THE INFLUENCE OF THE CONTINENTAL SHELF ON THE TIDES OF THE ATLANTIC COAST OF THE UNITED STATES

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ABSTRACT

Examination is made of the hypothesis that the tide along the eastern coast of the United States results from a co-oscillation of water over the Continental Shelf engendered by motion at the continental slope, the barrier against which the oceanic tidal wave is reflected. Using methods which are effective in analyzing tides in embayments, the range of tide and the time of high water over the continental slope have been estimated from data on the tide at the coast and from the topography of the off-lying shelf. Estimates indicate that the time of high water and the mean range of tide along the continental slope are much more uniform than those at corresponding positions along the coastline, as consideration of the velocity of waves in deep and shallow water demands. Estimated mean ranges along the 1000 m contour agree closely with those observed at oceanic islands in the Northwest Atlantic. The estimated velocities of tidal motion across the Shelf agree roughly with the velocities of tidal currents measured from anchored vessels. It is concluded that the variation in local characteristics of the tide along this coast may be explained by the influence of the topography of the Continental Shelf upon tidal motion. The general application of these results is discussed.

INTRODUCTION

Present knowledge of tidal movement in oceanic basins is based almost exclusively on observations made along the coasts. It is only at oceanic islands that the ocean tide has been observed unaffected by the disturbing influence of continental boundaries. Consequently charts of cotidal lines of the oceans, such as those prepared by Whewell (1833), Harris (1904), Sterneck (1920) and Die-

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The author's interest in tidal phenomena began during a visit to Friday Harbor at Professor Thompson's invitation in 1948, and his first publication on the subject contains a consideration of the tides in that region. The present paper is contributed in grateful recognition of the stimulus received from Professor Thompson at that time.

trich (1944), assume of necessity that the timing of the ocean tide is related to the time of high water at the coast. It has been recognized, of course, that shallow water distorts tidal behavior. Whewall wrote in 1836: ". . . the velocity of the tide-wave . . . is very much less near the shore than it is in the open ocean: perhaps we may even consider the velocity of the tide-wave in littoral regions as a quantity of a different order, and governed by different laws from its velocity in the open ocean . . .". Since then the special laws which relate the tide at the coast to the tide of the deep ocean do not seem to have received the attention they deserve.

The influence of embayments on tidal behavior is well understood. at least in situations of simple topography. In narrow embayments the tide may be considered to be a co-oscillation engendered by the oscillation in the "outer sea". This co-oscillation approaches a standing wave in form (Doodson and Warburg, 1941). The influence of reflection at the head of an embayment is such that it augments the amplitude of the tide to a degree which depends on the relation of the time required for the wave to traverse the length of the bay to the period of the wave. Amplification is maximal when this time is $^1/_4$ period. The effect of friction is to delay the time of high water relative to that in the "outer sea" (Redfield, 1950). Fleming (1938), who has employed these principles in examining the tides in the Gulf of Panama, has extended his examination out to the continental slope so as to draw some conclusions about the effect of the Shelf's profile on the velocities of the tidal currents developed.

In the present paper the hypothesis is presented that along the eastern coast of the United States the tide on the Continental Shelf may be treated as a co-oscillation strictly analogous to that occurring in embayments. This co-oscillation is engendered by the ocean tide at the continental slope, which forms the effective barrier to the oceanic oscillation. The tidal wave on the Shelf is delayed in crossing the shallow water and is augmented by reflection at the coast so as to produce the varied elevations and tidal intervals observed along its length.

When the profile of the continental margin is considered, it is evident that the principal barrier to the oceanic oscillation must be the continental slope, since at this region the depth decreases abruptly from about 3,000 to 100 m. Tidal motion over the Continental Shelf must have negligible influence on the oceanic oscillation because of the relatively small volume of water involved. However, the ocean tide will give rise to a wave moving across the Continental Shelf which is reflected at the coast. The properties of the tide observed at the coast are derived from the oceanic oscillation after modification by the character of the co-oscillation across the shallow water of the Shelf. See Fig. 1.

By analogy with tides in embayments, it may be expected that the tide at the coast will be amplified by reflection to a degree increasing with the width of the Shelf up to the point where the time required for a wave to cross it equals $^1/_4$ period and that the time of high water will be delayed. If the oceanic wave front is nearly parallel to the continental slope and if the shelf is of variable width, it may be expected, because of the higher velocity of waves in deep water, that high water will occur more nearly simultaneously at the slope than along the shore. For the same reason, the gradients in elevation should be smaller at the slope than along the coast.

