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# ON SOME PROPERTIES OF THE SPECTRUM OF WIND-GENERATED OCEAN WAVES<sup>1</sup>

By

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## ABSTRACT

The resonance theory of wave generation predicts that, for a given scalar wave-number  $\kappa$  in the two-dimensional instantaneous wave spectrum  $\Phi(\kappa)$ , resonance maxima should occur at angles  $\alpha_c$  such that  $\kappa U \cos \alpha_c = (g\kappa)^{1/2}$ , where  $U$  is the wind speed at a height of approximately  $\kappa^{-1}$  above the surface. Measurements made in the Stereo Wave Observation Project confirm the existence of these resonance maxima near the critical angles predicted by the theory. Other measurements obtained in this project confirm the existence of equilibrium ranges in both the wave-number and frequency spectra. These comparisons suggest a composite description of the wave spectrum in terms of an equilibrium range at high frequencies  $\omega$  or wave-numbers  $\kappa$ , a range of low values of  $\omega$  or  $\kappa$  which is described by a linear theory, and an intermediate transition range.

## 1. INTRODUCTION

Recently two different mechanisms have been proposed for the generation of waves by wind. One of these theories (Miles, 1957) considers the waves that are produced as a result of instability of the air-water interface when the velocity profile is assumed to have the logarithmic form associated with a turbulent wind. In the other approach (Phillips, 1957; 1958b), turbulence in the air is regarded as imposing on the water surface a random convected distribution of fluctuating pressure. It has been found that waves grow most rapidly by means of a selective resonance between the Fourier components of the fluctuating pressure distribution convected across the water surface by

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This article by Dr. Phillips and the one by Dr. Cox which precedes it, both pertinent to the same area of interest, were submitted at approximately the same time, and since it is the policy of this Journal to encourage discussion, I asked each author if he would be willing to prepare a constructive comment on the other's article. Both authors gladly agreed to my proposal, and the Comments which appear after each article resulted. To each author, many thanks.

Y. H. Olsen

the mean wind and those components of the wave field having both the same wave number and the same velocity as the convected pressure fluctuations. Each theory appears to be capable of explaining a number of observed phenomena and each leads to a rate of energy transfer from wind to waves that is of the correct order of magnitude.

It is likely that the two theories are complementary in the sense that waves may be raised by either mechanism or by both operating together according to the particular conditions prevailing, or, more specifically, according to the intensity and scale of the turbulence present in the air stream. It is clear, therefore, that experimental conditions should be carefully controlled; if one mechanism is found to be important in generating ocean waves, it will not necessarily be dominant in a laboratory experiment, and vice versa. Few laboratory studies in the past have included measurements of turbulence in the air stream, and this omission has probably contributed to the frequent conflict in results of different observers and of observations in the laboratory and in the field. This point has been emphasized clearly by Ursell (1956) in his well known review of the subject.

However, the primary interest of oceanographers is in natural conditions that prevail at sea, and an evaluation of the relative roles played by the two processes under these circumstances is a matter of some importance. One objective of this paper is to show how certain detailed properties of the low wave-number or of the low frequency part of the sea spectrum, as observed in the Stereo Wave Observation Project (1957), can be explained simply by the resonance theory. In the higher wave-number and frequency ranges, effects of nonlinear interactions between the wave components become increasingly important and the linear wave generation theories become invalid. It might be expected that a statistical equilibrium can be attained over these ranges of large wave-number and frequency (Phillips 1958a); deductions based on this expectation are compared with the S.W.O.P. data in Section 3.

## 2. THE OCCURRENCE OF RESONANCE WAVES

The fundamental expression given by the resonance theory for the asymptotic form at large wind durations,  $t$ , of the two-dimensional instantaneous spectrum,  $\Phi(\boldsymbol{\kappa})$ , of the sea-surface displacement is

$$\Phi(\boldsymbol{\kappa}) \sim \frac{\kappa^2 t}{4\sqrt{2\rho^2 n_2^2}} \int_{-\infty}^{\infty} \Pi(\boldsymbol{\kappa}, \tau) \exp \{-i(n_1 - n_2)\tau\} d\tau \quad (1)$$

