A NOTE ON TIDAL FRICTION[1]

By

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ABSTRACT

The Moon's gravitational torque on the ocean tide has been computed on the basis of Dietrich's cotidal charts. The result is -4.5×10^{23} dynes cm as compared to Heiskanen's (1921) value of -2.9×10^{23} dynes cm based on Sterneck's (1920) charts. Both values are subject to great uncertainty. The observed deceleration of the Moon implies a torque of $(-3.9 \pm 1.0) \times 10^{23}$ dyne cm, which is consistent with both foregoing calculations. It is concluded that the deceleration of the Moon's orbital motion and the associated deceleration of the Earth's rotation *may* be due to oceanic tidal dissipation. The solar decelerating torque is estimated at 0.31 times that of the lunar torque. The forces associated with the decelerating tidal torque give rise to a minute steady oceanic circulation.

INTRODUCTION

The observed position of the Sun and Moon do not agree precisely with their ephemerides. The discrepancies are due to (1) variations in the rate of rotation of the Earth and (2) variations in the orbital motion of the Moon over and above those included in the ephemerides. From simultaneous observations of the discrepancies in longitude of the Moon and Sun (or Moon and Mercury), the two effects can be separated according to Murray's (1957) scheme. Observations since 1680 indicate a constant deceleration[2] in the Moon's orbital motion by $11''.2$ century^{-2} (in addition to the $6''.1$ century^{-2} accel-

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[2] If the Moon's motion is referred to a time standard based on the Earth's rotation (such as a sundial), the velocity, angular and linear, does in fact increase. If it is referred to an "absolute" time standard, the motion is decelerated. The term "secular acceleration" used in astronomic literature refers to the former concept. The present discussion refers all acceleratious, etc. to absolute time.

eration due to planetary perturbations allowed for in the ephemerides). The modern value for the deceleration is not inconsistent with discrepancies inferred from Babylonian eclipses and other ancient observations (Munk and MacDonald, 1960).

The only theory that has been advanced to account for the anomalous lunar deceleration is that it is associated with tides on the Earth (Jeffreys, 1952: chap. 8). The astronomic evidence can give no information as to whether the atmosphere, the oceans or the bodily tides may be responsible. Atmospheric tides are rather well determined from barograms at many weather stations; however, the resulting lunar[3] torque is too small by a factor of 100 to account for the lunar deceleration. Bodily tides could be responsible if they lagged by 1° with respect to their equilibrium configuration (Munk and MacDonald, 1960), but this value is too small to be checked against measurements of Earth tides with gravimeters. Indirect evidence based on the anelastic properties of the solid Earth indicate that the bodily tide may be an important factor, but apparently not the principal factor. This leaves the ocean tide. The observations are barely good enough to make a rough estimate possible. The existence of steady tidal stresses and of a resulting westward oceanic drift, first mentioned by Hertz (1883), was subsequently discussed by G. H. Darwin (1898: 276). New observations conducted in southern oceans during the International Geophysical Year should lead to a substantial improvement in this estimate.

TIDAL FORCE AND TORQUE

Let ψ designate the tide-producing potential. This can be written as a sum of three groups of terms (see, for example, Doodson, 1921),

$$\psi_S = -\tfrac{1}{2} B \frac{gr^2}{a} \sin^2\theta \sum_S C_i \cos(\sigma_i t + \varepsilon_i + 2\varphi)$$

$$\psi_D = -\tfrac{1}{2} B \frac{gr^2}{a} \sin 2\theta \sum_D C_i \cos(\sigma_i t + \varepsilon_i + \varphi) \qquad (1)$$

$$\psi_L = -\tfrac{1}{2} B \frac{gr^2}{a} (\tfrac{1}{2} - \tfrac{3}{2}\cos^2\theta) \sum_L C_i \cos(\sigma_i t + \varepsilon_i),$$

[3] This lunar atmospheric torque must not be confused with a thermally induced solar atmospheric torque that is much larger and of opposite sign.

corresponding to the semidiurnal, diurnal, and long-period constituents, respectively. Here

$$B = \frac{3}{2}\frac{M}{E}\frac{a^3}{D^3} = 0.843 \times 10^{-7}$$

is a dimensionless constant, M the mass of the Moon, E the mass of the Earth, a the Earth's radius, D^{-1} the mean inverse distance of the Moon, r the distance from the Earth centroid, θ the colatitude, φ the east longitude, C_i the constituent coefficients, σ_i the frequencies, and ε_i the epochs depending on the time origin.

Eq. (1) includes only potentials of degree two; higher harmonics, proportional to higher powers of a/D, have been neglected. Both lunar or solar terms are to be included in the expansion.