Cotidal charts of the Atlantic indicate that the eastern coast of the United States lies in a broad area in which the M_2 component, which dominates the Atlantic tide, occurs at about 12 hours after the moon's transit at Greenwich. Harris (1900) proposed that the semidiurnal tide along this coast was due to a standing oscillation of ocean water across a nodal line extending northeastward from the Windward Islands. The American coast forms a barrier, almost at right angles to the assumed direction of motion, against which this wave is reflected.

Along the coast the range of tide decreases progressively from nearly seven feet in Georgia to about half that value at Cape Hatteras, north of which the tidal range increases to about four and one-half feet at New York. The time of high water varies in a parallel way. The width of the Continental Shelf between Georgia and New York varies in a manner similar to the tidal characteristics.

Concepts which are effective in the analysis of tides in embayments have been used to deduce the properties of the tidal wave across the Continental Shelf from data obtained at the coast and from the topography of the Shelf. The agreement of the times of high water estimated for positions along the continental slope and of the amplitudes at these positions and at oceanic islands provides a criterion of the applicability of the procedures employed and of the validity of the hypothesis on which they are based. The outcome explains the variation in tidal behavior observed along the coast.

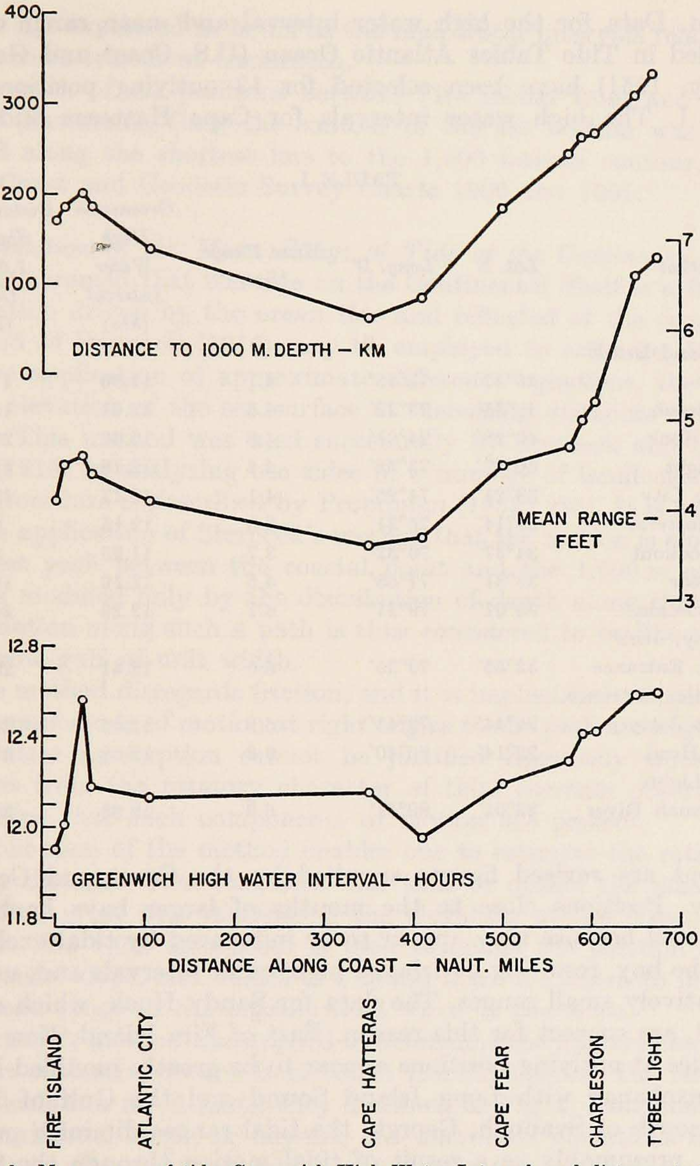

Figure 1. Mean range of tide, Greenwich High Water Interval, and distance to 1,000 m contour for selected outlying stations along the eastern coast of the United States.

Data. Data for the high water interval and mean range of tide recorded in Tide Tables Atlantic Ocean (U.S. Coast and Geodetic Survey, 1951) have been selected for 13 outlying positions. See Table I. The high water intervals for Cape Hatteras and Cape

TABLE I.