(Phillips, 1957: eqs. 4.3 and 4.9), where  $\boldsymbol{\kappa}$  is the two-dimensional wave-number vector,  $\rho$  the water density, and  $n_2 = (g\kappa)^{\frac{1}{2}}$  the angular fre-



quency of gravity waves of wave-number  $\kappa = |\mathbf{k}|$ . The frequency  $n_1 = \kappa \cdot U = \kappa U \cos \alpha$  say, where  $U$  is the convection velocity of the pressure fluctuations or the wind velocity at a height of approximately  $\kappa^{-1}$  above the surface. Thus the angle  $\alpha$  is the angle between the direction of the wind and the direction of propagation of the waves of wave-number  $\kappa$ . The function  $\Pi(\kappa, \tau)$  is the spatial Fourier transform of the covariance between the fluctuating pressures,  $p$ , at points on the sea surface separated by a distance  $\mathbf{r}$  and by a time interval  $\tau$ , measured in a frame of reference moving with velocity  $U$ :

$$\Pi(\mathbf{k}, \tau) = (2\pi)^{-2} \int p(\mathbf{x}, t) p(\mathbf{x} + \mathbf{r}, t + \tau) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} .$$

The expression for the wave spectrum in the fetch-limited case is similar to (1) (Phillips, 1958b), and the following arguments apply equally to this case also.

The integral in (1) is a maximum for a given value of the scalar wave-number  $\kappa$  when the oscillatory behaviour of the integrand vanishes; that is, when  $n_1 = n_2$  or at angles  $\alpha_c$  such that

$$\kappa U \cos \alpha_c = (g\kappa)^{\frac{1}{2}} , \quad (2)$$

$$\cos \alpha_c = \frac{g^{\frac{1}{2}}}{\kappa^{\frac{1}{2}} U} . \quad (3)$$

Condition (2) will be known as the resonance condition, and waves whose wave numbers and directions of propagation under a given wind field satisfy (2) will be called resonance waves. The resonance theory then makes the simple prediction that, for a given scalar wave-number of the spectrum, there will be resonance peaks in the directional distribution at angles  $\pm \alpha_c$  given by (3).

This result can also be obtained by more intuitive physical arguments. The integral in (1), as was shown by Phillips (1957: section 4.1), can be represented as

$$I = 2\Pi(\kappa) \theta(\kappa, V) , \quad (4)$$

where  $\Pi(\kappa) = \Pi(\kappa, 0)$  is the usual *instantaneous* spatial pressure spectrum and where  $\theta(\kappa, V)$  represents the integral time scale of the pressure fluctuations when observed in a frame of reference moving with speed  $V = c(\kappa)/\cos \alpha$  in the wind direction where  $c(\kappa) = (g/\kappa)^{\frac{1}{2}}$  is the speed of free surface waves of wave-number  $\kappa$ .  $\theta(\kappa, V)$  is therefore the time scale of pressure fluctuations acting upon such components of the wave field; the slower the pressure fluctuations experienced by the wave, the larger the time scale,  $\theta$ , and the faster the wave growth. The function  $\theta(\kappa, V)$  is largest for values of  $\kappa$  asso-

ciated with components of the wave field propagating at such a speed that they just keep up with the convected pressure distribution; that is, the time variations in pressure fluctuations result only from the relatively slow time evolution of the turbulence as it is carried along with the wave and not from its spatial pattern being swept past the wave. The time scale, hence the rate of growth, is therefore a maximum for values of  $\kappa$  such that

$$U = c(\kappa)/\cos \alpha , \quad (5)$$

which is exactly equivalent to the resonance condition (2).

This prediction can be tested directly by examination of the S.W.O.P. data, which were obtained by taking stereo-photographs of a part of the surface of the North Atlantic under fairly well defined meteorological conditions. A contour map of the instantaneous surface configuration was constructed, and a Fourier analysis of this gave the two-dimensional wave-number spectrum. The frequency spectrum was measured at the same time by using a wave pole. The observational difficulties encountered and the computational labour involved were immense, but the results, whatever their shortcomings, are probably the most complete to date. Fig. 1 represents a number of sections of the observed wave spectrum for constant values of the scalar wave-number  $\kappa$ , plotted as functions of the angle  $\alpha$  between the direction of propagation of the wave and the direction of the wind. These data are taken from table 11.6 of the S.W.O.P. report (1957); the ordinates represent the function  $\kappa\Phi(\kappa, \alpha)$  in the units  $\text{m}^3$  to provide some comparison of the relative contributions of each wave-number range to the mean square surface displacement

$$\bar{\xi}^2 = \int_0^\infty \int_{-\pi/2}^{\pi/2} \kappa\Phi(\kappa, \alpha) d\alpha d\kappa .$$

The black arrows indicate the angles at which resonance waves would be expected for the various wave-numbers. It is evident that maxima do occur near positions predicted by the theory, though some scatter is apparent. This is probably the result of both sampling errors and lack of steadiness in the wind field, the asymmetries in the observed distributions presumably being a measure of these uncertainties. It is difficult to see how these maxima could occur as a result of any other mechanism; for example, the instability theory would predict maxima at  $\alpha = 0$ . The characteristic double peaked form of these distributions for wave-numbers  $\kappa > 0.1 \text{ m}^{-1}$  offers good evidence that the resonance mechanism plays a considerable part in the generation of these ocean waves.