The force per unit mass, $\boldsymbol{F} = -\nabla\psi$, can be considered as constant from the (tidally disturbed) sea surface ($z = \eta$) to the sea bottom ($z = -h$). The total force per unit area on a water column equals

$$\boldsymbol{\tau} = \varrho\, \boldsymbol{F}(\theta, \varphi; t)\,[\eta(\theta, \varphi; t) + h(\theta, \varphi)],$$

where ϱ is water density, assumed constant. The time-average of this stress equals

$$\bar{\boldsymbol{\tau}} = \varrho\, \overline{\boldsymbol{F}\eta} \tag{3}$$

plus a term $h\varrho\bar{\boldsymbol{F}}$, causing a permanent tidal deformation which is customarily absorbed in the gravity field. The quantity $\bar{\boldsymbol{\tau}}$ as defined by (3) will be referred to as the *steady tidal stress.*

Strictly speaking, the time-average should be taken for each mass particle following its tidal trajectory. Actually, we have taken time-averages in fixed columns. It turns out that the steady tidal stress is distributed vertically depending on the vertical distribution of the tidal current. In a frictionless homogeneous ocean the tidal current, and hence the steady tidal forces, would be uniform from top to bottom.

The tidal elevation above mean sea level can be expanded in a form similar to (1), with the components

$$\begin{aligned}
\eta_S &= \sum_S H_i \cos(\sigma_i t + \varepsilon_i + 2\varphi - \varkappa_i) \\
\eta_D &= \sum_D H_i \cos(\sigma_i t + \varepsilon_i + \varphi - \varkappa_i) \qquad (4) \\
\eta_L &= \sum_L H_i \cos(\sigma_i t + \varepsilon_i - \varkappa_i),
\end{aligned}$$

where H_i and $\varkappa_i$ are empirical functions of θ, φ, to be determined from tide data. The essential feature is that the σ_i's are the same in (1) and (4); this follows because the response of the sea surface to the tide-producing force $\boldsymbol{F}$ contains the same frequencies as $\boldsymbol{F}$. The ε_i's are the same by appropriate definitions of $\varkappa_i$. In the product (3) of the summations, common frequencies have non-zero means. We note that $\boldsymbol{F}$ can be precisely computed for any geographic position according to (1); in the case of η we must depend on interpolation between widely separated tide stations. Combining (1), (3) and (4) we find that the steady tidal stress $\bar{\tau}$ has the components

$$\bar{\tau}_\theta = B\varrho g \left[\tfrac{1}{4}\sin 2\theta \sum_S C_i H_i \cos\varkappa_i + \tfrac{1}{2}\cos 2\theta \sum_D C_i H_i \cos\varkappa_i - \tfrac{3}{8}\sin 2\theta \sum_L C_i H_i \cos\varkappa_i\right]$$

$$\bar{\tau}_\varphi = B\varrho g \left[-\tfrac{1}{2}\sin\theta \sum_S C_i H_i \sin\varkappa_i - \tfrac{1}{2}\cos\theta \sum_D C_i H_i \sin\varkappa_i\right] \tag{5}$$

$$\bar{\tau}_r = B\varrho g \left[\tfrac{1}{2}\sin^2\theta \sum_S C_i H_i \cos\varkappa_i + \tfrac{1}{2}\sin 2\theta \sum_D C_i H_i \cos\varkappa_i + \left(\tfrac{1}{4} - \tfrac{3}{4}\cos^2\theta\right) \sum_L C_i H_i \cos\varkappa_i\right].$$

The *steady tidal torque* about the Earth's axis of rotation is

$$\bar{N} = a^3 \int_{\text{oceans}} \bar{\tau}_\varphi \sin^2\theta \, d\theta \, d\varphi. \tag{6}$$

In passing, let us consider the significance of the other torque components, $\bar{L}$, $\bar{M}$. If the Earth had finite strength, these components would lead to a minute permanent displacement of the pole of rotation. If, on the other hand, the Earth were to yield as a fluid to prolonged stress, no matter how minute, polar wandering would result. Eventually the pole would reach a position (if any) for which $\bar{L}$ and $\bar{M}$ are zero. The trouble with this intriguing hypothesis for polar wandering is that the torques involved are small, $\bar{L}$ and $\bar{M}$ being of the same order as $\bar{N}$, 10^{23} dyne cm. The distribution of continents and oceans, if isostatically balanced, are associated with a torque of the order of 10^{30} dyne cm (Munk and MacDonald, 1960: chap. 12). It should be remembered that the time-average was taken relative to geographical co-ordinates, and hence these components rotate with the Earth. If the time-average were to be taken relative to a nonrotating system, the steady torque components $\bar{L}$ and $\bar{M}$ would be fixed in space and would cause a slight anomaly in the Earth's precession rate. The reader can find a discussion in Darwin's works (for example, see Darwin, 1898).