Station	*Lat. N*	*Long. W*	*Mean Range (feet)*	*Greenwich High Water Interval (hrs)*	*Distance from Shore to 1,000 m Depth (km)*
Fire Island Breakwater	40°37′	73°18′	4.1	11.90	170
Long Beach	40°35′	73°39′	4.5	12.01	185
Sandy Hook	40°28′	74°01′	4.6	12.56	200
Sea Bright	40°22′	73°58′	4.4	12.18	185
Atlantic City	39°21′	74°25′	4.1	12.13	139
Cape Hatteras	35°14′	75°31′	3.6	12.15	62
Cape Lookout	34°37′	76°32′	3.7	11.95	85
Cape Fear	33°51′	77°58′	4.5	12.19	185
Cape Romain	33°01′	79°21′	4.7	12.29	245
Bull Bay, Jack Creek Entrance	32°56′	79°35′	5.0	12.41	263
Charleston Entrance, North Jetty	32°44′	79°48′	5.2	12.42	267
Hilton Head	32°14′	80°40′	6.6	12.58	310
Tybee Light, Savannah River	32°02′	80°51′	6.8	12.59	334

Lookout are revised figures supplied by the Coast and Geodetic Survey. Positions close to the mouths of larger bays have been eliminated because they appear to be influenced by tidal exchanges with the bay, resulting in delayed high water intervals and, usually, in relatively small ranges. The data for Sandy Hook, which are included, are suspect for this reason. East of Fire Island, New York, the tides at outlying positions appear to be greatly modified by the tidal exchange with Long Island Sound and the Gulf of Maine, while south of Svannah, Georgia the tidal ranges diminish progressively, presumably as a result of tidal motion through the Straits of Florida.

Elevations are expressed as mean range of tide in feet, assumed to equal approximately twice the amplitude of the M_2 component.

Times are expressed in hours as the high water intervals relative to the moon's transit at Greenwich.

For each of six positions between Fire Island Inlet and Tybee Light (Savannah, Ga.), the contour of the sea bottom was determined along the shortest line to the 1,000 fathom contour, using U.S. Coast and Geodetic Survey Charts 1000 and 1001.

Estimation of the Mean Range of Tide at the Continental Slope. If it is assumed that the tide on the Continental Shelf is a free co-oscillation driven by the ocean tide and reflected at the coast, the method of Sterneck (1915) may be employed to estimate, by successive application of approximate difference equations, the maximum elevation of the sea surface at increasing distances from the coast. This method was used successfully by Sterneck and by Defant (1919) in analyzing the tides in a number of landlocked seas. The procedure is described by Proudman (1953: 233). It is assumed in this application of Sterneck's method that the motion is along the shortest path between the coastal point and the 1,000 m contour and is modified only by the distribution of depth along that path. The motion along such a path is thus considered to be like that in a narrow gulf of unit width.

The method disregards friction, and it is implied in its application that components of motion at right angles to the path are negligible. The latter assumption cannot be justified rigorously since it is evident from the rotatory character of tidal currents observed at lightships that such components of motion are present.

Application of the method enables one to estimate the ratio η/η_0 at any distance from the coast and thus to obtain the maximum elevation, η, at that distance if the elevation at the coast, η_0, is known. The 1,000 m contour is used to define the position of the continental slope, and values of η found there are taken to indicate the mean range of the oceanic tidal wave at the slope.

Sterneck's method was applied to determine the relative elevation, η/η_0, at successive intervals of 10 km from the shore. The result of the calculation for Atlantic City is shown in Fig. 2. Note that over the continental slope, or beyond, the maximum elevations change very little with distance, which shows that the arbitrary depth of 1,000 m used to define the position of reflection of the oceanic wave might be varied considerably without changing significantly the estimate of the range of the oceanic tide. The maximum elevation at the 1,000 m contour is 0.66 that at the coast. Since the mean range at Atlantic City is 4.1 feet, the mean range at the 1,000 m contour off Atlantic City is estimated to be 2.71 feet.

Table II shows results of similar calculations for the six selected positions. The mean range at the 1,000 m contour is estimated to increase from 2.71 feet at Fire Island to 3.36 feet at Cape Hatteras and then to decrease to 2.79 feet at Tybee Light. This is an extreme difference of 0.65 feet as compared to 3.2 feet for the tidal ranges along the coast. The measured mean range of tide in nearby oceanic islands, Sable Island, Bermuda, Eleuthera, San Salvador, *et al.* is given as 2.6 feet, which is close to the ranges estimated for the 1,000 m contour off Atlantic City and Savannah.