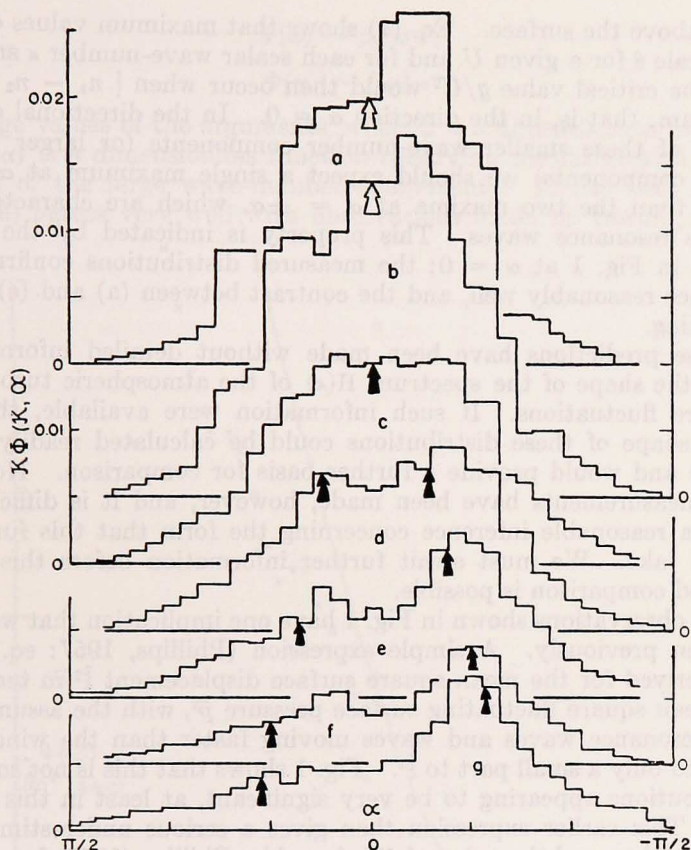


Figure 1. Sections of the weighted instantaneous wave spectrum  $\kappa\Phi(\kappa, \alpha)$  for fixed wave-numbers  $\kappa$ , observed in the Stereo Wave Observation Project. The wave-numbers and wavelengths associated with each section are:

Distribution	(a) $\kappa = 0.053\text{m}^{-1}$	$\lambda = 118\text{m}$
	(b) $\kappa = 0.063\text{m}^{-1}$	$\lambda = 100\text{m}$
	(c) $\kappa = 0.086\text{m}^{-1}$	$\lambda = 73\text{m}$
	(d) $\kappa = 0.099\text{m}^{-1}$	$\lambda = 64\text{m}$
	(e) $\kappa = 0.112\text{m}^{-1}$	$\lambda = 56\text{m}$
	(f) $\kappa = 0.128\text{m}^{-1}$	$\lambda = 49\text{m}$
	(g) $\kappa = 0.143\text{m}^{-1}$	$\lambda = 44\text{m}$

The curve (c) corresponds to a wave-number of  $0.086\text{ m}^{-1}$ , and the resonance angle for this wave-number under the observational wind speed is zero. For wave-numbers smaller than this,  $g^{1/2}/\kappa^{1/2} U > 1$ , so that from eq. (3) there are no real resonance angles, since the propagation velocity of these waves is greater than the wind velocity at a height

of  $\kappa^{-1}$  above the surface. Eq. (1) shows that maximum values of the time scale  $\theta$  for a given  $U$  and for each scalar wave-number  $\kappa$  smaller than the critical value  $g/U^2$  would then occur when  $|n_1 - n_2|$  is a minimum; that is, in the direction  $\alpha = 0$ . In the directional distribution of these smaller wave-number components (or larger wavelength components) we should expect a single maximum at  $\alpha = 0$  rather than the two maxima at  $\alpha = \pm\alpha_c$  which are characteristic of true resonance waves. This property is indicated by the open arrows in Fig. 1 at  $\alpha = 0$ ; the measured distributions confirm our inference reasonably well, and the contrast between (a) and (e), say, is striking.