EVALUATION FROM COTIDAL CHARTS

We assume that the cotidal contours ($\varkappa_i$ = constant) are identical for all semidiurnal constituents and that the co-range charts differ, one from another, by a constant factor equal to the ratio of the corresponding equilibrium tides. Thus

$$\begin{aligned} \varkappa_i(\theta, \varphi) &= \varkappa_S(\theta, \varphi) \\ H_i(\theta, \varphi) &= \frac{C_i}{C_S} H_S(\theta, \varphi) \end{aligned} \tag{7}$$

for each of the semidiurnal constituents. The subscript $_S$ refers to an important semidiurnal constituent for which satisfactory cotidal charts are available. Eq. (7) is essentially the "inference" method for evaluating the lesser constituents from short tide records. Similar relations are assumed for diurnal and long-period constituents, but application to the long-period components is doubtful (see section on long-period tides). The errors inherent in the approximation (7) are smaller than those associated with existing cotidal charts.

With this scheme, the only summations to be performed are of the type

$$\lambda_S = \sum_S C_i^2; \tag{8}$$

and similarly for λ_D, λ_L. Eq. (5) can then be written

$$\bar{\tau}_\theta = B\varrho g \left[\frac{1}{4}\frac{\lambda_S}{C_S} \sin 2\theta \,(H_S \cos \varkappa_S) + \frac{1}{2}\frac{\lambda_D}{C_D} \cos 2\theta \,(H_D \cos \varkappa_D) - \frac{3}{8}\frac{\lambda_L}{C_L} \sin 2\theta \,(H_L \cos \varkappa_L) \right]$$

$$\bar{\tau}_\varphi = B\varrho g\left[-\frac{1}{2}\frac{\lambda_S}{C_S}\sin\theta\,(H_S\sin\varkappa_S)-\frac{1}{2}\frac{\lambda_D}{C_D}\cos\theta\,(H_D\sin\varkappa_D)\right] \tag{9}$$

$$\bar{\tau}_r = B\varrho g\left[\frac{1}{2}\frac{\lambda_S}{C_S}\sin^2\theta\,(H_S\cos\varkappa_S)+\frac{1}{2}\frac{\lambda_D}{C_D}\sin 2\theta\,(H_D\cos\varkappa_D)\right.$$
$$\left.+\frac{\lambda_L}{C_L}\left(\frac{1}{4}-\frac{3}{4}\cos^2\theta\right)(H_L\cos\varkappa_L)\right].$$

Values of H_S and H_D have been interpolated for every ten degree square of the oceans from Dietrich's (1944) chart of the tidal range of the constituents M_2 and K_1, respectively. Values of $\varkappa_S$ and $\varkappa_D$ were read off Dietrich's cotidal charts for the same two constituents. Accordingly, we take $C_S = 0.9085$ and $C_D = 0.5305$, corresponding to these two constituents. The interpolated values are tabulated in the Appendix. The combined lunar and solar horizontal tidal stress is illustrated in Fig. 1.

THE DARWIN DEVELOPMENT

In order to use the more familiar tidal constituents from Darwin's development (see for example, Shureman, 1941), we would have to allow for their slowly varying amplitude over the Moon's nodal cycle. This is done by introducing the node factor $f_i(t)$ for each constituent. The only modification required is that λ must be written

$$\lambda_S = \sum_S \overline{f_i^2}\,C_i^2; \tag{10}$$

and similarly for λ_D and λ_L. The mean values of f_i^2 can be calculated by using eqs. (65) through (80), (206), (215), (227), (235), and the formula at top of page 156 in Shureman (1941). The values are as follows:

Constituent	f
Mf	1.172
MSf, $2SM$, MS, M_2, N_2, $2N_2$, ν_2, λ_2, μ_2	1.0014
Sa, Sa, P_1, S_2, R_2, T_2	1.0000[4]
K_1	1.0188
O_1, Q_1, $2Q_1$, ϱ_1, σ_1	1.0356
J_1, MP_1, SO_1, χ_1, θ_1	1.0402

[4] The node factors of the purely solar constituents have the constant value unity.