The first criterion, that the oceanic oscillation has almost uniform elevations over great distances, is thus confirmed by the computation. Actually, the results show that the effect of the bottom topography

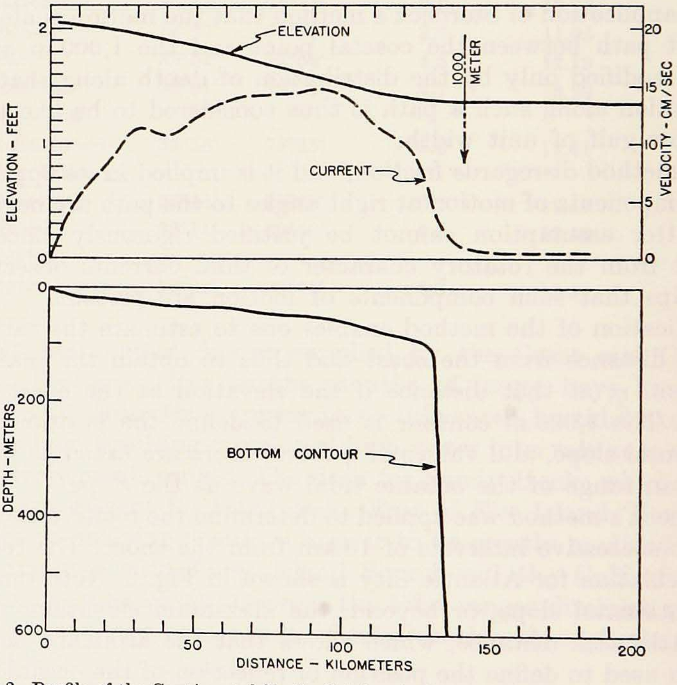

Figure 2. Profile of the Continental Shelf off Atlantic City and the distribution of estimated elevations at high water and maximum velocities of onshore and offshore tidal currents across this Shelf.

TABLE II. ESTIMATES OF THE MEAN RANGE OF TIDE AT THE 1,000 M CONTOUR

Station	*Bearing of Section-True*	*Mean Range at Coast (feet)*	η/η_0 *at 1,000 m Contour*	*Mean Range at 1,000 m Contour (feet)*
Fire Island	133°	4.1	0.66	2.71
Atlantic City	132°	4.1	0.66	2.71
Cape Hatteras	112°	3.6	0.93	3.36
Cape Lookout	120°	3.7	0.89	3.28
Cape Romain	118°	4.7	0.64	3.00
Tybee Light	105°	6.8	0.41	2.79

is *more* than enough to account for differences in the tidal range at the coast, since the greatest elevations of the oceanic tide are estimated to be off the capes of North Carolina, where the coastal range is in fact least.

Estimation of the Time of High Water at the Continental Slope. To account for the differences in high water intervals along the coast by the proposed hypothesis, it is necessary to introduce the assumption that the tidal wave is damped as it crosses the Continental Shelf. Otherwise high water would occur simultaneously, both along the coast and at the continental slope.

For a wave undergoing reflection and damping in a channel of uniform width, it may be deduced that

$$\eta/\eta_0 = {}^1/_2 (\cosh\ 2\mu x + \cos 2kx), \tag{1}$$

where η/η_0 is the ratio of the elevation at high water at the position at which η is measured relative to that at the barrier from which the wave is reflected, kx is the phase difference due to the position at which η is measured relative to the barrier, k is 360°, and μ is a damping coefficient defined so that the attenuation per period is $e^{-\mu}$ (Redfield, 1950). Using this relation, the phase difference between the tidal wave at the 1,000 m contour and the adjacent coast may be estimated for any chosen value of the damping coefficient μ from the values of η/η_0 obtained by the Sterneck method and given in Table II.

Under similar conditions, the time of high water at any distance from the reflecting barrier is given by

$$\sigma t_H = \tan^{-1} (- \tan kx\ \tanh\ \mu x), \tag{2}$$

where σt_H is the time angle of high water at a point where the phase difference is kx. Having obtained the value of kx at the 1,000 m contour by eq. (1), the time of high water there relative to the time at the coast may be obtained, assuming the same value for the damping coefficient, μ, in both steps of the calculation. For example, at Atlantic City η/η_0 is found to be 0.66 at the 1,000 m contour. Assuming $\mu = 1.0$, kx is found to be $-50°$, using eq. (1). Applying this value of kx to eq. (2), σt_H is found to be $-9.2°$, or -0.32 hours. The Greenwich High Water Interval for the tide at Atlantic City is 12.13 hours; consequently it is 11.81 hours at the continental slope.