These predictions have been made without detailed information about the shape of the spectrum  $\Pi(\kappa)$  of the atmospheric turbulent-pressure fluctuations. If such information were available, the detailed shape of these distributions could be calculated readily from eq. (1) and would provide a further basis for comparison. No relevant measurements have been made, however, and it is difficult to make a reasonable inference concerning the form that this function should take. We must await further information before this more detailed comparison is possible.

The observations shown in Fig. 1 have one implication that was unforeseen previously. A simple expression (Phillips, 1957: eq. 4.13) was derived for the mean square surface displacement  $\bar{\xi}^2$  in terms of the mean square fluctuating surface pressure  $\bar{p}^2$ , with the assumption that resonance waves and waves moving faster than the wind contributed only a small part to  $\bar{\xi}^2$ . Fig. 1 shows that this is not so, such contributions appearing to be very significant, at least in this situation. This earlier expression then gives a serious underestimate of wave heights, and the value of  $\bar{p}^2$  inferred by Phillips (1957) from some meagre atmospheric turbulence data is probably larger than the values actually present.

### 3. THE EQUILIBRIUM RANGES

If the fetch and duration of the wind are both sufficiently large, the wave field develops to such an extent that nonlinear interactions among the components over certain ranges begin to play an important role. The sea surface is then characterized by the appearance of sharp wave crests which may be unable to maintain their attachment and so degenerate into whitecaps. Consideration of this process suggests the existence of equilibrium ranges (Phillips, 1958a) in the wave-number and frequency spectra of high values of  $\kappa$  and  $\omega$ , over which the form of the spectrum is determined by the gravitational acceleration  $g$  alone. On dimensional grounds, therefore, we have

$$\Phi(\omega) \sim \beta g^2 \omega^{-5}, \quad (6)$$

$$\Phi(\kappa) \sim f(\alpha) \kappa^{-4} \quad (7)$$

for large values of the arguments, where  $\beta$  is a dimensionless constant and  $f(\alpha)$  is a dimensionless function specifying the directional distribution of the large wave-number components. It has been shown that (6) agrees very well with observations made by Burling (1955)

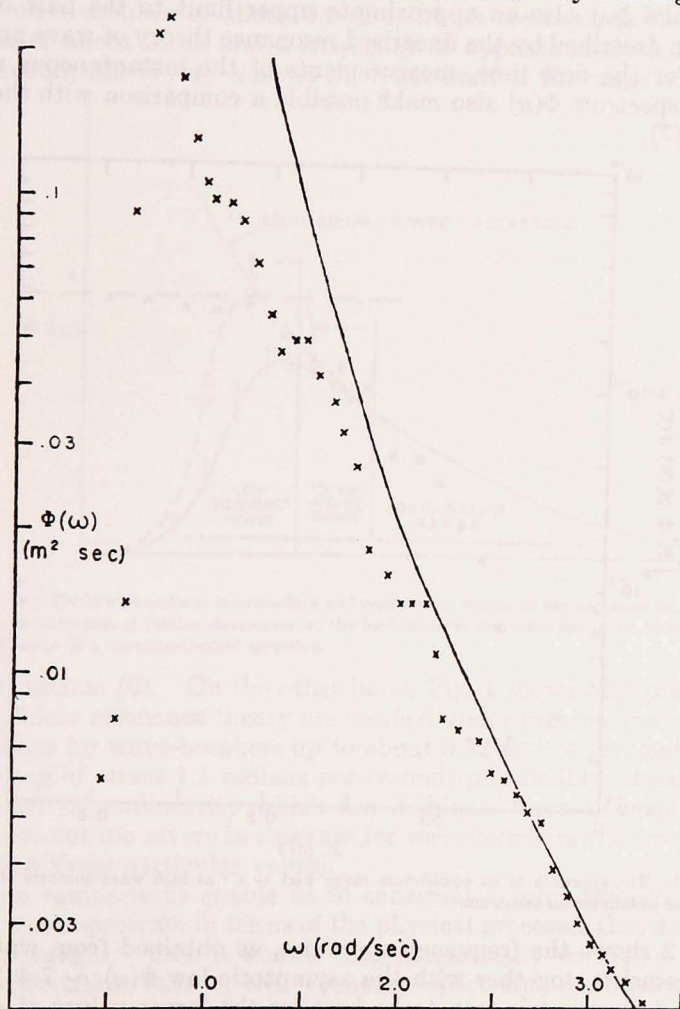


Figure 2. The frequency spectrum  $\Phi(\omega)$  from the S. W. O. P. wave-pole data. The solid curve represents the equilibrium range spectrum  $\Phi(\omega) \sim 7.4 \times 10^{-3} g^2 \omega^{-5}$ .



on Staines Reservoir in Middlesex over a frequency range of about 4 to 12 radians per second; the dimensionless constant  $\beta$  was found to be  $7.4 \times 10^{-3}$ .