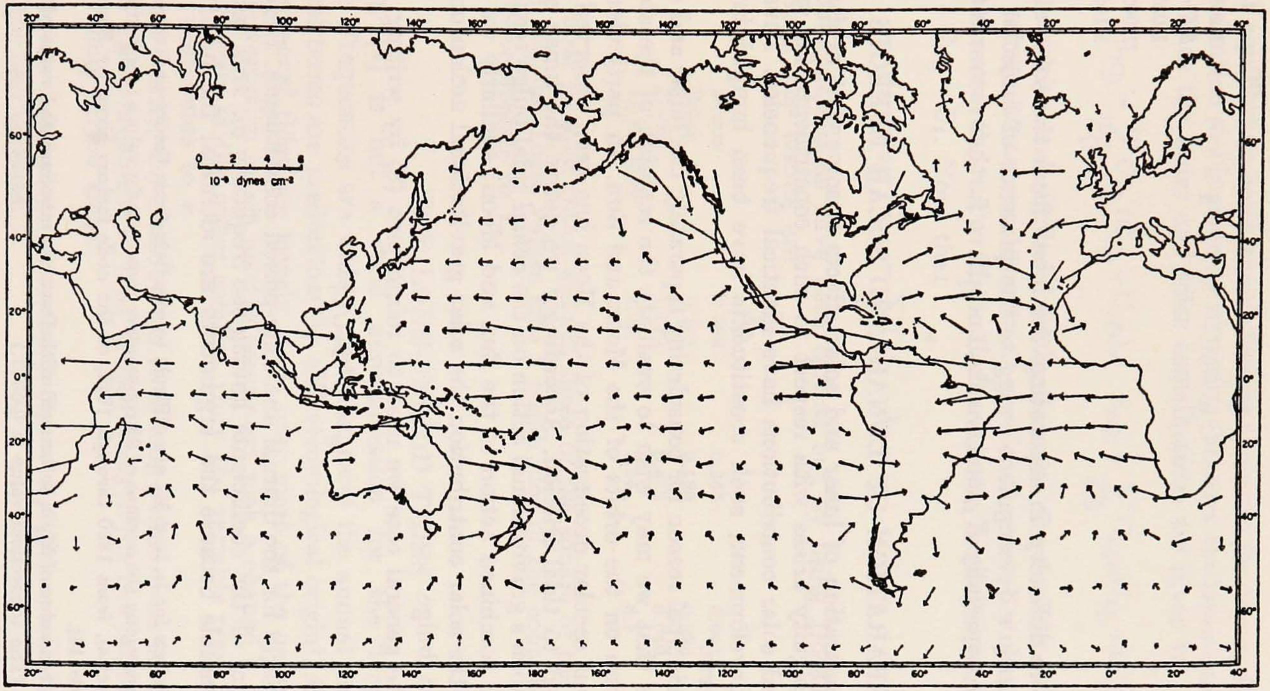

Figure 1. Horizontal component of the steady tidal stress from semidiurnal tides. The vectors are drawn from the center of each 10° square for which the stress could be computed.

Constituent	f
OO_1, KQ_1	1.421
M_1	2.574[5, 6]
K_2	1.0876
L_2	1.002[5]
KJ_2	1.192

There is difficulty in separating the lunar from the solar effects with Darwin's development owing to the presence of lunisolar constituents, especially K_1, and we shall not have further recourse to it.

SEPARATION OF LUNAR AND SOLAR EFFECTS

The separation of lunar and solar effects is somewhat arbitrary. The ambiguity arises with respect to such constituents for which lunar and solar contributions have identical frequencies. Thus far in the development, such constituents have been lumped into a single term.

The principal reason for considering separately the lunar and solar effects is that we may wish to evaluate the reaction of the steady tidal stress on the orbits of the Moon and Sun. In particular, an anomalous secular deceleration of the Moon has been observed and attributed to tidal friction. Accordingly we define the lunar effect as the Moon's gravitational action on the *actual* tidal bulge resulting from the combined action of the Sun and Moon; similarly we consider as the solar contribution the solar gravitational action on the combined bulge.

For the general case we need to reinterpret (8) by writing

$$\lambda_S = \sum_S C_i C_i'; \tag{11}$$

and similarly for the diurnal and long-period constituents. Here C_i' is the sum of the coefficients having the frequency σ_i. This procedure is possible because the arguments are identical for lunar and

[5] The values for L_2 and M_1 are difficult to calculate from theory and were evaluated by averaging the squares of 93 consecutive mean yearly values (approximately 5 nodal cycles), from 1850 through 1942, of the node factors given by Shureman (1941: table 14).