Lacking information on the damping coefficient to be applied over the Continental Shelf, values characteristic of large embayments have been employed. It has been found that a damping coefficient of $\mu = 1.0$ satisfies the conditions in Long Island Sound and the Bay of Fundy; 1.5 to 2.0 in the Straits of Juan de Fuca and Georgia (Redfield, 1950). Results of estimations using $\mu = 0.5$, 1.0, 2.0 are presented in Table III.

Estimates obtained with $\mu = 0.5$ are the most satisfactory. The extreme differences in Greenwich High Water Interval at the 1,000 m

TABLE III. Estimates of Greenwich High Water Interval at the 1,000 m Contour

Station	$\mu = 0.5$	$\mu = 1.0$	$\mu = 2.0$
Fire Island	11.75	11.58	11.14
Atlantic City	11.98	11.81	11.37
Cape Hatteras	12.13	12.10	12.04
Cape Lookout	11.95	11.87	11.77
Cape Romain	12.13	11.96	11.49
Tybee Light	12.18	11.69	—

contour are found to be 0.43 hours, or 0.22 hours if the early time at Fire Island Inlet is left out of account. This compares with a difference of 0.69 hours at the corresponding coastal positions. Moreover the trend is progressive, high water occurring earliest at the northeastern part of the slope, latest toward the southwest. Using $\mu = 1.0$, the spread of times is slightly less, 0.38 hours, but high water is estimated to occur latest in the middle section of the coast, which appears to be unreasonable. A damping coefficient of 2.0 gives results which are quite unsatisfactory. The spread in values at 1,000 m is nearly as great as that at the coast, and high water occurs distinctly later off the North Carolina capes than to the north or south. No solution can be obtained for Tybee Light, since the theory does not provide for such reduced values of η/η_0 as are estimated by the Sterneck method.

Although the assumption of the value of the damping coefficient is quite arbitrary, it may be concluded that, by choosing a reasonable value for this coefficient, the second criterion of the hypothesis may be confirmed; namely, that the oceanic oscillation attains maximum elevations almost simultaneously over great distances along the continental slope. The topography of the slope is consequently such as to account for the variation in both tidal range and time of high water along the eastern coast of the United States.

Cotidal Lines of Continental Shelf. The mean range of tide and the high water interval may be estimated for any point on the Continental Shelf by precisely the same procedure as that employed for the 1,000 m contour. Such estimates were made for points where the sections cross the 50, 100 and 400 m contours. The resulting values, together with those for the 1,000 m contour and for the coastal points of observation, have been combined in Figs. 3 and 4 in order to construct cotidal charts for mean range and time of high water on the Continental Shelf.

The approximate character of these charts is recognized. However, they serve to give a co-ordinated picture of the estimated results and probably show in a general way the character of the tidal motion. The ocean tide appears to approach the coast at right angles to the trend of the continental slope from a direction of 110°. South of Cape Hatteras, where the coast line falls away toward the west, the crest of high water appears to cross the Shelf obliquely and becomes progressively delayed in reaching the coast. The widening Shelf leads to substantial increase in tidal ranges along this part of the coast.

Tidal Currents on the Continental Shelf. The hypothesis under examination considers that the tidal motion across the Shelf is rectilinear and perpendicular to the 1,000 m contour. It thus ignores any components of motion parallel to the coast. In contrast, observations from lightships and other anchored vessels show that the tidal currents are rotary except near the entrances to the larger bays (Haight, 1942). The current roses are so oriented, however, that at about three hours before high water, when maximal flow