The S.W.O.P. data enable us to compare (6) with observation over a range of frequencies that extends to much smaller values than those in Burling's measurements. We are interested in discovering not only the lower limit to the range of  $\omega$  over which the asymptotic law (6) is valid but also an approximate upper limit to the part of the spectrum described by the linearised resonance theory of wave generation. For the first time, measurements of the instantaneous wave-number spectrum  $\Phi(\kappa)$  also make possible a comparison with the prediction (7).

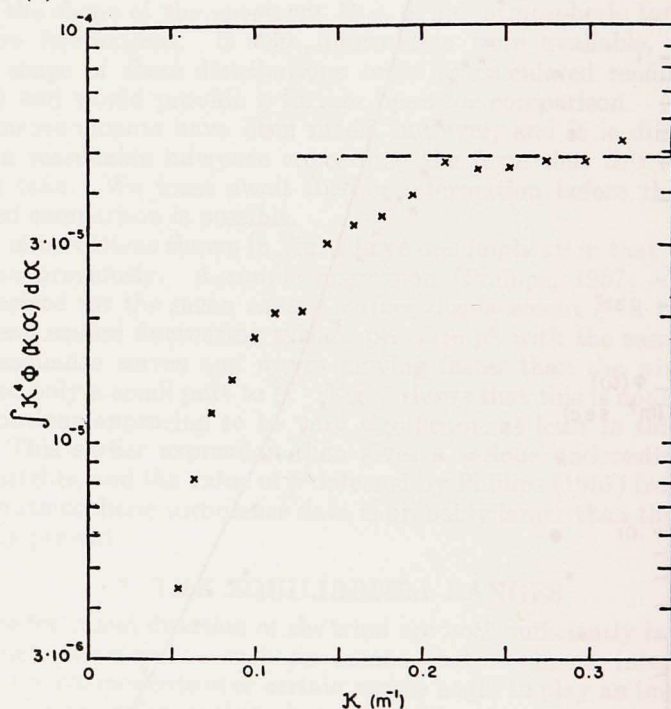


Figure 3. The approach to an equilibrium range  $\Phi(\kappa) \sim \kappa^{-4}$  at high wave-numbers of the two-dimensional instantaneous spectrum.

Fig. 2 shows the frequency spectrum, as obtained from wave-pole measurements, together with the asymptotic law  $\Phi(\omega) \sim 7.4 \times 10^{-3} g^2 \omega^{-5}$ . Agreement is seen to be best for the larger values of  $\omega$ , with the departure of the measured spectrum from the relation (6) becoming serious for frequencies below about 1.5 radians per second.

Fig. 3, derived from data given in table 11-6 of the S.W.O.P. report, represents, as a function of wave-number, values of the spatial spectrum  $\Phi(\kappa, \alpha)$  at a fixed  $\kappa$ , integrated over angles  $\alpha$  from  $-\pi/2$  to  $\pi/2$  and then multiplied by  $\kappa^4$ . According to eq. (7), this function should tend asymptotically to a constant for large  $\kappa$ . Fig. 3 gives a strong indication that this is so, the constant being approximately  $5 \times 10^{-5}$ . The asymptotic value appears to be attained in these measurements by a wave-number of about  $0.2 \text{ m}^{-1}$ , which corresponds to a wavelength of about 30 m and a frequency of approximately 1.4 radians per second; this is very close to the lower limit of the range of validity

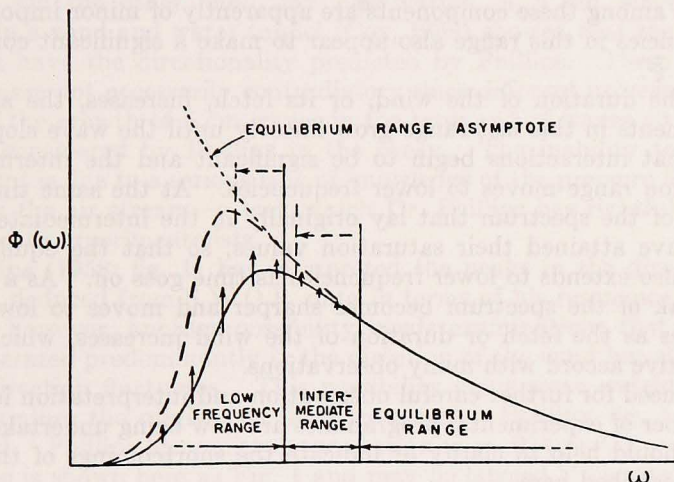


Figure 4. The low frequency, intermediate and equilibrium ranges of the spectrum  $\Phi(\omega)$ . Arrows indicate the direction of further development; the broken curve indicates the shape of the spectrum at a later time in a duration-limited situation.