[6] The large value of M_1 is to be used with Darwin's miscomputed value of $C(M_1)$ of 0.0209. (The theoretical value is 0.0317.)

solar equilibrium constituents having the same frequency. The lunar contribution to λ is given by summing (11) over the lunar constituents etc. Thus the lunar and solar contributions are given by

$$\lambda_{S☽} = \sum_{S☽} C_{i☽}(C_{i☽} + C_{i\odot}), \quad \lambda_{S\odot} = \sum_{S\odot} C_{i\odot}(C_{i☽} + C_{i\odot}) \qquad (12)$$

respectively; and similarly for λ_D, λ_L, with the understanding that for purely lunar constituents $C_{i☽} = C_i$, $C_{i\odot} = 0$, for purely solar constituents $C_{i\odot} = C_i$, $C_{i☽} = 0$, whereas for luni-solar constituents $C_{i☽} + C_{i\odot} = C_i$. Note that

$$\lambda_S = \lambda_{S☽} + \lambda_{S\odot} = \sum_S C_i^2, \quad \lambda_D = \ldots, \quad \lambda_L = \ldots.$$

The summations have been carried out by using Doodson's expansion. The results are

	λ_S	λ_D	λ_L
Lunar	0.888	0.353	0.041
Solar	0.222	0.120	0.005
Total	1.110	0.473	0.046
Lunar/solar	4.00	2.94	8.2

The lunar to solar ratios are not those of the square of the equilibrium heights. These values give a rough idea of the contribution of the three species to $\bar{\tau}$.

THE LONG-PERIOD TIDES

Eq. (7) is not a valid approximation for the long-period tides. Frequencies are widely separated, and the annual and semiannual terms are predominantly of meteorological origin[7]. Moreover the seasonal tides are largely steric over most of the oceans; that is, the variation in sea level is associated with a change in the specific volume of the water column. Only the variable mass per unit area contributes to $\bar{\tau}$.

In all events, the contribution from long-period tides is relatively small. The component $\bar{\tau}_\varphi$ (and consequently $\bar{N}$) is not influenced at all by long-period tides.

[7] The meteorological contribution to the S_1- and S_2-tides has a small effect on $\bar{\tau}$ and has been neglected.

THE STEADY TIDAL STRESS

Fig. 1 shows the horizontal component of $\bar{\tau}$ arising from semidiurnal tides only. A predominance of westward stresses is apparent. These act in the sense required to decelerate the Earth's angular velocity. An equivalent chart for diurnal tides (not presented here) has the same general appearance, but the stresses are roughly one-fourth of the semidiurnal stresses. The existence of steady tidal stresses and of a resulting westward oceanic drift has been discussed by Darwin (1898: 276).

TABLE I. Tidal Torque on the Earth in Units of 10^{23} Dyne cm

	Heiskanen's Lunar Semid.	*Lunar*			*Solar*			*Total*
		Semid.	*Diurnal*	*Total*	*Semid.*	*Diurnal*	*Total*	
Pacific								
+	3.5	2.7	0.0	2.7	0.7	0.0	0.7	3.4
—	*-5.4*	*-4.0*	*-0.8*	*-4.8*	*-1.0*	*-0.3*	*-1.2*	*-6.0*
Total	-1.9	-1.3	-0.8	-2.1	-0.3	-0.3	-0.5	-2.6
Atlantic								
+	1.7	1.6	0.1	1.7	0.3	0.0	0.3	2.0
—	*-2.5*	*-3.0*	*-0.1*	*-3.1*	*-0.7*	*0.0*	*-0.7*	*-3.8*
Total	-0.8	-1.4	0.0	-1.4	-0.4	0.0	-0.4	-1.8
Indian								
+	2.3	1.3	0.1	1.4	0.3	0.0	0.3	1.7
—	*-2.5*	*-2.1*	*-0.3*	*-2.4*	*-0.6*	*-0.1*	*-0.7*	*-3.1*
Total	-0.2	-0.8	-0.2	-1.0	-0.3	-0.1	-0.4	-1.4
Grand Total	-2.9	-3.5	-1.0	-4.5	-1.0	-0.4	-1.3	-5.8

The irrotational component of the stress field would result in a steady distortion of the sea surface relative to the geoid by the order 10^{-4} cm. There is no associated circulation. The rotational component of the stress field causes a steady circulation and surface displacement. We might make an estimate by analogy to the steady wind-driven circulation. Typical wind stresses are of the order of several dynes cm^{-2}, or 10^5 times the steady tidal stresses. In the western boundary zone, where the currents are strongest, the observed transport per unit width is of the order 10^7 cm^2 sec^{-1} and is concentrated near the surface. The maximum surface displacement is about 10^2 cm. Accordingly, we might expect the steady tidal transport and the surface displacement to be 10^2 cm^2 sec^{-1} and 10^{-3} cm, respectively. If the velocity is distributed uniformly from top to bottom, its maximum might attain 10^{-3} cm sec^{-1}. This is altogether negligible. Observed steady deep currents in the western boundary zone are of the order of one cm sec^{-1} (Swallow and Worthington, 1957). Away from the western boundary zone, the steady currents are probably an order of magnitude less.

