x}\right)-\varrho\frac{dS}{dx}\right\}, \quad (24)$$

since $\bar{u}\langle\langle U_0$ in general. Also, k can be replaced by g/C^2, where C is Chezy's constant and

$$\cos^{-1}\left(-\frac{\overline{u}}{U_0}\right) = \frac{\pi}{2} + \frac{\overline{u}}{U_0} + 0\left(\frac{\overline{u}}{U_0}\right)^3 ; \qquad (25)$$

hence, from (24), the drift is given by

$$\overline{u} = \frac{\pi\, C^2 h}{4\, U_0\, \varrho}\left\{ \frac{1}{2}\, h\left(-\frac{\partial \varrho}{\partial x}\right) - \varrho\frac{dS}{dx} \right\} . \qquad (26)$$

When approximate values of the various quantities are inserted, (26) gives $\overline{u} = 0.3$ ft/sec, in reasonable agreement with the model results.

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