of the relation (6). On the other hand, Fig. 1 shows that predictions of the linear resonance theory are verified under present observational conditions for wave-numbers up to about  $0.12 \text{ m}^{-1}$  corresponding to a frequency of about 1.1 radians per second; presumably, then, the restrictions of nonlinearity which are dominant over the equilibrium range are not too severe in this case for wave-numbers and frequencies less than these particular values.

These comparisons enable us to construct a composite description of the wave spectrum in terms of the physical processes that determine various ranges. This is shown diagrammatically in Fig. 4 for the easily illustrable case of the frequency spectrum; corresponding remarks can be made about the wave-number spectrum. At large frequencies, the spectrum has an equilibrium range described by (6), in which nonlinear wave interactions prevent further development;



any added energy is lost by wave breaking. Contributions from these frequencies to the mean square surface displacement  $\xi^2$  (or to the wave height) are small. At lower frequencies there is an intermediate range in which wave interactions are appreciable but not dominant. This is difficult to describe analytically, since the wave growth cannot be specified by a linear theory and since the spectrum does not possess the simple properties of the equilibrium range. However, this range does make a significant contribution to  $\bar{\xi}^2$ , as Fig. 2 shows. Finally, at low frequencies the spectrum can be described by a linear generation theory such as the resonance theory, since interactions among these components are apparently of minor importance. Frequencies in this range also appear to make a significant contribution to  $\bar{\xi}^2$ .

As the duration of the wind, or its fetch, increases, the spectral components in this last range grow linearly until the wave slopes are such that interactions begin to be significant and the intermediate transition range moves to lower frequencies. At the same time, the values of the spectrum that lay originally in the intermediate range may have attained their saturation values, so that the equilibrium range also extends to lower frequencies as time goes on. As a result, the peak of the spectrum becomes sharper and moves to lower frequencies as the fetch or duration of the wind increases, which is in qualitative accord with many observations.

The need for further careful observation and interpretation is clear. A number of experimental programmes are now being undertaken and these should help to clarify or indicate the shortcomings of the concepts described here.

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## COMMENTS ON DR. PHILLIP'S PAPER<sup>1</sup>

By

CHARLES S. COX

Dr. Phillips (1958) has found evidence to support his "resonance" theory of wave generation in the wave directionality measurements computed from the Stereo Wave Observation Project data (S.W.O.P., 1957). On the other hand, my observation of 5 cm wavelength waves in a wind and water tunnel (Cox, 1958) showed that the waves did not have the directionality predicted by Phillips. These observations are not necessarily contradictory since different processes may control the growth of 5 cm waves in the tank and of waves 40 m and longer considered by Phillips in the ocean. The inability to settle this point is due to a serious lack of knowledge of the pressure fluctuations in the air stream—a lack which Dr. Phillips has rightfully emphasized to experimentalists.

Phillips (1958: fig. 1) has interpreted the peaks in the directional spectra deduced from S.W.O.P. data in terms of his resonance mechanism; however, another possibility needs examination: that waves are generated predominantly in the direction of the wind but that the wind direction fluctuates. This possibility gains some weight when one examines the original sections from which Phillips' fig. 1 is abstracted. Phillips has omitted one section,  $\kappa = .074 \text{ m}^{-1}$ ,  $\lambda = 85 \text{ m}$ , and this is shown here as Fig. 1 and may be labeled  $b'$  because it belongs in the sequence between  $b$  and  $c$  of Phillips' fig. 1. There are two features on this section which call for attention.

First, a peak centered at an azimuth of  $-45^\circ$  is not explained by the resonance mechanism since these waves are traveling faster than the wind. One gets the impression, on comparing  $a$ ,  $b$ ,  $b'$ , and  $c$ , that this peak is connected with the left-hand peak in  $d$ ,  $e$ ,  $f$ , and  $g$ . If so, there is no definite trend to the azimuth of this peak since the peak tends to move to the right in the sequence  $a$ ,  $b$ , and  $b'$ , while it moves to the left in  $c$ ,  $d$ ,  $e$ ,  $f$ , and  $g$ .