In this calculation we have not allowed for distortion of the sea bottom. Such a distortion arises directly from the tide-generating force as well as from the variable load of the ocean tide. The load correction would be difficult to apply, if at all possible; one would have to evaluate the contribution of the ocean tide to the spherical surface harmonic of second degree (as a function of time), and apply the appropriate Love numbers (Munk and MacDonald, 1960). In related geophysical problems such corrections have been of the order of 20 to 30 %.

THE STEADY TIDAL TORQUE

Table I summarizes the torque calculation. Integration (6) was carried out separately for the Sun and Moon, for semidiurnal and diurnal tides, and for the three major oceans. For each of these 12 calculations the positive and negative contributions to the torque integral were evaluated separately, as shown. In general the positive and negative contributions are of the same order, which implies a considerable uncertainty in the sum. It is encouraging that in each of the 12 instances the sum is negative or zero.

Our results for the lunar semidiurnal tide agree well with Heiskanen's (1921) calculation. He neglected the lunar diurnal effect, which we find to be only one fourth of the total lunar effect. The combined lunar torque of $-4.5 \times 10^{+23}$ dyne cm is to be compared with the value $-3.9 \times 10^{+23}$ inferred from the astronomically observed deceleration of the Moon. The agreement is better than we have any right to expect. The chief uncertainty has to do with the poorly known tidal amplitudes in the open sea.

In a similar manner, solar torque must be associated with a deceleration of the Sun's motion around the Earth, but because of the large mass and distance of the Sun, this effect is far too small to be observed.

APPENDIX. Values of H and $\varkappa$, Deduced from Dietrich's Catidal Charts, which were used
Square Indicated by Co-Ordinates of its Midpoint. The Four Values for Each

	5E	15E	25E	35E	45E	55E	65E	75E	85E	95E	105E	115E	125E	135E	145E	155E	165E
75N	35	30	40	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	010	030	050	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	05	06	06	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	155	195	205	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65N	40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	330	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	175	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55N	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20	20
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	100	075
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30	30
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	155	145
45N	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30	15	20
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	090	095	085
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30	30	28
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	165	145	125
35N	—	—	—	—	—	—	—	—	—	—	—	—	—	50	30	20	15
	—	—	—	—	—	—	—	—	—	—	—	—	—	180	090	085	090
	—	—	—	—	—	—	—	—	—	—	—	—	—	22	22	20	20
	—	—	—	—	—	—	—	—	—	—	—	—	—	185	165	160	145
25N	—	—	—	—	—	40	40	—	—	—	—	50	50	40	20	15	20
	—	—	—	—	—	290	280	—	—	—	—	170	190	195	110	090	095
	—	—	—	—	—	45	45	—	—	—	—	20	20	18	18	16	14
	—	—	—	—	—	055	065	—	—	—	—	205	215	205	190	185	180
15N	—	—	—	—	40	40	35	30	45	80	—	—	50	40	15	20	35
	—	—	—	—	210	235	200	310	245	290	—	—	185	195	170	095	100
	—	—	—	—	35	39	35	30	12	11	—	—	15	15	16	14	10
	—	—	—	—	045	055	065	045	355	005	—	—	215	210	205	205	210
05N	55	—	—	—	60	35	20	20	20	30	—	—	50	40	20	15	35
	110	—	—	—	115	120	070	015	140	190	—	—	180	210	150	095	105
	10	—	—	—	28	28	20	10	08	08	—	—	15	15	17	16	10
	005	—	—	—	045	055	055	045	325	305	—	—	215	215	215	215	225
05S	45	45	—	—	80	50	45	25	20	25	40	75	—	—	10	05	30
	095	105	—	—	110	115	060	040	070	105	210	290	—	—	110	100	115
	10	08	—	—	18	20	10	02	04	10	20	20	—	—	25	24	15
	025	040	—	—	045	055	085	075	250	245	245	255	—	—	195	205	220

in the Computations. The Values Represent a Sort of Subjective Average over the Ten Degree Square are Arranged in the Following Order: H, k for M_2; H, k for K_1