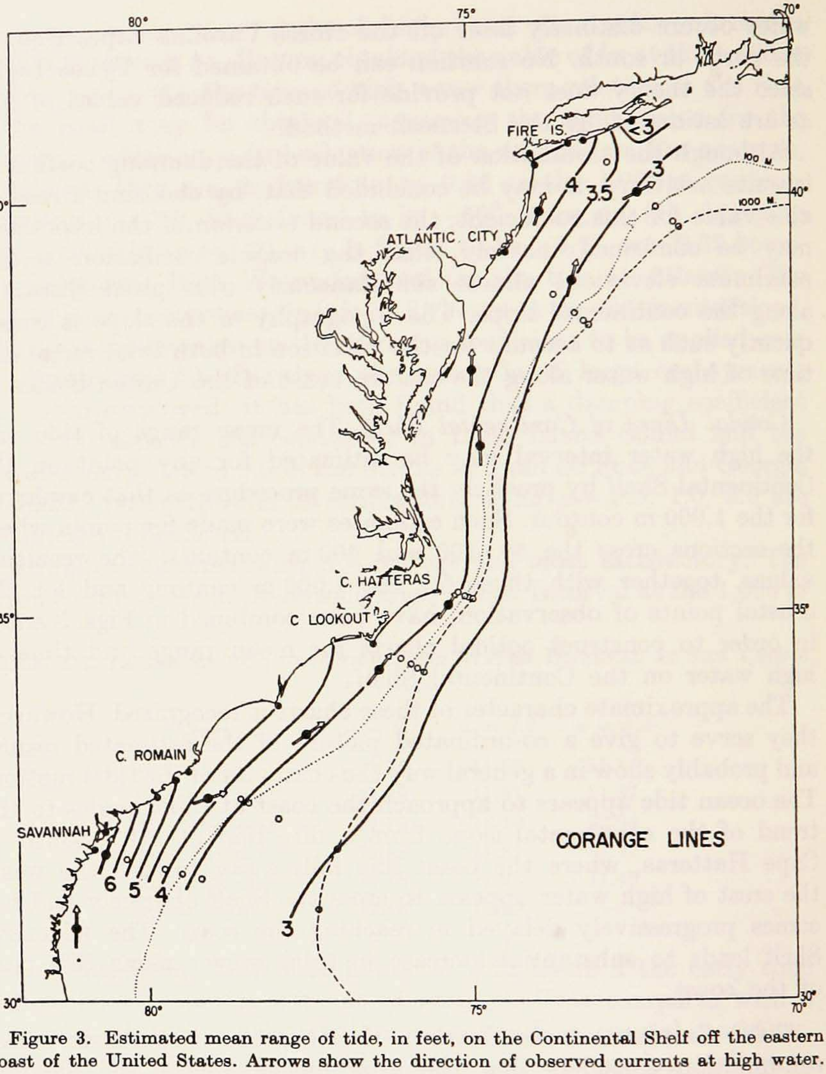

Figure 3. Estimated mean range of tide, in feet, on the Continental Shelf off the eastern coast of the United States. Arrows show the direction of observed currents at high water.

onto the Shelf is expected, the direction of flow is approximately normal to the coast. At high water, when the change in elevation is zero, the currents flow northeasterly, parallel to the shore line.

These relations are illustrated in Figs. 3 and 4, in which arrows indicate the direction of motion observed at anchored vessels. In Fig. 4 the direction of current when the shoreward component of

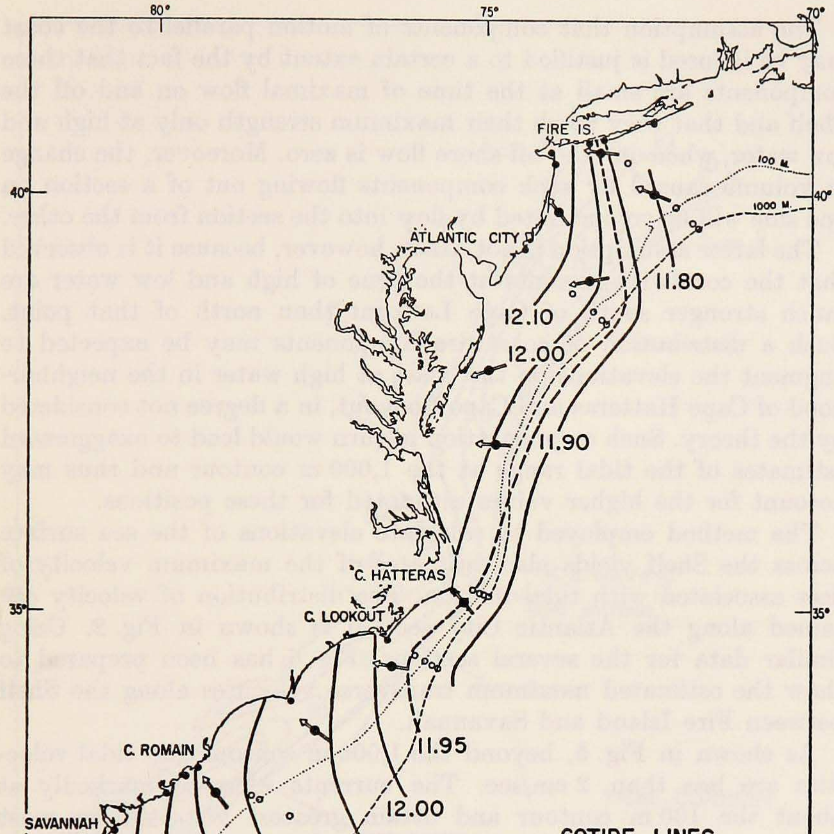

Figure 4. Estimated time of high water, in hours after the moon's transit at Greenwich, on the Continental Shelf of the United States, made on the assumption that the damping coefficient, μ, is 0.5. Arrows show the direction of observed currents three hours before high water.