Second, note in Fig. 1 the peak centered at an azimuth of  $60^\circ$ . A similar feature exists on all of the original sections corresponding to  $a$ - $f$ , but Phillips does not indicate this peak, presumably because one of the authors of S.W.O.P. considered that it was due to swell from a distant source. On the other hand, another author states "For the

<sup>1</sup> Contribution from Scripps Institution of Oceanography, New Series.



entire period [of five days preceding the stereo-photographs] there appears to have been little or no possibility of a contribution to the waves at the ship by wind fields in other parts of the North Atlantic. Therefore, the ship was essentially in the generating area at all times and no swell should be recorded." This statement and the observations that the spectral peak has a rather broad azimuth and frequency range suggest that the generation is local.

Weather maps from S.W.O.P. (fig. 10.3) show: 1) that wind directions were highly variable for a period of 12 to 42 hours before stereo-photographs were taken; 2) that wind observations to the west of the

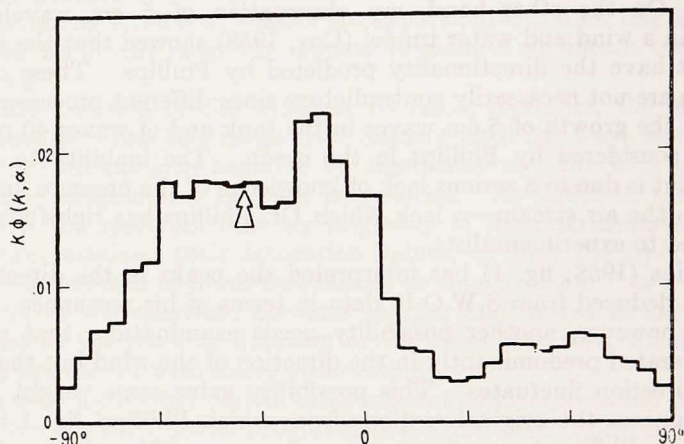


Figure 1. The angular variation of ocean waves of wavelength 85 m according to measurements by S.W.O.P. The ordinate is proportional to the estimated spectral density, the abscissa is the azimuth of the direction of the waves measured in degrees eastward of true north. The estimated wind direction is indicated by arrow and establishes the origin  $\alpha = 0$  of Phillips' fig. 1.

stereo observation point showed strong westerly winds from 42 to 36 hours before the stereo-photo time; 3) that lack of information prevents determining whether these winds persisted until 28 hours before stereo-photographs were exposed. If they did, then waves of 85 m wavelength, traveling at their group velocity, would reach the photograph area in time. These waves would arrive from an azimuth of about  $-90^\circ$ ; owing to an indeterminacy of  $180^\circ$ , these waves would appear on the directionality sections at an azimuth of  $+90^\circ$ . One can therefore identify the observed peak in this way, although identification is not certain. What then of the peaks discussed by Phillips? I believe there is a possibility that they can be interpreted in a similar way and that the wind observations are certainly not sufficient to rule out this possibility.

The considerations above suggest that definitive experiments must be designed to measure not only the parameters of the turbulence in the wind stream but also the large-scale meteorological effects of variable wind direction.

In the second part of his paper Phillips has found new data (also from S.W.O.P.) which support his dimensional arguments for an "equilibrium" range in the wave spectrum where the frequency spectrum has the form

$$\Phi(\omega) \sim \beta g^2 \omega^{-5},$$

with  $\beta = 7.4 \times 10^{-3}$ , independent of wind speed. The notation follows Phillips' usage. The dimensional argument is independent of any theory of wave generation. It is gratifying to find that experimental data now supply information down to much lower frequencies than those available earlier (due to Burling).

Wind and wave tank observations (Cox, 1958: fig. 4) suggest that the equilibrium range is limited at the high-frequency end by a sharp cut-off at an angular frequency of  $\omega_1 = 85 \text{ sec}^{-1}$ , independent of wind speed, followed by a capillary spectral peak for still higher frequency waves.