175E	175W	165W	155W	145W	135W	125W	115W	105W	95W	85W	75W	65W	55W	45W	35W	25W	15W	5W
—	—	—	—	—	—	—	—	—	—	65	45	45	45	—	—	40	40	40
—	—	—	—	—	—	—	—	—	—	280	300	310	315	—	—	310	000	090
—	—	—	—	—	—	—	—	—	—	35	35	35	35	—	—	05	05	05
—	—	—	—	—	—	—	—	—	—	155	165	160	155	—	—	095	105	115
20	20	20	—	—	—	—	—	—	—	—	—	45	45	40	40	40	40	25
110	130	150	—	—	—	—	—	—	—	—	—	330	250	150	130	190	060	350
40	40	40	—	—	—	—	—	—	—	—	—	25	25	10	10	07	05	07
175	185	195	—	—	—	—	—	—	—	—	—	125	125	105	095	090	100	115
20	20	25	90	100	100	100	—	—	—	—	—	60	30	20	10	35	55	60
085	090	120	300	325	350	350	—	—	—	—	—	020	180	180	290	080	090	140
35	40	40	40	40	40	40	—	—	—	—	—	15	15	08	05	04	10	10
155	155	145	115	110	110	115	—	—	—	—	—	115	115	100	085	080	080	115
20	20	20	35	80	90	100	—	—	—	—	60	55	45	25	25	40	60	60
080	060	070	350	300	310	340	—	—	—	—	230	260	250	000	320	025	050	080
27	30	35	36	36	37	35	—	—	—	—	06	07	07	05	04	05	07	07
125	105	090	085	090	100	115	—	—	—	—	105	115	110	090	070	055	055	065
15	15	15	10	25	60	40	40	—	—	20	40	50	60	50	60	65	70	60
085	085	085	105	160	240	265	270	—	—	190	210	230	250	275	310	000	020	050
23	23	25	26	29	31	32	31	—	—	07	05	05	05	03	04	05	06	06
125	095	075	080	085	090	095	095	—	—	125	120	120	125	105	040	030	030	050
20	15	15	20	25	35	40	45	40	—	—	20	30	60	65	50	40	65	—
095	100	100	110	130	160	210	240	250	—	—	215	230	250	275	300	320	350	—
15	17	17	16	18	20	18	18	17	10	10	09	07	05	04	04	06	06	—
150	105	075	080	080	085	085	080	065	290	275	135	145	170	205	305	345	000	—
45	35	25	25	25	35	40	35	25	80	80	—	05	65	85	75	40	40	—
110	115	120	115	130	150	175	220	270	040	310	—	230	160	160	175	190	220	—
09	10	10	10	10	11	13	15	16	16	10	10	10	05	04	04	05	05	—
205	155	075	080	080	085	075	055	045	015	005	180	165	190	215	275	315	345	—
50	45	30	15	15	25	30	20	20	75	85	85	—	80	90	90	65	60	55
120	135	140	130	115	135	165	195	300	035	065	100	—	130	150	155	165	180	110
08	08	07	05	05	06	08	10	12	13	13	13	—	05	08	05	05	10	12
235	225	165	085	075	075	035	015	355	340	335	345	—	215	225	265	305	335	355
55	45	30	05	15	25	25	20	10	35	50	—	—	—	100	90	60	35	40
130	155	180	050	085	125	150	180	180	095	085	—	—	—	160	140	140	155	105
09	07	05	02	02	04	05	07	08	09	11	—	—	—	08	05	05	07	10
235	245	240	235	275	315	315	305	315	325	320	—	—	—	225	265	290	330	355