flow is maximal, approximately three hours before high water, is shown superimposed on the cotidal lines. In Fig. 3 the direction of flow at high water is shown superimposed on contours indicating the relative elevation of the sea surface at that time.

The assumption that components of motion parallel to the coast may be ignored is justified to a certain extent by the fact that these components are small at the time of maximal flow on and off the Shelf and that they reach their maximum strength only at high and low water, when on- and off-shore flow is zero. Moreover, the change in volume caused by such components flowing out of a section on one side will be compensated by flow into the section from the other.

The latter assumption is not exact, however, because it is observed that the coastwise currents at the time of high and low water are much stronger south of Cape Lookout than north of that point. Such a distribution of coastwise components may be expected to augment the elevations on the coast at high water in the neighborhood of Cape Hatteras and Cape Lookout, in a degree not considered by the theory. Such augmentation in turn would lead to exaggerated estimates of the tidal range at the 1,000 m contour and thus may account for the higher values estimated for these positions.

The method employed to calculate elevations of the sea surface across the Shelf yields also estimates of the maximum velocity of flow associated with tidal motion. The distribution of velocity obtained along the Atlantic City section is shown in Fig. 2. Using similar data for the several sections, Fig. 5 has been prepared to show the estimated maximum transverse velocities along the Shelf between Fire Island and Savannah.

As shown in Fig. 5, beyond the 1,000 m contour the tidal velocities are less than 2 cm/sec. The currents increase markedly at about the 100 m contour and attain greatest velocities in most regions a short distance inside this contour. These relations are similar to those estimated by Fleming (1938) for the Gulf of Panama. The highest velocities occur off Savannah relatively near the coast because of the local bottom topography. This section and that off Cape Romain show also some influence of the relatively shoal water over the Blake Plateau in accelerating the oceanic tidal movement. Between the 1,000 and 100 m contours the current increases from 2.5 to 5 cm/sec as the Continental Shelf is approached.

The estimated velocities may be compared with direct measurements of surface tidal currents made with current poles from anchored vessels. See Table IV. The mean hourly velocities and directions of such tidal currents, from which the nontidal current has been eliminated, are given by Haight (1942). The maximum flood and ebb velocities have been averaged to give the values for mea-

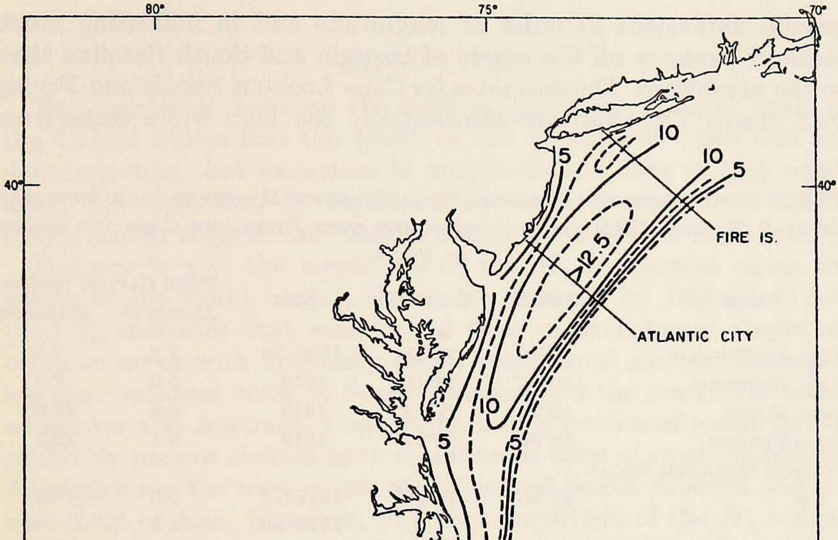

Figure 5. Estimated maximum onshore and offshore tidal current, in centimeters per second, on the Continental Shelf off the eastern coast of the United States.

sured currents listed in Table IV. No correction has been made for the departure in direction from a path normal to the coast. Such correction would decrease the values up to 20 %. The estimated currents at the position of the vessels are taken from Fig. 5 and are given to the nearest 2.5 cm/sec.