The contribution of capillary waves to the mean square slope in the open sea can be estimated with the help of the expression for the equilibrium spectral density. Let  $\sigma^2$ ,  $\sigma_1^2$  be the contributions of all waves and of gravity waves respectively to the mean square slope. By a known formula

$$\sigma_1^2 = \int_0^{\omega_1} \kappa^2 \Phi(\omega) d\omega.$$

Because of the factor  $\kappa^2$ , the contribution by long waves to the integrand is minor. Consequently it should be possible to replace the spectrum function by its equilibrium expression, provided one starts the integration at a suitable lower limit  $\omega_0$ . From many studies it seems plausible that the lower limit should be taken such that the longest waves run at a speed nearly equal to  $U$ , the wind speed measured at "anemometer level." This gives

$$\omega_0 = cg/U,$$

where  $c \doteq 1$ . Taking the appropriate gravity wave formula for  $\kappa$ , one finds

$$\sigma_1^2 = \int_{\omega_0}^{\omega_1} \beta \omega^{-1} d\omega = \beta \ln \left( \frac{\omega_1 U}{cg} \right).$$



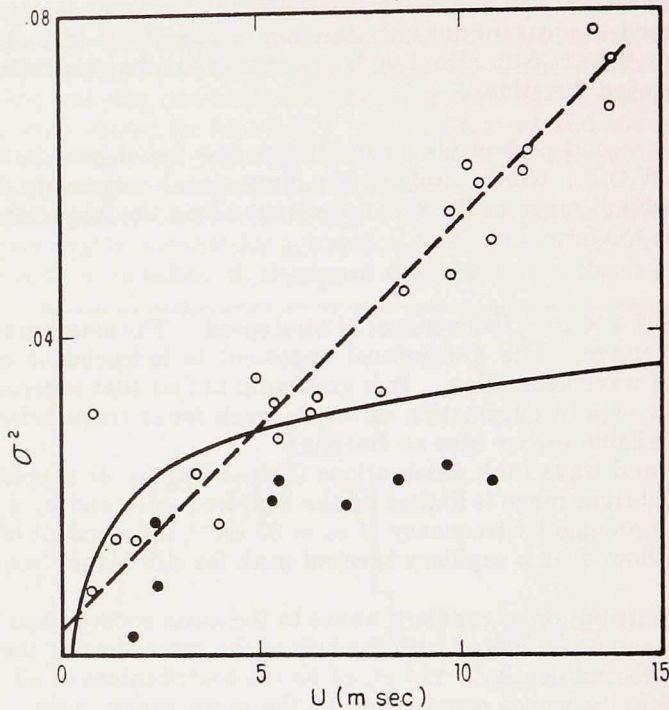


Figure 2. Mean square slope,  $\sigma^2$ , vs. wind speed  $U$  measured 41 feet above sea level according to Cox and Munk (1954). The open circles represent measurements over clean water surfaces; the solid circles, measurements over water covered with an oil slick. The dashed curve is a least square fit to the open circles; the solid curve represents  $\sigma_1^2$ , the contribution to mean square slopes by gravity waves according to Phillips' form of the equilibrium spectrum. The difference  $\sigma^2 - \sigma_1^2$  must represent the contribution by capillary waves.

Fig. 2 compares  $\sigma_1^2$  (computed with  $c = 1$ ) with  $\sigma^2$  as measured in the open sea (Cox and Munk, 1954). If  $c = \frac{1}{2}$  or 2, the computed curve would be moved up or down by only .005. For winds below 4 m sec<sup>-1</sup>, the computed value of  $\sigma_1^2$  lies above  $\sigma^2$ . Since this is an impossibility, one concludes that the equilibrium waves are not fully developed at such low winds. It is only for winds above 6 m sec<sup>-1</sup> that the curve for  $\sigma_1^2$  falls significantly below the one for  $\sigma^2$  and that an appreciable contribution from capillaries can occur. Observations of mean square slope where the ocean was covered by an oil slick fit into this picture well because detailed estimates (Cox and Munk, 1954) indicate that all capillaries and some of the gravity waves should be removed by such a slick; therefore the mean square slope in a slick

should be slightly less than  $\sigma_1^2$ , as is indeed found. Observations in the wind and wave tank are also consistent with a rapid growth of capillaries for winds above a critical windspeed. (The speed measured in the tank was  $2.5 \text{ m sec}^{-1}$ ; see Cox, 1958: table II; the windspeed measured at an anemometer level of 40 feet would have been approximately  $5.5 \text{ m sec}^{-1}$ .)

Conclusions of this commentary are:

First, additional experiments are necessary before directional properties of ocean waves become known. In this work it will be necessary to pay particular attention to large-scale fluctuations of wind direction over the generating area.

Second, the form of the equilibrium spectrum proposed and verified by Dr. Phillips is at harmony with measurements of water slopes both with and without oil slicks. An indirect calculation based on comparison of the analytical form of the equilibrium spectrum with observations of slopes in the open sea suggests that capillary waves become important only for winds above  $6 \text{ m sec}^{-1}$ .

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