APPENDIX

	5E	15E	25E	35E	45E	55E	65E	75E	85E	95E	105E	115E	125E	135E	145E	155E	165E
15S	45	45	—	80	80	30	45	30	20	20	35	90	100	—	20	60	40
	085	095	—	115	090	070	035	040	060	095	150	275	310	—	260	250	170
	06	06	—	05	12	10	07	05	05	10	18	22	22	—	40	27	18
	055	060	—	035	055	085	155	205	230	245	255	265	305	—	175	180	205
25S	40	45	—	70	35	15	30	30	20	15	15	25	—	—	—	60	45
	055	070	—	120	140	110	030	045	060	090	170	280	—	—	—	250	250
	05	05	—	03	05	04	04	04	05	09	17	20	—	—	—	22	15
	095	090	—	275	135	155	195	210	145	245	255	280	—	—	—	155	175
35S	30	40	50	40	25	20	25	30	20	10	05	10	10	15	25	40	60
	030	060	095	130	180	320	015	030	050	075	210	310	320	330	350	250	260
	04	05	05	05	04	03	04	04	06	10	15	17	19	20	13	16	10
	125	135	175	175	180	190	205	220	240	260	295	315	350	015	055	120	130
45S	20	25	30	25	20	20	30	40	20	15	10	10	10	15	30	40	65
	025	055	110	180	240	305	350	015	035	040	360	350	310	315	325	290	300
	05	04	04	04	04	04	04	05	08	12	17	20	20	20	08	07	06
	115	135	155	170	200	235	255	275	295	305	320	330	345	005	035	095	115
55S	10	05	10	20	20	20	30	35	30	20	15	15	15	15	30	35	30
	025	055	110	250	280	310	335	355	010	020	360	350	310	320	330	355	020
	07	06	05	04	04	04	05	07	10	14	20	22	25	26	14	10	10
	080	075	025	260	295	315	320	330	330	335	335	345	355	005	025	030	030
65S	15	15	15	20	20	20	25	25	25	20	20	20	20	20	30	30	25
	240	230	245	260	270	280	315	340	350	010	165	350	315	325	340	360	020
	08	07	06	04	05	05	07	10	13	16	20	23	25	28	30	20	17
	035	015	005	345	335	345	350	355	360	005	360	355	360	005	005	010	015
75S	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	000
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	015

ontinued)

75E	175W	165W	155W	145W	135W	125W	115W	105W	95W	85W	75W	65W	55W	45W	35W	25W	15W	5W
0	50	30	15	15	25	20	10	10	20	30	30	—	—	—	75	50	35	35
65	185	220	290	035	100	135	160	180	200	200	210	—	—	—	105	120	125	090
0	06	05	04	03	04	05	06	07	09	10	14	—	—	—	04	03	02	05
20	240	255	265	275	285	295	305	315	305	310	315	—	—	—	195	185	120	065
5	55	30	20	15	25	20	20	20	20	35	35	—	—	35	45	40	30	30
30	210	235	290	010	070	120	160	195	220	235	255	—	—	050	060	060	050	050
8	06	04	03	03	03	04	07	08	10	11	11	—	—	06	05	04	04	04
10	245	260	275	280	285	295	305	315	320	325	325	—	—	130	135	130	125	120
0	60	40	20	15	20	20	10	10	20	40	50	—	30	30	40	35	30	25
30	200	240	305	355	025	065	120	190	240	265	285	—	220	275	310	330	350	015
7	05	04	04	04	05	05	08	12	12	13	13	—	03	04	04	04	05	05
05	260	275	280	285	295	300	305	315	325	325	335	—	115	105	100	105	105	115
5	55	35	20	15	15	15	10	10	20	40	40	05	40	40	35	25	20	15
30	160	210	310	350	020	035	055	015	220	325	340	230	185	220	260	295	325	355
5	05	05	05	05	07	08	12	15	16	16	15	10	05	05	04	05	05	06
55	280	280	290	295	300	310	315	320	335	340	350	295	050	065	075	080	090	100
5	25	15	15	10	10	10	10	15	20	40	45	45	45	40	25	20	20	15
60	090	090	040	010	025	040	050	060	050	050	060	170	155	180	145	240	270	300
8	07	08	09	11	12	14	17	18	19	20	20	18	12	08	05	06	06	07
35	305	300	295	300	310	315	320	335	345	000	015	025	035	045	055	065	065	075
0	15	15	15	15	15	15	15	15	20	20	25	30	40	35	30	25	20	20
35	040	060	050	035	040	045	060	075	090	105	120	140	145	165	190	205	225	240
6	16	17	17	18	19	21	24	26	27	28	30	30	15	12	10	09	08	08
55	335	330	315	320	320	325	335	345	350	000	005	000	000	005	015	025	030	035
5	10	10	10	10	10	10	10	10	10	15	—	—	30	20	25	20	20	—
25	040	045	050	050	050	060	070	085	095	115	—	—	140	155	175	195	210	—
9	19	19	20	20	22	24	26	28	30	30	—	—	15	14	12	10	09	—
15	020	015	355	350	345	350	355	000	005	010	—	—	355	000	005	010	015	—

Both lunar and solar torques contribute appreciably to the deceleration of the Earth's rotation. The total value is

$$\frac{\bar{N}}{I} = \frac{-5.8 \times 10^{23}}{.81 \times 10^{45}} = -7.1 \times 10^{-22} \text{ rad sec}^{-2},$$

where I is the Earth's principal moment of inertia. This is of the same order as the observed deceleration since Egyptian times. An exact comparison is impossible because of much larger effects arising from variations in the Earth's moment of inertia.

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