Comparison of the measured currents with those estimated shows general agreement in order of magnitude and in indicating much stronger currents off the coasts of Georgia and South Carolina than to the northward. The estimates for Cape Lookout Shoals and Frying Pan Shoals Lightship are substantially too high while those from

TABLE IV. Onshore and Offshore Tidal Currents Measured from Anchored Vessels (Haight, 1942: table 5) compared with Estimated Currents shown in Fig. 5

Vessel	*Lat. N*	*Long. W*	*Date*	*Tidal Current (cm/sec)* *Measured*	*Estimated*
Fire Island l.s.	40°28′	73°11′	1938–39	7.2	7.5
u.s.s. Cardinal	40°16′	73°15′	1919	5.9	7.5
u.s.s. Finch	40°04′	72°43′	1919	6.4	10.0
u.s.s. Falcon	39°04′	72°25′	1919	3.1	12.5
Winter Quarter Shoal l.s.	37°55′	74°56′	1912–13	5.0	10.0
			1918–20	3.6	
u.s.s. Brant	37°05′	74°51′	1919	9.3	10.0
Diamond Shoal l.s.	35°05′	75°19′	1921	1.8	7.5
Cape Lookout Shoals l.s.	34°20′	76°25′	1912	21.0	7.5
			1918–19	9.0	
Frying Pan Shoals l.s.	33°34′	77°48′	1912	23.5	10.0
			1918–21	16.0	
u.s.s. Long Island	32°42′	79°06′	1919	15.0	17.5
Charleston l.s.	32°41′	79°44′	1912	21.6	15.0
			1915–16	16.2	
			1921	15.7	
Martin's Industry l.s.	32°06′	80°28′	1912	26.7	25.0
			1916	22.9	
Savannah l.s.	31°57′	80°40′	1912	21.1	25.0
			1916	15.7	

positions north of Cape Hatteras are generally too low. Where measurements have been made at two or three different periods at the same position, the agreement between successive periods is not better than that between the estimate and the nearest measurement. It seems probable that lack of agreement is due to inadequacies in measurement quite as much as to faults in the estimations, and it may be concluded that the estimated currents have some validity, if not great precision, when applied to tides of the Continental Shelf.

DISCUSSION

The correlation between the tidal range on the Atlantic coast of the United States and the width of the Continental Shelf and the demonstration that variations in amplitude and time of high water along the coast may be explained by the influence of the Shelf's configuration suggest that similar relations may exist more widely.

Examination of the amplitude of the M_2 component along the coasts of the world oceans, shown for example by Dietrich (1944: tafel 3), indicates that semidiurnal tides of exceptional amplitude occur in areas with unusually broad continental shelves. Examples are the southeast coast of South America and the northwest coasts of Europe and Australia. Conversely, long stretches of coasts having relatively narrow shelves have semidiurnal tides of small amplitude. Examples are the west coasts of Africa and South America and the east coast of Asia. However, when the amplitude of the M_2 component at a large number of coastal positions throughout the world is compared with the width of the off-lying continental coast, say, to the 1,000 m contour, no satisfactory correlation is obtained. The greatest amplitude of the M_2 tide occurring at any width of shelf does increase with the width at a rate of about one foot per 20 nautical miles. However, at any one width, amplitudes of any value less than the maximum may occur. It is only when the width of the shelf is greater than 150 nautical miles that the smaller amplitudes are not to be found. It is evident that factors other than the width of the shelf influence the amplitude of the tide at the coast.

Among such factors may be listed: the amplitude of the ocean tide itself; the angle of approach of the oceanic wave; the actual profile of the bottom across the shelf, which may be relatively deep or shallow within the limits on the shelf; the influence of local embayments in the coast line; and the period of the tidal component in question. Understanding of the relation of the tide, as observed along the coast, to the oscillation of the off-lying deep water consequently requires a detailed examination of these factors as they apply to any region in question. The coast under consideration in the present paper is particularly favorable for such studies because the ocean tide occurs so nearly simultaneously along its length that tidal movement on the Shelf may be dealt with as a transverse oscillation. The system of motion in other regions is no doubt much more complicated, but it is hoped that the application of principles similar to those employed in the present case may be effective in clarifying the relation of the coastal tide to that of the ocean basins.

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